We (partially) have a WDML
What we really need is Semantics!
Then we can do Search and Much More®

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June 1. 2012, WDML 2012

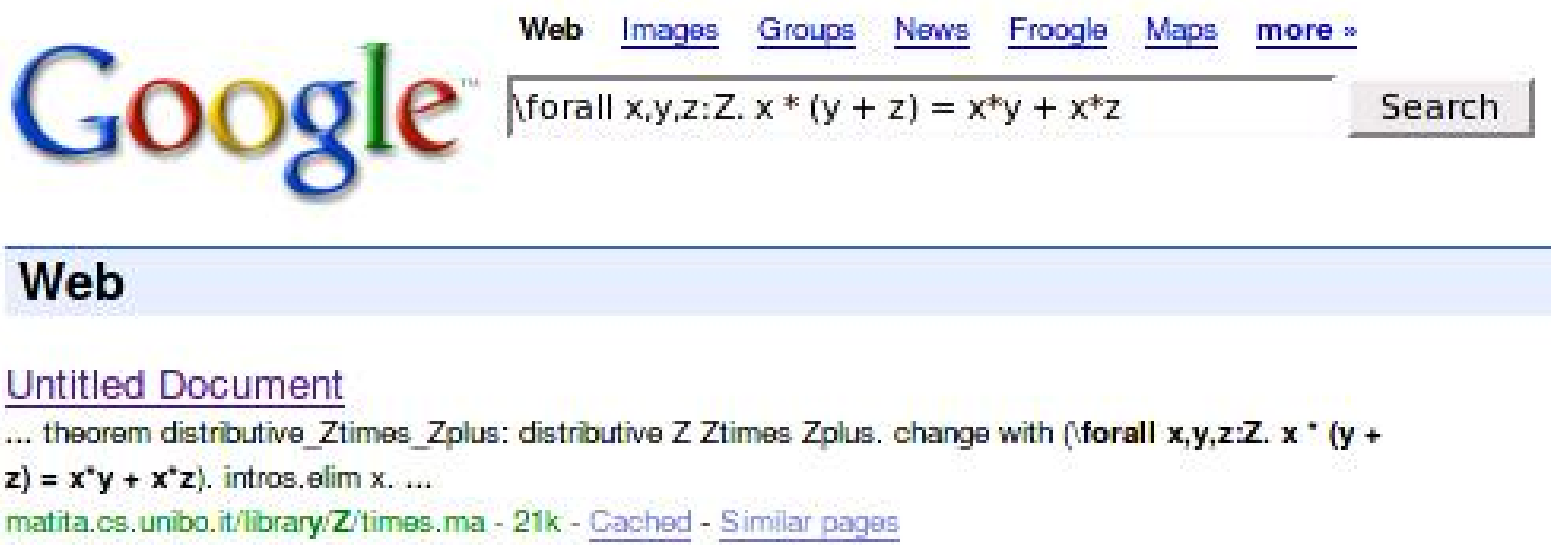
More Mathematics on the Web

- The Connexions project (http://cnx.org)
- Wolfram Inc. (http://functions.wolfram.com)
- Eric Weisstein's MathWorld (http://mathworld.wolfram.com)
- Digital Library of Mathematical Functions (http://dlmf.nist.gov)
- Cornell ePrint arXiv (http://www.arxiv.org)
- Zentralblatt Math (http://www.zentralblatt-math.org)
- ...
- Question: How will we find content that is relevant to our needs
- Idea: try Google (like we always do)
- Scenario: Try finding the distributivity property for $\mathbb{Z}$

$$(\forall k, l, m \in \mathbb{Z}.k \cdot (l + m) = (k \cdot l) + (k + m))$$

Searching for Distributivity

Searching for Distributivity



Searching for Distributivity

Of course Google cannot work out of the box

- Formulae are not words:
  - a, b, c, k, l, m, x, y, and z are (bound) variables.
    (do not behave like words/symbols)
  - where are the word boundaries for "bag-of-words" methods?
- Idea: Need a special treatment for formulae (translate into "special words")
  Indeed this is done ([MY03, MM06, LM06, MG11])
  . . . and works surprisingly well (using Lucene as an indexing engine)
- Idea: Use database techniques (extract metadata and index it)
  Indeed this is done for the `Coq`/`HELM` corpus ([AGC+06])
- Our Idea: Use Automated Reasoning Techniques
  (free term indexing from theorem prover jails)

