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Abstracts

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Section 1

Logic and Foundations

Axiom of Choice and Euclid Axiom 8

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1. The problem of infinity in mathematics is one of those “eternal problems” to which the science comes back on each coil of its development. Ancient greek mathematician Euclid (nearby 340–287 B.C.) introduced the Axiom 8 “And the Whole is more then its Part” [1, I, p. 15]. On the boundary of XIX and XX centuries founders of the sets theory rejected Euclid’s Axiom 8 by ignoring (or explicitly); an equivalency between any set and its own part has referred to as characteristic property of infinite sets. P. Cohen considered the existence of infinite collections as the most important issue, he recognized the necessity of inserting infinity axiom and emphasized, that “...by tradition the attitude to infinite sets was the criterion of discrepancy between mathematicians” [2, IV.13].

We think, at an axiomatization of the sets theory Euclid’s Axiom 8 was incorrectly transformed into “an axiom of infinity”. Any formulation of Axiom infinity [3, S. 11], as well as Peano’s Axiom of natural numbers set infinity [3, S. 12], is of potential character. Now, we formulate first statement.

Hypothesis 1. *Any infinite set is of potential nature.*

2. Remaining within the framework of Halmosh sets naive theory [3], we proved Euclid’s Axiom 8 in the form of following theorem.

Theorem 1. $B \subset A \Rightarrow (\forall \varphi : A \rightarrow B \exists (a, q), a, q \in A : a \neq q \ \& \ \varphi(a) = \varphi(q)).$

Theorem 1 has the following canonically short form: $B \subset A \Rightarrow \neg(A \sim B)$, in particular, $q \notin A \Rightarrow \neg(A \sim (A \cup \{q\}))$. This theorem demonstration contains,

in particular, references to Axiom of Choice [3, S. 15], the concept of exact rearrangement of set A and concept of the maximal element of chains family formed from set A subsets and others too.

Remaining within Theorem 1, now we formulate the following two suggestions:

Hypothesis 2. *Euclid Axiom 8 is unprovable without Axiom of Choice.*

Statement 1. *The power sets theory and continuum hypothesis ([2]–[3]) has new way of development with Euclid’s Axiom 8.*

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Effective Eilenberg Machines

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 68Q10, 68Q45

We revisit the idea of Eilenberg [1] to use automata labelled with binary relations as a general computational model. The Eilenberg machines model has interesting properties. First, It is general because it unifies most of the devices having a finite-state control (including Turing machines). Second, it is expressive thanks to the following modularity property: an Eilenberg machine defines a characteristic relation that shall be used in another Eilenberg machine. We study the effective fragment of Eilenberg machines for which we propose a simulation engine [2, 3]. The correction of the simulation engine is proved using a proof assistant. This approach of *effective Eilenberg machines* may be used to solve properly problems which use multiple stages of traditional finite-state machines.

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Another Arithmetization: and Godels Second Incompleteness Theorem

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 03B10, 03B25, 03B53

Here we present a discussion to arithmetize a consistency statement for any formal system with a recursive set of axioms, a finite number of finitary and recursive rules of inference including modus ponens. In informal language, the consistency of S shall be expressed by the sentence: there are no meaningful formulae y and z where z is the negation of y such that both are theorems of S . we then show that the formalized consistency statement for the arbitrary system S cannot be proved in a certain sub-theory T of Peanos arithmetic. This arithmetic theory T with $+$ and $.$, is much stronger than Q . It will follow that the consistency of Q formalized in the manner we do, cannot be proved in T , let alone in Q itself, which is a sub-theory of T .

When formalizing the notion of theorem-hood in any arithmetic system with $+$ and $.$, we have to refer to sequences in the formal language, since a proof is a sequence of formulae. In the construction of the proof-predicate the usual method is to allot (recursively) a single Godel number (g.n.) to each formula and then a single g.n. to a sequence of formulae (forming a proof, say of the last formula). This is what we have done. The novelty in our present construction is that instead of codifying sequences by single natural numbers, we use a number triple. This device enables us, as we shall see, to do away with the need to find a defining formula for the recursive function x^y . With a Godel triple to codify a sequence, our proof-predicate is a formula $\text{ProvS}(c,d,l,y)$ with 4 free variables, 3 of which are thought of as always appearing in a bunch. $\text{ProvS}(c,d,l,y)$ says that the triple $\langle c, d, l \rangle$ yields a sequence of expressions (i.e. their g.n.'s) constituting a proof in S of the formulae with g.n.y. Thus the triple $\langle c, d, l \rangle$

merely indicates a sequence of expressions without mentioning anything about it being finite or infinite. We shall exploit this to our advantage, using an infinite sequence of expressions to constitute a proof of its last member.



Analogy of Hilbert's Tenth Problem in Observers Mathematics

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This work considers analogy of Hilbert's tenth problem in a setting of arithmetic, algebra, geometry, topology provided by Observer's Mathematics (see [1], [2], [3]). Certain results and communications pertaining to solution of this problem are provided. Let W_n be a set of all finite decimal fractions of length $2n$, hence, visually W_n can be described as $W_n = \{\underbrace{\star \cdots \star}_n . \underbrace{\star \cdots \star}_n\}$. The following

Theorem is proved:

For any positive integers $m, n, k \in W_n$, $n \in W_m$, $m > \log_{10}(1 + (2 \cdot 10^{2n} - 1)^k)$, from the point of view of the W_m -observer, there is an algorithm that takes as input a multivariable polynomial $f(x_1, \dots, x_k)$ of degree q in W_n and outputs YES or NO according to whether there exist $a_1, \dots, a_k \in W_n$ such that $f(a_1, \dots, a_k) = 0$.

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Sequential Two-Level Formalization of Mathematical Theories

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 03B30, 03B40

Following step method of Kolmogorov and Markov, and ideas of sequential logic, we propose non-axiomatic set-theoretical formalization of mathematical theories in the form of two-level sequential (nonformula) calculus, representing provably complete and provably consistent foundations of classical set-theoretic mathematics. Preliminary results are reflected in a number of publications; see, in particular, references below.

This result is obtained by using of non-logical algorithmic undecidable (with the law of excluded middle) sequential calculus of Church Lambda-conversion and deductive sequential Gentzen constructions without postulated logic cut rule and with two postulated Lambda-cut rules, introduced by the first author.

Note that, following Gentzen, the basic object of our research of theories is deductive sequents (deducibilities) instead of formulas, though such sequents also are constructed only from formulas.

We give details of constructions, beginning with the alphabet, and corresponding proofs.

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Modal Logics of Forcing

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There is a natural forcing interpretation of modal operators: the possibility operator \diamond is interpreted as “there is a forcing extension such that” and the necessity operator \square is interpreted as “for all forcing extensions”. What are the validities of modal logic in this interpretation? The answer depends on the model of set theory. In [1], Hamkins provided a model in which the set of modal validities is the modal logic **S5**, but it is easy to see that this is not true in every model of set theory.

In [2], Hamkins and the author determined that the set of modal formulas φ such that every substitution instance of φ is provable in ZFC is exactly the modal logic **S4.2**. In this talk, we shall discuss this result and further applications of the technique used. In particular, we shall be looking at modal logics for restricted classes of forcing (e.g., the modal logic of collapse forcings) and modal logics in which \diamond is interpreted as the converse of forcing possibility: “there is a ground model such that”.

This talk is reporting on joint work with Joel Hamkins (CUNY).

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Computer-Aided Proofs

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2000 MATHEMATICS SUBJECT CLASSIFICATION. MSC Primary 03B35; Secondary 68T15;

Recent advances in the field of ‘Interactive Proof Checking’ with the associated development of powerful tools such as ‘Proof Assistants’ have given rise to an

interesting consequence – viz. the practical feasibility of importing techniques developed in the computer science community and redeploying them to improve the main activity of the working mathematician, namely the process of proof development. At the core of such redeployed techniques lie the notions of formal systems, formal reasoning, and formal proofs. However the process of formalizing mathematics is a highly non-trivial task, and gives rise to a number of challenging and interesting issues which need to be addressed in order to make the discipline of computer-aided proofs more prevalent in the future [1].

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Four Errors in Cantor’s Proofs on the Uncountability of Real Number Set and The Foundation of Mathematics

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 03B35, 03E99

The ancient Zeno’s Paradoxes disclosed the serious defects in the infinite theory and its relating limit theory. From than on, people have been trying very hard to solve the problems. But till now Zeno’s Paradoxes are still on and new versions of Zeno’s Paradox can still be found in modern mathematics. Based on the logical deduction, the author in this article proves the following conclusion: because of the long existing defects in the foundation of mathematics, Cantor unconsciously made four serious but concealed mistakes in his proofs on the uncountability of real number set and turned the proofs into a kind of typical mathematical magic, such a proof and the related result are not scientific at all. During the last two decades, the author has published more than 20 papers in different academic Chinese journals to discuss the foundation of mathematics[1], [2], [3], [4], [5].

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Herbrand Consistency and Bounded Induction

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Herbrand Consistency of a theory is defined to be the propositional satisfiability of the set of its all Skolem instances. This is a weaker notion than the standard (Hilbert style) consistency, resembling much to cut-free consistency. Cut-Free Consistency, and in particular Herbrand Consistency, had been suggested for Π_1 -separating the hierarchy of bounded arithmetics. It was proved in [1] that

- (1) for a bounded formula $\theta(\bar{x})$ if the theory $(\mathbb{I}\Delta_0 + \Omega_2) + \exists \bar{x} \in \log^3 \theta(\bar{x}) + \text{HCon}(\mathbb{I}\Delta_0 + \Omega_2)$ is consistent, then so is the theory $(\mathbb{I}\Delta_0 + \Omega_2) + \exists \bar{x} \in \log^4 \theta(\bar{x})$, where \log^n is the cut $\{x \mid \exists y[y = \exp^n(x)]\}$; and
- (2) for any m, n , there exists a bounded formula $\eta(\bar{x})$ such that the theory $(\mathbb{I}\Delta_0 + \Omega_m) + \exists \bar{x} \in \log^n \eta(\bar{x})$ is consistent, but the theory $(\mathbb{I}\Delta_0 + \Omega_m) + \exists \bar{x} \in \log^{n+1} \eta(\bar{x})$ is not consistent.

Thus one gets a model-theoretic proof for $\mathbb{I}\Delta_0 + \Omega_2 \not\vdash \text{HCon}(\mathbb{I}\Delta_0 + \Omega_2)$ by putting $m = 2, n = 3$. In other words, by (1) in the presence of Herbrand Consistency of the theory, one can shrink any $(\log^3 -)$ witness of any bounded formula logarithmically, but this cannot be done for *all* bounded formulas by (2); thus the theory cannot derive its own Herbrand Consistency.

The above proposition (1) was modified and generalized to the case of $\mathbb{I}\Delta_0 + \Omega_1$ in Chapter 5 of [2]. Here we further modify (1) and (2) to the case of $\mathbb{I}\Delta_0$: for the cuts $\mathcal{I} = \{x \mid \exists y[y = \exp(\omega_1^2(x))]\}$ and $\mathcal{J} = \{x \mid \exists y[y = \exp^2(x^4)]\}$ we show that

- (3) for a bounded formula $\theta(\bar{x})$ if the theory $\mathbb{I}\Delta_0 + \exists \bar{x} \in \mathcal{I} \theta(\bar{x}) + \text{HCon}(\mathbb{I}\Delta_0)$ is consistent, then so is $\mathbb{I}\Delta_0 + \exists \bar{x} \in \mathcal{J} \theta(\bar{x})$, where $\mathbb{I}\Delta_0$ is the theory $\mathbb{I}\Delta_0$ augmented with the additional axiom $\forall x \exists y(y = x \cdot x)$; and

- (4) for any m, n , there exists a bounded formula $\eta(\bar{x})$ such that $\mathbb{I}\Delta_0 + \exists \bar{x} \in \mathcal{I}\eta(\bar{x})$ is consistent, but $\mathbb{I}\Delta_0 + \exists \bar{x} \in \mathcal{J}\eta(\bar{x})$ is not consistent.

These two theorems immediately imply that $\mathbb{I}\Delta_0 \not\vdash \text{HCon}(\mathbb{I}\Delta_0)$. We note that the proof of (2) above in [1] works straightforwardly for (4) since the relation $2^x \in \mathcal{I} \iff x \in \mathcal{J}$ holds for \mathcal{I} and \mathcal{J} , just like the way for \log^n and \log^{n+1} .

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Some Properties of Images And Inverse Images of L-Intuitionistic or L-Vague Fuzzy Subsets

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The aim of this paper is basically to study some of the standard properties of the intuitionistic L-fuzzy images and intuitionistic L-fuzzy inverse images of intuitionistic L-fuzzy or L-vague fuzzy subsets of a set under a crisp map which not only play a crucial role in the study of both Intuitionistic L-Fuzzy Algebra and Intuitionistic L-Fuzzy Topology but also are necessary for the individual/exclusive development of intuitionistic L-Fuzzy or L-Vague Set Theory.



Section 2

Algebra

Cohomologically Trivial Modules over Finite p -groups

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20J06

Let A be an abelian group and G be a group acting on A as a group. Then A is a G -module and A is called a cohomologically trivial G -module whenever the Tate cohomology $H^n(X, A)$ is zero for all integers n and all $X \leq G$.

Let G be a finite p -group and N be a normal subgroup of G . Then $Z(N)$ can be viewed as a $\frac{G}{N}$ -module whenever G/N acts by conjugation on $Z(N)$, i.e., $z^{gN} := z^g = g^{-1}zg$ for all $g \in G$ and all $z \in Z(N)$. We show that $Z(N)$ is *never* a cohomologically trivial $\frac{G}{N}$ -module whenever G satisfy one of the following properties:

- (1) G is of nilpotency class 2 and G/N is not cyclic.
- (2) p is odd, G is of nilpotency class 3 and G/N is not cyclic.
- (3) p is odd, G/N is not cyclic and all elements of order p of $G/Z(G)$ are contained in the center of $G/Z(G)$.

P. Schmid [2] has proved that for a finite regular p -group G , $Z(N)$ is never a cohomologically trivial G/N -module for all normal subgroups N of G such that G/N is not cyclic. He then proposed the question [2, p. 3] of whether there is a non-regular finite p -group G such that $Z(\Phi(G))$ is a cohomologically trivial $\frac{G}{\Phi(G)}$ -module, where $\Phi(G)$ is the Frattini subgroup of G . Later, Schmid [3, p. 363] has announced that in [1] the existence of (non-regular) p -groups G such that $Z(\Phi(G))$ is a cohomologically trivial $\frac{G}{\Phi(G)}$ -module was shown. But

the author noted that all groups considered in [1] are finite p -groups of class 4 with $p \neq 2, 3$, so these groups are all finite regular p -groups and they cannot be candidate for Schmid's question. This latter has been kindly confirmed by P. Schmid [4].

We prove that groups G with `IdSmallGroup [256,i]` in small group library of GAP [5], where $i \in \{298, \dots, 307\}$, are such that $Z(\Phi(G))$ is a cohomologically trivial $\frac{G}{\Phi(G)}$ -module. This research was in part supported by a grant from IPM.

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A Characterization of $C_n(q)$ by the Set of Orders of Maximal Abelian Subgroups

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Let G be a finite group and $\pi(G)$ be the set of all prime divisors of $|G|$. The prime graph $\Gamma(G)$ of G is the graph whose vertex set is $\pi(G)$ and two distinct primes p and q are joint by an edge if and only if G contains an element of order pq . Characterization of finite groups by the set of orders of their maximal abelian subgroups, has first been considered by Wang [2]. A finite group G is said to be *characterizable by the set of orders of its maximal abelian subgroups*, if G is uniquely determined by the orders of its maximal abelian subgroups. Let $M(G) = \{|N| : N \text{ is a maximal abelian subgroup of } G\}$. According to [1], $B_n(q)$ is characterizable by the set of orders of its maximal abelian subgroups. Here, using $M(C_n(q))$ and $\Gamma(C_n(q))$, we study whether $C_n(q)$ is characterizable by the set of orders of its maximal abelian subgroups.

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Derivations on Prime Near Ring

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A near ring N is called 3-prime (resp. 3-semiprime) if for any nonzero elements $x, y \in N$, $xNy \neq 0$ (resp. $xNx \neq 0$). An additive mapping $d : N \rightarrow N$ is called a derivation on N if $d(xy) = xd(y) + d(x)y$ holds for all $x, y \in N$. An additive mapping $d : N \rightarrow N$ is called a (σ, τ) - derivation on N if there exist automorphisms $\sigma, \tau : N \rightarrow N$ such that $d(xy) = \sigma(x)d(y) + d(x)\tau(y)$ holds for all $x, y \in N$. Since E.C.Posner [1] established a very striking result which states that if R is a prime ring admitting a nonzero centralizing derivation, then R must be commutative, many authors have investigated the properties of derivations of 3-prime and 3-semiprime rings. Being important ring theory tools (see for example [2]) these results are one of the sources of the development of theory of differential identities and the theory of Hopf Algebras acting on rings. In this talk we discuss near rings admitting a nontrivial (σ, τ) - derivation and some generalizations of Posner's Theorem that we explored recently.

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On Jordan*-derivations in Rings with Involution

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 16N60, 16W10, 16W25.

Let R be an associative ring. An additive mapping $d : R \rightarrow R$ is called a *-derivation if $d(xy) = d(x)y^* + xd(y)$ holds for all $x, y \in R$. An additive mapping $d : R \rightarrow R$ is called a Jordan *-derivation if $d(x^2) = d(x)x^* + xd(x)$ holds for all $x \in R$. The study of Jordan *-derivations has been motivated by the problem of representability of quasi-quadratic functionals by sesquilinear ones. It turns out that the question of whether each quasi-quadratic functional is generated by some sesquilinear functional is intimately connected with the structure of Jordan *-derivations (viz., [Bull. Austral. Math. Soc. 37(1988), 27-29] and [Proc. Amer. Math. Soc. 91(1987), 133-136], where further reference can be looked).

In this lecture, we will present some new developments and generalization about these mappings and related concepts. Moreover, some examples and counter examples will be discussed for questions raised naturally. (This is a joint work with Ajda Fosner).



On Symmetric Generalized (α, β) -biderivations in Rings

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Let R be an associative ring and let α, β be endomorphisms of R . An additive mapping $d : R \rightarrow R$ is said to be an (α, β) -derivation if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$ holds for all $x, y \in R$. An additive mapping $F : R \rightarrow R$ is said to be a generalized (α, β) -derivation if there exists an (α, β) -derivation $d : R \rightarrow R$ such that $F(xy) = F(x)\alpha(y) + \beta(x)d(y)$ holds for all $x, y \in R$. A symmetric biadditive mapping $D : R \times R \rightarrow R$ is called a symmetric (α, β) -biderivation if $D(xy, z) = D(x, z)\alpha(y) + \beta(x)D(y, z)$ holds for all $x, y, z \in R$. A symmetric biadditive mapping $G : R \times R \rightarrow R$ is said to be a symmetric generalized (α, β) -biderivation if there exists a symmetric (α, β) -biderivation $D : R \times R \rightarrow R$ such that $G(xy, z) = G(x, z)\alpha(y) + \beta(x)D(y, z)$ holds for all $x, y, z \in R$. A symmetric generalized (α, β) -biderivation with $D = 0$ is called an α -left bimultiplier. A symmetric biadditive mapping $J : R \times R \rightarrow R$ is called a symmetric generalized Jordan (α, β) -biderivation on R if there exists a symmetric (α, β) -biderivation

$D : R \times R \longrightarrow R$ such that $J(x^2, z) = J(x, z)\alpha(x) + \beta(x)J(x, z)$ holds for all $x, z \in R$. It is straight forward to see that every symmetric generalized (α, β) -biderivation on R is a symmetric generalized Jordan (α, β) -biderivation, but the converse need not be true in general. In the present talk we shall discuss the conditions under which every symmetric generalized Jordan (α, β) -biderivation becomes symmetric generalized (α, β) -biderivation. Moreover, we shall also find the conditions under which every symmetric generalized (α, β) -biderivation on R turns out to be an α -left bimultiplier of R . Finally some more related result shall also be discussed.



Gelfand Pairs in the Alternating Groups

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Let H be a subgroup of a finite group G . The permutation representation of G on the set of cosets G/H is said to be multiplicity-free if all its irreducible constituents are distinct, and the subgroup H is called a multiplicity-free subgroup of G . The multiplicity-free condition is equivalent to the commutativity of the algebra of bi- H -invariant complex-valued functions on the double coset space $H \backslash G / H$ under convolution product. In this latter setting, G/H is a finite symmetric space and (G, H) is called a Gelfand pair. We determine all Gelfand pairs (A_n, H) in the alternating groups A_n , for all n . We also give some decompositions of multiplicity-free permutation characters of A_n . The symmetric group case was first investigated by Saxl, and recently completed by Godsil-Meagher and Wildon. Part of this paper is joint work with Raissa T. Relator.

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Associated Prime Ideals of Weak σ -rigid Rings and their Extensions

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Let R be a ring and σ an endomorphism of R . Recall that in [1], σ is called a rigid endomorphism if $a\sigma(a) = 0$ implies $a = 0$ for $a \in R$, and R is called a σ -rigid ring. For example, let $R = \mathbf{C}$, the field of complex numbers and $\sigma : R \rightarrow R$ be the map defined by $\sigma(a + ib) = a - ib$; a, b real numbers. Then it can be seen that R is a σ -rigid ring.

We also recall from [2] that R is said to be a weak σ -rigid ring if $a\sigma(a) \in N(R)$ if and only if $a \in N(R)$ for $a \in R$, where $N(R)$ is the set of nilpotent elements of R .

Let now R be a right Noetherian ring which is also an algebra over \mathbf{Q} (\mathbf{Q} the field of rational numbers). Let σ be an automorphism of R and δ a σ -derivation of R . Let further σ be such that R is a σ -rigid ring. In this paper we study the associated prime ideals of Ore extension $R[x; \sigma, \delta]$ and we prove the following in this direction:

Let R be a semiprime right Noetherian ring which is also an algebra over \mathbf{Q} . Let σ and δ be as above. Then P is an associated prime ideal of $R[x; \sigma, \delta]$ (viewed as a right module over itself) if and only if there exists an associated prime ideal U of R with $\sigma(U) = U$ and $\delta(U) \subseteq U$ and $P = U[x; \sigma, \delta]$.

We also prove that if R be a right Noetherian ring which is also an algebra over \mathbf{Q} , σ and δ as usual such that $\sigma(\delta(a)) = \delta(\sigma(a))$ for all $a \in R$ and $\sigma(U) = U$ for all associated prime ideals U of R (viewed as a right module over itself), then P is an associated prime ideal of $R[x; \sigma, \delta]$ (viewed as a right module over itself) if and only if there exists an associated prime ideal U of R such that $(P \cap R)[x; \sigma, \delta] = P$ and $P \cap R = U$.

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Prime Graph of a Ring

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We consider associative ring R (need not be commutative) and define the notion: prime graph of R . We present few illustrations and obtain fundamental results related to a prime graph. Further, it is if R is a semiprime ring then R is a prime ring if and only if the prime graph of R is a tree. We observe several properties of prime graph with respect to the properties like zero divisors, nilpotent elements in R .

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A Different Approach to Mathieu Groups

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The traditional approaches to the first five sporadic (finite simple) (Mathieu) groups M_{11} , M_{12} , M_{22} , M_{23} and M_{24} are via Steiner triple systems or through

the binary Golay code. In our approach, we recall the extra involutory automorphism of the alternative group $A_6 \approx PSL_2(9)$, so the normal extension S_6 is accompanied by $SL_2(9)$ and M_{10} , all of order 720. There must be a route from M_{10} to the other (simple) Mathieu groups, as e.g. $M_{10} \subset M_{11}$; we explore this route in our presentation. Our source of inspiration are [1], Essay 7, and [2].

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On the Discriminant of a General Polynomial in Terms of Elementary Symmetric Function

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In this paper, we discuss 1. How many terms are there in the discriminant Δ_n in terms of elementary symmetric functions; 2. How to determine the coefficients of Δ_n in terms of elementary symmetric functions; 3. How to transform any symmetric function in terms of ‘simple’ symmetric functions into that in terms of elementary symmetric functions. For problem 3, ‘simple’ symmetric function means symmetric function generated by a monomial such as $U_{k_1 k_2 \dots k_n} = \frac{1}{l_1! l_2! \dots l_r!} \sum x_{i_1}^{k_1} x_{i_2}^{k_2} \dots x_{i_n}^{k_n}$, which is generated by $x_1^{k_1} x_2^{k_2} \dots x_n^{k_n}$, r is the number of different k , denoted by $k_1^*, k_2^*, \dots, k_r^*$, l_i is the number of elements of the set $\{j | k_j = k_i^*\}$, $1 \leq i \leq r$; and \sum go over all the permutations i_1, i_2, \dots, i_n of $1, 2, \dots, n$, $N = \sum_{j=1}^r k_j$ is the order of the polynomial. We use d_n to express the number of terms in Δ_n , at the moment, to calculate the first n terms of the sequence $\{d_n\}$, we need to iterate $n \times n^2 \times n^2$ matrices, this should be greatly improved. About the problem 3, for each N , arrange all the U -terms of order N into a vector \vec{U}_N , and the corresponding S -terms form another vector \vec{S}_N , then there exists a (triangular) transitional matrix \sum_N such that $\vec{S}_N = \sum_N \vec{U}_N$, and $\vec{U}_N = \sum_N^{-1} \vec{S}_N$, we have an iterative procedure to calculate the transitional matrix \sum_N . Some numerical results for problem 2, we find out that there exists a (coefficient) sequence which covers all the coefficients of Δ_n for all n . The coefficients of Δ_n are the section of the first d_n terms of the

sequence. And the sequence is

$$1, -4, -4, 18, -27, -4, 16, 18, -80, -6, 144, -27, -128, -192, 256, \dots, \\ -46656, 381024, -926100, 600250, -444529, 1728720, -840350, 806736, \\ 518616, -1512630, 1411788, -605052, 1176490, 705894, -823543, \dots,$$

We see that the first 5 terms are the coefficients of Δ_3 , the first 16 terms are the coefficients of Δ_4 , another 15 terms are the coefficients of the last 15 terms in Δ_7 .

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On Factorization of Finite Groups

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A group G is called factorizable if there are proper subgroups A and B such that $G = AB$, otherwise G is called a non-factorizable group. In this paper we will investigate the involvement of the alternating and the symmetric groups in a factorization of a finite group G . We also obtain some results when G is a non-factorizable group.



Wreath Sum of Near-rings-Revisited

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In this paper we want to revisit the structure of wreath sum of near-rings to establish some elegant structure of so-called wreath sum of Near-ring groups

and discuss some related topics. It is observed here, how some examples of wreath sum of near-rings may lead us to the notion of what may be called the wreath sum of near-ring groups. It is interesting to note that, we have hardly met any examples as yet in this context (especially in case of near-rings). Such justifiable examples may lead us to or help in exploring some new areas and the projected expectation has been possibly reached by what the author here likes to claim. In contrast to the product, in the structure of wreath sum of near rings, an observation leads us to an implication that same set of wreath sum of near rings structure may have different explanation when the product is defined in an innovative way. It is noticeable that a subset of such a set of so-called wreath sum with a specific algebraic structure need not inherit it in the other context. This observation opens a new outlet to the fact that even in case of a particular near ring we may have more than one near-ring group structure of the same wreath sum though the sets may not have isomorphic algebraic structure. This very observation motivates us to extend the concept of wreath sum of near-rings to that of near-ring groups. The paper also contains some important and stimulating results relating to homomorphism and ideal structures of such systems. Such results include structure of wreath sum of near ring groups, inheritance of Noetherian and Goldie characters. Interestingly, wreath sum of two left Goldie as well as right Goldie near rings appear as so respectively, subject to the analogous restriction in each case.



A Study on Fuzzy Ideals of N -Groups

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 16Y30,03E72, 16Y99

Using the idea of the new sort of fuzzy subnearring of a near-ring, fuzzy subgroups and their generalizations defined by various researchers, we try to introduce the notion of $(\epsilon, \epsilon \vee q)$ -fuzzy ideals of N -groups. These fuzzy ideals are characterized by their level ideals and some other related properties are investigated.

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Application of Finite Field in Coding Theory

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One of the interesting and important applications of finite field has been the development of coding theory. Since $x^2 + x + 1$ is an irreducible polynomial in $Z_2(x)$, $Z_2(x) / \langle x^2 + x + 1 \rangle$ is a finite field having four elements, $0, 1, \alpha, \alpha + 1$. we use $Z_2(x) / \langle x^2 + x + 1 \rangle$ to construct a $(5, 2)$ linear code that will correct any single error. For a $(5, 2)$ linear code the parity check matrix for the code $0, 1, \alpha, \alpha + 1$, is
$$\begin{bmatrix} 1 & 0 & 1 & 1 & \alpha \\ 0 & 1 & \alpha & \alpha + 1 & \alpha + 1 \end{bmatrix}$$



On Prime Filters in Normal Almost Distributive Lattices

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The concept of an Almost Distributive Lattice(ADL) was introduced by Swamy and Rao [3] as a common abstraction of the existing ring theoretic and lattice theoretic generalizations of a Boolean algebra while characterizing the set of all global sections of sheaves over locally Boolean spaces algebraically. The concept of an ideal in an ADL was introduced [3] analogous to that in a distributive lattice and it was observed that the set $PI(R)$ of all principal ideals of R forms a distributive lattice. This enables us to extend many existing concepts from the class of distributive lattices to the class of ADLs such as pseudo-complementation on an ADL [4] and Stone ADL [5]. Later, Normal ADLs were studied by Rao and Ravi Kumar [1] and α -ideals in an ADL were studied by Rao and Sambasiva Rao [2]. In this talk we give characterization of a normal ADL topologically in terms of prime filters and maximal filters. Also, we introduce the concept of a B -normal ADL R , where B is Birkhoff centre of R as a generalization to the concept of a normal ADL. We prove that an ADL R in which the intersection of maximal filters is the set of all maximal elements is B -normal iff the space $Max_F R$ of maximal filters with the hull-kernel topology is a Boolean space. Also, it is proved that R is B -normal iff $Max_F R$ is homeomorphic to $Spec_F B$, the set of all prime filters of B .

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Frobenius Endomorphisms on Matrix Spaces

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The theory of Frobenius endomorphisms, i.e., transformations preserving different properties and invariants dates back to the works by Frobenius, Schur, and Dieudonné, and is an intensively developing part of algebra nowadays.

The problem can be formulated as follows. Let $T : M_n(R) \rightarrow M_n(R)$ be a certain transformation on matrices of a fixed order n over a certain ring R . Let us consider a subset $S \subseteq M_n(R)$, or a matrix functional $\rho : M_n(R) \rightarrow Q$, where Q is a given set (ρ can be a determinant, trace, rank, permanent, etc.), or a matrix property \mathcal{P} (nilpotence, idempotence, singularity, etc.), or a matrix relation \mathcal{R} (similarity, commutativity, order, etc.). It is assumed that one of the following holds: the transformation T preserves the set S , or the functional ρ , or the property \mathcal{P} , or the relation \mathcal{R} , which means that $X \in S$ implies $T(X) \in S$; $\rho(X) = \rho(T(X))$ for all $X \in M_n(R)$; if X satisfies \mathcal{P} , then $T(X)$ satisfies \mathcal{P} also; and XRY implies $T(X)\mathcal{R}T(Y)$, correspondingly.

Such maps are usually called *Frobenius endomorphisms*.

The main problem is to characterize all Frobenius endomorphisms preserving one of S , ρ , \mathcal{P} , or \mathcal{R} , possibly under some additional assumptions such as linearity, additivity, bijectivity, etc.

In this talk we give an overview of the development of this field including our recent results.



Lojasiewicz Inequality at Infinity for Polynomials in Two Real Variables

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We propose different types of Lojasiewicz inequality at infinity for polynomials in two real variables. The formulas for the Lojasiewicz exponents are given.

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Some Extensions of Semicommutative Compatible Ideals in Ore Extensions

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Throughout this note R always denotes an associative ring with unity. $R[x; \alpha, \delta]$ will stand for the Ore extension of R , where α is an endomorphism and δ an α -derivation of R , that is, δ is an additive map such that $\delta(ab) = \delta(a)b + \alpha(a)\delta(b)$ for all $a, b \in R$. An ideal I is called α -compatible if for all $a, b \in R$, $ab \in I \Leftrightarrow a\alpha(b) \in I$. Moreover, an ideal I is said to be δ -compatible if for each $a, b \in R$, $ab \in I \Rightarrow a\delta(b) \in I$. An ideal I is called semicommutative, if whenever $ab \in I$ implies $arb \in I$ for each $r \in R$. In this note we study connection between semicommutative compatible ideals of R and related ideals of Ore extension $R[x; \alpha, \delta]$ and skew power series $R[[x; \alpha]]$. Also we investigate the relationship of the prime radical and the upper nil radical of R with the prime radical and the upper nil radical of the Ore extension $R[x; \alpha, \delta]$ and skew power series $R[[x; \alpha]]$.

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What is a Finitely Related Structure, Categorically

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As opposed to the finitely presented and the finitely generated cases, the concept of finitely related algebra in a variety is essentially dependent on the signature on which the variety is defined. More specifically, there exist equivalences (even isomorphisms) between varieties, seen as categories, sending a finitely related algebra to one which is not.

A solution to this problem is to define a *categorically* finitely related algebra as one which is (classically) finitely related with respect to its *canonical* theory. Given a variety, its canonical theory is the dual of the full subcategory on its *perfectly presentable* objects (see [1], 8.12). It is also the idempotent completion of any of its (algebraic) theory. The categorically finitely related objects of a variety are then the algebras of the form $\coprod_I A_i$, where the A_i are finitely presentable and all but at most one of them are projective.

Another suggestion would be to include the retractions of these objects in the definition of the categorically finitely related objects. We note that they are the same than the retractions of the (classical) finitely related objects. This weaker concept has a simple categorical definition, as those objects X for which every morphism $f: X \rightarrow \text{Colim}_I C_i$ to the colimit of a filtered diagram made of surjective homomorphisms factorizes through one of the colimit morphisms $c_i: C_i \rightarrow \text{Colim}_I C_i$. All this, and what follows, generalize to locally finitely presentable categories (changing surjective and projective for strong epi and strong epi projective, respectively). This concept is the natural counterpart of the categorical definition of finitely presentable and finitely generated objects: indeed, the injectivity of the canonical maps $\text{Colim}_I(\text{Hom}(X, C_i)) \rightarrow \text{Hom}(X, \text{Colim}_I C_i)$ actually follows from its surjectivity in every finitely accessible category; note however that this is not true if one restricts to the diagrams made of strong epimorphisms.

Finally, we note that in any locally finitely presentable category, the equation

$$\textit{finitely presentable} = \textit{finitely generated} + \textit{categorically finitely related}$$

holds for both meanings of “categorically finitely related” suggested above.

