INTRODUCTION
We investigate extensions of the lattice Boltzmann method towards fluid-structure interaction problems. Focusing on a D2Q9 model, we consider a LBGK evolution
\[(1) \quad f_i(n+1,j+1,c_j) = f_i(n,j) + h f_i^n(c_j) - f_i^n(2c_j - c_i, u) + f_i^\text{eq}(2c_j - c_i, u)\]
on a regular h-spaced lattice, with equilibrium function
\[f_i^n = f_i^\rho + f_i^\text{c} - c_i, u + f_i^\text{eq}(2c_i - c_j, u)\]
where \(f_i^\rho, f_i^\text{c}, c_i, u\) are weights depending on the particular LBM realization.

Asymptotic Analysis
The numerical solution of (1) can be predicted using an asymptotic expansion:
\[(2) \quad F_i = f_i^0 + h f_i^1 + h^2 f_i^2(2)\]
whose coefficients can be defined [5] inserting the ansatz (2) into (1), as functions of pressure and velocity (solution of Navier-Stokes equations).

From (3), the prediction can be written as a sum of equilibrium+ non-equilibrium, the latter depending on velocity gradients
\[(4) \quad F_i = f_i^0 (1 + h^2 c_i^2, p, h\alpha) + f_i^\text{eq}(2) (\nabla u)\]

MOVING BOUNDARY LBM
An additional rule (refill) is needed to initialize the nodes entering the fluid domain (fig.1).
The \(\text{EQ}+\text{NE} \text{ refill}\) initializes new fluid nodes approximating the interior prediction (4):
\[f_i(\text{new node}) = f_i^n (1 + h^2 c_i^2, p, h\alpha) + f_i^\text{eq}(2)\]
extrapolating equilibrium (pressure and velocity) and non-equilibrium part.

According to (4), a low order approximation of non-equilibrium is sufficient: it can be copied from a neighbor

BENCHMARK, RESULTS & DISCUSSION
We consider a channel flow past a moving disk, whose motion is constrained by a spring (fig.3). Forces on the obstacle are computed with (6), and Newton equations are integrated (explicit Euler) for velocity and position of the disk.

At a time when vertical force is maximum, we approximate the local interface stresses comparing
\[\begin{align*}
\text{CME:} & \quad \sigma_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \delta_{ij} \frac{1}{3} \nabla \cdot u \right), \\
\text{me:} & \quad \sigma_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),
\end{align*}\]
within LBM, forces on a solid obstacle can be efficiently computed using the Momentum Exchange Algorithm (MEA) [6] (fig.2).

Within LBM, forces on a solid obstacle can be efficiently computed using the Momentum Exchange Algorithm (MEA) [6] (fig.2).

At each boundary node, the momentum exchanged along each LB-link is computed (using the post- 
collision distribution pointing into the solid):
\[\hat{f}_{ij}^\text{ME} = f_{ij}^0 (f_{ij}^0 (c_j - c_i, \kappa))\]
the sum of the contributions (5) along the boundary is used to approximate hydrodynamic force.

Corrected Momentum Exchange
Using (3), we find that the following correction is needed to obtain a Galilean invariant (in relevant orders) force computation:
\[\hat{f}_{ij}^\text{CME} = \hat{f}_{ij}^\text{ME} (c_j - c_i, \kappa)\]

Accuracy Results [2,3]
- Corrected ME provides an accurate (first order in \(h\)) global force evaluation
- CME is consistent also for Lees-Edwards BC (periodic in Galilean-transformed systems), useful tool in suspension simulations [7]
- Local interface stresses are approximated only up to order \(h^0\)

Local Boundary Forces
At selected points of the interfaces (●), two extrapolation methods are investigated (fig.6): (A) ME-based extrapolation: stresses are extrapolated from (6), using the expansion of the momentum exchanged (7) (B) pop-based extrapolation: LB distributions are approximated on the boundary.

(A) is more efficient within the LBM (no need of off-grid extrapolation), while (B) might be better in terms of stability, since it can be combined with different extrapolation rules.

FORCE EVALUATION

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