Instantiation Queries

- Application: Find partially remembered formulae
- **Example 1** An engineer might face the problem remembering the energy of a given signal $f(x)$
  - Problem: hmmmm, have to square it and integrate
  - Query Term: $\int_{\boxed{min}}^{\boxed{max}} \boxed{f}(x)^2 dx$ ($\boxed{i}$ are search variables)
  - One Hit: Parseval's Theorem $\frac{1}{T}\int^{T_0} s^2(t)dt = \sum_{k=-\infty}^{\infty} \|c_k\|^2$ (nice, I can compute it)
- This works out of the box (has ween working in `MathWebSearch` for some time)
- Another Application: Underspecified Conjectures/Theorem Proving
  - during theory exploration we often have some freedom
  - express that using metavariables in conjectures
  - instantiate the conjecture metavariables as the proof as the proof dictates

  applied e.g. in Alan Bundy's "middle-out reasoning" in proof planing

Generalization Queries

- Application: Find (possibly) applicable theorems
- **Example 2** A researcher wants to estimate $\int_{\mathbb{R}^2} |\sin(t)\cos(t)|dt$ from above
  - Problem: Find inequation such that $\int_{\mathbb{R}^2} |\sin(t)\cos(t)|dt$ matches left hand side.
  - e.g. Hölder's Inequality: ($\boxed{i}$ are universal variables)

$$\int_{\boxed{D}} \left|\boxed{f}(x)\boxed{g}(x)\right| dx \leq \left(\int_{\boxed{D}} \left|\boxed{f}(x)\right|^p dx\right)^{\frac{1}{p}} \left(\int_{\boxed{D}} \left|\boxed{g}(x)\right|^q dx\right)^{\frac{1}{q}}$$

  - Solution: Take the instance

$$\int_{\mathbb{R}^2} |\sin(x)\cos(x)| dx \leq \left(\int_{\mathbb{R}^2} |\sin(x)|^p dx\right)^{\frac{1}{p}} \left(\int_{\mathbb{R}^2} |\cos(x)|^q dx\right)^{\frac{1}{q}}$$

Problem: Where do the index formulae come from in particular the universal variables (we'll come back to that later)

Where do the universal variables come from

- Problem: we need to have e.g. Hölder's Inequality in the index:

$$\int_D \left|f(x)g(x)\right| dx \leq \left(\int_D \left|f(x)\right|^p dx\right)^{\frac{1}{p}} \left(\int_D \left|g(x)\right|^q dx\right)^{\frac{1}{q}}$$

  - How do we know what symbols are "universal" (to be instantiated?)
  - what is their scope (when are different occurrences of f different?)
  - we have no sources with explicit quantifiers, but ([Wikipedia])

    Let (D, Σ, μ) be a measure space and let $1 \leq p, q \leq \infty$ with $1/p + 1/q = 1$. Then, for all measurable real- or complex-valued functions f and g on D, ...

- Solution: Use techniques from computational linguistics and integrate them into the indexing pipeline. (we have started a bit on the arXiv)

What is the Point (I want to make)

- Search/Navigation is the prerequisite and killer application for a WDML
- Search/Navigation/Access has to be semantic (that's how humans think)
- Generally:The WDML needs a semantic Layer (Then we can access it)

  - The Elephant in the Room: Semantization
  - Possible Answers:
    - for new documents: write with semantic annotations (e.g. in sTEX)
    - for legacy documents: extract semantics with linguistic methods (or retype)
  - If you are interested, contact me

- BTW: NTCIR-10 (Information Retrieval Challenge like TREC) has a Math Pilot Task in 2013! (see http://ntcir-math.nii.ac.jp/)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