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Approximate Symmetry in Certain Quasigroups Derived from the Dihedral Group

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Groups arise from symmetry, understood through transitive group actions. Such actions may be defined equally well for quasigroups, where they lead to concepts of approximate symmetry. Applications in biology and other fields are now raising the problem of developing a rigorous theory of approximate symmetry. A quasigroup is defined as a set Q equipped with a multiplication, not necessarily associative, such that in the equation $x \cdot y = z$, knowledge of any two of the elements x, y, z of Q specifies the third uniquely. The body of the multiplication table of a finite quasigroup is a Latin square. Nonempty associative quasigroups are groups: [2] and [4].

In this paper, we consider the usual direct product G of the dihedral group of degree 4 and the cyclic group of order 2. By changing some intercalates of the body of the multiplication table of the group G , we get various quasigroup structures on the set G . We study homogeneous spaces derived from such a quasigroup and show how each action matrix acts on an orbit contained in the homogeneous space. Action matrices show the approximate symmetry.

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New Identities In Universal Osborn Loops

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A question associated with the 2005 open problem of Michael Kinyon (Is every Osborn loop universal?), is answered. Two nice identities that characterize universal(left and right universal) Osborn loops are established. Numerous new identities are established for universal(left and right universal) Osborn loops like CC-loops, VD-loops and universal weak inverse property loops. Particularly, Moufang loops are discovered to obey the new identity $[y(x^{-1}u) \cdot u^{-1}](xu) = [y(xu) \cdot u^{-1}](x^{-1}u)$ surprisingly. For the first time, new loop properties that are weaker forms of well known loop properties like inverse property, power associativity and diassociativity are introduced and studied in universal(left and right universal) Osborn loops. Some of them are found to be necessary and sufficient conditions for a universal Osborn to be 3 power associative. For instance, four of them are found to be new necessary and sufficient conditions for a CC-loop to be power associative. A conjugacy closed loop is shown to be diassociative if and only if it is power associative and has a weak form of diassociative.

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Orthogonal Derivations in Γ -rings

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Let (M, Γ) be a Γ -ring. An additive mapping $D : M \rightarrow M$ is called a generalized derivation if there exist a derivation $d : M \rightarrow M$ such that $D(x\alpha y) = D(x)\alpha y + x\alpha d(y)$ holds for all $x, y \in M$ and $\alpha \in M$. It is denoted by (D, d) . Two generalized derivations (D, d) and (G, g) of M are called orthogonal if $D(x)\Gamma M \Gamma G(y) = (0) = G(y)\Gamma M \Gamma D(x)$ for all $x, y \in M$. Suppose (D, d) and (G, g) be generalized derivations on a Γ -ring M . In this paper, we prove some necessary and sufficient conditions for the generalized derivations to be orthogonal. Infact, we prove that if (D, d) and (G, g) are generalized derivations of M , then for all $x, y \in M$ and $\gamma \in \Gamma$, the following conditions are equivalent: (i) (D, d) and (G, g) are orthogonal, (ii) (D, d) and (G, g) satisfy the following relations: (a) $D(x)\gamma G(y) + G(x)\gamma D(y) = 0$, (b) $d(x)\gamma G(y) + g(x)\gamma D(y) = 0$, (iii) $D(x)\gamma G(y) = d(x)\gamma G(y) = 0$, (iv) $D(x)\gamma G(y) = 0$ and $dG = dg = 0$, (v) (DG, dg) is a generalized derivation. We also prove that if (D, d) and (G, g) are generalized derivations of M , then the following conditions are equivalent: (i) (DG, dg) is a generalized derivation, (ii) (GD, gd) is a generalized derivation, (iii) D and G are orthogonal, also G and d are orthogonal.

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On Prime Ideal Principle in Lattices

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Keywords. Distributive lattice, 0-distributive lattice, prime ideal, semiprime ideal.

In this paper, we introduce a prime ideal principle in lattices and use it to prove that certain ideals in lattices are prime ideals. This also extends the results of Stone, Gorbunov and Tumanov, Rav etc. on prime ideals.

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Translation Plane of Order 3^4

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The construction of translation planes of order q^{t+1} through t-spread sets over a Galois field $\text{GF}(q)$ of order q , where q is a power of a prime has attracted many of the researchers. To construct and study the translation planes of order q^{t+1} , it has become a practice to construct t-spread sets over $\text{GF}(q)$.

The authors have constructed and classified all translation planes of order 3^4 which admit a collineation group isomorphic to a meta-cyclic group of order 20 as a subgroup of their $(\infty, [0,0])$ -homology group. For this purpose the authors have constructed all 3-spread sets over $\text{GF}(3)$ which admit a meta-cyclic group

$$\mathbf{G} = \langle x, y \mid x, y \in \text{GL}(4,3), x^5 = I, y^2 = -I, y^{-1}xy = x^{-1} \rangle$$

of order 20 in their left nuclei by Rao and Davis procedure of constructing spread sets, where

$$x = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 \\ 1 & 0 & 2 & 2 \end{pmatrix}, \quad y = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 2 & 0 & 1 & 2 \\ 1 & 2 & 2 & 2 \end{pmatrix}$$

As a result 456, 3-spread sets over $\text{GF}(3)$ are obtained and these 3-spread sets are partitioned into ten isotopic classes.

The aim of this paper is to study the translation plane π of order 3^4 associated with the representative 3-spread set \mathbf{C} over $\text{GF}(3)$ of the first isotopic class and its translation complement, where

$$\mathbf{C} = \{0\} \cup \mathbf{G} \cup A_1 \mathbf{G} \cup A_2 \mathbf{G} \cup A_3 \mathbf{G}$$

where A_1, A_2, A_3 are 4×4 matrices, $A_1 = (0001, 0010, 0212, 2121)$, $A_2 = (0010, 0112, 2001, 2200)$, $A_3 = (0100, 0021, 1111, 2221)$.

The conjugacy collineation group and autotopism group of π are determined. It is shown that the kernel K of π is $\text{GF}(3)$ and the autotopism group itself is the translation complement. Further, the translation complement is of order 160 and divides the set of ideal points into orbits of lengths 1, 1, 40, 40.

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p -Power Points and Modules of Constant p -power Jordan Type

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Keywords. shifted cyclic subgroup, p -point, p -power point, restriction, modular representation, constant Jordan type, constant p -power Jordan type, wild representation type

Let $k[G]$ be the group algebra of a finite abelian p -group G over an algebraically closed field of characteristic p . We define p -power points, shifted cyclic subgroups of $k[G]$ and give a characterization of these. Using p -power points we define modules of constant p -power Jordan type as a generalization of modules of constant Jordan type. Endotrivial $k[G]$ -modules, $k[G]$ -modules with equal image property are examples of constant p -power Jordan type modules. We give examples of non-isomorphic modules of constant p -power Jordan type having the same constant Jordan type.

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On Artinian Modules Over Duo Rings

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The well-known method, which reduces the study of Artinian modules over commutative rings to the study of Artinian modules over quasi-local rings, is extended to the study of these modules over duo rings. The duals of Krull Intersection Theorem and Nakayamas Lemma are proved for Artinian modules over a large class of duo rings. We also give an upper bound for the length of a duo ring R in terms of the length of a faithful R -module. This generalizes a well-known result of Schur concerning the cardinality of a maximal linearly independent set of commuting matrices.

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Reference Points and Roughness

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We introduce the notion of a reference point and provide local approximations for a subset of the universe. The notion of a reference point naturally gives rise to a rough approximations framework, wherein several approximations are possible on the same set. Also, we present an extension to the decision theoretic rough set model by using reference points.

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Lower Bounds for the Number of Conjugacy Classes of Finite Groups

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We present a lower bound for the number of conjugacy classes of a finite group in terms of the largest prime divisor of the group order. We also present examples for which this bound is best possible and discuss recent progress on the conjecture that these examples are the only ones meeting the bound.

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Generalization of Prime Ideals and Prime Submodules

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Suppose that R is a commutative ring with identity and M a unitary R -module. we introduce the concept of a 2-absorbing submodule which is a generalization of prime submodule. A non-zero proper submodule M_0 of an R -module M is called a 2-absorbing submodule of M if whenever, $r_1, r_2, r_3 \in R$, $m \in M$ and $r_1 r_2 r_3 m \in M_0$, then $r_1 r_2 m \in M_0$ or $r_1 r_3 m \in M_0$ or $r_2 r_3 m \in M_0$. M is said to be a 2-absorbing, if $\{0\}$ is a 2-absorbing submodule of M . It is shown that if M_0 is a 2-absorbing submodule of M , then either $Rad(M_0)$ is a primes ideal of R or $Rad(M_0) = P_1 \cap P_2$ where P_1, P_2 are the only distinct prime ideals of R minimal over $(M_0 : M)$. All 2-absorbing submodules of a module M over a Valuation domain or Dedekind domain R are completely discribed. It is shown that a Noetherian domain R is a Dedekind domain, if and only if for every 2-absorbing submodule M_0 of M , $(M_0 : M)$ is either a maximal ideal of R or m^2 for some maximal ideal m of R or $m_1 m_2$ where m_1, m_2 are some maximal ideals of R . Finally we introduce a relation between strongly irreducible submodules of M and 2-absorbing submodules.

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Generalized (σ, τ) -Higher Derivations in Prime Rings

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Let R be a ring and σ, τ be endomorphisms of R . Suppose that U is a Lie ideal of R such that $u^2 \in U$ for all $u \in U$. A family $D = \{d_n\}_{n \in \mathbb{N}}$ of additive mappings $d_n : R \rightarrow R$ is said to be (σ, τ) -higher derivation of R if $d_0 = I_R$, the identity map on R and $d_n(ab) = \sum_{i+j=n} d_i(\sigma^{n-i}(a))d_j(\tau^{n-j}(b))$ holds for all $a, b \in R$ and for each $n \in \mathbb{N}$. A family $F = \{f_n\}_{n \in \mathbb{N}}$ of additive mappings $f_n : R \rightarrow R$ is said to be generalized (σ, τ) -higher derivation (resp. generalized Jordan (σ, τ) -higher derivation) of U into R if there exist a (σ, τ) -higher derivation $D = \{d_n\}_{n \in \mathbb{N}}$ of R such that; $f_0 = I_R$, the identity map on R and $f_n(uv) = \sum_{i+j=n} f_i(\sigma^{n-i}(u))d_j(\tau^{n-j}(v))$ (resp. $f_n(u^2) = \sum_{i+j=n} f_i(\sigma^{n-i}(u))d_j(\tau^{n-j}(u))$) holds for all $u, v \in U$ and for each $n \in \mathbb{N}$. In the present paper we shall obtain the conditions under which every generalized Jordan (σ, τ) -higher derivation of U into R is a generalized (σ, τ) -higher derivations of U into R .

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Prolongations of Valuations to Finite Extensions

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Let $K = \mathbb{Q}(\theta)$ be an algebraic number field with θ in the ring A_K of algebraic integers of K and $f(x)$ be the minimal polynomial of θ over the field \mathbb{Q} of rational numbers. For a rational prime p , let $\bar{f}(x) = \bar{g}_1(x)^{e_1} \dots \bar{g}_r(x)^{e_r}$ be the factorization of the polynomial $\bar{f}(x)$ obtained by reducing coefficients of $f(x)$ modulo p into a product of powers of distinct irreducible polynomials over $\mathbb{Z}/p\mathbb{Z}$ with $g_i(x)$ monic. Dedekind proved that if p does not divide $[A_K : \mathbb{Z}[\theta]]$, then $pA_K = \wp_1^{e_1} \dots \wp_r^{e_r}$, where \wp_1, \dots, \wp_r are distinct prime ideals of A_K , $\wp_i = pA_K + g_i(\theta)A_K$ having residual degree equal to the degree of $\bar{g}_i(x)$. He also proved that p does not divide $[A_K : \mathbb{Z}[\theta]]$ if and only if for each i , either $e_i = 1$ or $\bar{g}_i(x)$ does not divide $\bar{M}(x)$ where $M(x) = \frac{1}{p}(f(x) - g_1(x)^{e_1} \dots g_r(x)^{e_r})$ (See [1, Theorem 6.1.4],[2]). Our aim is to give a weaker condition than the one given by Dedekind which ensures that if the polynomial $\bar{f}(x)$ factors as above over $\mathbb{Z}/p\mathbb{Z}$, then there are exactly r prime ideals of A_K lying over p , with respective residual degrees $\deg \bar{g}_1(x), \dots, \deg \bar{g}_r(x)$ and ramification indices e_1, \dots, e_r . In this paper, the above problem has been dealt with in a more general situation when the base field is a valued field (K, v) of arbitrary rank and $K(\theta)$ is any finite extension of K .

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Quasirecognition of $L_{32}(2)$ By Its Prime Graph

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Let G be a finite group and Ω be some properties of G . This is a natural question to ask about the number of finite groups H (up to isomorphism) such that H has the same properties Ω . If G is the only group with properties Ω , we say that G is recognizable by properties Ω . A finite nonabelian simple group P is called quasirecognizable by Ω , if each finite group G with properties Ω has a unique composition factor isomorphic to P . If n is an integer, then we denote by $\pi(n)$ the set of all prime divisors of n . Let G be a finite group. The set $\pi(|G|)$ is denoted by $\pi(G)$. Also the set of orders of the elements of G is denoted by $\pi_e(G)$. It is clear that the set $\pi_e(G)$ is closed and partially

ordered by divisibility, hence it is uniquely determined by $\mu(G)$, the subset of its maximal elements of G . The prime graph $\Gamma(G)$ of a group G is the graph whose vertex set is $\pi(G)$ and two distinct primes p and q are joined by an edge (we write $p \sim q$) if and only if G contains an element of order pq .

Hagie in [1] determined finite groups G satisfying $\Gamma(G) = \Gamma(S)$, where S is a sporadic simple group. Previously finite groups with the same prime graph as a *CIT* simple group are determined. It is proved that if $q = 3^{2n+1}$ ($n > 0$), then the simple group ${}^2G_2(q)$ is uniquely determined by its prime graph. Also it is proved that if $p > 11$ is a prime number and $p \not\equiv 1 \pmod{12}$, then $PSL(2, p)$ is uniquely determined by its prime graph. Finite groups with the same prime graph of ${}^2F_4(q)$ and $F_4(q)$ where q is even are determined in (see the references of [2]). In this paper we consider the prime graph of $L_{32}(2)$ and determined finite groups with the same prime graph as $L_{32}(2)$.

As the main result of this paper we prove that:

Main Theorem. The simple group $L_{32}(2)$ is quasirecognizable by prime graph, in fact if G is a finite group such that $\Gamma(G) = \Gamma(L_{32}(2))$ then $G/O_2(G) \cong L_{32}(2)$.

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Analogue of Eakin Sathaye Theorem over Rees Algebra

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Let F be the usual fibre cone of the Rees ring over the closed point. By the graded version of Noether Normalization, F is a finitely generated graded module over a standard graded polynomial ring in s variables over the base field k , where s is the analytic spread of the ideal I of the local ring R , with R having the infinite residue field k . The hypotheses of the Eakin-Sathaye theorem (see [1]) imply that s is at most r . Hence the ideal I certainly has a reduction generated by r elements, and we can lift these elements from r generic linear forms in F .

In the following $\mu(R[I^n t])$ denotes the minimal number of generators of $R[I^n t]$ over R . The main goal of this paper is to prove Eakin Sathaye theorem in the Rees algebra setting. We prove the following:

Let R be a local Noetherian ring with infinite residue field. Let I be an ideal in R and $R[It]$ denotes the Rees algebra of I . Let $n \geq 1$ and $r \geq 0$ be integers such that, $\mu_R(R[I^n t]) <^{n+r} C_r$ and F be the usual fibre cone of the Rees ring over the closed point. Then there exists an ideal $J \subset I$ such that $R[It]$ is a finitely generated module over $R[Jt]$ i. e. for some choice of f_1, \dots, f_r in the degree 1 component F_1 of F , homogeneous generators of F as a finitely generated homogeneous module over the standard graded affine algebra $k[f_1, \dots, f_r]$ lie in degree at most $n - 1$.

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Line Graphs and Quasi-total Graphs

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We introduce the concepts 1-quasitotal graph and 2-quasitotal graph. It is proved that if G is a graph consist of exactly m connected components $G_i, 1 \leq i \leq m$, then $L(G) = L(G_1) \oplus L(G_2) \oplus \dots \oplus L(G_m)$ where $L(G)$ denotes the line graph of G , and \oplus denotes the ring sum operation on graphs. The number of connected components in G is equal to the number of connected components in $L(G)$ and also if G is a cycle of length n , then $L(G)$ is also a cycle of length n . Further, it is proved that $Q_1(G) = G + L(G)$ where $Q_1(G)$ denotes 1-quasitotal graph of a given graph G ; and the conditions: (i) $|E(G)| = 1$; and (ii) $Q_2(G)$ contains unique triangle, are equivalent for a given graph G , where $Q_2(G)$ denotes the 2-quasitotal graph of G .

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On Equinormalizable Deformations of Isolated Singularities

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In this talk we study equinormalizable deformations of isolated, possibly non-reduced singularities. This has been initiated by Teissier ([3]) in the 1970's for deformations of reduced curve singularities over the germ of the complex plane

C where he proved that such a deformation is equinormalizable if and only if the deformation is δ -constant. Afterwards Teissier and Raynaud generalized the δ -constant criterion to deformations of isolated reduced curve singularities over normal base spaces of arbitrary dimension ([4]). Recently, Chiang-Hsieh and Lipman ([2]) corrected the argument given by Raynaud and Teissier and extended it to the case of deformations of reduced projective spaces of arbitrary dimension.

Equinormalizable deformations of isolated (not necessarily reduced) curve singularities over smooth 1-dimensional base spaces were studied by Brücker and Greuel ([1]) in the 1990's. The main aim of this talk is to generalize the results of Brücker and Greuel to deformations of isolated, possibly non-reduced curve singularities over arbitrary normal base spaces. For deformations of isolated (not necessarily reduced) singularities of arbitrary dimension, we define the δ -invariant of these singularities and give a similar criterion for the equinormalizability. We also give a relation between equinormalizable as well as δ -constant deformations and deformations of the normalization of isolated singularities.

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Minimum Rank of Line Graphs of Some Graphs

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The classic minimum rank problem involves real symmetric matrices described by a graph. The minimum (symmetric) rank of a simple graph G is the smallest possible rank among all symmetric matrices whose (i, j) th entry (for $i \neq j$) is nonzero whenever $\{i, j\}$ is an edge in G and is zero otherwise: [1] and [3].

The line graph $L(G)$ of a graph G is constructed by taking the edges of G as vertices of $L(G)$, and joining two vertices in $L(G)$ whenever the corresponding edges in G have a common vertex: [4]. Applying results from the minimum rank problem, we investigate the minimum rank of line graphs of some graphs, and compare minimum ranks of $L(G)$ and G : [1], [2] and [3].

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Prüfer v -multiplication Domains and Related Domains of the Form $A + B[\Gamma^*]$

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Let $A \subseteq B$ be an extension of integral domains, Γ be a nonzero torsion-free grading monoid with $\Gamma \cap -\Gamma = \{0\}$, $\Gamma^* = \Gamma - \{0\}$, S be a (saturated) multiplicative subset of A , and let $R = A + B[\Gamma^*]$. In this talk, we study when R is a PvMD, a GCD-domain or a GGCD-domain. First, we show that if A is a

proper subring of $B = A_S$, then R is a PvMD (resp., GCD-domain, GGCD-domain) if and only if A is a PvMD (resp., GCD-domain, GGCD-domain), Γ is a valuation semigroup and S is a t -splitting (resp., splitting, d -splitting) set of A . Second, we prove that if A is a field, then R is a PvMD (resp., GCD-domain, GGCD-domain) if and only if $A = B$ and Γ is a PvMS (resp., GCD-semigroup, GGCD-semigroup). Finally, we prove that if A is not a field and $qf(A) \subseteq B$, then $R = A + B[\Gamma^*]$ is a PvMD (resp., GCD-domain, GGCD-domain) if and only if A is a PvMD (resp., GCD-domain, GGCD-domain), Γ is a valuation semigroup and $B = qf(A)$, where $qf(A)$ is the quotient field of A .

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First Order Infinitesimals in the Category of Smooth Functors

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Infinitesimal spaces are important objects in the category of smooth functors. A C^∞ -ring [4] is a ring A in which we can interpret every smooth map $\mathbb{R}^m \rightarrow \mathbb{R}$ as an operation $A^m \rightarrow A$ in such a way that projections, composition and identity are preserved. Furthermore, a map between two such C^∞ -rings is a ring homomorphism which preserves this additional structures, a “ C^∞ -homomorphism”. The resulting category \mathbb{L} of formal C^∞ -varieties is the opposite of the category of finitely generated C^∞ -rings and C^∞ -homomorphisms. Now consider the category $\mathbf{Sets}^{\mathbb{L}^{op}}$ of contravariant set-valued functors, the so-called smooth functors. Notice that by Yoneda embedding [1]

$$Y : \mathbb{L} \hookrightarrow \mathbf{Sets}^{\mathbb{L}^{op}}, \quad Y(\ell A) = \mathbb{L}(-, \ell A),$$

\mathbb{L} may be identified with the full subcategory of $\mathbf{Sets}^{\mathbb{L}^{op}}$ and we usually just write ℓA for $Y(\ell A)$. In particular, we have *the first-order infinitesimals*

$$D = \ell(C^\infty(\mathbb{R})/(x^2))$$

and the *smooth line* $R = \ell C^\infty(\mathbb{R})$ in $\mathbf{Sets}^{\mathbb{L}^{op}}$ that will play an important role in this talk.

The aim of this research, among other things, is to establish an explicit description of the automorphisms group object of D in terms of the object of units of the smooth real line R .

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0-Primitivity in Matrix Near-rings

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Some light is shed on the (still) open problem of whether the 0-primitivity of a matrix near-ring $M_n(R)$ ($n > 1$) over a zero-symmetric near-ring R with identity implies that R is also 0-primitive. Positive results are given in the finite case, but negative results exist in the realm of generalised matrix near-rings ([1]).

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Lifting of Generators of an Ideal over Laurent Polynomial Ring

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Let R be a commutative Noetherian ring and I be an ideal of the Laurent polynomial ring $R[X, X^{-1}]$ that contains a doubly monic polynomial such that $\dim(R[X, X^{-1}]/I) = 0$. Let I/I^2 is generated by $n \geq 2$ elements over $R[X, X^{-1}]/I$. Define $I(1) = \langle \{f(1) : f \in I\} \rangle$. Then any set of n generators of $I(1)$ over R can be lifted to a set of n generators of I over $R[X, X^{-1}]$.

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On G -domains

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It is shown that a domain R with $k - \dim R = 1$ has infinite number of prime ideals (i.e., maximal ideals) if and only if the rank of every maximal ideal in $R[x]$ is 2. We also extend the well-known characterization of Noetherian G -domains to G -domains with *acc* on radical ideals and *dcc* on prime ideals.

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Alternating Units in Integral Group Rings

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In this work, we search for free subgroups in the unit group $\mathcal{U}(\mathbb{Z}G)$ of the group ring $\mathbb{Z}G$, using an alternating unit and another unit, either bicyclic or alternating.

Let G be a group containing $x \in G$ an element of odd order n , and $c \in \mathbb{N}$, $1 \leq c < 2n$ such that $(c, 2n) = 1$. Then, according to [4, Lemma 10.6], the element

$$u_c(x) := \sum_{i=0}^{c-1} (-x)^i = 1 - x + x^2 - \dots + x^{c-1} \in \mathbb{Z}G$$

is a unit in $\mathbb{Z}G$, called an *alternating unit*. In the present work, we also extend the definition of alternating units to even values of c .

Just like some other types of units in group rings [2], [3], alternating units defined in a homomorphic image of $\mathbb{Z}G$ may be lifted to alternating units in $\mathbb{Z}G$. So the research technique must involve studying the behavior of pairs formed by an alternating unit and another unit (either bicyclic or alternating) in group rings $\mathbb{Z}H$, with H minimal groups that could be counter-examples to the result. As a partial result, we classify such groups as well.

In a similar investigation [1], Gonçalves and del Rio show that in the integral group ring $\mathbb{Z}G$, with G a nonabelian group with order coprime with 6, there always exists a pair formed by a Bass cyclic unit and a bicyclic unit, such that the subgroup they generate is not 2-related. This work has motivated ours, as Bass-cyclic units and alternating units behave similarly under group ring representations.

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Groups in which Elements of same Order Outside a Normal Subgroup N are in the same Coset of N

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We prove that if G is a finite group in which elements of same order outside a normal subgroup N form a coset of N , then G/N is elementary abelian 2-group. Also, if G is a finite group in which elements of same order outside a normal subgroup N are conjugate, then $G/G'N$ is an elementary abelian 2-group and all elements with the same order in $G - G'N$ lie in the same coset of $G'N$.

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Probabilistic Algorithms and Results for Group-theoretic Matrix Multiplication

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A finite group G can realize, via its regular group algebra $\mathbb{C}G$, the tensor (bilinear map) $\langle m, p, q \rangle$, of size mpq , describing $m \times p$ by $p \times q$ matrix multiplication (over \mathbb{C}), if G has a triple of subsets $S, T, U \subseteq G$, of sizes m, p, q satisfying the *triple product property* (TPP): $s's^{-1}t't^{-1}u'u^{-1} = 1_G \iff s' = s, t' = t, u' = u$, for all $s', s \in S, t', t \in T, u', u \in U$, ([2]). This leads to the bound $\omega \leq 3 \frac{\log[\sum_{\rho} \mathfrak{R}(\langle d_{\rho}, d_{\rho}, d_{\rho} \rangle)]}{\log(mpq)}$ for the exponent $\omega \in [2..3]$ measuring asymptotic complexity of matrix multiplication, where the d_{ρ} are the complex irreducible character degrees of G , and $\mathfrak{R}(\langle d_{\rho}, d_{\rho}, d_{\rho} \rangle)$ measures the minimal multiplicative complexity of $d_{\rho} \times d_{\rho}$ matrix multiplication, and $\sum_{\rho} \mathfrak{R}(\langle d_{\rho}, d_{\rho}, d_{\rho} \rangle)$ measures the minimal multiplicative complexity of group algebra multiplication in $\mathbb{C}G$, ([3]). This bound is proportional to the largest size d^* of any complex irreducible character of G , and inversely proportional to the realized tensor sizes mpq of G , (pp. 51-53, [3]). Generally, $mpq < |G|^{\frac{3}{2}}$, and $mpq = |G|$ if G is abelian, ([2]). The problem is to identify finite groups with small degrees but large sized tensors, for improving the current best bound, $\omega < 2.39$, ([2]).

The author's results: (1) a group G realizes $\mathcal{O}(|G|^2 \cdot 4^{|G|})$ distinct TPP triples; (2) the probability $P_{tpp,G}((S, T, U))$ of a subset triple (S, T, U) of G being a TPP triple is $\mathcal{O}(\frac{|G|^3}{2^{|G|}})$, showing that TPP triples, and, therefore, tensors, are "rare" in large finite groups; (3) an efficient, probabilistic search algorithm for maximal TPP triples of G (written and run on the GAP computer algebra system, v. 4) on the "sample space" of "candidate TPP triples" of G , including an exact TPP checking subalgorithm of complexity $\mathcal{O}(|G|^6)$, with a success probability proportional to running time t (minutes); (4) a computational cum probabilistic proof that *no* alternating group or dihedral group lead to better upper bounds for $\mathfrak{R}(\langle 3, 3, 3 \rangle) \in [14..28]$, or for $\mathfrak{R}(\langle 4, 4, 4 \rangle) \in [29..47]$.

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An Elementary Solution to Classical Problems in Number Theory and Algebra

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An algorithm for factorization of Fermat's numbers, a proof for the infinitude of Mersenne Primes and developing cosine trisection quartics and its application to trisection of angles and an approximate real expression for $\tan 20^\circ$ (and in general any $\tan(\theta/3)$) are discussed.



Some Dimension Conditions in Rings with Finite Dimension

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The aim of the present paper is to obtain some interesting results related to the concept "finite dimension" in the theory of associative rings R with respect to two sided ideals. It is known that if an ideal H of R has finite dimension, then there exist uniform ideals U_i , $1 \leq i \leq n$ of R such that the sum $U_1 \oplus U_2 \oplus \cdots \oplus U_n$ is essential in H . This n is independent of choice of uniform ideals and we call it as dimension of H (we write $\dim H$, in short). We obtained some important relations between the concepts complement ideals and essential ideals. Finally, we obtained that $\dim(R/K) = \dim R - \dim K$ for a complement ideal K of R . Some necessary examples were included.

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Automorphism Group of a Witt Type Lie Algebra and Jacobian Conjecture

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We will show that there is a class of transcendental numbers which are solutions of the equations of the F -algebra $F[e^{\pm x_1}, \dots, e^{\pm x_m}, x_1, \dots, x_n]$. It is well known that if we find the automorphism group of the Weyl algebra, then the Jacobian conjecture can be solved [5] and [6]. Zhao's results of his paper imply that if every non-zero endomorphism of $W^+(2)$ is surjective, then the Jacobian conjecture holds [8]. In this work, we will find the automorphism group of the Witt type algebra $W^+(2)$ and prove that the Jacobian conjecture holds on the polynomial ring $F[x, y]$ [1], [3], and [4]. We also prove that the algebra $W(2)$ has a non-zero endomorphism such that the endomorphism is not surjective [7]. We will show some interesting open problems on Weyl algebra and Lie algebra [2] and [3].

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Counting the Number of Distinct Fuzzy Subgroups Some of the Dihedral Groups

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Keywords. Fuzzy subgroups, Dihedral group, Equivalence relation

In this paper, by using of an equivalence relation on fuzzy subgroup, we determine the number of distinct fuzzy subgroups some of the dihedral groups. A fuzzy subset of a set X is a mapping $\mu : X \rightarrow [0, 1]$. Fuzzy subset μ of a group G is called a fuzzy subgroup of G if

$$(G_1) \mu(xy) \geq \mu(x) \wedge \mu(y) \forall x, y \in G;$$

$$(G_2) \mu(x^{-1}) \geq \mu(x) \forall x \in G.$$

The set of all fuzzy subgroup of a group G denoted by $F(G)$. Let G be a group and $\mu \in F(G)$. The set $\{x \in G | \mu(x) > 0\}$ is called support of μ and denoted by $\text{supp}\mu$. Let G be a group, and $\mu, \nu \in F(G)$. Define the relation \sim on $F(G)$ as follows: $\forall \mu, \nu \in F(G)$, $\mu \sim \nu$ if and only if for all $x, y \in G$, $\mu(x) > \mu(y)$ if and only if $\nu(x) > \nu(y)$ and $\mu(x) = 0$ if and only if $\nu(x) = 0$.

We say two fuzzy subgroups are distinct if they are not equivalent. The set of all fuzzy subgroups μ of G such that $\mu(e) = 1$ denoted by $F_1(G)$. The number of equivalence classes \sim on $F_1(G)$ will be denoted by r_G^* .

Theorem. Let G be a finite group and H be a subgroup of G . Then the number of distinct fuzzy subgroups of G such that their support is exactly equal to H is $\frac{r_H^* + 1}{2}$.

Theorem. Let G be the dihedral group of order 2^n . If $n \geq 3$, then $r_G^* = 2^{n+2} + 2^{n-1} + \sum_{i=1}^{n-2} 2^i r_{(D_{2^{n-i}})}^* - 3$.

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On the Distance of a Polynomial Near-ring Code

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For a polynomial $f(x) \in \mathbb{Z}_2[x]$ it is natural to consider the near-ring code generated by the polynomials $f \circ x, f \circ x^2, \dots, f \circ x^k$ as a vectorspace. It is a 16 year old conjecture that for the polynomial $f(x) = x^n + x^{n-1} + \dots + x$ the distance of this code is n .

The conjecture is equivalent to the following purely number theoretical problem. Let $\underline{m} = \{1, 2, \dots, m\}$ and $A \subset \mathbb{N}$ be an arbitrary finite subset of \mathbb{N} . Show that the number of products that occur odd many times in $\underline{n} \cdot A$ is at least n . Among others we prove that for $A = \underline{k}$ this number is at least n and the distance of the code is at least $n/(\log n)^{0.223}$.

While proving the case $A = \underline{k}$ we use different methods depending on the size of k (respect to n). For example, for $k \leq 1.34 \log n$ we apply inclusion-exclusion principle. The most difficult part is when the interval $(k, n]$ contains at most one prime. In this case the key idea is to estimate the number of elements of the set $\{1^2, 2^2, \dots, k^2\}$ which have a divisor in $(k, n]$. Furthermore, in all cases we use several estimates on the distribution of primes.



On Abel Maps for Singular Curves

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The d -th Abel map of a smooth curve associates to a d -tuple of points of the curve the line bundle associated to the d -tuple of points. This map has the remarkable property that its fibers are projectivized linear systems. Recently, the problem of constructing a resolution for Abel maps of a singular curve has been considered by many authors. The problem has been solved if the curve is irreducible in [1], if $d = 1$ in [2] and [3], if the curve is of compact type in [4]. The general problem is still open. It is expected that the study of the fibers of these resolutions should give interesting results on limit linear series on singular curves. In this talk we will show how to get a resolution of the d -th Abel map for curves of compact type and for the 2-nd Abel map for stable curves, giving some applications to limit linear series on singular curves. This is a joint work with Juliana Coelho and Eduardo Esteves.

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Generalized Weil Representations for Classical Groups with a Bruhat Presentation

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Jorge Soto-Andrade

Let A be a ring with an involution $*$. Then the groups $GL_*^\varepsilon(2, A)$ and $SL_*^\varepsilon(2, A)$ are a (tamely) non commutative version of the general linear and specially linear groups over a field, consisting of 2×2 matrices with coefficients in A , that satisfy certain commuting relations which involve $*$.

Several times these groups afford Bruhat like presentations (symplectic groups and orthogonal groups are examples of the groups under consideration) for different choices of the involutive ring.

We define a (generalized) Weil representation of the group $G = SL_*^\varepsilon(2, A)$ for a general finite ring A in the case when G has a Bruhat presentation with simple minimal relations. The representation is defined on the generators in such a way that the linear automorphisms associated to them, satisfy those minimal relations.



Symmetric Ideals in Free Group Rings and Simplicial Homotopy

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20C07; 55Q40

The purpose of this paper is to use methods of simplicial homotopy for the characterisation of subgroups determined by certain ideals, here called symmetric ideals, in free group rings.

{This is joint work with Roman Mikhailov, Steklov Mathematical Institute, Moscow.}



Combinatorics of Words over Semigroups

Gabriella Pluhár

The number of n -variable expressions, polynomials over a finite algebraic structure can be very informative about the structure itself. For example, it is well-known that over a finite field every function is a polynomial, so for the p element field F_p the number of n -variable polynomials is p^{p^n} . For the abelian group Z_p every polynomial can be written in the form of $x_1^{k_1} \dots x_2^{k_2}$, therefore the number of polynomials is only p^n .

For groups it was proved in the 1960's that for a finite group G this number is exponential, if G is nilpotent and double exponential, if G is not nilpotent.

We investigate the number of expressions for semigroups. As a first step, we proved that the logarithm of the number of expressions over idempotent

semigroups is asymptotically $\frac{4}{(k-3)!}n^{k-3} \log n$, where k depends on the semigroup. A semigroup is idempotent if it satisfies $x^2 = x$. This growth was not experienced before. We continue with the semigroups closest to groups, with the so-called completely regular semigroups. We show the following: Let V be the class of semigroups defined by the identity $x^3 = x$. Then the number of polynomials is greater than $2^{2^{\dots^2}}$, where there are n -many 2-s in the formula.



Finite Non-solvable Groups Having a Unique Irreducible Character of a Given Degree

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If G is a finite simple Chevalley group, then G has an irreducible complex character χ whose degree is the order of a p -sylow subgroup of G , and all other irreducible complex characters are in the principal p -block (such a χ is called the Steinberg character). For simple groups G , this property seems to hold only if G is a finite simple Chevalley group. The conjecture, then, is that the above property about the irreducible complex characters of G forces G to be a finite simple Chevalley group. The simple group $\text{PSL}(2, q)$, however, is the only known simple group which satisfies the following stronger hypothesis:

(H) *There is a unique irreducible complex character χ of degree $m > 1$ and every other irreducible complex character is such that its degree is relatively prime to m .*

So the problem of classifying all finite groups G satisfying the hypothesis (H) can be viewed, at least in the case when G is simple, as a first step towards classifying finite simple Chevalley groups by their Steinberg character. Solvable groups satisfying the hypothesis (H) have been classified in a separate paper [1]. In this short communication, we study non-solvable groups satisfying the hypothesis (H). Assuming that χ is faithful, it is shown that derived group G' is a non-abelian simple group and that when $\chi(1) = p$, p an odd prime, G itself is a non-abelian simple group, and is such that its p -sylow subgroup P is a cyclic group of order p and equals its centralizer. From this, it follows that (when $\chi(1) = p$) all involutions in G are conjugate.

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On Generalized Jordan Triple Derivations on Rings

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Let R be an associative ring, and $F : R \rightarrow R$ is an additive mapping. F is called a Jordan triple derivation if $F(xyx) = F(x)yx + xF(y)x + xyF(x)$ for all $x, y \in R$ is fulfilled [1], and F is called generalized Jordan triple derivation if $F(xyx) = F(x)yx + xf(y)x + xyf(x)$ with some Jordan triple derivation f for all $x, y \in R$ is fulfilled [2]. In the present paper, we studied some other types of generalized Jordan triple derivations on rings R .

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Recursive Neural Networks for Processing Directed Graphs

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Keywords. Directed acyclic graphs, cyclic graphs, encoding network, recursive equivalence, recursive neural networks.

While neural networks are very successfully applied to the processing of fixed-length vectors and variable-length sequences, since in great variety of real-world problems such as molecular biology and chemistry, pattern recognition, document processing the information is naturally incoded in the relationships among the basic entities, the principle subject is the efficient processing of structured objects of arbitrary shape (like logical terms, trees, or graphs). Here

we explain methods of processing for directed acyclic and directed cyclic graphs with the recursive neural network in such a way that directed acyclic graphs are processed by unfolding the recursive network into an encoding network and directed cyclic graphs map into a recursive-equivalent tree.



Connes Subgroups and Graded Semirings

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Primary 19Y60; Secondary 19Y99

We started this paper with the aim to find out the validity of results proved by S.Montgomery and D.S. Passman [1] (regarding the connection between the Connes subgroups and the ideal structure of a graded rings) for semirings. In the absence of additive inverses in semirings, the conditions to find out the validity of the results of ring theory become complicated, thus one needs a weaker version of of additive inverses, i.e. cancellation of elements. If a semiring is additively cancellative, then its ring of differences exists and becomes an imprtant tool to find out the validity of the results of rings for semirings, e.g. see [3].

Let K be an additively cancellative commutative semiring and R an additively cancellative K -semialgebra graded by a finite group G . Then there exists an extension semiring S (known as smash product), with same 1 , which comes from the study of semi-Hopf algebras [2]. If R is additively cancellative, then so is S and hence its ring of differences also exists. R embeds in its ring of differences where as S embeds in its ring of differences. These embeddings become useful as the results of Montgomery and Passman are valid for these rings of differences, thereby providing us with an incisive technique for analyzing these results for R and S .

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Some Characterization of Regular Groupoid Lattices

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The notion of regularity was introduced by J. Von Neumann in his paper [11]. And A.H. Clifford and G.B. Preston [1], L. Kovács [2], S. Lajos [4] [5] [6] [8] [9], J. Luh [3], and O. Steinfeld [10] [12] have characterized many results on regular rings and semigroups by means of their left ideal, right ideals, and quasi-ideals. In this paper we will show some results of regular groupoid-lattices which can be considered as common generalization of regular rings and semigroups.

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On The Finiteness Properties of Extension Functor and Local Cohomology Modules

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Let R denote a commutative Noetherian ring (with identity), I an ideal of R , and M a finitely generated R -module. In [5] L.J. Ratliff, Jr., conjectured about the asymptotic behaviour of $\text{Ass}_R R/I^n$ when R is a Noetherian domain. Subsequently, M. Brodmann [2] showed that if R is Noetherian and M is a finitely generated R -module, then $\text{Ass}_R M/I^n M$ is ultimately constant for large n . In [4], Melkersson and Schenzel asked whether the sets $\text{Ext}_R^i(R/I^n, M)$ become stable for sufficiently large n . In this paper we show that, for all $i \geq 0$, the sets of prime ideals $\text{Ass}_R \text{Ext}_R^i(R/I^n, M)$, $n = 1, 2, \dots$, become independent of n , for large n , whenever I is principal, which is an affirmative answer to the above question in the case I is principal. Also, it is shown that, if I is generated by an R -regular sequence and $\text{Ext}_R^i(R/I, M)$ is Artinian, then the set $\bigcup_{n=1}^{\infty} \text{Ass}_R \text{Ext}_R^{i+1}(R/I^n, M)$ is finite. Another aim of this paper is to show that, if $x \in I$ is a regular element on $R/\Gamma_I(R)$ and $M/\Gamma_I(M)$, then for every $i, j \geq 0$, the R -module $\text{Ext}_R^i(R/xR, H_I^j(M))$ is I -cofinite, whenever $\dim R/I \leq 1$.

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Some Characterizations of Submodules of *QTAG*-Modules

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A module M over an associative ring R with unity is a *QTAG*-module if every finitely generated submodule of any homomorphic image of M is a direct sum of uniserial modules. There are many fascinating concepts related to these modules of which h -pure submodules and N -high submodules are very significant. A submodule N of M is semi h -pure in M if it is contained in a h -pure submodule of M and the minimal h -pure submodule of M containing N is the h -pure hull of N in M . Here we characterize the h -pure hulls of the submodules $N \subset M$ as S -high submodules of M , where S is a subocle of M . We also show that for two h -pure hulls L and K (of N in M) $f_{M/K}(t) = f_{M/L}(t) = 0$, $\forall t \geq o$.

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Mathematical Theory of Concepts: Lattices of (sub)classes, Distance

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Concepts are couples of sets O and A . (O, A) gives any concept, that is the assignment, of the object O to the set A of their (common) attributes. The lattice is created by two algebraic operations: “intersection of couples-concepts” as the multiplication of the Boolean Algebra-Lattice and “symmetric-difference (!) of couples-concepts” as the addition (!) of the Boolean Algebra-Lattice (see ICM94). There are, also, two other operations (the “union of two concepts” and the “complement of a concept”). Intersection and union (which cannot play the role of multiplication) express similarities, while the other two operations express dissimilarities. Definition 1. The complement of the concept (O, A) is the concept (O^C, A^C) , where O^C and A^C are the usual set-theoretic complements of O and A , respectively. Definition 2. The symmetric-difference of two concepts (O_1, A_1) and (O_2, A_2) , is the concept $D = (O_1 + O_2, (A_1 + A_2)^C)$, where $O_1 + O_2$, $A_1 + A_2$ are the usual set-theoretic symmetric-differences of O_1 and O_2 , A_1 and A_2 , respectively. The non - common objects $O_1 + O_2$ have the common attributes $A_1 \cap A_2$, but they may have, also, others ‘out of’ $A_1 \cup A_2$ (that is, in the complement of $A_1 \cup A_2$, which is the **fuzzy factor** in the **definition** or comparison of concepts). Definition 3. We define distance $d(X, Y)$ of two sets X and Y , the non-negative integer expressing the number of elements of the set $X + Y$, that is of their symmetric-difference. So, $d(X, Y) = n(X + Y)$. The operation complement can not be expressed by the two predefined operations “union” (\cup) and “intersection” (\cap), which have the meaning of “common”. So, the complement gives the different, not the common, the variety. The symmetric-difference of two concepts is proved to be a **“distance”** between them (in the mathematical sense!). So, ***an object is not defined but just compared to another object, through one or more concepts. Our world consists of similarities for objects and of equivalences for concepts.*** We are free to construct classes as thin as possible. **It is proved that every class is a sublattice, every concept is, also, a sublattice and that all classes are the elements of a new lattice!** Taking as concepts only the standardized ones, fits well with (not intelligent. . .) robots, but not with human beings and real life. Our system of concepts is open, enriched with differences, distances, classes of concepts and lattices of classes!(see ICM94). All concepts are accepted, not only the standardized. ***How many*** such concepts can we get? Can we ***enumerate*** them? It depends on the complement of a concept, that is on the concept (O^c, A^c) . There is ***a space beyond our knowledge(?)*** Even if we make standardization of the

concept (O, A) (which means that O is the maximum set of objects with the minimum set A of common attributes), when we “go out of” O , nothing is sure for the attributes of O^c . Many people think that, if we make standardizations, then everything will be sure in our world. Unfortunately, we can be sure only for a finite number of objects, but we cannot be sure for all the others.



The Higher Moments Dynamics on the Stochastic SIS Model

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The stochastic SIS - Susceptible, Infected and Susceptible - model is a well known mathematical model, studied in several contexts, and a special case of more complex models [4]. In an epidemiological context, this model is used for endemic infections that do not confer immunity and can be interpreted as a birth-and-death process with a finite state space, correspondent to the number of infected individuals $I(t) \in \{0, 1, 2, \dots, N\}$ at time t . Since the state $I(t) = 0$ is the only one absorbing, the stationary distribution of the SIS model is degenerated and the interest goes to compute the quasi-stationary distribution that does not have explicit form [1, 2].

Many authors worked on the SIS model considering, only, the dynamical evolution of the mean value and the variance of the infected individuals. In this study, we derive recursively the dynamic equations for all the moments of the infected quantity and, using the moment closure approximation [3], we develop a recursive formula to compute the equilibria manifold of the m moment closure ODEs. We discover that the stable equilibria of the m moment closure ODEs can be used to compute a good approximation of the quasi-stationary mean value of infecteds for relatively small populations size N and also for infection rates β relatively close to its critical values by taking m large enough.

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Krull Dimension in Power Series Rings

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Let R be a commutative ring with identity. We denote by $\dim R$ the Krull dimension of R . It is shown in [10] that if $\dim R = n$, then $n + 1 \leq \dim R[X] \leq 2n + 1$ where $R[X]$ is the polynomial ring over R . For the power series ring $R[[X]]$, we show that $\dim R[[X]]$ can be uncountably infinite even if $\dim R$ is finite.

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Homotopy Batalin-Vilkovisky Algebras

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We present an explicit quasi-free resolution BV_∞ of the operad BV that encodes Batalin-Vilkovisky algebras, and thus a notion of *homotopy Batalin-Vilkovisky algebras*, or BV_∞ -algebras, with good homotopy properties. We compare our notion with other definitions in the literature [1, 5]. In order to provide our quasi-free resolution we introduce a general theory of Koszul duality for pre-operads defined by quadratic and linear relations, extending the now classical theory of [2] for purely quadratic operads and that of [3, 4] for algebras. The operad BV is not purely quadratic but is Koszul in our sense, allowing us to prove a Poincaré-Birkhoff-Witt Theorem and give the explicit small quasi-free resolution BV_∞ .

With this resolution we can introduce deformation theory and homotopy theory of BV -algebras, and of BV_∞ -algebras, and we also develop an obstruction theory for algebras over any Koszul operad P with only quadratic and linear relations.

As applications we prove that any topological conformal field theory carries a BV_∞ -algebra structure that lifts the BV -algebra structure on homology. The same result is proved for the singular chain complex of the double loop space of a topological space endowed with an action of the circle. We also extend

a conjecture of Lian-Zuckerman, showing that certain vertex algebras have an explicit BV_∞ -algebra structure, and prove the cyclic Deligne conjecture with the operad BV_∞ .

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Invertibility and Dedekind Finiteness in Structural Matrix Rings

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An example in [4] suggests that Dedekind finiteness may play a crucial role in a characterization of the structural subrings $M_n(\theta, R)$ of the full $n \times n$ matrix ring $M_n(R)$ over a ring R which are closed with respect to taking inverses. It turns out that $M_n(\theta, R)$ is closed with respect to taking inverses in $M_n(R)$ if all the equivalence classes with respect to $\theta \cap \theta^{-1}$, except possibly one, are of size less than or equal to p (say) and $M_p(R)$ is Dedekind finite. Another purpose of this paper is to show that $M_n(\theta, R)$ is Dedekind finite if and only if $M_m(R)$ is Dedekind finite, where m is the maximum size of the equivalence classes (with respect to $\theta \cap \theta^{-1}$). This provides a positive result for the inheritance of Dedekind finiteness by a matrix ring (albeit not a full matrix ring) from a smaller (full) matrix ring.

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On The Additive and Multiplicative Structure of Semirings

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Let $(S, +, \cdot)$ be a semiring. A semiring is said to satisfy Integral multiple property (*IMP*), if for every a in S there exists a positive integer n such that $a^2 = na$ and n depends on the element a . Additive and multiplicative structures play an important role in determining the structure of semirings. In [2], Satyanarayana proved that if a totally ordered (t, o) semirings contains multiplicative identity, then $(S, +)$ is non-negatively ordered or non-positively ordered. Examples are given that the converse of this not true. We prove that if a totally ordered semiring satisfies *IMP*, then $(S, +)$ is non-negatively (non-positively) ordered if and only if (S, \cdot) is non-negatively (non-positively) ordered, we also prove that if S is a totally ordered semiring with *IMP*, then $(S, +)$ is *O*-Archimedean if and only if (S, \cdot) is *O*-Archimedean. A semiring is called a divided semiring, if (S, \cdot) is a group. We study the conditions that $(S, +)$ is positively ordered in the strict sense or negatively ordered in strict sense in divided semirings.

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Convolution Rings and Some Applications

Stefan Veldsman

Convolution types and convolution rings were introduced for two reasons. Firstly, they provide a convenient tool to describe that which is common to a wide variety of ring constructions (eg. polynomial rings, power series rings, matrix rings, group rings, incidence algebras, necklace rings and many more). Secondly, the convolution type separates the construction method from any algebraic considerations. This is done in order to identify and describe the properties of the parameters of the convolution type that will force certain properties on the constructed rings. For example, a matrix convolution type is defined and when imposed on a given ring, the resulting convolution ring is just the matrix ring. The non-commutativity of the matrix ring is independent of the underlying base ring and the reasons for the non-commutativity is to be found in the construction method.

In this talk, these ideas will be illustrated with special reference to the arithmetical rings based on the Cauchy, Dirichlet and Lucas products respectively.



About Krasner's and Vuković's Paragraduations

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In our common papers [1]–[3] as well as in our monograph [4], M. Krasner and myself first introduced a theory of a para- and extra- graduations which generalize a classical graduations as defined by Bourbaki, as well as some earlier results of M. Krasner. Our aim was to introduce the structures which have, in each of the three cases (groups, rings modules), the property of closure with respect to direct sum and direct product in the sense that the support of the homogeneous parts of this product is Cartesian restricted product, resp. Cartesian product of the homogeneous parts of components. The characterization axioms of para- and extra- graded groups give a way to three study methods of this groups which are in principle equivalent: non-homogeneous, semi-homogeneous, and homogeneous. In my presentation I will speak about extra- and para-graded groups from semi-homogeneous aspect.

If H is a subset of the group G and $x \in H$, let $H(x) = \{y \in H; xy \in H\}$. Then, H is homogeneous part of the group G in relation to the paragraduation

E iff H satisfies the system of the following three axioms:

1. If $x \in H$, $g(x) = \{y \in H; H(y) \supseteq H(x)\}$ is invariant subgroup of the group G .

(The group G is very saturated if $x \in g \Rightarrow g(x) \subseteq g$).

2. If $A \subseteq H$ is a subset of the set A such that $\forall x, y \in A \quad xy \in H$, then there exists a very saturated subgroup g of G with $g \subseteq H$ such that $A \subseteq g$, i.e. $x \in g \Rightarrow g(x) \subseteq g$.

3. a) H generates the group G , and

b) H generates G with the system of the relations R .

A pair $(G, H \subseteq G)$ for which H satisfies axioms 1-3 defines the paragradaution of the group G up to the equivalence and is called a paragraded group from the aspect of the semi-homogeneity.

A paragradaution which belongs to this class of equivalence can be constructed canonically.

A paragraded group (G, H) is an extragraded group iff the part 3.b) of the axiom 3. is replaced with:

4. Let $u_1, u_2, \dots, u_n \in H$ such that $H(u_i)$ are incomparable in pairs (as to the relation of inclusion \subset), and let $x_1, x_2, \dots, x_n, x'_1, x'_2, \dots, x'_n$ be elements of H such that $\forall i \in \{1, 2, \dots, n\} : H(x_i) \cap H(x'_i) \subseteq H(u_i)$, then $x_1 x_2 \dots x_n = x'_1 x'_2 \dots x'_n$ means that $\forall i \in \{1, 2, \dots, n\} : H(x_i^{-1} x'_i) \subseteq H(u_i)$ (or better said, $x_i^{-1} x'_i \in H$ because $x_i, x'_i \in g(u_i)$).

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On Special Homomorphic Images of $SU(1, D)/[U(1, D), U(1, D)]$

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Let F be a field. For a central division F -algebra D of index $d > 1$, the reduced Whitehead group $SK_1(D)$ of D is defined. Similarly, if F/K is a quadratic field extension ($\text{char}(F) \neq 2$) and D is endowed with F/K -involution τ , then the reduced unitary Whitehead group $SUK_1(D, \tau)$ of D is defined.

Both groups play an important role in the study of the structure of simple simply connected isotropic algebraic groups and algebraic K -theory ([1]).

Passing to anisotropic groups of type A_n one needs to look at groups $SU(1, D)$:

$$U(1, D) = \{d \in D^* \mid dd^\tau = 1\}, \quad SU(1, D) = U(1, D) \cap SL(1, D).$$

By analogy with the isotropic case one can define the following group

$$SUK_1^{an}(D, \tau) = SU(1, D)/[U(1, D), U(1, D)],$$

$[U(1, D), U(1, D)]$ denotes the commutator subgroup of $U(1, D)$ which is still slightly studied. Even the problem of non-triviality of $SUK_1^{an}(D, \tau)$ is settled ([2], [3]) only in some particular cases which are strongly based on the special structure of D . In this situation it is interesting to find a characteristic of non-triviality of $SUK_1^{an}(D, \tau)$ which does not depend on that structure.

Such a characteristic is contained in the following

Theorem. There exists an epimorphism

$$SUK_1^{an}(D, \tau) \longrightarrow SUK_1(D, \tau)/{}_2SUK_1(D, \tau),$$

where ${}_2SUK_1(D, \tau)$ is the 2-torsion part of $SUK_1(D, \tau)$.

Corollary. If the period of $SUK_1(D, \tau) > 2$, then $SUK_1^{an}(D, \tau) \neq \{0\}$.

Corollary. If d is odd and $SUK_1(D, \tau) \neq \{0\}$, then $SUK_1^{an}(D, \tau) \neq \{0\}$.

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Some Characterization Results in Multiplication Modules

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Keywords. Dedekind rings, multiplication modules, uniform modules, prime submodule.

Let R be a commutative ring with identity and M be a finitely generated generalized multiplication R -module. It will be proved that any submodule N of M is the intersection of its isolated \mathfrak{p} -primary components, where \mathfrak{p} runs over the minimal prime ideals of $(N :_R M)$. Using this fact we prove that if Q is a primary submodule of M , then $Q = \sqrt{Q :_R M}^t M$ for some positive integer t . Also a criteria for which a finitely generated multiplication R module to be a generalized multiplication module will be investigated.



Section 3

Number Theory

Bloch-Kato Conjecture for Convolution L -functions

M. Agarwal

We give evidence for the Bloch-Kato conjecture for the convolution L -function of two elliptic modular forms. Let f be a new cuspform of weight 2 and g be a new cuspform of weight $k+2$, $k \in \{2, 4, 6, 8, 12\}$, of level $\Gamma_0(q)$ for an odd prime q such that they are ordinary at p and have residually absolutely irreducible Galois representations mod p for p an odd prime different from q . Under some additional conditions on p we show that if

$$p^n \mid L^{\text{alg}}(2 + k/2, f \times g) \implies p^n \mid \#H_f^1(G_{\mathbf{Q}}, \rho_f \times \rho_g(-k/2 - 1))$$

This is carried out by studying congruences between Yoshida lift of f, g and stable forms on $GSp(4)$. This is a report on joint work with Krzysztof Klosin.



A Note on Iwasawa μ -Invariants of Elliptic Curves

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Suppose that E_1 and E_2 are elliptic curves defined over rationals and p is an odd prime where E_1 and E_2 have good ordinary reduction. In this paper, we prove that if $E_1[p^i]$ and $E_2[p^i]$ are isomorphic as Galois modules for $i = \mu(E_1)$, then $\mu(E_1) \leq \mu(E_2)$. If the isomorphism holds for $i = \mu(E_1) + 1$, then both the curves have same μ -invariants. We also discuss one numerical example.

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Fermat's Last Theorem

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Outline of this Proof of Fermat's Last Theorem

Sophie Germain developed a proof of Case I based on the idea that at least one auxiliary prime can be found for every prime $n > 2$ which shows that a solution under Case I is impossible. Study of her ideas is contained in Appendix XVI and XVII. Sophie Germain's method fails

- i) because it is difficult to prove that an auxiliary prime exists for every prime n , even though this is a reasonable conjecture
- ii) It only addresses Case I and does not address Case II

This proof, developed after studying the work of Sophie Germain, begins with a proof of Case I based on a fascinating property of the prime factors of the sum of two relatively prime n^{th} powers. All the possible prime factors of $(x^n + y^n)$, excluding the prime factors contained in $(x+y)$, are of the form; $p = 1 + 2 * n * k$. (k is any integer which makes p a prime). All the primes (p) of this form behave as auxiliary primes for all primes n in a similar way that was envisaged by Sophie Germain.

For each possible value of p ,

- i) $(-y/x)$ must be an n^{th} power residue in $\text{mod}(p)$
- ii) $(-y/x)$ must be a specifically implied n^{th} root of 1 in $\text{mod}(p)$ and
- iii) $(-y/x)$ cannot be 1 $\text{mod}(p)$

For every prime n , all primes p which are of the form $1+2^n \cdot k$ and are factors of z not contained in $(x+y)$ imply that the three concurrent conditions above contradict each other.

Under Case II, the same approach as in Case I is demonstrated to be still valid. If we take z as the variable divisible by n , we know x and y cannot be divisible by n . Since p is defined as all the prime factors of $(x^n + y^n)$ excluding primes contained in $(x+y)$, the proof as in Case I still holds. Under Case II, if z is the variable divisible by n , $(x+y)$ must include n as a prime factor, hence n is excluded from p by definition.

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Shifted Primes and Semismooth Numbers

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Turning around an unsolved problem of Erdős and Odlyzko [3], first a related problem is solved. Let $k \neq 0$ be an integer and $P = \{p_1, \dots, p_l\}$ a set of primes. Then the number of primes $p \leq x$ such that $p+k = \alpha \cdot u$, where α is supported by P and u is squarefree, as $x \rightarrow \infty$, is given by

$$\prod_{p \nmid k\wp} \left(1 - \frac{1}{p^2 - p}\right) \cdot \text{Li}(x) + O\left(\frac{x}{\log^H x}\right)$$

where $\wp := p_1 \cdots p_l$ and $H > 0$ is a real number. Then the question of multiple shifts of primes having the given shape $\alpha \cdot u$ is explored [1, 2].

Then this idea is related to the concept of smooth number [4, Chapter 7]. Let p_n be the n 'th prime. A number m is **p_n -semismooth** (compare [5]) if $m = \alpha \cdot u$ where α is supported by $\{p_1, \dots, p_n\}$ and u is squarefree. Then the count of these numbers up to x is given by

$$\Theta_n(x) = \prod_{p > p_n} \left(1 - \frac{1}{p^2}\right) \cdot x + O(\sqrt{x} \log^n x).$$

These ideas enable us to develop a theory with has elements in common with those of smooth numbers and squarefree numbers. For example we will address

the question of semismooth numbers in short intervals and define **semismooth** integer sequence, a very large subset of the set of all integer sequences. The maximum number of consecutive p_n -semismooth numbers is $p_{n+1}^2 - 1$ and this is assumed an infinite number of times.

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Generalized Difference Operator of the n^{th} Kind and its Applications in Number Theory (Part - I)

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In this paper, the authors extend the theory of the generalized difference operator Δ_ℓ to the Generalized difference operator of the n^{th} kind $\Delta_{\ell_1, \ell_2, \dots, \ell_n}$ for the positive reals $\ell_1, \ell_2, \dots, \ell_n$. Also present the discrete version of Leibnitz Theorem, Binomial Theorem, Newton's formula with reference to $\Delta_{\ell_1, \ell_2, \dots, \ell_n}$. Also by defining its inverse to establish a formulae for the sum of general partial sums of the higher powers of arithmetic progression in number theory.

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On a Sierpiński's Problem

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11A41, 11A67.

Let $s \geq 2$ be an integer. Denote by μ_s the least integer so that every integer $l > \mu_s$ is the sum of exactly s integers > 1 which are pairwise relatively prime. In 1964, Sierpiński [7] asked a determination of μ_s . Let $p_1 = 2, p_2 = 3, \dots$ be the sequence of consecutive primes. In 1965, P. Erdős [4] proved that $p_2 + p_3 + \dots + p_{s+1} - 2 \leq \mu_s < p_2 + p_3 + \dots + p_{s+1} + C$, where C is an absolute constant. We solve this problem completely.

As a corollary, we prove that if $p_{s+2} - p_{s+1} > 1100$, then

$$\mu_s = \sum_{i=2}^{s+1} p_i + 1100.$$

In particular, the set of integers s with

$$\mu_s = \sum_{i=2}^{s+1} p_i + 1100$$

has the asymptotic density 1.

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Probability of Integers Being Prime & Usage of Prime Pairs in Cryptography

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20K

The Riemann Zeta function $\zeta(s) = \sum_{i=1}^{+\infty} \frac{1}{n^s}$ converges absolutely for $s > 1$. In this section, I firstly show that $\zeta(s)$ is the reciprocal of $\sum_{i=1}^{+\infty} \frac{\mu(n)}{n^s}$, $\mu(n)$ being the divisor function for 'n'. An important result is that the value of the Riemann Zeta function for $s = 2$ is $\frac{\pi^2}{6}$. The proof of this result is also shown in

this section. It can be used in finding the mean value of the Euler Phi Function : $\Phi(x) = \sum_{n \leq x} \varphi(n) = \frac{3x^2}{\pi^2} + O(x \log x)$.

The above result is used in showing that the probability of any two positive integers being relatively prime is almost $\frac{6}{\pi^2}$. Also in this section I show that if a positive integer 'n' is equal to the product of two distinct primes 'p' & 'q' [$p > x, q > x$], the probability that a randomly chosen positive integer up to 'x' & relatively prime to 'n' is greater than $(1 - \frac{1}{x})^2$. Notably the probability is greater than 0.99, if 'x' is 200 or more. This idea is widely used in public key cryptography where large distinct primes are chosen so that it becomes almost practically impossible to factorize 'n' creating the cryptosystem to be secure.

It is an open problem to determine whether the number of twin prime pairs ($p, p + 2$) is finite or infinite. One of the largest known twin prime pairs is 1,000,000,009,649 & 1,000,000,009,651. However it will be feasible in choosing only one large prime instead of two, if we work with twin prime pairs. Knowledge of 'p' implies knowledge of 'q', since $q = p + 2$.

This enhances the security of the cryptosystem, since less amount of data needs to be protected.

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On Additive Complements

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Two infinite sequences A and B of non-negative integers are called additive complements, if their sum contains all sufficiently large integers. Let $A(x)$ and $B(x)$ be the counting functions of A and B . Motivated by a problem of Hanani and Erdős [2],[3], Danzer [1] conjectured that for additive complements A and B , if

$$\limsup_{x \rightarrow \infty} \frac{A(x)B(x)}{x} \leq 1,$$

then

$$A(x)B(x) - x \rightarrow +\infty \quad \text{as} \quad x \rightarrow +\infty.$$

In [8], Sárközy and Szemerédi proved this conjecture.

In this talk, we prove that for additive complements A and B , if $\limsup_{x \rightarrow \infty} \frac{A(x)B(x)}{x} < \frac{5}{4}$ or $\limsup_{x \rightarrow \infty} \frac{A(x)B(x)}{x} > 2$, then $A(x)B(x) - x \rightarrow +\infty$.

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Transfer Inequalities for Diophantine Exponents

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Consider a system of linear equations $\Theta \mathbf{x} = \mathbf{y}$, where Θ is an $n \times m$ real matrix. The supremum of real numbers γ , such that for each t large enough the inequalities $0 < |\mathbf{x}|_\infty \leq t$, $|\Theta \mathbf{x} - \mathbf{y}|_\infty \leq t^{-\gamma}$ admit a solution in $\mathbf{x} \in \mathbb{Z}^m$, $\mathbf{y} \in \mathbb{Z}^n$, is called the *uniform Diophantine exponent* of Θ and is denoted by $\alpha(\Theta)$. If the words “for each t large enough” are substituted by “there are arbitrarily large values of t for which”, we get the *individual Diophantine exponent*, which is denoted by $\beta(\Theta)$.

Our first result to be discussed at the talk is the following theorem improving results of Jarník [1] and Apfelbeck [2].

Theorem 1. *For all positive integers n, m , not equal simultaneously to 1, we have*

$$\alpha(\Theta^\tau) \geq \begin{cases} \frac{n-1}{m-\alpha(\Theta)}, & \text{if } \alpha(\Theta) \leq 1, \\ \frac{n-\alpha(\Theta)^{-1}}{m-1}, & \text{if } \alpha(\Theta) \geq 1. \end{cases}$$

Our second result improves the theorem of Dyson [3] and generalizes to the case of arbitrary n, m the theorem of Laurent and Bugeaud [4].

Theorem 2. *For all n, m , not equal simultaneously to 1, we have*

$$\begin{aligned} \beta(\Theta^\tau) &\geq \frac{n\beta(\Theta) + n - 1}{(m-1)\beta(\Theta) + m}, \\ \beta(\Theta^\tau) &\geq \frac{(n-1)(1+\beta(\Theta)) - (1-\alpha(\Theta))}{(m-1)(1+\beta(\Theta)) + (1-\alpha(\Theta))}, \\ \beta(\Theta^\tau) &\geq \frac{(n-1)(1+\beta(\Theta)^{-1}) - (\alpha(\Theta)^{-1} - 1)}{(m-1)(1+\beta(\Theta)^{-1}) + (\alpha(\Theta)^{-1} - 1)}. \end{aligned}$$

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Multilinear Exponential Sums in an Arbitrary Finite Field Under Optimal Entropy Condition on the Sources

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Jean Bourgain

2000 MATHEMATICS SUBJECT CLASSIFICATION. 11T23

Recently J. Bourgain [1] proved that for a given $0 < \delta < \frac{1}{4}$ and $r \in \mathbb{Z}_+$ there is $\delta' > (\frac{\delta}{r})^{Cr}$ such that if p is sufficiently large prime and subsets $A_1, A_2, \dots, A_r \subset F_p$ satisfy $|A_i| > p^\delta, i = 1, 2, \dots, r$ and $\prod_{i=1}^r |A_i| > p^{1+\delta}$ then there is an exponential sum bound

$$\left| \sum_{x_1 \in A_1, \dots, x_r \in A_r} e_p(x_1 \dots x_r) \right| < p^{-\delta'} |A_1| \dots |A_r|.$$

In my joint paper with J. Bourgain we extending this result to the case of an arbitrary finite field. These results with the sketches of their proofs will be presented at the communication.

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Newer Facet of Prime Number Theory

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The newer facet of prime number theory has been studied by defining prime and composite numbers in the backdrop of Sieve of Eratosthenes, as follows.

An integer $n > 1$ which can be obtained by the multiplication of two integers, other than 1 and n , is a composite number, otherwise it is a prime. Using these definitions, following results have been obtained.

Even integers $2n$ represent prime and composite numbers according as $n = 1$ and $n > 1$. Odd integers $2n + 1$ will represent composite numbers (having factors $2m+1$ and $2k+1$) and primes according as $n =$ and $\neq 2mk+m+k$, where $n, m, k \geq 1$. These findings represent the distribution of prime and composite numbers through even and odd integers [1].

It is possible to find out n , in terms of m, k , for which $an + b$, subject to $(a, b) = 1$ (refer to Dirichlet's prime number theorem) will represent prime and composite numbers [2].

It is also possible to obtain $\pi(x)$, the number of primes up to any given odd integer x , by writing $\pi(x) = \frac{(x+1)}{2} - c(x)$ and obtaining $c(x)$, the number of odd composite numbers up to x by applying the above findings [2].

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On the Prime Geodesic Theorem for Hyperbolic Manifolds with Cusps

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Let X_Γ be a d -dimensional real hyperbolic manifold with cusps given as locally symmetric space $\Gamma \backslash G/K$, where $G = \mathrm{SO}_0(d, 1)$ is the rank-one semi-simple Lie group, $K = \mathrm{SO}(d)$ is the maximal compact subgroup of G and Γ is a discrete co-finite torsion-free subgroup of G such that

$$\Gamma \cap P = \Gamma \cap N(P)$$

for $P \in \mathfrak{R}$, where \mathfrak{R} is the set of Γ -conjugacy classes of Γ -cuspidal parabolic subgroups in G and $N(P)$ is the unipotent radical of P . Referring to Fried [1] and using Ruelle zeta function instead of the Selberg zeta function, Park [3] proved the prime geodesic theorem for such a d -dimensional manifold X_Γ with the error term $O(x^{\frac{3}{2}d_0}(\log x)^{-\frac{1}{2}})$, $d_0 = \frac{d-1}{2}$. Having in mind that Randol [4] obtained $O(x^{\frac{3}{4}}(\log x)^{-1})$ for compact Riemann surfaces, we give an another proof of Park's prime geodesic theorem focusing on Randol's approach and using a meromorphic extension of the twisted Ruelle zeta function described in [2], [3] to achieve the improved error term $O(x^{\frac{3}{2}d_0}(\log x)^{-1})$ for $3 \leq d \leq 5$.

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On the Mean Square of $|\zeta(1 + it)|$

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Let $R(T) := \int_1^T |\zeta(1 + it)|^2 dt - \zeta(2)T + \pi \log T$ denote the error term in the mean square formula for $|\zeta(1 + it)|$. We derive a precise explicit expression for $R(t)$ which is used to prove that

$$\int_1^T R(t) dt = BT + O\left(\frac{T}{\log T}\right)$$

and

$$\int_1^T R^2(t) dt = CT + O\left(\frac{T}{\log T}\right)$$

for suitable constants B and $C > 0$.

These results improve on earlier upper bounds of Balasubramanian, Ramachandra and the author [1] for the integrals in question.

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A Conjecture on Integer Powers

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11D41

Some results [1, 2] and a bit of analysis convince this author to frame the following conjecture that relates to the powers of integers. For any positive integer n , the n^{th} power of an arbitrary positive integer can be expressed in infinite number of ways as the sum or difference of $(n + 1)$ number of other n^{th} powers of positive integers. When n equals 1, the conjecture is obvious. We will produce the proof of the conjecture with formulae to establish the cases for n taking values 2, 3 and 4. The structure of these results would tempt us to discover other formulae relating to higher values of n greater than 4. Possibly, the complete proof of this conjecture would open up our vision to add a new dimension to the understanding of the Diophantine problems and the related fields, known and unknown.

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Some Irreducibility Results for Truncated Binomial Expansions

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11C08, 11R09, 12E05

For positive integers $n > k$, let $P_{n,k}(x) = \sum_{j=0}^k \binom{n}{j} x^j$ be the polynomial obtained by truncating the binomial expansion of $(1+x)^n$ at the k^{th} stage. In

2007, Filaseta, Kumchev and Pasechnik (see [1]) considered the problem of irreducibility of $P_{n,k}(x)$ over the field \mathbb{Q} of rational numbers. In case $k = 2$, $P_{n,k}(x)$ has negative discriminant and hence is irreducible over \mathbb{Q} . Filaseta et al. pointed out that when $k = n - 1$, then $P_{n,k}(x)$ is irreducible over \mathbb{Q} if and only if n is a prime number. They also proved that for any fixed integer $k \geq 3$, there exists an integer n_0 depending on k such that $P_{n,k}(x)$ is irreducible over \mathbb{Q} for every $n \geq n_0$. In this paper, the authors prove the irreducibility of $P_{n,k}(x)$ over the field of rational numbers when $4 \leq 2k \leq n < (k + 1)^3$.

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Shift Radix Systems - Finiteness and Periodicity Properties

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11A63

For $\mathbf{r} = (r_0, \dots, r_{d-1}) \in \mathbf{R}^d$ define the *shift radix system* $\tau_{\mathbf{r}}$ by the function

$$\tau_{\mathbf{r}} : \mathbf{Z}^d \rightarrow \mathbf{Z}^d, \quad \mathbf{z} = (z_0, \dots, z_{d-1}) \mapsto (z_1, \dots, z_{d-1}, -[\mathbf{r}\mathbf{z}]),$$

where $\mathbf{r}\mathbf{z}$ is the scalar product of the vectors \mathbf{r} and \mathbf{z} (cf. Akiyama et al. [1]). If each orbit of $\tau_{\mathbf{r}}$ ends up at $\mathbf{0}$, we say that $\tau_{\mathbf{r}}$ has the *finiteness property*. It is well-known that each orbit of $\tau_{\mathbf{r}}$ ends up periodically if the polynomial $t^d + r_{d-1}t^{d-1} + \dots + r_0$ associated to \mathbf{r} is contractive. On the other hand, whenever this polynomial has at least one root outside the unit circle, there exist starting vectors that give rise to unbounded orbits. We want to present a number of recent results gained together with H.Brunotte, A.Pethő, P.Surer and J.Thuswaldner (cf. [2], [3] and [4]) on periodicity properties of the mappings $\tau_{\mathbf{r}}$ for the remaining situations of vectors \mathbf{r} associated to polynomials whose roots have modulus less than or equal to one with equality in at least one case. We show that for a large class of vectors \mathbf{r} belonging to the above class the ultimate periodicity of the orbits of $\tau_{\mathbf{r}}$ is equivalent to the fact that $\tau_{\mathbf{s}}$ is a shift radix system with finiteness property or has another prescribed orbit structure for a certain parameter \mathbf{s} related to \mathbf{r} . These results are combined with new algorithmic results in order to characterize vectors \mathbf{r} of the above class that give rise to ultimately periodic orbits of $\tau_{\mathbf{r}}$ for each starting value. In particular, we present the description of these vectors \mathbf{r} for the case $d = 3$. This leads to sets which seem to have a very intricate structure. For the instance

$d = 3$ we furthermore settle the conjecture positively that the fact that τ_r has the finiteness property implies that the polynomial $t^d + r_{d-1}t^{d-1} + \cdots + r_0$ is contractive.

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On Certain Products which are Never Squares

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11D41, 11L40

Let $p(x)$ be a polynomial in one variable with integer coefficients. We investigate the question when $\Pi(n) = p(1)p(2)\dots p(n)$ is a square. If $p(x)$ is an irreducible quadratic polynomial, it can be shown that for n large enough $\Pi(n)$ is not a square. However, obtaining a complete list of squares for all values of n requires certain effective character sum estimates, and this is a difficult problem to solve uniformly for all quadratic extensions. We concentrate on the latter question. In particular, we prove that for $p(x) = x^3 + 1$, $\Pi(n)$ is not a square for any n , and for $p(x) = x^2 + 5$, $\Pi(n)$ is a square if and only if $n = 4$.

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On Sums of Fibonacci Numbers Modulo p

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20K

Here, we show that for most primes p , every residue class modulo p can be represented as a sum of at most 32 Fibonacci numbers. We also look at the similar problem for composite moduli.

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Lagrange’s Method in Theory of Diophantine Equation: New Integer Differential Analysis

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The introduction of definition of derivative of one function in the real analysis is a revolutionary moment in the history of mathematics. For that reason we pose the question: Which is the respective interpretation of this concept in the Number theory? This paper is solution of this problem.

In the first section we have a possibility of interpreting Lagrange's method using the theory of Linear Differential Equations for obtaining the general solution of one Diophantine linear equation with two unknown quantities. Here we follow the idea of Lagrange literally.

In the second section we consider integer interpretations of some basic definitions and theorems by real analysis. This gives us the possibility to prove functional independence in the multitude of the integer numbers.

In section three, attention deserves T.3.2 about the presentation of X by the power of n as a sum of consecutive powers of X . This theorem is a result of the application of a Theory of Probability method (Two-dimensional distribution) for integers X and n . Hence, this theorem defines all possible cases for obtaining all possible values of X by the power of n . There is and the fundamental theorem as the new interpretation of Lagrange's method. This theorem answers of the question of how to introduce new indeterminate quantities in order to compare different addends from two equal sums. After all that, we can a possibility to prove "Fermat's Last Theorem".

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A Novel Polynomial Criterion and Algorithm for Twin Primes

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There exist a lot of publications on prime numbers, but in comparison nearly none on twin primes. Herein a novel criterion for a pair being a twin prime is presented [1, 2, 3, 4, 5].

Let $m \in \mathbb{N}$, $D(m) := \left\{ r \in \mathbb{Z} \mid \left\lfloor \frac{1-m^2}{5} \right\rfloor \leq r \leq \left\lfloor \frac{m^2-1}{7} \right\rfloor \right\}$, then

$\forall r \in D(m) \setminus \{0\} \sqrt{9r^2 - r + m^2} \notin \mathbb{Z} \iff (6m - 1, 6m + 1)$ is a twin prime.

The proof of this theorem will be presented at the conference. Also a novel short algorithm for computing all twin primes in a given intervall without performing any decomposition will be shown. The algorithm was implemented symbolically and numerically and the results agree with Ribenboim's computed twin primes table [5]. The computational time is in general $O(m)$ and under some certain circumstances $O(\sqrt{m})$.

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Applications of Multinomial Measure to the Digital Sums

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Let $p \geq 2$ be an integer and denote the p -adic expansion of $n \in \mathbf{N}$ by $n = \sum_{i \geq 0} \alpha_i(n)p^i$ ($\alpha_i(n) \in \{0, 1, \dots, p-1\}$) and set $s(n, l) = \sum_{i \geq 0} \mathbf{1}_{\{\alpha_i(n)=l\}}$ ($l = 0, 1, \dots, p-1$). The p -adic digital sum is defined by $s_p(n) = \sum_{l=1}^{p-1} ls(n, l)$.

Gelfond [2] investigated the distribution of $s_p(n)$ in arithmetic progressions, where $n \equiv l \pmod{m}$, $s_p(n) \equiv a \pmod{q}$ for $m(\geq 2), q(\geq 2), l, a \in \mathbf{Z}$ with $(q, p-1) = 1$. Noticing the relation between the distribution function of the multinomial measure and $s_p(n)$, we shall give explicit formulas of exponential sums related to Gelfond's theorem. As an application of those formulas, we shall obtain a simple expression of Newman-Coquet type summation formula related to the number of digits in a multiple of a prime number.

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A New Point in Lagrange's Spectrum Near L^C

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Here we investigate a new strategy to find a new point in Lagrange's spectrum near L^C (the origin of the Hall's ray) using a concept of slowly growing bilateral critical string.

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The Josephus Problem Generalized

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The purpose of this research is to present some algorithmic solutions of a generalization of the well known Josephus problem ([1], [2], [3], [5], [7], [8] and [9]) in which the elimination process consists of only one step by allowing it to have multiple steps. First, it is given an algorithm (Theorem 1) to solve a particular case of the Josephus problem generalized (JPG) by following the approach developed by Graham, Knuth and Patashnik in [4, p. 81]. Next, the JPG is solved in full (Theorem 2) and a recursion proved (Corollary 2). The technique used is based on modular arithmetic in accordance with some ideas introduced by Halbeisen and Hungerbuhler in [6], with few changes.

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Instant Evaluation of $\zeta(-n, \alpha)$, $\zeta_r(-n_1, -n_2, \dots, -n_r)$, $\zeta(n)$, $L(n, \chi)$

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11 M35

For the Hurwitz zeta function $\zeta(s, \alpha)$, we study $\zeta^{(r)}(s, \alpha)$, r -th order derivative with respect to s , as an analytic function of complex variable α , especially its power and Fourier series. We show that the only possible singularities of $\zeta^{(r)}(s, \alpha)$ are at the non-positive integral values of α . For integral $n \geq 0$, we show that $\zeta(-n, \alpha)$ is a polynomial, equal to $-\frac{1}{n+1}B_{n+1}(\alpha)$, $B_{n+1}(\alpha)$ being the Bernoulli polynomial of degree $(n+1)$. In particular for integral this results in instant evaluation of multiple zeta value $\zeta_\alpha(-n_1, -n_2, \dots, -n_r)$, defining this value as the result of repeated limits. Following functional equations, we also get instantly the values $\zeta(2n)$ and $L(n, \chi)$ for integral $n \geq 1$, where $\zeta(s) = \zeta(s, 1)$ and $L(s, \chi)$ is the Dirichlet L -series corresponding to a Dirichlet character χ , with $n \geq 1$ and χ both even or both odd. We also give insight into possible nature of $\zeta(2n+1)$. We treat Lerch's zeta function likewise.

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On Continued Fraction Expansions of the n -th Roots of the Solutions of Certain Quadratic Equations

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In the context of the achievement of a generalized continued fraction expansion of the n -th roots of the solution of a quadratic equation $x^2 - mx \pm 1 = 0$, a positive integer number, m , is fixed, the positive solution of $x^2 - mx - 1 = 0$, will be denoted by σ_m , and the positive solution of $x^2 - mx + 1 = 0$, $m = 3, 4, 5, \dots$, by τ_m . We can easily check that $\sigma_m = m + \frac{1}{m+} \frac{1}{m+} \dots$, and $\tau_m = (m-1) + \frac{1}{1+} \frac{1}{(m-2)+} \frac{1}{1+} \frac{1}{(m-2)+} \dots$. Indeed, the number σ_m belongs to the Metallic Number Family, [2], and the number τ_m does not belong to the Metallic numbers' class, however it is linked to it, [1]. By using sophisticated tools we may obtain generalized continued fraction for particular squared roots. For instance, $\sqrt{\sigma_2}$ can be expressed in terms of Dedekind eta function, since it coincides with the inverse of $\sqrt{2}\eta(z)\eta^2(4z)\eta^{-3}(2z)$, using the argument $z = \sqrt{-1}$, [3]. We directly obtain two different generalized expansions of the positive squared root of the number σ_m , $m = 1, 2, 3, \dots$. One of them, with complex coefficients, coincides with the real part of $1 + \frac{m-1+2i}{2+} \frac{m-1+2i}{2+} \dots$. The other one, with a fractal character, $\frac{1}{1-} \frac{\delta}{2-} \frac{\delta}{2-} \dots$, $\delta = 1 - (\sigma_m)^{-1}$ provides infinite sequences of rational approximations which converge to $\sqrt{\sigma_m}$. We evaluate the speed of convergence of both preceding formal expansions. We also obtain regular (simple) continued fractions for odd n -th roots of particular cases of σ_m . Among others results, that the cubic root of σ_m , $m = p^3 + 3p$, $p = 1, 2, 3, \dots$, is σ_p . In a similar way, we can consider, particular cases for even n -th roots of τ_m . For example, the positive quartic root of τ_m , $m = r^4 + 4r^2 + 2$, $r = 1, 2, 3, \dots$, is σ_r . Looking at the solutions of the first equation, we characterize the subset of σ_m , $m = 1, 2, 3, \dots$ which involves $\sqrt{2}$, obtaining a continued fraction with rational coefficients expressed in terms of leg-leg twin Pythagorean triples, which quickly converge to the considered σ_m .

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Proof of Fermat's Last Theorem (FLT)

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It took more than three centuries and a half to generate a proof of FLT as a supplementary result of the proof of *Taniyama-Shimura Conjecture* [4]. This proof, based on modern theory of elliptical curves, an idea undreamt of in the 17th century [1]. Acknowledging this proof as a great achievement, I still am of the view that Fermat possessed a logical proof other than this.

My search for such a proof led me to the following basic requirements.

- (i) The equation $x^n + y^n = z^n$ (1) is to be made into factorisable form for all n as $y^n = z^n - x^n$ which implies $\frac{y^n}{(z-x)} = (z^{n-1} + z^{n-2}x + \dots + zx^{n-2} + x^{n-1}) = P(z)$ (2). For natural number solution set (x, y, z) of equation (1), obviously $\frac{y^n}{(z-x)}$ should be a natural number and at the same time $P(z)$ is not divisible by $(z-x)$ [2].
- (ii) The set of natural numbers is to be properly partitioned for detailed analysis. The most appropriate partition [3], according to me is *Unit-Prime-Composite Partition* (UPC Partition) as $N = I \cup P \cup C$ where $I = \{1\}$ unit set, $P = \{2, 3, 5, 7, 11, \dots\}$ set of primes and $C = \{4, 6, 8, 9, \dots\}$ set of composite numbers.

The above partition leads us to 9 different cases on the key factors y and $(z-x)$ of $y^n = z^n - x^n$. For natural number solution set (x, y, z) of equation (1) provides $x \neq y$ therefore $x < y < z$. That is $(z-x) > 1$. Hence 3 cases having $(z-x) \in I$ can be ruled out. When $y \in I$, $\frac{y^n}{(z-x)}$ is fractional, no solution.

For remaining 4 cases obtained from *UPC Partition*, we can apply the method of contradiction. For the case $y \in P$ and $(z-x) \in P$, consider a natural number solution set (x, y, z) satisfying (1). Let $y = p \in P$ and $(z-x) = q \in P$. For integral values of $P(z)$ in (2) $p = q$, then $\frac{y^n}{(z-x)} = \frac{q^n}{q} = q^{(n-1)} = (z-x)^{(n-1)} = P(z)$, a contradiction. In the remaining 3 cases too we can prove that y and $(z-x)$ can not be determined as specified conditions of $P(z)$ in (2). And this proves ***Fermat's last theorem***.

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Interesting Properties Related with the Tribonacci Sequence

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In this paper we derive some interesting properties related with the sequence $\{T_n\}_{n \geq 1}$ of Tribonacci numbers defined by $T_1 = 0, T_2 = 1, T_3 = 1$ and $T_{n+3} = T_{n+2} + T_{n+1} + T_n$ for $n \geq 1$.

We first express T_n in simple explicit form and use it to derive the recursive formula for T_n to complete the successor and predecessor of any given Tribonacci number. We also derive nice bounds for T_n .

We then obtain the formula for the number of digits of T_n and also the value of the finite series

$$\sum_{i=1}^{\infty} \frac{T_i}{x^{(i+1)n}}, \text{ for } x \geq 2$$

Finally we consider the generalized Tribonacci sequence $\{t_n\}_{n \geq 1}$ having initial terms $t_1 = a, t_2 = b$ and $t_3 = c$ for non-zero relatively prime integers b, c . We show that this sequence does not have the gcd property.

$$(t_i, t_j) = t_{(i,j)}, \quad \text{for any } i, j \geq 1$$



Jacobi Sums and Cyclotomic Numbers of Order l^2

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Cyclotomic numbers had their origin in the work of C. F. Gauss. In his book *Disquisitiones Arithmeticae*, he had obtained cyclotomic numbers of order 3 and 4 while solving the problem of constructibility of regular polygons. For $e \geq 2$, Jacobi sums of order e are algebraic integers in the cyclotomic field $\mathbb{Q}(\zeta_e)$, $\zeta_e = \exp(2\pi i/e)$, and in 1935, L. E. Dickson (see [2]) used them in connection with Waring's problem. He showed that if Jacobi sums of order e are known then cyclotomic numbers of order e can be obtained.

It may be noted that an element α coprime to l in the cyclotomic ring $\mathbb{Z}[\zeta_l]$, l prime, can be determined uniquely if we know its prime ideal decomposition, absolute value and congruence modulo $(1 - \zeta_l)^2$. To determine an element in the ring $\mathbb{Z}[\zeta_{l^2}]$ which is coprime to l , the congruence is required modulo $(1 - \zeta_{l^2})^{l+1}$. Prime ideal decomposition and absolute value of Jacobi sums of order e are well-known. So it is required to find the appropriate congruences. The congruences for Jacobi sums of order l and $2l$ (l prime) are well-known and they have been used by Katre and Rajwade [4] and V. V. Acharya and S. A. Katre [1] for arithmetic characterization of Jacobi sums of order l and $2l$ respectively.

In the present paper ([5]) we find determining congruences for Jacobi sums $J(1, n)_{l^2}$ of order l^2 for a finite field \mathbb{F}_q , $q = p^r \equiv 1 \pmod{l^2}$ where $l > 3$ and p are primes. These are appropriate congruences useful in giving algebraic characterization of the Jacobi sums of order l^2 and they have been obtained in terms of cyclotomic numbers of order l . Our results sharpen the congruences given in [3]. We also obtain cyclotomic numbers $(h, k)_{l^2}$ of order l^2 in terms of the coefficients of the Jacobi sums $J(1, j)_l$ of order l and $J(1, n)_{l^2}$ of order l^2 .

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Combinatorial Configurations in Dense Subsets of Two-dimensional Grid

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A well-known Szemerédi's theorem [1, 2] on arithmetic progressions asserts that any sufficiently dense subset of the segment $\{1, 2, \dots, N\}$ contains an arithmetic progression of any length. Mutidimensional variants of the result was studied by various authors (see e.g. [3]–[6]). We prove that any sufficiently dense set of

two-dimensional grid $(\mathbf{Z}/5\mathbf{Z})^n$ contains a configuration of the form $\{(x, y), (x + d, y), (x + 2d, y), (x, y + d)\}$ with nonzero d .

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Defining Power Sums of n and $\varphi(n)$ Integers

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Let n be a positive integer and $\varphi(n)$ denotes the Euler phi function. It is well known that the power sum of n can be evaluated in closed form in terms of n . Also, the sum of all those $\varphi(n)$ positive integers that are coprime to n and not exceeding n , is expressible in terms of n and $\varphi(n)$. Although such results already exist in literature [1, 2], but here we have presented some new analytical results in these connections. Some functional and integral relations are derived for the general power sums. Two sample results are [3]:

$$\sum_{d < n, (d, n) = 1} d^k = \frac{n^{k+1}}{k+1} \sum_{m=0}^{\lfloor \frac{k}{2} \rfloor} C(k+1, 2m) B_{2m} n^{-2m} \prod_{p|n, p \text{ prime}} (1 - p^{2m-1}). \quad (1)$$

$$\mathbf{S}_k(x) = k \int_0^x \mathbf{S}_{k-1}(t) dt + x C_k, \quad C_k = 1 - k \int_0^1 \mathbf{S}_{k-1}(t) dt \quad (2)$$

for any real x where B_{2m} denote the $2m$ -th Bernoulli number.

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On the Solutions of Some Specific Exponential Diophantine Equations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11D61, 11D25, 11D59, 11J86

In this manuscript, we consider some specific classes of exponential Diophantine equations and give methods to obtain classes of solutions for each Diophantine equation. An exponential Diophantine equation is an equation $x^2 + C = y^n$ where C is a product of several prime powers. In literature, there are partial results concerning the solutions of these Diophantine equations. We discuss these equations in three different cases: $n = 3$, $n = 4$ and $n > 4$. The main tool used in calculations is MAGMA [3].

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Excess Continued Fractions Expansions

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It is well known that in the case of “simple” infinite continued fraction expansions of real numbers, the rational approximants have the property that if they are even, they increase with n increasing while if they are uneven, they decrease strictly with n increasing.

Looking for a quicker convergence of the rational approximants, we have found that if we consider the quadratic equation

$$x^2 - nx + 1 = 0 \quad (n \text{ is a natural number } \geq 3) \quad (3)$$

its solutions can be expressed as a continued fraction expansion of the form

$$x = n - \frac{1}{n - \frac{1}{n - \ddots}}$$

denoted by $x = [n-]$, which we call “excess continued fraction expansion”.

Applying this result to the well known Golden Mean $\phi = \frac{1+\sqrt{5}}{2}$ and the sequence of its successive powers $\phi^2, \phi^3, \phi^4, \dots$ it is easy to prove that its uneven powers have a purely periodic continued fraction expansion and its even powers satisfy a quadratic equation similar to (3), having consequently an excess continued fraction expansion which converges much faster than a

simple one. For other members of the Metallic Means Family like the Silver Mean, the Bronze Mean, the Copper Mean, the Nickel Mean, etc. (introduced by the author in 1998, see [1]), which have a purely periodic continued fraction expansion, the behavior is analogous to the one found for the Golden Mean. From the numerical point of view, this is a big advantage because not only the Metallic Means are frequently used in many applications, but also its powers, [2].

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Proof of Fermat’s Last Theorem

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20K

In 1994 the Englishman Andrew Wiles presented a proof of the Tamiyama-Shimura conjecture resulting from mapping Fermat’s L.T. into elliptic curves. The solution proposed herein is simple and understanding.

In the formula $\beta^\nu + \gamma^\nu = \alpha^\nu$ when $\nu > 2$ the number ν has an odd factor. For $\nu =$ prime number $\Rightarrow \beta^\nu + \gamma^\nu = (\beta + \gamma) \sum_{\kappa=1}^{\nu} (-1)^{\kappa+1} \beta^{\nu-\kappa} \gamma^{\kappa-1} = \alpha^\nu$. Setting $\beta + \gamma = \delta$ and $\sum_{\kappa=1}^{\nu} (-1)^{\kappa+1} \beta^{\nu-\kappa} \gamma^{\kappa-1} = \Delta$. According to the Euclidean division there exists exactly one $\Upsilon = \Delta - \delta\pi$. From $\Upsilon = \nu\gamma^{\nu-1} \Rightarrow \beta + \gamma = \alpha_1^\nu$ and $\sum_{\kappa=1}^{\nu} (-1)^{\kappa+1} \beta^{\nu-\kappa} \gamma^{\kappa-1} = \alpha_2^\nu$ where α_1, α_2 either are relatively prime numbers or have a common factor ν .

We probe the form $(\beta + \gamma)^{\nu-1} - \sum_{\kappa=1}^{\nu} (-1)^{\kappa+1} \beta^{\nu-\kappa} \gamma^{\kappa-1} = (\alpha_1^\nu)^{\nu-1} - \alpha_2^\nu$ and we prove that the right side of the above equation is not integer expression.

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Exact Results for the Frobenius Problem in Three Variables

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11D04

Given positive integers a_1, \dots, a_k , with $\gcd(a_1, \dots, a_k) = 1$, the Coin Exchange Problem of Frobenius asks for the largest positive integer N such that the equation

$$a_1x_1 + \dots + a_kx_k = N \tag{4}$$

has no solution in nonnegative integers x_1, \dots, x_k . This number is usually represented by $g(a_1, \dots, a_k)$, and it is well known that $g(a_1, a_2) = a_1a_2 - a_1 - a_2$. There are several results that pertain to the three variable and the more general case, including algorithms and results that apply to special cases.

The purpose of my talk is to present an old and unpublished result that gives a closed-form formula for $g(a_1, a_2, a_3)$. I will also briefly present results for the related problem of determining $n(a_1, a_2, a_3)$, that counts the number of N in (1) that are nonrepresentable by a_1, a_2, a_3 .

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A New Bound for the Sphere Problem

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The number $N(R)$ of lattice points inside a sphere of radius R is comparable to its volume, but to decide to what extent is still a matter of research. We are interested in discovering the smallest real number θ such that we have

$$|N(R) - \frac{4}{3}\pi R^3| < R^{\theta+o(1)}$$

when $R > 1$ goes to infinity. It has been conjectured that actually $\theta = 1$ and we know that $\theta \geq 1$. Gauss was already interested in this problem due to its relationship with the average behaviour of class numbers of binary quadratic forms, and proved $\theta \leq 2$ by geometrical means. Afterwards there have been several improvements due to Lipschitz, Mertens (by using Fourier Analysis), Chen, Vinogradov (by adding exponential sums estimates), F. Chamizo and H. Iwaniec [1], and finally D. R. Heath-Brown [4] (by introducing character sums, due to the relationship with class numbers), with $\theta \leq 1 + 5/16$.

Nowadays the problem is understood as a mixed one, involving correlations of exponentials and characters. After some related work [3], we were finally able to understand this mixture in a broader way, thus improving [2] the best known bound for θ .

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The Number of Sumsets in a Finite Field

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11B

I will report on our recent work [1], in which we show that there are $2^{p/2+o(p)}$ distinct sumsets $A + B$ in \mathbb{F}_p where $|A|, |B| \rightarrow \infty$ as $p \rightarrow \infty$. For the proof we use a Fourier-analytic method developed in [2] whenever $|A|$ and $|B|$ are large and a combinatorial argument otherwise.

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Higher Twists of Elliptic Curves with Positive Rank

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11G05, 14G05

Let us recall that if E/\mathbb{Q} is an elliptic curve with j -invariant 0, i.e. E is given by the equation $y^2 = x^3 + q$ for some $q \in \mathbb{Q}^*$, then the cubic twist of E by $d \in \mathbb{Q}^*$ has the equation $y^2 = x^3 + d^2q$. Moreover, if $d \in \mathbb{Q}^*$ then the sextic twist of E by d has the equation $y^2 = x^3 + dq$.

We prove that for any pair of elliptic curves $E_i : y^2 = x^3 + a_i$, where $a_i \in \mathbb{Z} \setminus \{0\}$ for $i = 1, 2$, the set

$$\mathcal{D}_3 = \{d \in \mathbb{Q} : \text{cubic twist of } E_i \text{ by } d \text{ has positive rank for } i = 1, 2\}$$

is dense in the set \mathbb{R} .

A slightly better result can be proved for sextic twists of elliptic curves with $j = 0$. More precisely, in [2] we prove that for any quadruple of elliptic curves $E_i : y^2 = x^3 + a_i$, where $a_i \in \mathbb{Z} \setminus \{0\}$ for $i = 1, 2, 3, 4$, the set

$$\mathcal{D}_6 = \{d \in \mathbb{Q} : \text{sextic twist of } E_i \text{ by } d \text{ has positive rank for } i = 1, 2, 3, 4\}$$

is infinite.

If now E/\mathbb{Q} is an elliptic curve with j -invariant 1728, i.e. E is given by the equation $y^2 = x^3 + px$ then the quartic twist of E by $d \in \mathbb{Q}^*$ has the equation $y^2 = x^3 + dpx$.

We prove that for any quadruple of elliptic curves $E_i : y^2 = x^3 + a_i x$, where $a_i \in \mathbb{Z} \setminus \{0\}$ for $i = 1, 2, 3, 4$, the set

$$\mathcal{D}_4 = \{d \in \mathbb{Q} : \text{quartic twist of } E_i \text{ by } d \text{ has positive rank for } i = 1, 2, 3, 4\}$$

is infinite.

In each case we construct a polynomial $d_m \in \mathbb{Q}[t]$ such that $d_m(\mathbb{Q})$ is contained in \mathcal{D}_m for $m \in \{3, 4, 6\}$.

Our results complement the Kuwata and Wang result given in [1] concerned the existence of simultaneous quadratic twists with positive rank of pairs of elliptic curves.

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On Pythagorean Triples of the form $(i, i + 1, k)$

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11B37, 11Y50

A Pythagorean triple is a triad of integers which satisfy Pythagoras' equation. In this paper, we shall consider triples of the form $(i, i + 1, k)$, and the recurrence relations governing them.

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On the Primes and the Prime Factorization of Composite Numbers in a Polynomial Sequence

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We discuss the prime numbers and the prime factor decomposition of composite numbers in a sequence $\{P(n)\}$ of polynomial with integer coefficient. Take the $P(n) = n^2 + n + 1$ as an example, there are two steps: 1. for each prime number, solve the congruence equation $r^2 + r + 1 \equiv 0 \pmod{p}$, $1 \leq r \leq p - 1$, we reserved those p which have solutions, the other p won't become the factors of the $n^2 + n + 1$; to this example, all the prime numbers which have been eliminated are those prime numbers in shape of $6m + 5$; 3, as a prime number, has a solution $r^{(3)} = 1$; 7 has two solutions $r_1^{(7)} = 2$, $r_2^{(7)} = 4$, all the prime numbers which in shape of $6m + 1$ have 2 solutions $r_1^{(p)}$, $r_2^{(p)}$. 2. Revise the Sieve method: $n \equiv 1 \pmod{3}$, testify that $P(n)$ is a multiple of 3, so it should be eliminated; $n \equiv r_1^{(p)} \pmod{p}$ or $n \equiv r_2^{(p)} \pmod{p}$, testify that $P(n)$ is a multiple of p , so it should be eliminated (however, if p itself is $P(n)$, it could be reserved). Through such means, the subsequence left 3, 7, 13, 43, 73, 157, \dots are all prime numbers, while the $P(n)$, which has been eliminated at the same time by p_1, p_2, \dots, p_u will have the prime factor decomposition: $P(n) = p_1^{k_1} p_2^{k_2} \dots p_u^{k_u}$. Although the majority of k equal to 1, k bigger than 1 is also possible. But the prime factor number 3's exponent could only be 1.

When we solve the congruence equation till $p \leq 2000$, we can obtain the complete segment when $P(n) \leq 4000000$. The last prime number in the segment is $P(1994) = 3978031$, and there are no representative prime factor number decomposition I want to mention, as follow: $P(1712) = 7 \times 13^2 \times 37 \times 67$, $P(1935) = 1753 \times 2137$, $P(1733) = 7^3 \times 8761$. All the decompositions above are all solved by hand, if we use the computer, we could find a plenty of huge prime numbers and the huge prime factor decomposition of the composite numbers.

The composition of $2^{67} - 1$ by the mathematician Cole can be used as the first item of the sequence $\{P(n) = \sum_{k=0}^{66} n^k\}$, and we use the similar revision of Sieve method, we can obtain the prime numbers and the prime number factor decomposition of composite numbers. This research effort is in progress, $P(2)$'s prime number decomposition, we have a simple method to figure it out only by hand and this method will merely cost a few hours to process. What we need to use are simply some rather little prime numbers, such as 3, 5, 7, 11, in order to eliminate all the non-square numbers in a second-order arithmetic, then we can obtain the result.

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Section 4

Algebraic and Complex Geometry

Residues of Logarithmic Differential Forms

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2010 MATHEMATICS SUBJECT CLASSIFICATION. 32S65, 14B05, 14F10, 58K45

The purpose of the talk is to give an elementary introduction to the theory of residue of logarithmic and multi-logarithmic differential forms [1, 2, 4], and to describe some of the less known applications of this theory, developed by the author in the past few years. In particular, we briefly discuss the notion of residue due to H. Poincaré, J. de Rham, J. Leray and K. Saito, and then obtain an elegant description of the modules of regular meromorphic differential forms in terms of residues of meromorphic differential forms logarithmic along hypersurface or complete intersections with arbitrary singularities. We also discuss a new method for computing the topological index of complex vector fields on hypersurfaces with arbitrary singularities [3], some applications to the theory of holonomic D -modules of Fuchsian and logarithmic types [5], and to the theory of Hodge structures on the logarithmic de Rham complex.

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Weighted Homogeneous Singularities and Rational Homology Disk Smoothings

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We classify the resolution graphs of weighted homogeneous surface singularities which admit rational homology disk smoothings. The nonexistence of rational homology disk smoothings is shown by symplectic geometric methods, while the existence is verified via smoothings of negative weights.



On the Real Nerve of the Moduli Space of Complex Algebraic Curves

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Primary 30F; Secondary 14H

The real locus $\mathcal{M}_g^{\mathbb{R}}$ in the moduli space \mathcal{M}_g of complex algebraic curves of genus g is covered by the strata $\mathcal{M}_g^{k,\varepsilon}$ each of which is determined by the real curves of the given topological type (k, ε) , where k stands for the number of connected components and $\varepsilon = \pm 1$ corresponds to the separability type of the smooth projective models. Furthermore, two such strata $\mathcal{M}_g^{k,\varepsilon}$ and $\mathcal{M}_g^{k',\varepsilon'}$ intersect if there is a complex algebraic curve of genus g having two real forms of the types (k, ε) and (k', ε') . We study the nerve $\mathcal{N}(g)$ corresponding to this covering, ([7] 3.1.6), called the *real nerve* of complex algebraic curves of given genus g . Some results concerning $\mathcal{N}(g)$ are known. First of all, by the results of Hurwitz and Weichold, it has $[(3g + 4)/2]$ points (c.f. [3]). By the results of Buser, Seppälä and Silhol [2], $\mathcal{N}(g)$ is connected and furthermore, it was shown by Costa and Izquierdo in [4] that given g and a type (k, ε) there exists a Riemann surface X of genus g having two symmetries σ, τ of the types (k, ε) and $(1, -1)$ respectively which means that $(1, -1)$ is a *spine* for $\mathcal{N}(g)$ for arbitrary g . We find both geometrical and homological dimension of $\mathcal{N}(g)$ and give some results concerning the global properties of $\mathcal{N}(g)$ for any even $g \geq 2$. We also present some results for odd values of g . The proofs were obtained by methods of combinatorial group theory, i.e. theory of non-euclidean crystallographic groups.

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Topology and Singularities of Free Group Character Varieties

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 14L30 14L24 57M07 14B05

Let \mathfrak{X} be the moduli of $SL(n, \mathbb{C})$, $SU(n)$, $GL(n, \mathbb{C})$, or $U(n)$ valued representations of a rank r free group. We compute the fundamental group of \mathfrak{X} and show that these four moduli otherwise have identical higher homotopy groups. We then classify the singular stratification of \mathfrak{X} . This comes down to showing the singular locus corresponds exactly to reducible representations if there exist singularities at all. Lastly, we show that the moduli \mathfrak{X} are generally not topological manifolds, except for a few examples we explicitly describe. This is joint work with C. Florentino (see [arXiv:0907.4720](https://arxiv.org/abs/0907.4720) and [arXiv:0807.3317](https://arxiv.org/abs/0807.3317)).



Moduli of Parabolic Higgs Bundles and Atiyah Algebroids

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 14D20

Higgs bundles, were introduced by Hitchin in [2]. Over a smooth compact Riemann surface the moduli space of Higgs bundles contains as a dense open subset the total space of the cotangent bundle to the moduli space of vector bundles. The induced complex symplectic form is part of a hyper-Kähler structure and extends to the whole moduli space, and the moduli space is equipped with an algebraically completely integrable system through the Hitchin map.

A natural generalization of vector bundles arises when one endows the vector bundle with a parabolic structure [6], i.e. with choices of flags in the fibers over certain marked points on the Riemann surface. One can talk of Higgs bundles as well, as was first done by Simpson [7].

We show that this space possesses a Poisson structure, extending the one over the dual of an Atiyah algebroid over the moduli space of parabolic vector bundles. By considering the case of full flags, we get a Grothendieck-Springer resolution for all other flag types, in particular for the moduli spaces of twisted Higgs bundles, as studied by Markman [5] and Bottacin [1] and used in the recent work of Laumon-Ngô [3]. We discuss the Hitchin system, and demonstrate that all these moduli spaces are integrable systems in the Poisson sense.

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Isomorphism Among the Families of Weighted $K3$ Hypersurfaces

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The aim of this talk is to explain our recent work on an establishment of correspondences among the 95 families of weighted $K3$ hypersurfaces [1].

It is well known that the projective plane \mathbf{P}^2 is birational to $\mathbf{P}^1 \times \mathbf{P}^1$ by blowing up two points P and Q in \mathbf{P}^2 and blowing down the strict transform of the line passing through these points to a point R in $\mathbf{P}^1 \times \mathbf{P}^1$. On the other hand, the complete anticanonical linear systems of \mathbf{P}^2 and $\mathbf{P}^1 \times \mathbf{P}^1$ are not isomorphic since the dimensions of them are different. However, these systems have the isomorphic sublinear systems under the birational transformation, that is to say, the cubic curves through P and Q in \mathbf{P}^2 and the $(2, 2)$ -curves through R in $\mathbf{P}^1 \times \mathbf{P}^1$. It is also the complete anticanonical linear system of the del Pezzo surface of degree 7.

We consider a two-dimensional analogue of such birational transformation. In our case, it is observed that some of the 95 families have the isometric Picard lattices. Then, a natural question arises; whether two generic weighted $K3$ hypersurfaces in different families themselves are isomorphic or not. That is a subtle question since the Picard lattices would not always specify the family of

$K3$ surfaces. We have made correspondences of weighted $K3$ hypersurfaces by explicitly constructing the monomial birational morphisms among the weighted projective spaces. In other words, all the weight systems having the isometric Picard lattices commonly possess an anticanonical sublinear system. We remark that since the birational transformations are given as monomial maps, the corresponding amoebas of $K3$ hypersurfaces are linearly isomorphic. We also confirm that the Picard lattice of the sublinear system we obtained is the same as those of the complete linear systems. Consequently, the number of the families of weighted $K3$ hypersurfaces is essentially 75.

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On Birational Invariants of Algebraic Curves on Irrational Ruled Surfaces

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Let S be a nonsingular irrational ruled surface S irregularity $q \geq 1$ and D a nonsingular curve on S . We shall study pairs (S, D) of projective non-singular irrational ruled surfaces S and curves D on S .

Define $P_m[D]$ to be $\dim H^0(S, \mathcal{O}(m(D + K_S)))$ ($m > 0$) and $\kappa[D]$ the $K_S + D$ dimension of S , which is denoted by $\kappa(D + K_S, S)$, where K_S indicates a canonical divisor on S . Both $P_m[D]$ and $\kappa[D]$ are invariants under birational transformations between pairs. The pair (S, D) is said to be *relatively minimal*, if every exceptional curve E of the first kind on S satisfies the inequality $E \cdot D \geq 2$ ($E \neq D$) (cf. [I1]). Moreover, (S, D) is said to be *minimal*, since S is an irrational ruled surface, the Albanese map $\alpha : S \rightarrow \mathbf{Alb}(S)$ gives rise to a subjective morphism $\alpha : S \rightarrow \alpha(S) = B$, which is a curve of genus q . Let F denote a general fiber of $\alpha : S \rightarrow B$. Then the intersection number $D \cdot F$ coincides with the mapping degree of $\alpha|_D : S \rightarrow B$, which is denoted by $\sigma(D)$.

The structure of relatively minimal pairs (S, D) with $\kappa[D] \leq 1$ have been precisely determined by [Ma]. If $\kappa[D] = 2$ then relatively minimal pairs are always minimal (cf. Proposition 7 in [Ma]). Introduce an invariant η to be $4g(D) - D^2 - 8q + 4$.

Theorem 1. *Suppose that $\kappa[D] = 2$. Then $\sigma(D) \leq \eta(\eta + 2)$ when $q = 1$, and $\sigma(D) \leq 2 + \eta/2(q - 1)$ when $q \geq 2$.*

Let τ denote $4\bar{g} - P_2[D]$, where $\bar{g} = g(D) - 1$.

Theorem 2. *Suppose that (S, D) is a minimal pair with $\kappa[D] = 2$ and $g(D) \geq 2$. Then $P_2[D] \geq 3\bar{g} + \bar{q}$, where $\bar{q} = q - 1$. Furthermore, we obtain the following estimates of $\sigma(D)$.*

(1) *Assume that $q > 1$. Then $\sigma(D) \leq 2(\bar{g} - \bar{q} - \tau)/3\bar{q} + 3$.*

(2) *Assume that $q = 1$.*

If $\tau > 0$ then $\sigma(D) \leq 2\bar{g}/\tau + 1$. If $\tau \leq 0$ then $\sigma(D) \leq 16(\bar{g} - \tau)^2 - \tau + 2\bar{g} + 2$.

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Remarks on Some Non-linear Heat Flows in Kähler Geometry

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In this talk, we clarify or simplify certain aspects of the Calabi flow and of the Donaldson heat flow.

In particular, in [1], the Calabi flow is studied as a flow of conformal factors $g_{ij}(t) \equiv e^{2u(t)} \hat{g}_{ij}(0)$,

$$\dot{u}(t) = \frac{1}{2} \Delta R \tag{1}$$

and the convergence of the conformal factors $u(t)$ in the Sobolev norm $\|\cdot\|_{(2)}$ is obtained. Although the convergence of the conformal factors established by Struwe [1] is only in the $\|\cdot\|_{(2)}$ norm, he states clearly that the convergence in arbitrary Sobolev norms, and hence in C^∞ , should follow in the same way. In the first part of this talk, we confirm that this is indeed the case.

Next we discuss the Donaldson heat flow. We shall show directly the C^0 boundedness of the full curvature tensor $F_{\bar{k}j}^{\alpha\beta}$ on $[0, \infty)$. Once again, our main technique is differential inequalities for the L^2 norms of the derivatives of $F_{\bar{k}j}^{\alpha\beta}$, in analogy with the methods of [3, 2] and the treatment of the Calabi flow that we used in the previous section.

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Higgs Bundles and Generalized Cayley Correspondence

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In 1846, Arthur Cayley ([2]) defined a transformation into matrices. In the special case of complex dimension 1, it can be seen geometrically as a correspondence between the Poincaré disc and the hyperbolic upper half-plane, both models of the symmetric space $SU(1, 1)/U(1)$.

This correspondence was generalized for any Hermitian symmetric space in [3]. Let G be a real non-compact Lie group of Hermitian type, and H its maximal compact subgroup. We will focus on the spaces G/H of tube-type. In this case, the Shilov boundary is a compact symmetric space H/H' and the Cayley transform is a tube domain over a self-dual cone. This cone is precisely H^*/H' , the non-compact dual of the Shilov boundary, and the space where it lives is endowed with a Jordan algebra structure.

It turns out that the Cayley correspondence shows up also in the moduli space of G -Higgs bundles over a compact Riemann surface. A G -Higgs bundle is a pair consisting of a holomorphic $H^{\mathbb{C}}$ -bundle and a holomorphic section of

the bundle associated to the isotropy representation. For the moduli space of G -Higgs bundles with maximal Toledo invariant we have that

$$\mathcal{M}_{\max}(G) \cong \mathcal{M}_{K^2}(H^*),$$

where $\mathcal{M}_{K^2}(H^*)$ denotes the moduli space of K^2 -twisted H^* -Higgs bundles. This result was proved recently for the classical groups, using the classification theorem of Lie groups, in [1].

In this talk we present the case of the exceptional symmetric domain of tube type and show an intrinsic and general proof, which reveals the important role played by the Jordan algebra structure.

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Hilbert Schemes of r -points for Irreducible Simple Plane Curve Singularities

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Let k be an algebraically closed field of characteristic zero. We consider the following three types of irreducible plane curve singularities:

$$A_{2d} : x = t^2, y = t^{2d+1} \quad E_6 : x = t^3, y = t^4 \quad E_8 : x = t^3, y = t^5.$$

They are called *irreducible simple singularities*. We denote by X one of them. The local rings of A_{2d} , E_6 and E_8 are defined by $\mathcal{O}_{1,2d} := k[[t^2, t^{2d+1}]]$, $\mathcal{O}_2 := k[[t^3, t^4]]$ and $\mathcal{O}_3 := k[[t^3, t^5]]$, respectively. We write \mathcal{O} for $\mathcal{O}_{1,2d}$ or \mathcal{O}_2 or \mathcal{O}_3 . The set $\Gamma := \{\text{ord}(f) \mid f \in \mathcal{O}\}$ is called the *semigroup* of \mathcal{O} . Fix the notations as follows.

$$\bar{\mathcal{O}} := \mathbb{C}[[t]], \quad \bar{I}(n) := \{f \in \bar{\mathcal{O}} \mid \text{ord}(f) \geq n\}, \quad I(n) := \bar{I}(n) \cap \mathcal{O}$$

The positive integer $\delta := \dim_k(\overline{\mathcal{O}}/\mathcal{O}) = \#(\mathbb{N} \cup \{0\} \setminus \Gamma)$ is called the δ -invariant of Γ . An element W of $\text{Gr}(\delta, \overline{\mathcal{O}}/\overline{I}(2\delta))$ is said to be *good space*, if it is an \mathcal{O} -submodule. The set

$$\mathcal{M} := \{W \in \text{Gr}(\delta, \overline{\mathcal{O}}/\overline{I}(2\delta)) \mid W \text{ is good}\}$$

is called the *PS-space* of X . For an ideal I of \mathcal{O} , we define the *codimension* of I by $\tau(I) := \dim_k \mathcal{O}/I$. The set

$$\mathcal{M}_r := \{I \mid I \text{ is an ideal of } \mathcal{O} \text{ with } \tau(I) = r\}$$

is called the *Hilbert scheme of r -points* on X .

Consider the map $\phi_r : \mathcal{M}_r \rightarrow \mathcal{M}$ defined by $\phi_r(I) = t^{-r}I / I(2\delta)$. In [1], Prof. Pfister and Steenbrink proved that the map ϕ_r is injective. It is also shown that, for $r \geq 2\delta$, the map ϕ_r is bijective (i.e. $\phi_r(\mathcal{M}_r) = \mathcal{M}$). Let $\psi : \text{Gr}(\delta, \overline{\mathcal{O}}/\overline{I}(2\delta)) \rightarrow \mathbb{P}^N$ be Plücker embedding where $N = \binom{2\delta}{\delta} - 1$. For certain classes of curve singularities, Prof. Pfister and Steenbrink studied the structure of $(\psi \circ \phi_r)(\mathcal{M})$ in [1]. For irreducible simple singularities, we study that of $(\psi \circ \phi_r)(\mathcal{M}_r)$ where $1 \leq r \leq 2\delta - 1$.

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Section 5

Geometry

From Lichnerowicz to Basic Lichnerowicz Cohomology

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 53C12; 53C15.

Let \mathcal{F} be a n -dimensional foliation on a closed manifold M . The aim of this work is to introduce and give some properties of the Lichnerowicz basic cohomology which is an important generalization of the basic cohomology.

In [1, 2] we have showed the Leray-Hirsch theorem for basic and vertical forms. In this note we will generalize this theorem for all differential forms.

We will also prove that many properties of the usual basic cohomology still have their analogues within the Lichnerowicz basic cohomology.

We will also compute this new cohomology for some foliations. In particular, we will show that the Lichnerowicz basic cohomology of 0 degree of any connected manifold is trivial which is not the case for the basic cohomology.

The Gysin sequences [4] are used to give the relationship between the Lichnerowicz cohomology [3] and the Lichnerowicz basic ones.

We will use this sequence to give some properties of Lichnerowicz basic cohomology for given foliation.

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On the Gaussian Curvature of Complete Spacelike Surfaces in Lorentzian Products

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Given Σ^2 a compact spacelike surface immersed in a Lorentzian product space $M^2 \times \mathbb{R}_1$, we establish an integral formula which allows us to derive some interesting consequences in terms of the Gaussian curvature of the surface. For instance, when M^2 is either the sphere \mathbb{S}^2 or the real projective plane \mathbb{RP}^2 , we characterize the slices of the trivial totally geodesic foliation $M^2 \times \{t\}$ as the only complete spacelike surfaces with constant Gaussian curvature in the Lorentzian product $M^2 \times \mathbb{R}_1$.

On the other hand, we show that our results are no longer true when $M^2 = \mathbb{H}^2$ is the hyperbolic plane. In fact, we give examples of complete spacelike surfaces with constant Gaussian curvature $K \leq -1$. However, using the abstract theory of Codazzi pairs, we show that there exists no complete spacelike surface in $\mathbb{H}^2 \times \mathbb{R}_1$ with constant Gaussian curvature $K > -1$.

These results are part of a joint work with Luis J. Alías and Juan A. Aledo and are contained in [1] and [2].

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On the Existence of Affine Maximal Maps

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In affine surfaces theory, Blaschke (see [4]) found that the Euler-Lagrange equation of the equiaffine area functional is of fourth order and nonlinear. He also showed that this equation is equivalent to the vanishing of the affine mean curvature, which led to the notion of *affine minimal surface* without a previous study of the second variation formula. But Calabi proved in [5] that, for locally strongly convex surfaces, the second variation is always negative and since then, locally strongly convex surfaces with vanishing affine mean curvature are called *affine maximal surfaces*.

After Calabi's work this class of surfaces has become a fashion research topic and it has received many interesting contributions.

In this poster we present the resolution of the problem of existence and uniqueness of affine maximal surfaces containing a regular analytic curve and with a given affine normal along it, see [2]. As applications we give results about symmetries, characterize when a curve in \mathbf{R}^3 can be a geodesic of a such surface and study helicoidal affine maximal surfaces, that is, surfaces invariant under a one-parametric group of equiaffine transformations. We obtain new examples with an analytic curve in its singular set, which have been studied in [3]. To do that, we introduce the notion of affine maximal map which allows us to analyze global problems regarding to affine maximal surfaces admitting some natural singularities.

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On the Isotropy Constant of Polytopes

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A convex body K is said to be isotropic if it satisfies the following conditions: $|K| = 1$, $\int_K x dx = 0$ and $\forall \theta \in S^{n-1} \int_K \langle x, \theta \rangle^2 dx = L_K^2$.

This constant L_K , independent of the vector θ is called the isotropy constant of K . It is not known if there exists an absolute constant bounding from above the isotropy constant of any convex body. This problem is known as the slicing problem.

Conjecture (Slicing problem) There exists an absolute constant C such that for any convex body

$$L_K \leq C$$

Since any convex body can be approximated by polytopes this conjecture is true for any convex body if and only if it is true for polytopes. Thus we study the isotropy constant of polytopes.

We prove the following theorem, which gives a positive answer for the slicing problem for polytopes with few vertices:

Theorem [ABBW] Let K be an n -dimensional convex polytope with N vertices. Then

$$L_K \leq C \sqrt{\frac{N}{n}}.$$

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Invariant Structures on Homogeneous k -symmetric Spaces

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It is known that classical Kähler (K) and nearly Kähler (NK) structures were provided with remarkable collections of homogeneous examples, namely, Hermitian symmetric spaces (A.Borel, A.Lichnerovich et al.) and homogeneous 3-symmetric spaces (N.A.Stepanov, J.A.Wolf, A.Gray, V.F.Kirichenko and others), respectively. *Metric f -structures* ($f^3 + f = 0$, K.Yano) is a natural generalization of almost Hermitian structures and metric almost contact structures. Important classes of metric f -structures such as *Kähler* (Kf), *Hermitian* (Hf), *Killing* ($Killf$), *nearly Kähler* (NKf), G_1f -*structures* (G_1f) have been introduced and intensively studied since the 1980s in the framework of *generalized Hermitian geometry* [1]. These classes include the corresponding Gray-Hervella classes K , H , NK , G_1 in Hermitian geometry. In comparison with almost Hermitian structures, the metric f -structures had not been provided with invariant examples up to the middle 1990s. A rich collection of canonical f -structures was discovered on *regular Φ -spaces*, in particular, on *homogeneous k -symmetric spaces* [2]. It gave the opportunity to present wide classes of invariant above mentioned f -structures (see, e.g., [3],[4]). Here the particular role belongs to the canonical f -structures on naturally reductive 4- and 5-symmetric spaces. Besides, four canonical f -structures on homogeneous

6-symmetric spaces were also completely studied. Recently, some general results for canonical f -structures on arbitrary k -symmetric spaces were obtained.

Many particular examples of both semisimple and solvable types were investigated in detail. They are the flag manifolds $SU(3)/T_{max}$, $SO(n)/SO(2) \times SO(n-3)$, the 6-dimensional generalized Heisenberg group and some others. Specifically, we present first invariant Killing f -structures with non-naturally reductive metrics.

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Expanding the Mandelbrot Set into 3D and 4D

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When in 1980 Benoit Mandelbrot described the $z \rightarrow z^2 + c$ formula, many mathematicians and programmers tried to expand the Mandelbrot Set into the third dimension. But all of them where stopped by the non-equivalence in 3D to the 2D complex product $(a + bi) \cdot (c + di)$, something that was well known since times of mathematician W. R. Hamilton. Also, as the 80's computers where not able to produce the calculations needed to represent an image of that kind, all research moved towards other fractal fields. It was in 2007 when the search was recovered by means of a controversial algorithm using a triplex algebra structure based on Spherical Coordinates $\{\rho, \phi, \theta\}$ (module, longitude and latitude). Although, from a strict mathematical point of view, the process is not correct, the stunning images of the 3D set, especially when raised to higher polynomials $z \rightarrow z^n + c$ soon became an iconic fractal named Mandelbulb. The expansion of the Mandelbrot Set in 4D by means of quaternions is also possible. Recent experiments reveal that adequate projecting surfaces provide a fascinating group of 4D Mandelbrot Set projections into 3D.

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The Spectrum of the Martin-Morales-Nadirashvili Minimal Surfaces is Discrete

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S. T. Yau, revisiting Calabi conjectures on minimal surfaces, in his Millennium Lectures [4], [5], wrote: *It is known [3] that there are complete minimal surfaces properly immersed into a [open] ball. . . . Are their spectrum discrete?* Although, it is not clear that the Nadirashvili's complete bounded minimal surface [3] is *properly immersed*, F. Martín and S. Morales in [1] and [2], constructed, for every open convex subset B of \mathbb{R}^3 , complete proper minimal immersions of the unit disk \mathbb{D} into B . In this presentation we are going to show that the spectrum of a complete submanifold properly immersed into a ball of a Riemannian manifold is discrete, provided the norm of the mean curvature vector is sufficiently small. In particular, the spectrum of complete minimal surfaces properly immersed into a ball of \mathbb{R}^3 , (Martin-Morales-Nadirashvili minimal surfaces), is discrete.

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Use Convexity, not ‘Compactness’, to Decide if Legislative Districts are Nicely Shaped

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In this paper we examine how closely a polygonal planar set is to being convex. This is accomplished by considering the ratio of the area of a largest convex set contained in the original polygon to the area of the convex hull of the set. Algorithms for determining the convex hull and for determining a largest convex set interior to the polygon are exhibited. After defining when such sets are *nearly convex* we then use this result to decide when legislative districts are *nicely shaped*. We show that this method for measuring the shape of legislative districts is better than, or as good as, other techniques in the literature. This is accomplished by pointing out flaws in the other methods and by examining examples of district shapes found in the literature.

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New Calabi-Bernstein Results for Maximal and Constant Mean Curvature Spacelike Graphs

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Keywords. Maximal surface equation, constant mean curvature spacelike surface equation, Calabi-Bernstein problem, Generalized Robertson-Walker spacetime.

Uniqueness and non-existence results of entire solutions to the maximal surface equation and to the constant mean curvature spacelike surface equation on certain complete Riemannian surfaces are obtained.

This is a joint work with Alfonso Romero and Rafael M. Rubio.



Some Properties of Biwave Maps

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Biwave maps generalize wave maps. We study biwave maps and equivariant biwave maps. We obtain the formulations for equivariant biwave maps into various spaces by applying eigen maps between spheres. We compute the biwave fields of inclusions into warped product manifolds and construct examples of biwave maps. We finally investigate the stress bi-energy tensors and the conservation laws of biwave maps and discuss some applications.

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Ricci Soliton and Ricci Flow on Some Type of Almost Contact Manifolds

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In the present paper we study the behaviour of gradient Ricci soliton and Ricci soliton on some almost contact metric manifolds which are $N(k)$ η -Einstein, Kenmotsu and trans-Sasakian manifolds respectively. This paper also deals with the global curvature derivative estimates of Ricci flow for pseudo-projectively flat, quasi conformally flat and conharmonically flat η -Einstein manifolds respectively.

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Certain Conditions on the Second Fundamental Form of CR Submanifolds of Maximal CR Dimension of Complex Hyperbolic Space

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We investigate n -dimensional real submanifolds M of Kähler manifolds \overline{M} when the maximal holomorphic tangent subspace is $(n - 1)$ -dimensional.

Besides the submanifold structure, represented by the second fundamental tensor h of M in \overline{M} , there is another geometric structure, an almost contact metric structure (F, u, U, g) , naturally induced from the almost complex structure of the ambient space.

We study certain conditions on the structure F and on h of CR submanifolds of maximal CR dimension in complex hyperbolic space, we obtain a complete classification of submanifolds M which satisfy these conditions and we characterize several important classes of these submanifolds. Since, in general, F is not a contact structure, we also give the condition for F to be the contact one and we obtain some characterizations of contact submanifolds.

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Isoparametric Submanifolds in Hilbert Space

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In finite dimensions, a submanifold of Euclidean space is called *isoparametric* if: (a) its normal bundle is flat; and (b) the shape operators along any parallel normal vector field are conjugate. It follows from theorems of Dadok [1], Palais-Terng [2] and Thorbergsson [4] that every isoparametric submanifold in Euclidean space of codimension different from two is a principal orbit of the isotropy representation of a symmetric space.

In infinite dimensions, one works in the category of proper Fredholm submanifolds in Hilbert space and defines such a submanifold to be isoparametric if it satisfies conditions (a) and (b) above. Terng [3] has constructed very interesting examples of homogeneous isoparametric submanifolds in Hilbert space, principal orbits of the so called $P(G, H)$ -actions, which are essentially isotropy representations of affine Kac-Moody symmetric spaces.

In this talk, we will explain our proof that every isoparametric submanifold in Hilbert space of type $\tilde{A} - \tilde{D} - \tilde{E}$ and codimension different from one is a principal orbit of a $P(G, H)$ -action.

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Combinatorial Yamabe Flow on Hyperbolic Surfaces with Boundary

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In trying to develop the piecewise linear conformal geometry, Luo [6] studied the combinatorial Yamabe problem for piecewise flat metrics on triangulated surfaces. Glickenstein [3] unified the theory of combinatorial Yamabe flow of piecewise flat metrics with the theory of circle packing on surfaces.

We consider the combinatorial Yamabe flow on hyperbolic surfaces with boundary. The length of boundary components is uniquely determined by the combinatorial conformal factor. And the space of the length of boundary components is identified. The combinatorial Yamabe flow is a gradient flow of a concave function. The sum of the square of the length of boundary components is decreasing along the flow. We also study the long time behavior of the flow.

The main result is obtained by applying a variational principle. A concave energy function is constructed using the derivative cosine law which is developed in [7, 5]. The approach of variational principle of studying polyhedral surfaces was introduced by Colin de Verdière [2] in his proof of Andreev-Thurston's circle packing theorem. For related works, see [8, 1, 9, 4] and others.

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Euler-Lagrange Equation and Regularity for Flat Minimizers of the Willmore Functional

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The Willmore energy of an immersed surface $u : \Sigma \rightarrow R^3$ is given by $\frac{1}{4} \int_{\Sigma} |H|^2 d\mu_g$, where H denotes the mean curvature of u , and μ_g denotes the area measure induced by the metric $g_{ij} = \partial_i u \cdot \partial_j u$. Over the past decades there has been considerable interest in the properties of critical points of this functional, the so-called Willmore surfaces, cf. e.g. [4]. Recently, there has been growing interest in constrained versions of the Willmore functional, c.f. e.g. [1]. Such constrained versions are also highly relevant for applications. e.g. for the Helfrich model of biological membranes or for Kirchhoff's plate theory in nonlinear elasticity, cf. [2].

We present recent results about critical points of the Willmore functional constrained to the set of all isometric immersions $u : S \rightarrow R^3$ of a given flat surface $S \subset R^2$. This corresponds to the highly degenerate nonconvex pointwise constraint

$$\partial_i u \cdot \partial_j u = \delta_{ij} \text{ for } i, j = 1, 2. \quad (1)$$

This problem arises naturally in three dimensional nonlinear elasticity, cf. [2].

We derive the Euler-Lagrange equation satisfied by local minimizers of the Willmore functional under the constraint (1); it is of considerable interest in its own. We then prove an optimal regularity result for solutions to that equation. It shows that they are C^∞ away from a singular set Σ_1 , but that surprisingly they are exactly C^3 away from a certain subset $\Sigma_0 \subset \Sigma_1$. A careful analysis of the local behaviour near Σ_0 reveals an interesting logarithmic scaling behaviour of the mean curvature. We also obtain a rather explicit description of the geometry of the singular sets. These results were announced in [3].

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On Geodesic E-convex Sets, Geodesic E-convex Functions and E-epigraphs

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In this paper, we introduce a new class of sets and a new class of functions called geodesic E-convex sets and geodesic E-convex functions on a Riemannian manifold, which are the extension of convex sets and geodesic convex functions defined by Udriste [9]. The concept of E-quasiconvex functions on R^n is extended to geodesic E-quasiconvex functions on Riemannian manifold and some of its properties are investigated. Afterwards, we generalize the notion of epigraph called E-epigraph and a characterization of geodesic E-convex functions in terms of its E-epigraph is discussed. Some properties of geodesic E-convex sets are also studied.

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Ricci Semi-symmetric and Ricci-pseudosymmetric Mixed Super Quasi-Einstein Manifolds

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Quasi-Einstein, generalized quasi-Einstein, super quasi-Einstein and mixed generalized quasi-Einstein manifolds are generalization of Einstein manifolds. The object of the present work is to study mixed super quasi-Einstein manifolds which is generalization of all of these manifolds. First we study the nature of the associated 1-forms A and B of mixed super quasi-Einstein manifolds. After that it is shown that a mixed super quasi-Einstein manifold can not be Ricci semi-symmetric. Finally we find the curvature characterization of a Ricci-pseudosymmetric mixed super quasi-Einstein manifold.

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On Local Geometry of Carnot–Carathéodory Spaces Under Minimal Assumptions on Smoothness

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This talk is dedicated to the new approach to investigation of a local geometry of Carnot–Carathéodory spaces under minimal assumptions on smoothness of basis vector fields. The main point is that, opposite to the well-known methods, we study such geometry without Campbell–Hausdorff formula and Gromov’s Theorem [1] on convergence of “rescaled” basis vector fields to vector fields constituting a nilpotent graded Lie algebra (or, by another words, nilpotentized vector fields).

This approach allows us to extend geometric results of [2] to all equiregular Carnot–Carathéodory spaces with $C^{1,\alpha}$ -smooth basis vector fields $\{X_i\}_{i=1}^N$, $\alpha > 0$ (i. e., we do not need here to assume that conditions of [2, Remark 2.2.19] hold). We remark that in [2] main results are established under assumption that the depth M of a space \mathcal{M} equals 2 or Gromov’s Theorem is true. Nevertheless, in [2], one of the basic geometric facts is proved independently from Gromov’s Theorem.

The goal of the talk is to show that all the other geometric results of [2] (estimates in comparison of local geometries, Rashevskii–Chow Theorem, Ball–Box Theorem, etc) including the main one (see Theorem 1) can be proved without using Gromov’s Theorem on convergence since its validity is still unknown for Carnot–Carathéodory spaces of depth more than 2 with $C^{1,\alpha}$ -smooth basis vector fields, $\alpha \in [0, 1)$. In Theorem 1, $\{X_i^u\}_{i=1}^N$ ($\{X_i^{u'}\}_{i=1}^N$) is the base on local Carnot group at u (u') [2, Definitions 2.1.11, 2.1.21], $\text{deg}X_i$ is a degree of a vector field X_i , $i = 1, \dots, N$ [1], [2, Definition 2.1.1], ρ is a Riemannian distance, and d_∞^u ($d_\infty^{u'}$) is a sub-Riemannian distance in local Carnot group at u (u') [2, Definition 2.1.27].

Theorem 1. *Consider in a neighborhood \mathcal{U} of a Carnot–Carathéodory space \mathcal{M} points u, u', v , $w_\varepsilon = \exp\left(\sum_{i=1}^N w_i \varepsilon^{\text{deg}X_i} \widehat{X}_i^u\right)(v)$, and $w'_\varepsilon = \exp\left(\sum_{i=1}^N w_i \varepsilon^{\text{deg}X_i} \widehat{X}_i^{u'}\right)(v)$. Then, $\max\{d_\infty^u(w_\varepsilon, w'_\varepsilon), d_\infty^{u'}(w_\varepsilon, w'_\varepsilon)\} = \varepsilon \Theta \left(\left[\log_{\rho(u, u')^{\frac{\alpha}{M}}} \varepsilon^{\frac{M-1}{M}} \right] + 1 \right) \cdot \rho(u, u')^{\frac{\alpha}{M}}$. (Here Θ is bounded uniformly in $u, u', v \in \mathcal{U} \subset \mathcal{M}$, $\{w_i\}_{i=1}^N$, and $\varepsilon > 0$).*

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A Theory of Face Polytopes

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Define a fan to be a finite set of pointed polyhedral cones such that the intersection of any two cones in this set is the common face of both. Consider a polytope P such that the origin lies the interior of P . Denote by \mathcal{F} any set of cones spanned by proper faces of P . The set \mathcal{F} is a fan. We say that \mathcal{F} is a face fan of the polytope P and P is a face polytope of the fan \mathcal{F} (see, for example, [1]).

The theory of face polytopes considers polytopes as face polytopes, that is, not by themselves but together with their face fans. This consideration allows to define a new sum operation for polytopes. Also, the theory studies thoroughly whether a given fan is a face fan of some polytope. This study discovers two important new features of fans. They are a characteristic cone and a family of transfer cones.

Investigating the sums of polytopes and also the characteristic cones and the families of transfer cones of fans finds new surprising facts on both fans and polytopes. Among them: existence of fans consisting of three cones in \mathbb{R}^3 that are not face fans; possibility to combine facets of every simplicial polytope in a convex manner; separation theorems for more than two sets; mean theorems for polytopes.

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M-Modular Pairs in Multiplicative Lattices

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The notion of a multiplicative lattice is introduced by Ward and Dilworth [6] in their study of abstract formulation of ideal theory of commutative rings. A **multiplicative lattice** is a complete lattice L with a commutative, associative multiplication satisfying the following properties for all $a, b, b_i \in L$, (i) $a(\bigvee_i b_i) = \bigvee_i(ab_i)$, (ii) $0 \leq a \wedge b$ and (iii) $a1 = a$.

There are many papers in which properties of such lattices are studied; see, Anderson [3], Alarcon et. al. [1].

A pair a, b of elements in a lattice L is called a **modular pair**, in notation, $(a, b)M$ if $(c \vee a) \wedge b = c \vee (a \wedge b)$ for every $c \leq b$.

Birkhoff [2] p. 109 posed the following problem " How to define modular pairs in a general poset?". As an attempt to answer this problem, many researchers for example Thakare, Wasadikar and Maeda [5] have defined modular pairs in semilattices and obtained their properties. Further, Thakare, Pawar and Waphare [4] have defined modular pairs in posets and obtained many results.

We define a **modular pair** in a multiplicative lattice using the multiplication operation. We show by an example, that if we consider the multiplication operation instead of the meet operation, then a modular relation with respect to these operations need not hold in a multiplicative lattice. This motivates us to introduce and study the concept of an m-modular pair, dual m-modular pair, m-covering property in multiplicative lattices. We obtain some results and give a characterization of m-covering property.

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Existence of Proper Contact CR -product and Mixed Foliate Contact CR Submanifolds

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A submanifold M^{n+1} of a Sasakian manifold \bar{B}^{2m+1} with structure tensors (ϕ, ξ, η, g) is called a *contact CR submanifold* if there exists two differentiable distribution \mathcal{D} and \mathcal{D}^\perp on M such that $TM = \mathcal{D} \oplus \mathcal{D}^\perp \oplus \text{Span}\{\xi\}$, $\phi\mathcal{D}_x = \mathcal{D}_x$ and $\phi\mathcal{D}_x^\perp \subset T_x M^\perp$ for each $x \in M$, where \mathcal{D} , \mathcal{D}^\perp and $\text{Span}\{\xi\}$ are mutually orthogonal to each other. A contact CR submanifold is said to be *proper* if neither $\dim\mathcal{D} = 0$ nor $\dim\mathcal{D}^\perp = 0$. A contact CR submanifold is said to be *mixed foliate* if (a) $\mathcal{D} \oplus \text{Span}\{\xi\}$ is integrable and (b)

$h(X, Y) = 0$, $X \in \mathcal{D}$, $Y \in \mathcal{D}^\perp$, where h is the second fundamental form of M . A contact CR -submanifold M is called a *contact CR -product* if (a) $\mathcal{D} \oplus \text{Span}\{\xi\}$ is integrable and (b) M is locally a Riemannian product $M^\top \times M^\perp$, where M^\top and M^\perp are leaves of $\mathcal{D} \oplus \text{Span}\{\xi\}$ and \mathcal{D}^\perp , respectively.

In 1982, Bejancu provided that there is no proper contact CR -product in Sasakian space form $\bar{B}(c)$ with constant ϕ -holomorphic sectional curvature $c < -3$.

The purpose of this paper is (1) to investigate some properties concerning with ϕ -holomorphic bisectional curvature \bar{H}_B and prove Theorem A which yields Bejancu's result; (2) to show Theorem B as an existence theorem of mixed foliate proper contact CR submanifolds in $E^{2m+1}(-3)$

Theorem A. Let \bar{M} be a Sasakian manifold with $\bar{H}_B < -2$. Then every contact CR -product in \bar{M} is either an invariant submanifold or an anti-invariant submanifold. In other words, there exists no proper contact CR -product in any Sasakian manifold with $\bar{H}_B < -2$.

Theorem B. Let M be a mixed foliate proper contact CR submanifold of the standard Sasakian space form $E^{2m+1}(-3)$. If $h(X, Y) \in \phi\mathcal{D}^\perp$, $X, Y \in \mathcal{D}^\perp$, then for a point $x \in M$ there exists a unique complete totally geodesic invariant submanifold M' of $E^{2m+1}(-3)$ such that $x \in M'$ and $T_x M' = T_x M \oplus \phi\mathcal{D}_x^\perp$.

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On the Semi-Riemannian Bumpy Metric Theorem

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Keywords. Closed geodesics, bumpy metrics, semi-Riemannian manifolds

We prove the semi-Riemannian bumpy metric theorem using equivariant variational genericity. The theorem states that, on a given compact manifold M , the set of semi-Riemannian metrics that admit only nondegenerate closed geodesics is generic relatively to the C^k -topology, $k = 2, \dots, \infty$, in the set of metrics of a given index on M . A higher order genericity Riemannian result of Klingenberg and Takens is extended to semi-Riemannian geometry.



On Duality Principle and Osserman Condition for Algebraic Curvature Tensors

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Let \mathcal{R} be an algebraic curvature tensor, i.e., the tensor which satisfies the same symmetries as the curvature tensor of a pseudo-Riemannian manifold equipped with Levi-Civita connection. Let $\mathcal{R}_X(Y) = \mathcal{R}(Y, X)X$ be the corresponding Jacobi operator.

For a real number λ we say that it satisfies the duality principle if for all mutually orthogonal unit vectors X, Y holds

$$\mathcal{R}_X(Y) = \varepsilon_X \lambda Y \quad \implies \quad \mathcal{R}_Y(X) = \varepsilon_Y \lambda X.$$

If the duality principle holds for all real numbers then we say that duality principle holds for the algebraic curvature tensor \mathcal{R} .

We extend the notion of duality principle to all type of vectors, and investigate relations between duality principle and Osserman condition for the simple algebraic curvature tensors. Also, we study algebraic curvature tensors provided by a Clifford structure and give some interesting examples.

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Extrinsic Geometric Flows on Foliated Manifolds

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We study (in common with P. Walczak, University of Lodz, Poland) deformations of Riemannian metrics on a manifold equipped with a codimension-one foliation subject to quantities expressed in terms of its second fundamental form. The local existence and uniqueness theorem is proved and the existence time of solutions for some particular cases is estimated. The key step of the solution procedure is to find (from a system of quasilinear PDE's) the principal curvatures of the foliation. Examples for extrinsic Newton transformation flow and extrinsic Ricci flow, and applications to foliations on surfaces are given.

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How to Make Quermassintegrals Differentiable. Solving a Problem by Hadwiger

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Let K, E be convex bodies in the Euclidean space \mathbb{R}^n , E with interior points, and let $\lambda \geq 0$. The (*relative*) *outer parallel body* of K (with respect to E) at distance λ is the Minkowski sum $K + \lambda E$. On the other hand, the (*relative*) *inner parallel body* of K (with respect to E) at distance λ , $0 \leq \lambda \leq r(K; E)$, is defined as the *Minkowski difference* $K \sim \lambda E = \{x \in \mathbb{R}^n : \lambda E + x \subseteq K\}$, where the *relative inradius* $r(K; E)$ is defined by $r(K; E) = \sup\{r \geq 0 : \exists x \in$

\mathbb{R}^n with $x + rE \subseteq K$. Now we write K_λ to denote the *full system of parallel bodies* of K ,

$$K_\lambda := \begin{cases} K \sim |\lambda|E & \text{for } -r(K; E) \leq \lambda \leq 0, \\ K + \lambda E & \text{for } 0 \leq \lambda < \infty. \end{cases} \quad (2)$$

The well known (*relative*) *Steiner formula* states that the volume of the outer parallel body $K + \lambda E$ is a polynomial of degree n in $\lambda \geq 0$,

$$V(K + \lambda E) = \sum_{i=0}^n \binom{n}{i} W_i(K; E) \lambda^i,$$

where the coefficients $W_i(K; E)$ are called the *relative quermassintegrals* of K . Then, writing $W_i(\lambda) := W_i(K_\lambda; E)$ for $-r(K; E) \leq \lambda < \infty$, we can speak about differentiability of the quermassintegrals.

Definition 1. A convex body $K \subset \mathbb{R}^n$ belongs to the class \mathcal{R}_p , $0 \leq p \leq n - 1$, if for all $0 \leq i \leq p$, and $-r(K; E) \leq \lambda < \infty$ it holds $'W_i(\lambda) = W'_i(\lambda) = (n - i)W_{i+1}(\lambda)$.

This is a natural definition, since from the concavity of the family (2) and the general Brunn-Minkowski theorem for relative quermassintegrals, it holds $'W_i(\lambda) \geq W'_i(\lambda) \geq (n - i)W_{i+1}(\lambda)$, for $i = 0, \dots, n - 1$.

We study the convex bodies lying in the classes \mathcal{R}_p , $0 \leq p \leq n - 1$. It is known that the volume is always differentiable and satisfies $'V(\lambda) = V'(\lambda) = nW_1(\lambda)$ for the full range $-r(K; E) \leq \lambda < \infty$, which implies that the class \mathcal{R}_0 is the set of all convex bodies. This problem was originally posed in dimension 3, and for E being the Euclidean ball, by H. Hadwiger in 1955, and it has been opened since then. We have obtained the characterization of the classes \mathcal{R}_{n-2} and \mathcal{R}_{n-1} , which closes the original Hadwiger problem in dimension 3.



A Study of Three-dimensional Quasi-Sasakian Manifolds

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The object of the present paper is to calculate the ϕ -sectional curvature of a three-dimensional quasi-Sasakian manifold and to show equivalence of some geometrical properties of the manifold depending on the ϕ -sectional curvature and ϕ -symmetry of the manifold. To illustrate the results examples are given. Existence of totally geodesic hypersurface of a three-dimensional quasi-Sasakian

manifold is established. Non-equivalence of totally geodesic and invariant submanifolds of quasi-Sasakian manifolds is shown. Finally we study submanifolds of three-dimensional quasi-Sasakian manifolds with recurrent, 2-recurrent and generalized 2-recurrent second fundamental form.



Theorems on bi-recurrent and bi-symmetric Sasakian Manifold

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 49Q15, 58A03, 5A05

C.E. Weatherburti [2], An introduction to Riemannian geometry and tensor calculus. K. Yano [3] Differential geometry on complex and almost complex spaces, Pergoman Press. B.B. Sinha [1] On H-curvature tensor is in Kaehler manifold, D.S. Negi and K.S. Rawat [4], studied integral inequalities in Kaehlerian manifold. In the present paper, we have studied the theorem on bi-recurrent and bi-symmetric sasakin manifold.

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Intrinsic Geometry of Cyclic Heptagons/Octagons via “New” Brahmagupta Formula

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Finding formulas for the area or circumradius of polygons inscribed in a circle in terms of side lengths is a classical subject (cf.[1]). For triangle/cyclic quadrilaterals we have famous Heron/Brahmagupta formulae. In 1994. D.P. Robbins found a minimal area equations for cyclic pentagons/hexagons by a method of undetermined coefficients (cf.[3]). This method could hardly be used for heptagons due to computational complexity (143307 equations). In [4], by using covariants of binary quintics, a concise minimal heptagon/octagon area equation was obtained as a fraction of two resultants which in expanded form has almost one milion terms. It is not clear if this approach could be effectively used for cyclic polygons with nine or more sides. In [6], by using Wiener-Hopf factorization approach, we have obtained a very explicit minimal heptagon/octagon circumradius equation in Pellian form with coefficients up to four digits. A non-minimal area equation is also obtainable by this method. Both methods are somehow external. But, based on our new intermediate Brahmagupta formula, we have succeeded also in finding an intrinsic proof of the Robbins formula for the area (and also for circumradius and area times circumradius) of cyclic hexagon based on an intricate direct elimination of diagonals (the case of pentagon was much easier cf. [5]). We also get a simple(st) system of equations for the area and area times circumradius of cyclic heptagons/octagons. It seems remarkable that our approach, with a help of Groebner basis techniques leads to minimal equations (for any concrete instances we have tested), what is not the case with iterated resultants approach.

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Structure and Rigidity of Self-similar Jordan Arcs in \mathbf{R}^n

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1. A *graph-directed iterated function system* \mathcal{S} with *structural graph* $\Gamma = \langle V, E, \alpha, \omega \rangle$ (see [1],[2]) is a finite collection of complete metric spaces $\{X_v\}_{v \in V}$ together with a collection of contraction maps $\{S_e : X_{\alpha(e)} \rightarrow X_{\omega(e)}\}_{e \in E}$. A family of compact sets $\{K_v\}_{v \in V}$ is called the *attractor* of the system \mathcal{S} if for every $v \in V$

$$K_v = \bigcup_{\omega(e)=v} S_e(K_{\alpha(e)}).$$

2. The system \mathcal{S} is called a *multizipper* (see [3]), if the following conditions are satisfied:

- a) for each $v \in V$ a finite sequence $z_0^v, \dots, z_{n_v}^v$ of points in X_v is specified,
- b) there is a bijection f of the set $P = \{(v, k) : v \in V, 1 \leq k \leq n_v\}$ to E and
- c) if $e = f(v, k)$, $u = \alpha(e)$ then $S_e : X_u \rightarrow X_v$ and $S_e(\{z_0^u, z_{n_u}^u\}) = \{z_{k-1}^v, z_k^v\}$.

3. The following theorem shows that each self-similar Jordan arc may be represented as the attractor of some multizipper:

Theorem 1. *Let Jordan arc γ be a component K_v of the attractor of a graph-directed iterated system \mathcal{S} of similarities in \mathbf{R}^n with strongly connected structural graph Γ . Then, either the arc γ is a component of the attractor of some multizipper \mathcal{Z} , or γ is a straight line segment, and in this case the system \mathcal{S} does not satisfy weak separation property and the self-similar structure (γ, \mathcal{S}) is rigid.*

4. The next theorem is crucial for finding the Hausdorff dimension of the arcs:

Theorem 2. *If all components K_v of the attractor of a multizipper \mathcal{Z} are quasiarcs, then \mathcal{Z} satisfies the open set condition.*

5. There are the examples of multizippers \mathcal{Z} in \mathbf{R}^3 , whose attractors are collections of Jordan arcs with unbounded torsion, and which do not satisfy the weak separation property.

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Complex Geometry of Nilmanifolds and Special Hermitian Structures

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The complex geometry of nilmanifolds provides a rich source of explicit examples of compact complex manifolds admitting additional special structures with interesting properties. Many authors have studied several aspects of this geometry from different points of view (see [2, 3, 4, 6, 7] and references therein). Here we focus mainly on the study of the invariant balanced Hermitian geometry of six dimensional nilmanifolds. An interesting fact [4] is that the associated Bismut connection [1] has holonomy contained in $SU(3)$. We show that such holonomy reduces to a proper subgroup if and only if the underlying complex structure is abelian.

As an application we provide explicit solutions of the Strominger system in heterotic supersymmetry. Fu and Yau first proved in [5] the existence of solutions of this system satisfying the anomaly cancellation condition with respect to the Chern connection on a Hermitian non-Kähler manifold given as a T^2 -bundle over a $K3$ surface. Here we show many new explicit compact solutions with non-flat instanton, which are a deformation of the solution found in [3] with flat instanton.

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On the Willmore Energy Under Infinitesimal Bending

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The Willmore energy is defined by the mean and the Gaussian curvature as $W = H^2 - G$. The Willmore energy is a quantitative measure of how much a given surface deviates from a round sphere. It presents a special case of so-called elastic bending energy, which determines the equilibrium shape of a membrane and describes a case of symmetric membrane, by taking bending rigidity as a constant. A membrane can be regarded as two-dimensional surface embedded in three-dimensional space, because its thickness is much smaller than its lateral dimension. The change of the Willmore energy can be considered under infinitesimal bending of membranes, as their special deformation.

In this work the Willmore energy of the surfaces is discussed and its visualization considering some examples is given. The behavior of that geometric magnitudes under infinitesimal bending of the surfaces is considered. Its variation under infinitesimal bending is determined. The stationarity condition for the Willmore energy is given.

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Balanced Metrics by Means of Evolution Equations

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Construction of new balanced Hermitian manifolds is an active area of research and several recent techniques have provided new examples. For instance, balanced Hermitian metrics are constructed on torus bundles over $K3$ surfaces and over complex Abelian surfaces in [5], whereas in [4] balanced metrics are

constructed on the family of non-Kähler Calabi-Yau threefolds obtained by smoothing after contracting $(-1, -1)$ -rational curves on a Kähler Calabi-Yau threefold.

Here we construct explicit new balanced Hermitian metrics in dimension 6 by means of appropriate evolution equations starting from a suitable structure on a 5-dimensional manifold. Evolution equations have been previously used to construct metrics of special holonomy G_2 in seven dimensions [6], to obtain metrics with $SU(3)$ -holonomy in six dimensions [1] and to get new nearly Kähler structures in dimension six [2]. These new balanced Hermitian metrics extend the ones constructed in [3] and have holonomy of the Bismut connection equal to $SU(3)$.

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On Graphs Derived from Posets

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Beck [2] introduced the notion of coloring in commutative rings. Several researchers defined graphs on various algebraic structures such as commutative rings, Anderson and Naseer [1], commutative semigroups, DeMeyer, McKenzie and Schneider [4] etc. There are many papers which interlink graph theory and lattice theory. The study of graphs related to lattices, namely, covering graphs and comparability graphs is well-known; see, Gedenova [6], Duffus and Rival

[5] and Bollobas and Rival [3] et. al. In a recent paper, Nimbhorkar et. al. [8] have studied graphs derived from a meet-semilattice with 0 and obtained properties of these graphs. Their work was generalized to posets with 0 by Halas and Julk [7].

In this paper we associate a graph with a poset P without 0. Let P be a poset and $S \subseteq P$. We write $S^l = \{x \in P \mid x \leq s \text{ for every } s \in S\}$. If $S = \{a, b\}$, then we write $(a, b)^l$ for S^l . We associate a simple graph, $\Gamma(P)$, with a poset P , whose vertices are those elements $x \in P$, for which, there is some $y \in P$ with the property $(x, y)^l = \emptyset$ and two distinct vertices x, y are adjacent if and only if $(x, y)^l = \emptyset$. It is shown that if the chromatic number of the associated graph is finite then the poset has only a finite number of minimal prime semi-ideals and moreover, the clique number is equal to the chromatic number. Some properties of this graph are obtained. These results generalize the results of Nimbhorkar et. al. [9] and Halas and Julk [7] to a larger class of posets.

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Hypersurfaces with Two Distinct Principal Curvatures in Space Forms

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 53C42, 53B30

We investigate hypersurfaces in space forms with two distinct principal curvatures and constant m -th mean curvature. It is well-known that when the two principal curvatures are non-simple, then the hypersurface is isometric to the Clifford type hypersurface. Thus the key is to study the case when one of the two principal curvatures is simple. In this situation, the constance of m -th mean curvature is equivalent to an ODE of order two, and we obtain some local and global classification results for such hypersurfaces by carefully analyzing the solution of ODE. The main technique is based on Otsuki's idea [1]. As the application of the classification results, we prove that any local hypersurface in the spheres or in Euclidean space of constant mean curvature and two distinct principal curvatures is an open part of a complete hypersurface of the same curvature properties. The corresponding result does not hold for m -th mean curvature when $m \geq 2$. We also obtain some global rigidity results for Clifford type hypersurfaces and obtain some non-existence results [2, 3, 4]. The same argument can be used to study spacelike hypersurfaces in Lorentzian space forms with two distinct principal curvatures and constant m -th mean curvature. We obtain some classification results of such spacelike hypersurfaces and some applications of the classification results [5, 6, 7].

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Li-Yau Type Gradient Estimates, Differential Harnack Inequalities and Monotonicity of Entropy Formulas on Complete Riemannian Manifolds with Negative Ricci Curvature

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On complete Riemannian manifolds with $\text{Ricci}(M) \geq -K$, $K \in \mathbf{R}$, we prove the new Li-Yau type gradient estimates for the positive solutions of the linear heat equation in [1] and of the Schrödinger operator in [3], which generalize the famous Li-Yau gradient estimates in [4]; and we prove the new Harnack inequalities for the positive solutions of the linear heat equation in [2], which generalize the Ni's Harnack inequality in [5] and [6] for the heat equation on manifolds with nonnegative Ricci curvature; and we also prove the monotonicity of corresponding entropy formulas in [2] for the linear heat equation on complete Riemannian manifolds with Ricci curvature bounded from below, which generalize the monotonicity of entropy formulas on manifolds with nonnegative Ricci curvature in [5] and [6]. As applications in [1]-[3], several parabolic Harnack inequalities are obtained and they lead to new estimates on heat kernels of manifolds with Ricci curvature bounded from below. These are joint works with Dr. Junfang Li.

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Section 6

Topology

$C_\infty(X)$ and Related Ideals

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We have characterized the spaces X for which the smallest z -ideal containing $C_\infty(X)$ is prime. It turns out that $C_\infty(X)$ is a z -ideal if and only if every zero-set contained in an open locally compact σ -compact set is compact. Some interesting ideals related to $C_\infty(X)$ are introduced and corresponding to the relations between these ideals and $C_\infty(X)$, topological spaces X are characterized. Finally we have shown that a σ -compact space X is Baire if and only if every ideal containing $C_\infty(X)$ is essential.

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Common Fixed Points of Nonsself Mappings in Convex Metric Spaces

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 47H10, 54H25.

In this paper, we extend the notions of reciprocal continuity and C_q -commutativity to nonsself setting besides observing equivalence between compatibility and ϕ -compatibility, and utilize the same to obtain some results on coincidence and common fixed points for two pairs of nonsself mappings in metrically convex metric spaces. As an application of our main result, we also prove a common fixed point theorem in Banach spaces besides furnishing several illustrative examples.



On the Group of Torsion Elements of $C(X)$

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Let X be Tychonoff space and $C(X)$ be the ring of real continuous functions on X . We consider the set of torsion elements of $C(X)$, denoted by $T(X)$, and find a close relation between $T(X)$ and zero dimensionality of X . Specially, we prove that if X and Y are two zero dimensional compact spaces, then X and Y are homeomorphic if and only if the rings generated by $T(X)$ and $T(Y)$ are isomorphic.

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The $K(1)$ Bousfield-Kan Spectral Sequence of Certain Toric Spaces

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In this talk we will show how the combinatorics of a certain class of simplicial complexes K manifests itself when computing the unstable $K(1)$ -completion of the associated Borel Space $B_T Z_K$ using a certain generalized composite functor spectral sequence constructed by the second author. Explicit calculations will be highlighted as well as obstructions to more generalized calculations.



Equivalence of Real Milnor Fibration for Quasi-homogeneous Singularities

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Primary 32S55; Secondary 57Q45, 32C40, 32S60

We show that for real quasi-homogeneous singularities $f : (R^m, 0) \rightarrow (R^k, 0)$, $m > k \geq 2$ with isolated singular point at the origin, the projection map of the Milnor fibration $S_\epsilon^{m-1} \setminus K_\epsilon \rightarrow S^1$ is given by $\frac{f}{\|f\|}$. Moreover, for these singularities the two versions of the Milnor fibration, on the sphere and on a “Milnor tube”, are equivalent.

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Artin Presentations and Fundamental Groups

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 57M05

In this work we consider links in S^3 which are the closure of pure 3- braids and we do integral surgery on these links. Applying the theory of Artin presentations [1], [2], we calculate the fundamental group of the 3-manifolds obtained by integral surgery on these links. We define an operation on the set of Artin presentations and show that this operation gives a group structure to the set, which is used to calculate the fundamental groups of the 3-manifolds mentioned above. In some cases these groups are shown to be nontrivial.

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Linearly Ordered Quasi F -spaces

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 54C40

We show that linearly ordered quasi F -spaces and linearly ordered almost P -spaces coincide. Linearly ordered P^+ (P^-)-spaces are introduced and an example of a P^+ -space without P^- -point is given. We also show that a linearly ordered space (LOTS) is sequentially connected if and only if it is connected without any almost P -point.

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The Moduli Space of Hex Spheres

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A surface is called *singular Euclidean* if it can be obtained from a finite disjoint collection of Euclidean triangles by identifying pairs of edges by Euclidean isometries. The surface is locally isometric to the Euclidean plane except at finitely many points, at which it is locally modeled on Euclidean cones. These singular points are called the *cone points*. For each cone point there is a *cone angle*, which is the sum of the angles of the triangles that are incident to the cone point.

Singular Euclidean surfaces arise in several contexts within mathematics:

- Masur and Tabachnikov studied them from the viewpoint of billiard flows on Euclidean polygons ([1]);

- Rivin investigated their relations with volumes in hyperbolic geometry ([2]);
- Troyanov analyzed them in the context of Riemann surfaces and their parameter spaces (*Teichmüller* and *Moduli* spaces) ([3]);

A *hex sphere* is a singular Euclidean sphere with 4 cones whose cone angles are (integer) multiples of $\frac{2\pi}{3}$ but less than 2π . Given a hex sphere, we consider its Voronoi decomposition centered at the two cone points with greatest cone angles. This decomposes the hex sphere into two cells, the Voronoi cells, which intersect along a graph. By cutting a Voronoi cell along a special shortest geodesic, the Voronoi cell becomes a polygon on the Euclidean plane ([4]). This polygon will be referred to as a *Voronoi polygon*. We prove that the Moduli space of hex spheres of unit area is homeomorphic to the the space of similarity classes of Voronoi polygons in the Euclidean plane. A corollary is that every hex sphere has an embedded, totally geodesic Euclidean annulus of positive width. In particular, every hex sphere has a simple closed geodesic.

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Open Book Decompositions of Links of Quotient Surface Singularities

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In this paper we write explicitly the open book decompositions of links of quotient surface singularities that support the corresponding unique Milnor fillable contact structures. The page-genus of these Milnor open books are minimal among all Milnor open books supporting the unique Milnor fillable contact structures. That minimal page-genus is called the Milnor genus. In this paper we also investigate whether the Milnor genus is equal to the support genus

for links of quotient surface singularities. We show that for many types of the quotient surface singularities the Milnor genus is equal to the support genus of the corresponding contact structure. For the remaining we are able to find an upper bound for the support genus.

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Path Topology and Simple Connectedness

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The present paper is focused on a study of path topology introduced in [1]. Motivated by the facts that the topology on 4-dimensional Euclidean space does not incorporate the causal structure of spacetime and that the homeomorphism group of it being vast is of no physical significance, in 1967 Zeeman [2] initiated the study of non-Euclidean topologies that incorporate the causal structure of spacetime by introducing the notion of fine topology. Hawking et al. [1] pointed out the following shortcomings and proposed path topology on strongly causal spacetime to overcome these shortcomings: (i) fine topology is difficult to deal with as no point of it has a countable basis, (ii) the homothecy group of Minkowski space may not be physically significant and (iii) the set of continuous paths in fine topology does not incorporate accelerating particles moving under forces in curved lines.

Spacetime is considered to be a connected, Hausdorff, paracompact, C^∞ real 4-dimensional manifold without boundary, with a Lorentzian metric and associated pseudo Riemannian connection. Path topology on strongly causal spacetimes is defined to be the finest topology satisfying the requirement that the induced topology on every timelike curve coincides with the topology induced from the standard (positive definite) metric topology. Path topology is Hausdorff, connected, locally connected, path connected and locally path connected but not regular, normal, locally compact or paracompact [1].

In this paper, the compact sets of strongly causal spacetime with path topology have been characterized. Further, its simple connectedness is explored.

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The Hexatangle II

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Following [3], we say that a tangle is a pair (B, A) , where B is the 3-sphere with the interiors of a finite number of 3-balls removed, and A is a disjoint union of properly embedded arcs in B such that A meets each component of B in four points. The hexatangle is a certain tangle having six boundary components and a projection into the plane with no crossings. By filling the boundary components of a tangle with rational tangles we get knots and links in the 3-sphere. By filling one of the components of the hexatangle with a certain rational tangle we get the pentangle, which is studied in [2]. In a previous work [1], we determined all the integral fillings on the hexatangle that produce the trivial knot. Now we consider arbitrary rational fillings of the hexatangle, and have a conjecture which says exactly when we can get the trivial knot. We show some partial results about this conjecture. The hexatangle is somehow the simplest tangle with no crossings for which the problem of the triviality of knots is difficult. We also consider fillings that produce other classes of links, like split links or composite links. The double branched cover of the hexatangle is a certain hyperbolic link L of six components in S^3 . Our problem is equivalent to determining all Dehn surgeries on L that produce the 3-sphere. This link is interesting, for many hyperbolic knots and links with exceptional surgeries are obtained by performing Dehn surgery on some components of L .

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Bounding Genus and the Spin Cobordism Category of 3-manifolds

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 57R57

In this talk, we generalize the notion of the bounding genus of homology 3-spheres to that in the cobordism category of 3-manifolds and give a lower bound by using a V-manifold version of the Furuta-Kametani-10/8-inequality.

The bounding genus is a homology cobordism invariant of homology 3-spheres introduced by Y. Matsumoto with 11/8-conjecture in 1982. By using the Seiberg-Witten theory, the author used a V-manifold version of Furuta's 10/8-inequality to determine the bounding genera for infinite families of Brieskorn homology 3-spheres [2]. This inequality is improved by M. Furuta and Y. Kametani with the quadruple cup products of the first cohomology of closed spin 4-manifolds [1].

Motivated by this, we introduced a category \mathcal{L}_3 of graded commutative rings and a homology functor $\Phi : \mathcal{C}_3 \rightarrow \mathcal{L}_3$ from the spin cobordism category \mathcal{C}_3 of 3-manifolds and constructed a non-associative algebra $\mathcal{R}_*(L)$ which obstructs to realize a morphism $(H \rightarrow L \leftarrow H') \in \mathcal{L}_3(H, H')$ through Φ [3]. Then we generalize the notion of the bounding genus (Φ -bounding genus) between two objects $M_1, M_2 \in \text{ob } \mathcal{C}_3$ and a morphism $L \in \mathcal{L}_3(H_*(M_1), H_*(M_2))$ with a set of quadruples Λ of 2-cycles on $M_1 \sqcup M_2$ as follows. The Φ -bounding genus $|(M_1, M_2)|_{L; \Lambda}$ is defined to be the minimum of $b_2^+(W)$ for all $W \in \mathcal{C}_3(M_1, M_2)$ satisfying $\text{Sign}(W) = 0$, $\Phi(W) = L$ with even/odd quadruple products of 3-cycles on $(W, \partial W)$ with boundary Λ . Now suppose M_i , $i = 1, 2$ has vanishing triple cup products and bounds a compact spin 4-V-manifold X_i with non-singular intersection pairing. If the w -invariants satisfy $\delta := w(M_1) - w(M_2) > 0$, then by using a V-manifold version of the Furuta-Kametani-10/8-inequality, the lower bound of Φ -bounding genus is:

$$|(M_1, M_2)|_{L; \Lambda} \geq \delta + 2m(X_1 \sqcup X_2, \Lambda) + 1 - b_2^-(X_1) - b_2^+(X_2),$$

where $m(X_1 \sqcup X_2, \Lambda)$ is the maximum number of linearly independent odd/even quadruple products of 3-cycles on $(X_1 \sqcup X_2, M_1 \sqcup M_2)$ with boundary Λ .

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Separation Axioms in a Bitopological Space and Their Consequences

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Study of Bitopological space is taking a rapid stride primarily for sake of unification and in a sense, of restoration of symmetry missing in a Topological space. To look into separation axiom in a Bitopological space has thus become an inevitable exercise.

In this paper we have studied graded classification of separation axioms sometimes affixed with weak and we have incorporated new separation axioms with a natural task to establish their mutual implications among new separation axioms. We have also cited appropriate illustrative examples either to support our contention or test the strength of implications. We have also dealt with several separation axioms in a Bitopological space.



A Note on the Nearly Additivity of Knot Width

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Let k be a knot in S^3 . In [1], H.N. Howards and J. Schultens introduced a method to construct a manifold decomposition of double branched cover of (S^3, k) from a thin position of k . In this talk, we will prove that if a thin position of k induces a thin decomposition of double branched cover of (S^3, k) by Howards and Schultens' method, then the thin position is the sum of prime summands by stacking a thin position of one of prime summands of k on top of

a thin position of another prime summand, and so on. Therefore, k holds the nearly additivity of knot width (i.e. for $k = k_1 \# k_2$, $w(k) = w(k_1) \# w(k_2) - 2$) in this case. Moreover, we will generalize the hypothesis to the property a thin position induces a manifold decomposition whose thick surfaces consists of strongly irreducible or critical surfaces in the submanifolds obtained by cutting double branched cover along thin surfaces.

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On Cohomological Rigidity of Toric Hyperkähler Manifolds

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Principal: 55N91; 57S25, Secondly: 14M25; 32S22; 37J05

Toric hyperKähler manifolds, introduced by Bielawski and Dancer in [1], are defined by the hyperKähler quotient of torus actions on quaternionic spaces. This manifold can be regarded as the hyperKähler analogue of the toric manifolds. For the toric manifolds, in [5], Masuda and Suh proposed the following problem motivated by the Masuda’s theorem in [4]:

Problem 1 (Cohomological rigidity problem). *Let M and M' be toric manifolds. Are they homeomorphic if $H^*(M) \simeq H^*(M')$?*

This problem is still open, and we can also ask this problem for toric hyperKähler manifolds.

In this contributed abstract, we introduce the following two theorems:

Theorem 1 ([2]). *Let $(M_\alpha, T, \mu_{\widehat{\alpha}})$ and $(M'_{\alpha'}, T, \mu'_{\widehat{\alpha}'})$ be triples of toric hyperKähler manifolds with torus actions and their hyperKähler moment maps. Then, $(M_\alpha, T, \mu_{\widehat{\alpha}})$ and $(M'_{\alpha'}, T, \mu'_{\widehat{\alpha}'})$ are weakly hyperhamiltonian isomorphic if and only if there is a weak $H^*(BT)$ -algebra isomorphism $f : H_T^*(M_\alpha; \mathbb{Z}) \rightarrow H_T^*(M'_{\alpha'}; \mathbb{Z})$ such that $f(\widehat{\alpha}) = \widehat{\alpha}'$.*

Theorem 2 ([3]). *Let M and M' be toric hyperKähler manifolds. Then, M and M' are diffeomorphic if and only if $\dim M = \dim M'$ and $H^*(M) \simeq H^*(M')$.*

Theorem 1 can be regarded as the hyperKähler analogue of the Masuda’s theorem in [4], and Theorem 2 gives the answer of the cohomological rigidity problem for toric hyperKähler manifolds.

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Almost Complex Quasitoric Manifolds

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We prove the existence of T^n -invariant almost complex structure on any quasitoric manifold with positive omniorientation. This is an answer to problem posed by M. Davis and T. Januskiewicz in their classical paper [1]. We also show that any such structure is equivalent to a canonical stably complex structure on quasitoric manifold with omniorientation [2]. The number of different T^n -invariant almost complex structures on quasitoric manifold is estimated by an invariant of underlying polytope, which may be described in purely combinatorial terms.

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Top Terms of Polynomial Traces in Kra's Plumbing Construction

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Let Σ be a surface of negative Euler characteristic together with a pants decomposition \mathcal{P} . Kra's plumbing construction endows Σ with a projective structure as follows. Replace each pair of pants by a triply punctured sphere and glue, or 'plumb', adjacent pants by gluing punctured disk neighbourhoods of the punctures. The gluing across the i^{th} pants curve is defined by a complex parameter $\tau_i \in \mathbb{C}$. The associated holonomy representation $\rho : \pi_1(\Sigma) \rightarrow PSL(2, \mathbb{C})$ gives a projective structure on Σ which depends holomorphically on the τ_i . In particular, the traces of all elements $\rho(\gamma)$, $\gamma \in \pi_1(\Sigma)$, are polynomials in the τ_i .

Generalising results proved in [1, 2] for the once and twice punctured torus respectively, we prove a formula giving a simple linear relationship between the coefficients of the top terms of $\rho(\gamma)$, as polynomials in the τ_i , and the Dehn-Thurston coordinates of γ relative to \mathcal{P} .

This could be applied to give a formula for the asymptotic directions of pleating rays in the Maskit embedding of Σ as the bending measure tends to zero.

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On δ sII Generalized-closed Sets and their Applications

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Keywords. Topological space, $\delta s\Pi g$ - closed sets, $\delta s\Pi g - T_{1/2}$, $\delta s\Pi g$ -continuous and $\delta s\Pi g$ -irresolute maps, compactness, $\delta s\Pi g$ O-compact spaces, S-weakly Hausdorff space.

In this paper a new class of sets called $\delta s\Pi$ generalized-closed sets (briefly $\delta s\Pi g$ -closed sets) is introduced and its properties are studied. Further the notions of $\delta s\Pi g - T_{1/2}$ space, $\delta s\Pi g$ -continuous and $\delta s\Pi g$ - irresolute mappings are introduced. We obtain some of their properties via the concept of $\delta s\Pi g$ -closed sets and to relate the concept to the classes of $\delta s\Pi g$ O-compact spaces. It is seen that a topological space (X, T) is S-weakly Hausdorff if and only if each singleton is δ -semi closed. Lastly some applications are shown with the above concepts

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a –Scattered Spaces

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The above topological spaces are introduced and studied. It is shown that every continuous image of a compact Hausdorff a –scattered space X (i.e., every subset A of X with $|A| \geq a$ has an isolated point relative to A and a is the least regular cardinal with this property) is b –scattered for some $b \leq a$. Consequently, if X is a compact Hausdorff a –scattered space, where $a \leq c$ and c is the cardinality of continuum, then $a = \aleph_0$ the first infinite cardinal and X is scattered. Surprisingly, it follows that in any compact Hausdorff space X , every non-empty subset has an isolated point if and only if every uncountable subset of X has an isolated point.

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On the Bounded Isometry Conjecture

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In [3], F. Lalonde and L. Polterovich study the isometries of the group of Hamiltonian diffeomorphisms with respect to the Hofer metric. They defined a symplectic diffeomorphism ψ to be bounded, if the Hofer norm of $[\psi, h]$ remains bounded as h varies on $\text{Ham}(M, \omega)$. The set of bounded symplectic diffeomorphisms, $\text{BI}_0(M)$, of (M, ω) is a group that contains all Hamiltonian diffeomorphisms.

F. Lalonde and L. Polterovich conjecture that these two groups are equal, $\text{Ham}(M, \omega) = \text{BI}_0(M, \omega)$ for every closed symplectic manifold. In [3], they prove this conjecture in the case when the symplectic manifold is a product of closed surfaces of positive genus; and in [2], F. Lalonde and C. Pestieau proved the conjecture for product of closed surfaces of positive genus and a simply connected manifold. Recently, Z. Han [1] prove this conjecture for the Kodaira-Thurston manifold.

We prove the bounded isometry conjecture for a closed symplectic manifold (M, ω) of dimension $2n$ that satisfies the following hypothesis:

- (a) There are open sets $U_1, \dots, U_l \subset M$ such that each U_k is symplectomorphic to $T^{2n} \setminus \{pt\}$ with the standard symplectic form up to a scalar.
- (b) The induced maps $j_{k,*} : H_c^1(U_k) \rightarrow H^1(M)$ are such that

$$H^1(M) = \bigcup_{k=1}^l j_{k,*}(H_c^1(U_k)).$$

Basically what we need for the symplectic manifold is to have several punctured torus that generate all the cohomology in degree one of M . This is needed in order to describe the whole flux on M in terms of the flux on U_k .

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Elliptic Topology and Bifurcation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 58E07, 35B32, 58J20

I will discuss the relationship between elliptic topology and bifurcation theory. More precisely, given a family $\{f_\lambda\}$ of Fredholm maps between Banach spaces parametrized by a finite CW -complex Λ and such that $f_\lambda(0) = 0$, I will show that bifurcation of nontrivial zeroes arise if, for some $\mu \in \Lambda$, $Df_\mu(0)$ is invertible and the index bundle of the family of linearizations $\{Df_\lambda(0)\}$ along the trivial branch is stably fiberwise homotopically nontrivial. Using the Atiyah-Singer family index theorem, Fedosov's formula and some classical results about J -homomorphism, I will obtain sufficient conditions for the appearance of nontrivial classical solutions of nonlinear elliptic boundary value problems bifurcating from a trivial branch.



On Unknotting Numbers

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In this presentation we give a bound for the unknotting number of any given knot. In many cases the given bound is exactly equal to the unknotting number. We have utilized quasitoric braid representation for a given knot in finding the bound.

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Trisecants for Knots

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Let K be a polygonal knot. We say that a line ℓ in \mathbf{R}^3 is an *n -secant line* for K if ℓ intersects K in at least n distinct points. If ℓ is an n -secant line for K and the intersection of ℓ with K consists of the points x_1, x_2, \dots, x_n , in that order along ℓ , no two of which lie in a common straight subarc of K , then $x_1x_2\dots x_n$ is an *n -secant* for K . We say that a 3-secant is a **trisecant** for K .

In [1], Erika Pannwitz proved that each point of K is the starting point of at least κ trisecants for K , where κ is the necessary number of boundary singularities for a disk bounded by K .

Let x be a point in K . We are interested in counting the number of trisecants for K having the point x in common. If we restrict x to appear only as a starting point for trisecants, we have found a lower bound on the number of trisecants in terms of the crossing number of the knot: a knot K with crossing number, $cr(K)$, has at least $\frac{2cr(K)+1}{3}$ trisecants.

We will discuss how to get this lower bound and some ideas that might lead us to find another lower bound for the number of trisecants if we allow x to appear not only as a starting point but also as a middle point.

A detailed study of trisecants appear in [1] and [2].

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A Counterexample to Ganea's Conjecture with the Minimum Known Dimension

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Let X be a topological space and denote by $cat X$ the Lusternik–Schnirelmann category [4] of X . Ganea [2] conjectured that $cat(X \times S^n) = cat X + 1$ for any finite CW -complex X and $n \geq 1$. Although this conjecture was proved in some particular cases, it has been disproved in general [3] with the lowest dimensional counterexample having dimension 10. Ganea [1] established that an upper bound to the category of a space is equivalent to the existence of sections of some fibrations associated to it. In this work, using Ganea's characterisation and a divisibility phenomenon for the Hopf invariants [5] of its attaching maps, a 7-dimensional CW -complex X such that $cat X = 2$ is constructed. In addition, an alternative cell decomposition of X is presented and by another divisibility phenomenon of its attaching maps, it is proved that $cat(X \times S^n) = 2$ for $n \geq 2$. Such space hence constitutes the minimum dimensional known counterexample to Ganea's conjecture on the Lusternik–Schnirelmann category of spaces.

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On Strongly αg^* -continuous Maps in Bitopological Spaces

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We introduce the notion of pair wise $s\alpha g^*$ -closure operator analogous to the $s\alpha g^*$ -closure in a topological space. We also introduce $s\alpha g^*$ -continuous maps in bitopological spaces by using $s\alpha g^*$ -closed sets of bitopological spaces and study some of their basic properties.

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On Two Topologies Associated with a Topology

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With every topology T on a set X , two finer topologies on X have been introduced in the past, viz., T^f , called the *front topology* ([1], [5]) and T^i , called

the *indiscrete generated refinement* of T ([3]). Let $X = (X, T)$ be a topological space, Δ_X be the diagonal of X , and P be the product topology on $X \times X$. It is known that X is T_0 iff Δ_X is P^f -closed ([2]) iff Δ_X is P^i -closed ([3]). This raises the question: *Given a topological space (X, T) , are T^f and T^i related in some way(s)?*

In this note, we establish some nice relationships between T^f and T^i , which include:

- $T^{ff} = T^i$.
- $T^f = T^i$ iff (X, T) satisfies the T_{D-0} -separation axiom (as defined in [4]).

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When the Set of Embeddings is Finite?

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Primary 57R52, 57R40; Secondary: 57R65.

This talk is on classification of embeddings of manifolds. Given a manifold N and a number m , we study the following question: *is the set of isotopy classes of embeddings $N \rightarrow S^m$ finite?* In case when the manifold N is a sphere the answer was given by A. Haefliger in 1966 [1]. The case when N is a disjoint union of spheres was treated by D. Crowley, S. Ferry and independently by the author in 2008. In this talk we consider the next natural case when N is a product of two spheres.

Theorem. Assume that $m > 2p + q + 2$ and $m < p + 3q/2 + 2$. Then the set of isotopy classes of smooth embeddings $S^p \times S^q \rightarrow S^m$ is infinite if and only if either $q + 1$ or $p + q + 1$ is divisible by 4, or there exists a point (x, y) in the set $U(m - p - q, m - q)$ such that $(m - p - q - 2)x + (m - q - 2)y = m - 3$.

Here $U(i, j) \subset \mathbb{Z}^2$ is a concrete subset defined in the talk, which depends only on the parity of i and j .

Our approach is based on a group structure on the set of embeddings [2] and a new exact sequence, which in some sense reduces the classification of embeddings $S^p \times S^q \rightarrow S^m$ to the classification of embeddings $S^{p+q} \sqcup S^q \rightarrow S^m$ and $D^p \times S^q \rightarrow S^m$, cf. [3].

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Butterflies: A New Representation of Links

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We define an interesting class of 3-balls (called butterflies), with faces identified by pairs. The identification space is S^3 , and the image of a preferred set of edges is a link. As motivation we give some examples. We prove that every link can be represented in this way (butterfly representation). The butterfly number of a link is also defined and we prove that this number and the bridge number of a link coincide. We give a partial extension of the Schubert theorem that classifies the two bridge links and we show how to associate a special triple of rational numbers $(p/n; q/m; s/l)$ to each 3-bridge link.

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Lattices in Complete Kac–Moody Groups

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20G99, 20E08, 22D05

A classical theorem of Siegel [3] states that the minimum covolume among lattices in $G = SL_2(\mathbf{R})$ is $\frac{\pi}{21}$, and determines the lattice which realises this minimum. In the nonarchimedean setting, Lubotzky [1] and Lubotzky–Weigel [2] constructed the lattice of minimal covolume in $G = SL_2(K)$, where K is the field $\mathbf{F}_q((t^{-1}))$ of formal Laurent series over \mathbf{F}_q .

The group $G = SL_2(\mathbf{F}_q((t^{-1})))$ has, in recent developments, been viewed as the first example of a complete Kac–Moody group of rank 2 over a finite field. Such Kac–Moody groups are locally compact, totally disconnected topological groups, which may be thought of as infinite-dimensional analogues of semisimple algebraic groups. Apart from the “affine case” $G = SL_2(\mathbf{F}_q((t^{-1})))$, these groups are nonlinear.

We determine the cocompact lattice of minimal covolume in such groups G . We also classify those lattices in G which act transitively on the edges of its associated Bruhat–Tits tree X , and show that in many cases, the cocompact lattice of minimal covolume in G is edge-transitive. Our methods include finite group theory, covering theory for graphs of groups and the dynamics of the G -action on X .

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Braids and Groups and Monoids Connected with Them

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We give a survey of recent results [1], [2], [3] related to some aspects of braids.

Inverse braid monoid describes a structure on braids where some strings of initial n may be deleted. It was shown [3] that many properties and objects based on braid groups may be extended to the inverse braid monoids. For

example an inclusion into a monoid of partial monomorphisms of a free group gives a solution of the word problem. Let ϵ_i denote the trivial braid with i th string deleted. We call a braid b on n strands *brunnian* if it satisfies the following equations:

$$\epsilon_i m = \epsilon_i \text{ for all } i = 1, \dots, n.$$

Let $\text{Brun}_n(M)$ be the group of brunnian braids on a surface M and $A_{i,j}[M]$ are the images of the canonical generators of the pure braid group by the homomorphism induced by the inclusion of a disc into a manifold M .

Theorem. [2] *Let M be a connected 2-manifold and let $n \geq 4$. Let*

$$R_n(M) = [\langle\langle A_{1,n}[M] \rangle\rangle, \langle\langle A_{2,n}[M] \rangle\rangle, \dots, \langle\langle A_{n-1,n}[M] \rangle\rangle]_S$$

be the symmetric commutator subgroup and $\langle\langle A_{i,j}[M] \rangle\rangle$ means a subgroup of pure braids normally generated by $A_{i,j}[M]$.

1. *If $M \neq S^2$ or $\mathbf{R}P^2$, then $\text{Brun}_n(M) = R_n(M)$.*
2. *If $M = S^2$ and $n \geq 5$, then there is a short exact sequence*

$$1 \rightarrow R_n(S^2) \rightarrow \text{Brun}_n(S^2) \rightarrow \pi_{n-1}(S^2) \rightarrow 1.$$

3. *If $M = \mathbf{R}P^2$, then there is a short exact sequence*

$$1 \rightarrow R_n(\mathbf{R}P^2) \rightarrow \text{Brun}_n(\mathbf{R}P^2) \rightarrow \pi_{n-1}(S^2) \rightarrow 1.$$

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Ideals in Stone-Čech Compactifications

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The operation of a discrete semigroup S can be naturally extended to the Stone-Čech compactification βS of S so that for each $p \in \beta S$, the right translation $\beta S \ni x \mapsto xp \in \beta S$ is continuous, and for each $a \in S$, the left translation $\beta S \ni x \mapsto ax \in \beta S$ is continuous. The semigroup βS has important applications to combinatorial number theory and to topological dynamics [2]. In [3] it was shown that if S is an infinite Abelian group, then βS contains $2^{2^{|S|}}$ closed two sided ideals, and in [1] this result has been extended to an arbitrary countably infinite group. We show that for every infinite semigroup S embeddable into a group, βS contains $2^{2^{|S|}}$ closed two sided ideals [4].

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Section 7

Lie Theory and Generalizations

Graded Characters of Minimal Affinizations of Quantum Groups

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Let \mathfrak{g} be a finite-dimensional simple Lie algebra over the field of complex numbers. Consider its loop algebra $\tilde{\mathfrak{g}}$ and the corresponding quantized enveloping algebras $U_q(\mathfrak{g})$ and $U_q(\tilde{\mathfrak{g}})$. We will refer to the latter as the quantum affine algebra of \mathfrak{g} . It is known that, unless \mathfrak{g} is of type A, there is no quantum group analogue of the evaluation maps $\tilde{\mathfrak{g}} \rightarrow \mathfrak{g}$. In particular, the concept of evaluation representations is not available in the context of quantum affine algebras in general and it turns out that not every representation of $U_q(\mathfrak{g})$ can be extended to one of $U_q(\tilde{\mathfrak{g}})$. To overcome this issue, V. Chari and A. Pressley introduced the concept of minimal affinizations of an irreducible finite-dimensional $U_q(\mathfrak{g})$ -module. Roughly speaking, a minimal affinization is a minimal enlargement of the given irreducible representation of $U_q(\mathfrak{g})$ which can be extended to a representation of $U_q(\tilde{\mathfrak{g}})$.

A special class of minimal affinizations is the one of Kirillov-Reshetikhin modules which are the minimal affinizations of the irreducible modules whose highest weights are multiples of the fundamental weights. These modules were originally introduced in the mathematical physics literature before the concept of minimal affinizations was defined. One problem of particular interest regarding minimal affinizations is that of describing their characters. In this talk I will present character formulas for certain minimal affinizations which were obtained by comparing the classical limit of the minimal affinizations with certain

graded modules for the underlying current algebra. This strategy was developed and successfully used by V. Chari and the speaker in [1, 2] to prove several character formulas for Kirillov-Reshetikhin modules which had been conjectured in the mathematical physics literature previously. More recently, the speaker has devised a way of extending this method to minimal affinizations having more general highest weights and obtained some formulas which appear not to have been conjectured before [3, 4].

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Asymptotic K -character of Nilpotent Orbits

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Let G be a complex reductive algebraic group and $K \subset G$ the fixed-point set of a regular involution on G . Denote by $\mathfrak{k} \subset \mathfrak{g}$ the corresponding Lie algebras, and by \mathfrak{p} the Cartan complement of \mathfrak{k} in \mathfrak{g} . Let $G_{\mathbb{R}}$ be a real form of G and $\mathfrak{g}_{\mathbb{R}}$ the Lie algebra of $G_{\mathbb{R}}$. Write $K_{\mathbb{R}} = G_{\mathbb{R}} \cap K$ and $\mathfrak{k}_{\mathbb{R}} = \mathfrak{g}_{\mathbb{R}} \cap \mathfrak{k}$. Denote by \mathcal{N}^* the cone of nilpotent elements in the dual Lie algebra \mathfrak{g}^* , and consider a $G_{\mathbb{R}}$ -orbit \mathcal{V} in $\mathcal{N}^* \cap \mathfrak{g}_{\mathbb{R}}^*$ and a K -orbit \mathcal{O} in $\mathcal{N}^* \cap \mathfrak{p}^*$ associated by the Kostant-Sekiguchi correspondence.

According to a conjecture of Vogan the asymptotic behaviour of multiplicities of $K_{\mathbb{R}}$ -types in the ring of regular functions $R[\overline{\mathcal{O}}]$ can be described in terms of the Liouville measure $\beta_{\mathcal{V}}$ on \mathcal{V} . More precisely, we consider the generalised function $J_{\mathcal{V}}$ defined as restriction of the Fourier transform of $\beta_{\mathcal{V}}$ to $\mathfrak{k}_{\mathbb{R}}$. On the other hand, $R[\overline{\mathcal{O}}]$ is a trace class representation of $K_{\mathbb{R}}$ [4], hence one can define the asymptotic $K_{\mathbb{R}}$ -character of \mathcal{O} as the limit

$$M_{\mathcal{O}}(X) = \lim_{t \rightarrow 0} t^d \text{Tr}(R[\overline{\mathcal{O}}])(\exp tX), \quad d = \dim_{\mathbb{C}} \mathcal{O}, \quad X \in \mathfrak{k}_{\mathbb{R}}.$$

Vogan's conjecture states that $J_{\mathcal{V}} = M_{\mathcal{O}}$ as generalised functions on $\mathfrak{k}_{\mathbb{R}}$. The conjecture was established by King for even nilpotent orbits and minimal nilpotent orbits [1, 2], and by Vergne for complex groups [4]. The aim of this work is to provide additional evidence for Vogan's conjecture. Relying on the results of Schmid and Vilonen [3] we show that Vogan's conjecture is true for the classical groups $GL(n, \mathbb{R})$, $SL(n, \mathbb{R})$, $SU^*(2n)$, $Sp(p, q)$ and $SO^*(2n)$.

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On Evolution Algebras

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Evolution algebras appear as a intrinsic and general mathematical structure of the stochastic processes and genetics.

Evolution algebras can be defined by generators and defining relations. It is notable that the generators set of an evolution algebra can serve as a basis of the algebra.

Let (A, \cdot) be an algebra over a field K . The algebra A is called an evolution algebra if it admits a basis $\{x_1, x_2, \dots, x_n\}$, such that

- $x_i \cdot x_j = 0, \quad i \neq j$
- $x_i \cdot x_i = \sum_{k=1}^n a_{i,k} x_k$ for any i .

The basis will be called a natural basis (see [1]).

In this work we will study various algebraic concepts in evolution algebras. For example, the notions of nilpotency and we will show the connections with graph theory.

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Tensor Products and Blocks of Finite-dimensional Representations of Quantum Affine Algebras at Roots of Unity

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The category of finite-dimensional representations of quantum affine algebras is not semisimple. For generic values of the quantization parameter, results of V. Chari [1] and M. Kashiwara [2] provide a way of obtaining indecomposable objects by giving sufficient conditions for a tensor product of simple objects to be highest-weight. In particular, a tensor product of fundamental representations can always be reordered in such a way that these conditions are satisfied. Furthermore, this property turned out to be one of the essential ingredients used to describe the block decomposition of the category.

In this talk, we will focus on a joint work with A. Moura [3] where we consider the root of unity setting. We prove an analogue of Chari's version of the aforementioned result on tensor products of simple modules. However, the result about tensor products of fundamental representations is no longer valid. We will discuss the techniques we used to overcome this issue for describing the blocks in the root of unity setting.

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Intuitionistic Fuzzy Lie Algebra over a Fuzzy Field

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The fuzzy algebraic structure play a prominent role in mathematics with wide applications in many other branches of science. After the introduction of fuzzy sets by Zadeh [4], several scholars studied fuzzy substructures of many algebraic structures. K.T. Atanassov [3] introduced the notion of intuitionistic fuzzy sets as generalization of fuzzy sets. In [2] we introduced fuzzy Lie algebra over a fuzzy field. In this paper we introduce the notion of intuitionistic fuzzy Lie algebra over a fuzzy field and give some results in this respect.

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Green Functions and Related Boundary Value Problems on the Heisenberg Group

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Using inversion with respect to ball of arbitrary radius, a modified Kelvin transform on the Heisenberg group \mathbb{H}_n is defined which gives explicit expressions for the Green's function and Poisson kernel for Korányi ball of arbitrary radius and annular domain. Solution of Dirichlet problem for union of two balls is discussed using schwarz alternating method.

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Splints of Root Systems of Lie Superalgebras

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In this paper we classify the splints of the root system of classical Lie superalgebras as a superalgebraic conversion of the splints of classical root systems. We hope in some cases splints will play a role in determining branching rules of a module over a complex classical Lie superalgebras when restricted to a subalgebras analogous to the case of classical Lie algebras.

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Irreducible p, q -representations of the Lie Algebra $gl(2)$ and p, q -Mellin Integral Transformation

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The theory of irreducible representations of Lie algebras has been a rich source of results in special function theory, [1]. In particular, the Mellin integral transformation [2] as well as its q -analogue [3] have been studied from the Lie algebraic point of view resulting in special function identities and recurrence relations. In the proposed talk, we discuss new models of the irreducible p, q -representations of the Lie algebra $gl(2)$. An integral transformation motivated by the p, q -gamma function is used to transform these models of $gl(2)$ and identities involving p, q -special functions are obtained.

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A Non-recursive Criterion for Weights of Affine Lie Algebra Representations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 17B65, 20C08, 17B67,

Let Λ be a dominant integral weight of level r for the affine Lie algebra g and let α be a non-negative integral combination of simple roots of height d . We address

the question of whether the weight $\eta = \Lambda - \alpha$ lies in the set $P(\Lambda)$ of weights in a highest weight module with highest weight Λ . We give a non-recursive criterion in terms of the coefficients of α modulo an integral lattice rM , where M is the lattice parameterizing the abelian normal subgroup T of the Weyl group. The criterion does not require the preliminary computation of a set no larger than the fundamental region for rM , consisting of the maximal weights with positive hubs and representatives of their images under the classical Weyl group W_0 associated with the Weyl group W of g . The criterion is a generalization to $r > 1$ of [Ka, 12.6.3].

The original motivation for this research was in investigation of the existence of the block H_α^Λ of the cyclotomic Hecke algebra $H_d^\Lambda(F, \xi)$, where $\xi \in F^*$ is an e -th root of unity. This question is typically settled by a recursive construction of the weights of blocks up to rank d or by the construction of a multipartition with content α . By the categorification result in [AM], such a block exists if and only if the corresponding weight η is in $P(\Lambda)$ for the affine Lie algebra $A_{e-1}^{(1)}$, so our non-recursive criterion above gives a criterion in terms of the residues of the coefficients of α modulo r . In this case the set to be computed is of order r^{e-1} .

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Motion Groupoids and Geometric Gelfand Models

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We introduce an intrinsic version (called here geometric induction) of the classical induction of representations from a subgroup H of a (finite) group G , which associates to any, not necessarily transitive, G -set X and any representation of its motion groupoid $M(X, G)$, a representation of the group G . We show that for $G = PGL(2, q)$, geometric induction applied to a suitable linear character of the motion groupoid of the G -set X consisting of symmetric matrices in G affords a “twisted natural representation”, which is a Gelfand Model for G [2, 1]. We conjecture moreover that for any finite group of Lie type G a canonical

G -set X may be found affording a Gelfand Model by geometric induction from a suitable linear character of its motion groupoid $M(X, G)$.

This work was partially supported by Fondecyt Grants 1070246 and 7040148.

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On the G -superalgebra Structure for Higher Syzygies of a Projective G -variety

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This is a report on some unpublished work by the author during the year 2003 (and before). Let G be an algebraic group(scheme), and V a G -module. (All vector spaces are finite dimensional over a fixed unmentioned field of characteristic zero). Let S denote the symmetric algebra $\text{Sym}^\bullet V$, and $I \subseteq S$ a homogeneous G -ideal with no (nonzero) element of S_1 . Let $X = \text{Proj } R = (\text{Aff } R)/\mathbf{G}_{\text{mult}}$, for $R := S/I$. Our focus is on the case of G connected, acting transitively on X ; but for the discussion of generalities it is important to let G even be the one-element group.

One is looking for an in-depth study of the minimal free resolution (MFR):

$$0 \leftarrow R \leftarrow S = (F_0) \leftarrow F_1 \leftarrow \cdots \leftarrow F_D \leftarrow 0;$$

where F_j (for $j \geq 1$) are built as in the textbooks. (Uniqueness of F_i 's is also 'clear', and the value of D has been a topic of intensive study in literature.)

The study of MFR needs detailed attention to the boundary maps $F_j \rightarrow F_{j-1}$; to that end we must write $F_j = \bigoplus_k M_{j,k} \otimes S(-k)$, where $S(-k)$ is the same as S except for a degree shift. Thus we need to find the G -modules $M_{j,k}$ (plus suitable morphisms between them, on which we have inadequate space to devote here).

MAIN RESULT: The module $M_{j,k}$ is isomorphic to the j -th cohomology of the finite complex (always of (S, G) -modules) $\mathcal{E}_k = \{\mathcal{E}_{k|j} \mid 0 \leq j \leq k\}$, whose j -th component $\mathcal{E}_{k|j} = E_{i,j} := R_i \otimes \wedge_j$ for $i = k - j$, and $\wedge_j = \wedge^j V$. This

is a generalization of the standard Koszul Complex (obtained for $I = (0)$). We discovered this using the philosophy of Generating Functions. The result, however, is a rewording of the well-known Koszul Cohomology notion of Mark Green (introduced in 1984 in two celebrated and intricate papers of the same title).

The fact that ‘Full Syzygy Space’ $M_{.,.}$ (defined as the direct sum of all G -modules $M_{j,k}$) has a natural supercommutative G -algebra structure (similar to the situation for De Rham cohomology), arises readily from the same assertion for an arbitrary mod-2 graded G -algebra (equipped with a special superderivation, such as our boundary map, whose ‘square’ is zero).

The prime example (the only ‘tractable’ one) with $G = GL(V)$ which the author could successfully tackle is the case of ‘Line Grassmannian’ (the space of all lines in \mathbb{P}^{n-1} , i.e. the $d = 2$ case of the d -Grassmannian variety consisting of all d -dimensional subspaces of affine n -space $\mathbb{A}^n = \text{Spec } S$). The net result, therein is the discovery of the role of the infinite family of ‘special hooks’ with leg-size exactly 3 more than the arm-size, defining (as anti-commutative generators of) the said G -superalgebra. Thus $M_{.,.} = \bigwedge^\bullet \{\text{the graded span of all such hooks}\}$.



Section 8

Analysis

Effectiveness of the Gregory-Type Set of Interpolatory Polynomials

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We discuss in the paper the effectiveness properties of the Gregory-Type set $p_0(z) = 1$, $p_k(z) = (z - a)(z - 2a^2)(z - 3a^3) \dots (z - ka^k)$; $k \geq 1$, for the cases $|a| < 1$, $|a| > 1$ and $|a| = 1$ where a is a given complex number.

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L_p – Inverse Theorem for Iterates of Bernstein-Durrmeyer Type Polynomials

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For $f \in L_p[0, 1]$, $1 \leq p < \infty$, the Bernstein-Durrmeyer type polynomial operators

$$P_n(f; x) = \int_0^1 W_n(t, x) f(t) dt,$$

where the kernel $W_n(t, x) = n \sum_{k=1}^n p_{n,k}(x) p_{n-1,k-1}(t) + (1-x)^n \delta(t)$, $\delta(t)$ being

the Dirac-delta function, were introduced by Gupta and Maheshwari [2] to study the approximation of functions of bounded variation. It turns out that the order of approximation by these operators is at best $O(n^{-1})$, however smooth the function may be. In order to speed up the rate of convergence by the operators P_n , we apply the technique of iterative combination as given below:

The iterative combination $T_{n,k} : L_p[0, 1] \rightarrow C^\infty[0, 1]$ of the operators P_n is defined as

$$T_{n,k}(f; x) = (I - (I - P_n)^k)(f; x) = \sum_{r=1}^k (-1)^{r+1} \binom{k}{r} P_n^r(f; x), \quad k \in N,$$

where $P_n^0 \equiv I$ and $P_n^r \equiv P_n(P_n^{r-1})$ for $r \in N$.

Assuming that $I = [a, b]$, where $0 < a < b < 1$, in [1] we proved that if $\omega_{2k}(f, \tau, p, I) = O(\tau^\alpha)$ as $\tau \rightarrow 0$ then

$$\|T_{n,k}(f, \cdot) - f\|_{L_p(I)} = O(n^{-\alpha/2}) \text{ as } n \rightarrow \infty, \text{ where } 0 < \alpha < 2k.$$

The aim of this paper is to establish the corresponding inverse theorem i.e. the characterization of the class of functions for which $\|T_{n,k}(f, \cdot) - f\|_{L_p(I)} = O(n^{-\alpha/2})$ as $n \rightarrow \infty$, where $0 < \alpha < 2k$.

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Use of Shear Construction To Study Harmonic Univalent Mappings with Directional Convexity

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Clunie and Sheil-Small [3] in 1984 discovered a general method, known as ‘shear construction’ for constructing harmonic mappings with specified properties. This method essentially produces a harmonic mapping onto a convex domain in one direction by “shearing” (or stretching, or translating) a given conformal mapping along parallel lines. For example, see [5]. In this paper we use shear construction to generate certain subclasses of harmonic univalent mappings with directional convexity because it allows us to study such functions by examining their related analytic univalent mappings. In this setting we find growth, distortion, and coefficient bounds for harmonic univalent mappings that are convex in both the directions of real axis and imaginary axis. For basic definitions and terminology, one may refer to [1], [2], and [4].

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Region of Variability for Spirallike Functions with Respect to a Boundary Point

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Let \mathcal{F}_μ denote the class of all non-vanishing analytic functions f in the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ with $f(0) = 1$, and for $\mu \in \mathbb{C}$, such that $\operatorname{Re} \mu > 0$ satisfying $\operatorname{Re} \left(\frac{2\pi}{\mu} \frac{zf'(z)}{f(z)} + \frac{1+z}{1-z} \right) > 0$ for $z \in \mathbb{D}$. Functions in the class \mathcal{F}_μ are called spirallike functions with respect to a boundary point, which has been studied extensively in [1]. For $\mu = \pi$, the class \mathcal{F}_μ reduces to the class of starlike functions with respect to a boundary point introduced by M. S. Robertson [8]. In this talk we discuss the following problem: For any fixed $z_0 \in \mathbb{D}$ and $\lambda \in \mathbb{D}$, we shall determine the region of variability $V(z_0, \lambda)$ for $\log f(z_0)$ when f ranges over the class $\mathcal{F}_\mu(\lambda) = \{f \in \mathcal{F}_\mu : f'(0) = \frac{\mu}{\pi}(\lambda - 1)\}$. Using Mathematica we also graphically illustrate the region of variability for several sets of parameters. For a recent investigation on region of variability problems we refer to [2, 3, 4, 5, 6, 7, 9].

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Nonspherical Partial Sums and the Pinsky Phenomenon

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Let C be a smooth bounded, strongly convex symmetric set in R^n and B be the open convex subset of R^n dual to C . For a given $g \in C^\infty(R^n)$ we define a piecewise smooth function f , supported on \overline{B} , as follows: $f(x) = g(x)$, if $x \in B$ and $f(x) = \frac{1}{2}g(x)$ if $x \in \partial B$. Consider a nonspherical partial sums of n -fold Fourier integrals

$$S_{\lambda C}f(x) = \int_{\lambda^{-1}\xi \in C} \hat{f}(\xi)e^{ix\xi}d\xi,$$

where $\hat{f}(\xi)$ is the Fourier transform of f :

$$\hat{f}(\xi) = (2\pi)^{-n} \int_{R^n} f(x)e^{-ix\xi}dx.$$

In 1997 Pinsky and Taylor [1] proved, that when $\lambda \rightarrow \infty$:

$$n \leq 2 \Rightarrow S_{\lambda C}f(x) \rightarrow f(x), \forall x \in R^n,$$

$$n \geq 3 \Rightarrow S_{\lambda C}f(x) \rightarrow f(x), \forall x \neq 0.$$

We establish necessary and sufficient conditions for piecewise smooth function f , which guarantee the convergence of its partial sums $S_{\lambda C}f(0)$ when $n \geq 3$. In case of the spherical partial sums this result coincides with a theorem of Pinsky [2], which is in the mathematical literature called “the Pinsky phenomenon”.

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On Some Invariant of Two-parametrical Families of Real Functions

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In this paper we continue the investigations from [1]. There we have considered in the region, $x \geq 0$, $y \geq 0$, two-parametrical families of differentiable functions $f(x, a, b)$, $a > 0$, $b > 0$, satisfying the following conditions:

1. $\int_0^\infty (x, a, b)dx < \infty$,
2. Product $xf(x, a, b)$ has only one local maximum for fixed parameters a and b .

Let $S(a, b)$ denote an area of the figure bounded by x-axis, y-axis and curve $y = f(x, a, b)$.

Let $P_{max}(a, b)$ denote the value of local maximum of the product $xf(x, a, b)$.

And finally a quotient $k_e := S(a, b)/P_{max}(a, b)$ is said to be *extremality coefficient*.

In [1] we showed that in many cases this coefficient is constant. Now we investigate general case of two-parametrical families possessing constant *extremality coefficient* and some physical interpretations of obtained results. In the last part of this paper we also study in the similar way the case of functions of two variables.

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Convolution Equation on Certain Hypergroups

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Let $(H, *)$ be a commutative hypergroup [1]. We shall consider the following general problem [2], [3]: Let $A_1, A_2, \dots, A_k \subset H$ and let a be a fixed element of H . Under some conditions on H we estimate the number of solutions of the convolution equation.

$$\delta_{x_1} * \delta_{x_2} * \delta_{x_3} * \dots * \delta_{x_k} = \delta_a \quad (x_i \in A_i, i = 1, 2, \dots, k)$$

In particular, decide whether or not a solution exists.

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Differentiation on Local Fields

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 43A75, 46S10

In the field of p -adic numbers Q_p , let S_γ be the sphere centered at 0 and with the radius equal to p^γ .

For $m \in N$, let $U_m = 1 + p^m Z_p$, where Z_p denotes the ring of p -adic integers.

There exists a sequence of multiplicative characters $(\theta_n)_{n \geq 0}$ on S_0 such that $\theta_0 \equiv 1$ and θ_n is trivial on U_{N+1} if $n = a_0 + \sum_{s=1}^N (p-1)a_s p^{s-1}$ for some $N \geq 0$, $0 \leq a_0 < p-1$ and $0 \leq a_s < p$.

Each character θ_n is extended to Q_p^* by the relation $\theta_n(x) = \|x\|^{-1/2} \theta_n(\|x\|x)$.

Departing from the traditional approach (cf., e.g., [2] and the literature there), we introduce a differentiation operator with multiplicative characters as its eigenfunctions and investigate basic properties of this derivative. Differentiation of test functions, regular distributions, multiplicative convolution and Fourier transform is considered.

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Multipliers of $A_p(0, \infty)$ with Order Convolution

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 43A22

The algebras $A_p(G)$ of elements in $L_1(G)$ whose Fourier transforms belong to $L_p(\hat{G})$ and the multipliers for these algebras have been studied by various authors. ([3-5]). Let $I = (0, \infty)$ be the locally compact, idempotent, commutative topological semigroup with the usual topology and max multiplication. Let \hat{I} be the maximal ideal space of $L_1(I)$. Then $\hat{I} = (0, \infty]$ [2]. Define for $p \geq 1$, $A_p(I) = \{f \in L_1(I); \hat{f} \in L_p(\hat{I})\}$ and the norm of $f \in A_p(I)$ as $\|f\| = \|f\| + \|\hat{f}\|$. The maximal ideal space of $A_p(I)$ is $(0, \infty)$ [1]. Though the algebras $L_1(G)$ and $A_p(G)$ show similarity the algebras $L_1(I)$ and $A_p(I)$ are dissimilar. The multipliers from $A_r(I)$ to $A_p(I)$ are studied in this note. A set of necessary and another set of sufficient conditions are found.

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Fixed Point Theorems for Certain Contractive Mappings in Cone Metric Spaces

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In the present paper we prove some common fixed point theorems by using the Reich and Rhoades type contractive conditions in complete cone metric spaces which generalize and extend the respective theorems of Morales and Rojas [1] and others.

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On Integrability with Weight of the Sum of Series with Respect to Multiplicative Systems

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Let $\{\psi_n(x)\}_{n=0}^{\infty}$ be periodic multiplicative Vilenkin-Price system defined with the help of a generating sequence $\{p_k\}_{k=1}^{\infty}$ of natural numbers p_k , $2 \leq p_k < \infty$, $m_n = p_1 p_2 \dots p_n$ [1].

Let $\theta \in (0, 1]$. By definition $G\overline{M}_\theta$ is the collection of all sequences $\{a_k\}$ such that

$$\sum_{k=n}^{\infty} |\Delta a_k| < K n^{\theta-1} \sum_{k=\lceil \frac{n}{c} \rceil}^{\infty} \frac{|a_k|}{k^\theta} < \infty$$

for some $c > 1$, where $\Delta a_k = a_k - a_{k+1}$, $[t]$ is a integer part of number t , $n \in N$, K positive constant independent on n .

Theorem. Let $1 \leq p < \infty$, $\{b_n \geq 0\} \in G\overline{M}_\theta$, $\theta \in (0, 1]$, $1 - \theta p < \alpha < 1$, $f(x) = \sum_{k=0}^{\infty} b_k \psi_k(x) \in L(0, 1)$.

Then

- a) if $\sum_{n=1}^{\infty} n^{\alpha+p-2} b_n^p < \infty$, then $|f(x)|^p x^{-\alpha} \in L(0, 1)$,
 b) if $|f(x)|^p x^{-\alpha} \in L(0, 1)$, then $\sum_{k=1}^{\infty} m_k^{\alpha+p-1} \left(\frac{1}{m_k} \sum_{n=m_k}^{m_{k+1}-1} b_n \right)^p < \infty$.

For trigonometric series similar result was given in [2].

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A Global Characterization of Tubed Surfaces in \mathbb{C}^2

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Let $M^3 \subset \mathbb{C}^2$ be a three times differentiable real hypersurface. The Levi form of M transforms under biholomorphism, and when restricted to the complex tangent space, the skew-hermitian part of the second fundamental form transforms under Möbius transformation. (The Möbius transformations are the automorphisms of $\mathbb{CP}^2 \supset \mathbb{C}^2$.) The surfaces for which these forms are constant multiples of each other were identified in previous work, provided the constant is not unimodular. Here it is proved that if the surface is assumed to be complete, and if the constant is unimodular, then the surface is tubed over a strongly convex curve. The converse statement is true, too, and is easily proved.

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What does the Uncertainty of Elements Mean

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 03B30

This paper puts forward the concepts of element and real number order. It points out that an element has the uncertainty of lower or higher order than

that of physical quantities, while the uncertainty described in both probability theory [1] and fuzzy set theory [2] is of the same order as that of physical quantities. In this paper, the possible values of an element are defined by a set of non-zero real numbers. The arithmetic operations of the elements can be formulated by defining the arithmetic operations of the sets. Functions and their finite precision calculus are then defined on the set of elements. The paper argues that the set of elements are closed under the arithmetic operations, and the ordered systems described by the elements and their operational relations are discrete and irreversible. Using the elements to express physical quantities can lead to the elimination of singularities in physics, the discretization of space-time and the breaking of the symmetry of time direction.

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Uniqueness of Normalized Solutions to Nonlinear Beltrami Equations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 30C62

We study $W_{loc}^{1,2}$ homeomorphisms f from the complex plane onto itself, that satisfy a general nonlinear Beltrami equation,

$$\bar{\partial}f(z) = \mathcal{H}(z, \partial f(z)),$$

and normalized by $f(0) = 0$ and $f(1) = 1$. Here $\mathcal{H}(z, w)$ is measurable as a function of z , and

$$|\mathcal{H}(z, w_1) - \mathcal{H}(z, w_2)| \leq k(z) |w_1 - w_2|,$$

with $\|k\|_{L^\infty} < 1$. Such solutions f always exist (see [1]), but they need not be unique. We will explain how uniqueness properties are related to the behavior of $k(z)$ as $|z| \rightarrow \infty$. This is a joint work with K. Astala, D. Faraco, J. Jääskeläinen and L. Székelyhidi.

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Two Valued Measure and Summability of Double Sequences

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In [1] and [2] Connor proposed two very interesting extensions of the concept of statistical convergence (see [5]) using a complete $0, 1$ valued measure μ defined on an algebra of subsets of N . The notion of statistical convergence was introduced for double sequences by Mursaleen and Eedely [7] (also by Móricz [6] who introduced it for multiple sequences). More references on double sequences can be seen in [3].

Here we first introduce the notions of μ -statistical convergence and convergence in μ -density (following the line of Connor [1]) using a two valued measure μ defined on an algebra of subsets of $N \times N$ and mainly investigate the inter-relationship between these two concepts. Next we focus on the Cauchy criteria and introduce the Cauchy conditions associated with the two types of convergence. Though one of them, namely μ -statistical Cauchy condition appeared in [2], the other Cauchy condition in μ -density and in particular the relation between these two concepts was never explored before. Finally we explore another relatively unexplored concept, namely, the divergence of double sequences of real numbers corresponding to the measure μ . We introduce a new property of the measure μ called (APO2) which plays the most important role all throughout the paper and show by example that this condition is strictly weaker than the condition (APO) of Connor [2].

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Scale Free Analysis and Applications: A Brief Report

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11A41, 26E30, 34F05

Can time (t) (that is to say, an independent real variable) change by inversions rather than simply by linear shifts (translations)? This is the central question, investigated over a period of about a decade, that now leads to the formulation of a *scale free analysis* accommodating inversions (jumps) $t \rightarrow t^{-1}$ as a valid mode of increments over and above the usual shifts $t \rightarrow t + \delta t$. This constitutes an extension of the ordinary real analysis to that of an infinite dimensional non-archimedean space accommodating nontrivial *valued infinitesimals* (and infinities). The so-called valued infinitesimals generate a countable number of nontrivial scales and transition between two such scales are performed by inversions. In this report we present a short review of the said formalism highlighting three different applications: (a) a re-interpretation of the so-called nonsmooth solutions of the scale free differential equation $t \frac{dx}{dt} = x$ [1] (b) formalism of an ultrametric analysis on a Cantor set [2, 3] and (c) an elementary proof of the Prime Number Theorem [4].

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On the Analytic Inverse of a Generalized Attenuated Radon Transform

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In X-Ray fluorescence computed tomography (xfct) a sample is irradiated with high intensity monochromatic synchrotron X-rays stimulating fluorescence emission detected by an outside detector. Part of the emission is absorbed by the sample. Mapping fluorescence emission density distributions has many important applications in medical imaging (malignancy analysis, for example).

A continuous mathematical model for xfct is given by the Generalized Attenuated Radon Transform

$$\mathcal{R}_{\text{xfct}}f(t, \theta) = \int_{\tau(t, \theta)} dx f(x) e^{-\mathcal{D}\lambda(x, \theta + \pi)} \int_{\Gamma} d\gamma e^{-\mathcal{D}\mu(x, \theta + \gamma)} \quad (1)$$

where f is the emission density, λ is a fixed transmission attenuation map, μ is the fluorescence attenuation map and Γ is the angle section for stimulated fluorescence rays starting from each point over the line $\tau(t, \theta) = \{x \in \mathbb{R}^2 : x \cdot \xi_\theta = t\}$. The operator \mathcal{D} is the divergent beam transform defined below, where we have used the notation $\xi_\theta = (\cos \theta, \sin \theta)$

$$\mathcal{D}a(x, \theta) = \int_0^\infty a(x + s\xi_\theta^\perp) ds. \quad (2)$$

In this work, following Fokas and co-authors approach [1] for the Attenuated Radon Transform, we develop a new analytic inverse for (1). By using an appropriate change of variables, we obtain *d-bar* equations that lead to a Riemann-Hilbert problem, whose solution, in turn, leads to the inversion formula (details in [2]). We also present some experiments with simulated and real data.

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Coincidences and Common Fixed Points in Intuitionistic Fuzzy Metric Spaces

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In the present paper, first we prove a coincidence theorem for a family of mappings on an arbitrary set with values in an intuitionistic fuzzy metric space. We further establish a common fixed point theorem. We generalize and extend some of the results of Jesic and Babacev [1] and Mishra, Singh and Chadha [2] to intuitionistic fuzzy metric spaces. Our results fuzzify and generalize several results on metric, fuzzy and Menger spaces.

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On Singular Integrals Defined on Non-doubling Measure Metric Spaces and Krein's Theorem

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In this talk we consider singular integral operators defined on non-doubling measure metric spaces of finite measure. We present necessary and sufficient

conditions for the boundedness of these operators on inhomogeneous Lipschitz spaces. Since in these context, Lipschitz spaces are continuously embedded in L^2 spaces, as an application of our result, we use Krein's theorem to obtain boundedness on L^2 when the singular integral and its adjoint are bounded operators on the Lipschitz spaces and these spaces are dense in L^2 . These results appeared in the reference below [1]

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Approximation of and by the Riemann Zeta-function

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It is possible to approximate the Riemann zeta-function by meromorphic functions which satisfy the same functional equation and satisfy (respectively do not satisfy) the analogue of the Riemann hypothesis.

In the other direction, it is possible to approximate meromorphic functions by various manipulations of the Riemann zeta-function.

This abstract is based on [3] which summarizes and extends works by my students [1], [2] and [7] and with co-authors [4], [5], and [6].

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Subsets of the Square with the Continuous and Order Preserving Fixed Point Property

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This study characterizes the convex sets whose complements in the unit square exhibit the fixed point property for continuous and order preserving mappings.

If every continuous and order preserving function on an ordered space has a fixed point, then the space is said to have the *cop fpp*.

Here we show that subsets of line have the cop fpp only if they already have the fixed point property for order preserving maps alone.

The analogous result for subsets of R^2 is not true. Hence our interest lies in identifying subsets of R^2 that have the cop fpp. This work characterizes the most basic category of such spaces, the complement of a convex set interior to the unit square in the plane. Characterization of such sets with the cop fpp requires verifying some set inclusions after a finite iterative construction.

The characterization presented here involves the shape and girth of the convex set, K , and has unexpected consequences. For example, let

$$K_\alpha = \{(x, y) \in [0, 10] \times [0, 10] : x - 2 < y < x + \alpha\}.$$

Then every continuous and order preserving mapping of

$$P_\alpha = [-1, 11] \times [-1, 11] - K_\alpha$$

has a fixed point if and only if

$$\alpha \in \left[-2, \frac{2}{5}\right] \cup \left(\frac{2}{3}, 1\right] \cup (4, 6].$$



Sandwich-type Theorems of Some Subclasses of Multivalent Functions Associated with a Differintegral Operator

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Let \mathcal{H} be the class of functions analytic in the open unit disk $\mathcal{U} := \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$ and \mathcal{A}_p be the subclass of \mathcal{H} consisting of functions of the form $f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n$ ($z \in \mathcal{U}$). For $f \in \mathcal{A}_p$, let

$$\mathcal{I}_{p,\delta}^\lambda f(z) = z^p + \sum_{n=p+1}^{\infty} \left(\frac{p+\delta}{n+\delta} \right)^\lambda a_n z^n \quad (\lambda \in \mathbb{R}, \delta > -p, z \in \mathcal{U}).$$

The operator $\mathcal{I}_{p,\delta}^\lambda$ includes and generalizes several previously studied familiar operators (for details see [1]). It is observed that $\mathcal{I}_{p,\delta}^\lambda$ behaves like an integral operator for $\lambda > 0$ and a differential operator for $\lambda < 0$. Thus $\mathcal{I}_{p,\delta}^\lambda$ can be thought of as a *fractional differintegral operator*. In the present paper subordination and superordination results of some subclasses of multivalent functions associated with $\mathcal{I}_{p,\delta}^\lambda$ are investigated. As consequence, differential sandwich-type theorems for the above classes are presented. Relevant connections of the results, which are presented in this paper, with various other known results are also pointed out.

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Recent Developments Regarding the Halo Conjecture

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The Halo Conjecture has long provided a fascinating open problem in the theory of differentiation of integrals. Recent progress towards the resolution of this conjecture will be discussed, in particular the theorem of Hagelstein and Stokolos that any density basis consisting of a homothety invariant collection of convex sets must necessarily differentiate L^p for sufficiently large p . Connections between this result, the work of Bateman and Katz on Kakeya sets and directional maximal operators, and improvements on the well-known theorem of Córdoba and Fefferman relating the L^p bounds of geometric maximal operators to those of certain multiplier operators will also be given.

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Quasianalytic Functions of Several Variables in the Sense of Gonchar

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Let $K \subset \mathbf{C}^n$ be arbitrary compact set and let $f(z)$ be a continuous function on K . By $\rho_m(f, K)$ we denote the least deviation of $f(z)$ from the best approximation of $f(z)$ on K by rational functions of degree less than or equal to m : $\rho_m(f, K) = \inf_{r_m} \|f - r_m\|_K$. Here $\|\cdot\|_K$ is the uniform norm and the infimum is taken over all rational functions of the form

$$r_m(z) = \frac{\sum_{\alpha \leq m} a_\alpha z^\alpha}{\sum_{\alpha \leq m} b_\alpha z^\alpha}, \quad \text{where } \alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \text{ is a multiindex.}$$

As usual, we denote by $e_m(f, K)$ the least deviation of function $f(z)$ on K from its polynomial approximation of degree less than or equal to m . Obviously, $\rho_m(f, K) \leq e_m(f, K)$ for each $m = 1, 2, \dots$. In ([1], [2]) Gonchar proved that if $K = [a, b] \subset \mathbf{R} \subset \mathbf{C}$, then the class of functions $R([a, b]) = \{f \in C[a, b]: \lim_{m \rightarrow \infty} \sqrt[m]{\rho_m(f, K)} < 1\}$ possesses one of the important properties of the class of analytic functions: if $\lim_{m \rightarrow \infty} \sqrt[m]{\rho_m(f, K)} < 1$ and $f(x) = 0$ on a set $E \subset [a, b]$ of positive logarithmic capacity, then $f(x) \equiv 0$ on $[a, b]$. By analogy with the class $B(K) = \{f \in C(K): \lim_{m \rightarrow \infty} \sqrt[m]{e_m(f, K)} < 1\}$, which is called the class of quasianalytic functions of Bernstein ([3],[4]), we call $R(K) = \{f \in C(K): \lim_{m \rightarrow \infty} \sqrt[m]{\rho_m(f, K)} < 1\}$ the class of quasianalytic functions of Gonchar. We consider the quantity $\rho_m^*(f, K) = \inf_{p_m, q_m} \|q_m f - p_m\|_K$, where p_m, q_m are polynomials with degrees less than or equal to m and $\|q_m\|_K = 1$ for all $m = 1, 2, \dots$. We introduce the following class of functions $R^*(K) = \{f \in C(K): \lim_{m \rightarrow \infty} \sqrt[m]{\rho_m^*(f, K)} < 1\}$. Clearly, $B(K) \subset R(K) \subset R^*(K)$. Our main theorem states that if $f \in R^*(K)$ and $f(x) = 0$ on a nonpluripolar set E , then $f(x) \equiv 0$ on K .

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A Canonical δ -hyperbolic Metric for Metric Spaces

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Suppose (Z, d) is a locally compact noncomplete metric space and let \overline{Z} be its metric completion. For $x \in Z$, we let $d_Z(x) = \text{dist}(x, M)$, where $M = \overline{Z} \setminus Z$. In geometric function theory the quantities

$$\frac{|dx|}{d_Z(x)} \quad \text{and} \quad \frac{d(x, y)}{d_Z(x)d_Z(y)}$$

are rather ubiquitous. They are used in the definitions of various metrics, such as the Poincaré, Barbilian and Apollonian metrics ([1],[2]), the hyperbolic cone metric ([3]), the j -metric and the quasihyperbolic metric ([4],[5]) and the hyperbolic metric of the hyperspaces ([6]).

We introduce the following distance function on Z

$$u_Z(x, y) = 2 \log \left(\frac{d(x, y) + \max\{d_Z(x), d_Z(y)\}}{\sqrt{d_Z(x)d_Z(y)}} \right)$$

and prove: (1) u_Z is a metric, (2) the space (Z, u_Z) is Gromov δ -hyperbolic with $\delta \leq \log 4$ and (3) the identity map $\text{id}_Z: (Z, d) \rightarrow (Z, u_Z)$ is a homeomorphism. The metric u_Z can be thought of as a canonical Gromov hyperbolic metric of Z since it appears that many Gromov hyperbolic metrics introduced in geometric function theory are special cases of u_Z (up to a quasiisometry). We verify this claim for the metrics mentioned above. In particular, this in combination with (2) gives alternative proofs of the Gromov hyperbolicity of these metrics.

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A Study of Functions Associated with Mock Theta Functions

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Ramanujan gave a number of Mock Theta Functions of different orders and C. Truedell defined the differential difference equations. We consider the q -analogue of the differential difference equation and call it q -differential difference equation. Then, we give certain generalized functions of different orders of Ramanujans Mock Theta Functions satisfying this equation. and discuss their properties. Further, We give inter-relationships between these generalized functions of different orders. We also give integral representations for these functions.

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A Generalized Common Fixed Point Theorem in Fuzzy Metric Space

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The study of common fixed points of mappings in a fuzzy metric space satisfying certain contractive conditions has been at the center of vigorous research activity. The concept of fuzzy sets was initiated by L. Zadeh [7] in 1965. With the concept of fuzzy sets, the fuzzy metric space was introduced by O. Kramosil and J. Michalek [6] in 1975. Also, S. Heilpern [3] in 1981 first proved a fixed point theorem for fuzzy mappings. Again, M. Grabiec [2] in 1988 proved the contraction principle in the setting of the fuzzy metric space. Moreover, A. George and P. Veeramani [1] in 1994 modified the notion of fuzzy metric spaces with the help of continuous t-norm, by generalizing the concept of probabilistic metric space to fuzzy situation. The concept of compatible mappings in metric space was introduced by G. Jungck [4] in 1988. Also, the notion of weakly compatible mappings in metric space was introduced by G. Jungck and B.E. Rhoades [5] in 1998. With these two compatible and weakly compatible concepts, there exist several interesting results in the literature on fixed point theorems in fuzzy metric space.

The main objective of this paper is to establish a common fixed point theorem in fuzzy metric space under the weak compatible conditions which generalizes and improves various similar results of fixed points.

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Characterization of Totally Bounded Subsets of Locally Compact Group G Through Almost Periodic Like Families

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In this paper We introduce the concept of almost periodic like families using the definitions of almost periodic functions that are given by J Nuemaan, W Maak and H Bohr and characterize the totally bounded subsets of locally compact groups.

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Reliability Analysis of Preventively Maintained System Using Finite Element Method

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In this paper finite element method [1] is proposed to solve the system of partial differential equations determining the reliability of the five unit preventively maintained system having variable failure and repair rates. The mathematical formulation of the five units system is carried out using supplementary variable technique [2, 3, 4, 5] and [6]. The long run availability and other parameters have also been computed. Certain conclusions based on this analysis are finally discussed.

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Inclusion Properties of a Subclass of Analytic Functions Defined by an Integral Operator

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In the present paper we introduce a new subclass of analytic functions in the unit disc defined by convolution $(f_\mu)^{-1} * f(z)$, where

$$f_\mu = (1 - \mu)z {}_2F_1(a, b; c; z) + \mu z (z {}_2F_1(a, b; c; z))'.$$

Several interesting properties of the class and integral preserving properties of the subclasses are also considered.

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Some Connections Among Generalized Hypergeometric Functions, q -hypergeometric Functions and Pathway Model and their Applications

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 33C05, 33C15, 33C20, 33C60, 33D15, 44A20

The idea of pathway model introduced by Mathai [4, 5] is used to obtain pathways among generalized hypergeometric functions. Some connections between generalized hypergeometric functions and basic hypergeometric functions (q -series) [6] are established. Possible application of the above pathways in applied analysis, reaction rate theory in astrophysics and micro-economics are examined [2, 1]. The paper also examines the possible extensions of the above idea to the multi-variate and matrix-variate cases. The behavior of the pathways are studied.

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Prolate Spheroidal Wavelet Coefficients, Frames and Double Infinite Matrices

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The study of continuous prolate spheroidal wave functions (PSWFs) has been an active area of research in both electrical engineering and mathematics. The PSWFs are those that are most highly localized simultaneously in both the time and frequency domain. This fact was discovered by Slepian and his collaborators and was presented in a series of articles [4], [5] and [8]–[10]. In this paper we define the double infinite matrix $A = a(m, n, k)$ and study the action of A on $f \in L^2(R)$ and on its prolate spheroidal wavelet coefficients. We also find the frame condition for A -transform of $f \in L^2(R)$ whose wavelet series expansion is known.

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A Parseval Equation and the Distributional Finite Generalized Hankel-Clifford Transformation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 44, 46F12

In this paper, we study the finite generalized Hankel-Clifford transformation on spaces of generalized functions (distributions) by developing a new procedure. We consider two finite generalized Hankel-Clifford type transformations $\hbar_{\alpha,\beta}$ and $\hbar_{\alpha,\beta}^*$ connected by the Parseval equation $\sum_{n=0}^{\infty} (\hbar_{\alpha,\beta} f)(n) (\hbar_{\alpha,\beta}^* \phi)(n) = \int_0^1 f(x) \phi(x) dx$. A space $S_{\alpha,\beta}$ of functions and a space $L_{\alpha,\beta}$ of complex sequences are introduced. $\hbar_{\alpha,\beta}^*$ is an isomorphism from $S_{\alpha,\beta}$ onto $L_{\alpha,\beta}$ when $(\alpha - \beta) \geq -\frac{1}{2}$. We propose to define the distributional finite generalized Hankel-Clifford transformation $\hbar'_{\alpha,\beta} f$ of $f \in S'_{\alpha,\beta}$ by $\langle (\hbar'_{\alpha,\beta} f), (\hbar_{\alpha,\beta}^* \phi)(n) \rangle_{n=0}^{\infty} = \langle f, \phi \rangle$ for $\phi \in S_{\alpha,\beta}$.

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A Note on Convex Combinations

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2010 MATHEMATICS SUBJECT CLASSIFICATION. 30 C 45

Let \mathcal{A} denote the class of normalized functions defined in the unit disc $\mathcal{U} = \{z : |z| < 1\}$. Using the Ruschewyeh derivative $D^\delta f$, we define the class $V_k^\lambda(\beta, \delta)$ which includes convex and starlike functions of order β , spirallike functions of order β , bounded boundary rotation etc. Certain radii results concerning linear combinations of analytic functions in this class are studied.

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Spectral Analysis on Self-similar Sets and Spectral Zeta Function of Fractals

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 30, 30B50

The Laplacian operator is one of the most important operators studied in the theory of analysis on manifolds. To define a differential operator like the Laplacian on fractals is not possible from the classical viewpoint of analysis. We construct the Laplacian on finitely-ramified self-similar fractals, such as the Sierpinski gasket and discuss its spectrum. The decimation method is a process that describes the relationship between the spectrum of the Laplace operator and the dynamics of the iteration of a certain polynomial on \mathbb{C} . Furthermore, we discuss the spectral zeta function of the Laplacian. Teplyaev discovered the product structure of the spectral zeta function in the case of Sierpinski gasket that involves a geometric part and a new zeta function of a polynomial induced by the decimation method. An interesting feature of the product structure is the cancellation phenomenon between the poles of the zeta function of a polynomial and the zeros of the geometric part of the spectral zeta function of the Laplacian. Initially, M. Lapidus illustrated a similar product structure for self-similar fractal strings. Briefly, we will discuss the renormalization map of several complex variables induced by a Sturm-Liouville operator on the interval studied by Sabot and the corresponding spectral zeta function.

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Continued Fractions and Linear Fractional Transformations. Why do Continued Fractions Converge so Well?

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 40A15

A continued fraction

$$\mathbf{K}(a_n/b_n) = \frac{a_1}{b_1 + \frac{a_2}{b_2 + \frac{a_3}{b_3 + \ddots}}} = \frac{a_1}{b_1} + \frac{a_2}{b_2} + \frac{a_3}{b_3} + \cdots$$

can be viewed as a sequence $\{S_n\}$ of linear fractional transformations constructed by compositions $S_n = s_1 \circ s_2 \circ \cdots \circ s_n$ where $s_k(w) = a_k/(b_k + w)$. The amazing convergence properties of continued fractions are connected to this fact. In this talk we shall look closer at what it is with the linear fractional transformations that makes this work so well.



Additive Functional Equations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 39B52, 39B82

In this paper, the author investigate the general solution and generalized Ulam-Hyers-Aoki-Rassias stability of a additive functional equation of the form

$$h\left(\frac{xy + zw}{u}\right) = h\left(\frac{xy}{u}\right) + h\left(\frac{zw}{u}\right) \quad (3)$$

with $u \neq 0$. As a particular case, when $u = 1$ the above equation (1) is transformed in to

$$h(xy + zw) = h(xy) + h(zw)$$

and investigate its generalized Ulam-Hyers-Aoki-Rassias stability.

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Discrete Fourier Transform and Jacobi θ Function Identities

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The discrete Fourier transform(DFT) is an important tool for applications in engineering and physics, and it is also a source of interesting mathematical problems. The trace of the DFT matrix is the well known Gauss sum up to normalization factor. (see Ref [1]). One of the important problems associated with DFT is to have wider class of eigenfunctions. Matveev [2] has proven beautiful consequences of the fact that the DFT Φ is a fourth root of unity i.e. $\Phi^4 = I$. Given any absolutely summable series g_n , he has constructed eigenfunctions of the DFT from the series g_n . This is then applied for the case when the series arises as the summands of a ν -theta function with characteristic (a,b) namely $\theta_{a,b}(x, \tau, \nu)$. This reduces to usual theta function when $\nu = 1$. We extend the result of Matveev to derive identities of classical Jacobi theta functions using properties of eigenvectors of the DFT $\Phi(2)$. We obtain an extended Watson addition formula and as a particular case of it Watson addition formula. We prove fourth order Riemann identity of theta functions from which all the well known classical fourth order identities of theta functions can be proved. All these classical identities are derived from eigenvectors of the DFT $\Phi(2)$. There is a natural extension of these identities corresponding to theta functions on $\frac{1}{3}Z$ using DFT $\Phi(3)$. We obtain corresponding version of Watson addition formula corresponding to DFT $\Phi(3)$. The method we use is conceptually simple and doesn't depend on the properties of zeros of theta functions and there infinite product representation.

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Fourier Multipliers on Totally Disconnected Groups

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We characterize atomic Hardy spaces on unbounded locally compact Vilenkin groups by means of a modified maximal function. The obtained Fourier multiplier theorem is more general than the corresponding results due to Kitada [4], Onneweer-Quek [5] and Daly-Phillips [3] that were proved under the boundedness assumption on the underlying group. Namely, we prove the following result:

Let $\phi \in L^\infty(\Gamma)$ and $\sup_N \int_{G_N^c} |(\phi - \phi_{N+1})^\vee(y)| dy = O(1)$, where $\phi_{N+1} = \phi 1_{\Gamma_{N+1}}$ and \wedge, \vee denote respectively the Fourier transform and the inverse Fourier transform. Then ϕ is a multiplier on H^1 .

In the compact case we prove a multiplier theorem providing conditions on Fourier coefficients.

We give an example that shows the sharpness of Quek's result on weak type multipliers for Lipschitz functions [6].

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A Coefficient Inequality for a Subclass of Carathéodory Functions Defined by Conical Domains

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 30C

For $0 \leq k < \infty$, let Ω_k be the conical domain in the complex plane defined by $\Omega_k = \{w \in C : w = u + iv, u^2 > k^2((u-1)^2 + v^2), u > 0\}$. Let $q_k(z)$ be the Riemann map of $\mathcal{U} := \{z \in C : |z| < 1\}$ onto Ω_k satisfying $q_k(0) = 1, q'_k(0) > 0$. Let $\mathcal{P}(q_k)$ be the class of analytic functions $h(z)$ subordinate in \mathcal{U} to $q_k(z)$ and represented by $h(z) = 1 + b_1z + b_2z^2 + \dots, (z \in \mathcal{U})$. Sharp estimates for $|b_2 - ub_1^2|, (-\infty \leq u < \infty)$, are found in this note. This result improves upon an estimate of Kanas both in terms of bounds and ranges of the parameter u , [S.Kanas, Coefficient estimates in subclasses of the Carathéodory class related to conical domains, Acta Math. Univ. Comeniana, LXXIV, 2(2005) 149-161].

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Inclusion Properties of a Certain Subclass of Strongly Close-to-convex Functions

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 30C45, 30C50

The purpose of this paper is to derive some inclusion and argument properties of a new subclass of strongly close-to-convex functions defined in the open unit disc. An integral operator is defined by convolution with a hypergeometric function. The subclass also extends to the class of quasi-convex and close-to-convex functions and α -spirallike functions of complex order.

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Hardy-type Inequalities via Superquadratic and Subquadratic Functions

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Primary 26D15, Secondary 26A51

Let the positive function ϕ be superquadratic for $p \geq 2$ and subquadratic for $1 < p \leq 2$, and such that $Ax^p \leq \phi(x) \leq Bx^p$ holds on \mathbb{R}^+ for some constants $A \leq B$. We derive a general class of new Hardy-type integral inequalities with power weights for $p \geq 2$. The inequalities are reversed for $1 < p \leq 2$, while equality holds for $p = 2$. The related dual inequalities are also derived and discussed. The main tool used in the proofs are some new results for superquadratic and subquadratic functions. The results obtained unify and extend several inequalities of Hardy-type known in the literature.



Some Characterization Properties of Classes of p -valent Analytic Functions Involving Certain Integral Operator

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 30C45, 30C55, 33C60

In this paper we study characterization properties of some classes of p -valent analytic functions involving a normalized repeated Erdelyi-Kober fractional integral operator $Ef(z) \equiv E_{(\beta_i),m}^{(\gamma_i),(\delta_i)} f(z)$ [2,4] for integer $m \geq 1$, $\delta_i \geq 0$, $\gamma_i \in \mathfrak{R}$, $\beta_i > 0$, $i = 1, 2, \dots, m$ and its operated function $E^j f(z)$. Specially we investigate some inclusion relations, class preserving properties of convolution of two functions and of an integral operator for these classes. In obtaining inclusion relations between these classes we use Briot- Bouquet differential subordination method in the form of Miller-Mocanu [3] and Hallenback-Ruscheweyh [1] lemmas.

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An Abstract Approach to Henstock Integral

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We present an approach to Henstock integral with a setup which is closer to classical measure theory. The Henstock integral on Euclidean spaces is a notion more general than Lebesgue integral. In particular for a mapping defined on a closed interval on real line every derivative is integrable and therefore the second fundamental theorem of calculus need not assume integrability of derivative. This is essential for giving complete treatment of differential calculus in Banach spaces as compared to the treatment given in (1) or (2).

Let X be a nonempty set. Consider an algebra of subsets of X . The members of the algebra are called as bases.

A tagged collection S is a collection of pair (e, E) where E is a base and e is in E . The same base can appear more than once. A measure m is a finitely additive nonnegative real valued function on collection of bases with $m(\phi) = 0$. A gauge δ is a nonnegative real valued function on X with $\delta(x) > 0$ for each $x \in X$.

A subset A of X is termed admissible if there exists a partition of A , that is a finite collection of Bases E_k such that $\cup E_k = A$ and E_k 's are pairwise disjoint.

Cousin's axiom: Given an admissible set A and a gauge δ there exists a tagged partition $\{(e_k, E_k)\}$ of A such that $m(E_k) > \delta(e_k)$ for each k . Such a partition is called a δ - fine partition.

Given a map $f : A \rightarrow Y$ (a Banach space) on an admissible set A we say f is integrable on A with integral as $L \in Y$, if given $\epsilon > 0$ there exists a gauge δ such that for all δ fine partitions P of A

$$|\sum f(e_k)m(E_k) - L| < \epsilon \dots (1)$$

A map f on a set X is integrable on X if in addition for each $\epsilon > 0$ there exists a real number M such that inequality (1) holds for each admissible set A with $m(A) \geq M$.

We can show that Henstock's lemma, usual linearity properties as well as Cauchy criterion holds.

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Determination of Jumps by Fourier-Jacobi Coefficients

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We say that a function w is a generalized Jacobi weight and write $w \in GJ$ if

$$w(t) = h(t)(1-t)^\alpha(1+t)^\beta |t-x_1|^{\delta_1} \dots |t-x_M|^{\delta_M}$$

$$h \in C[-1, 1], h(t) > 0, (|t| \leq 1), \omega(h; t; [-1, 1])t^{-1} \in L[0, 1],$$

$$-1 < x_1 < \dots < x_M < 1, \alpha, \beta, \delta_1, \dots, \delta_M > -1.$$

By $\sigma(w) = (P_n(w; x))_{n=0}^\infty$ we denote the system of algebraic polynomials $P_n(w; x) = \gamma_n(w)x^n +$ lower degree terms with leading positive coefficients $\gamma_n(w)$, which are orthonormal on $[-1, 1]$ with respect to the weight $w \in GJ$.

Theorem 1. Let f be a function of bounded p -variation, i.e. $f \in \mathcal{V}_p, p > 1$, such that $fw \in L[-1, 1], w \in GJ$. Then the sequence $(a_n(w; f)P'_n(w; x))$ is (C, α) , $\alpha > 1 - \frac{1}{p}$ summable to $\frac{(1-x^2)^{-\frac{1}{2}}}{\pi}(f(x+0) - f(x-0))$ for every $x \in (-1, 1), x \neq x_1, \dots, x_M$, where $a_n(w; f)P'_n(w; x)$ is the n -th term of the differentiated Fourier-Jacobi series of f .

By a theorem of Avdispahić [1], there exist the following inclusion relations between the classes $\mathcal{V}_p, \Lambda BV$ and $V[\nu]$ of generalized bounded variation in the sense of Wiener, Waterman and Chanturiya

$$\{n^\alpha\}BV \subset \mathcal{V}_{\frac{1}{1-\alpha}} \subset V[n^\alpha] \subset \{n^\beta\}BV, \quad \text{for } 0 < \alpha < \beta < 1.$$

Therefore, we have

Theorem 2. If f belongs to $\{n^\beta\}BV$ or $V[n^\beta]$, then the claim of Theorem 1 is valid for $(C, \alpha), \alpha > \beta$.

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Continuity of Pseudo-Differential Operator $h_{\mu,a}$ Involving Hankel Translation and Hankel Convolution on Some Gevrey Spaces

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The Pseudo-Differential Operator (p.d.o.) $h_{\mu,a}$ associated with the Bessel Operator involving the symbol $a(x, y)$ whose derivatives satisfy certain growth conditions depending on some increasing sequences is studied on certain Gevrey spaces. The p.d.o. $h_{\mu,a}$ on Hankel translation τ and Hankel convolution of Gevrey functions is continuous linear map into another Gevrey spaces.



A Note on Certain Classes of Transformations

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The present paper deals with the derivation of a new classes of transformation formulae involving several variables in terms of the multiple q -series identities.

The results obtained, besides being capable of unifying, and providing extensions to various transformations and reduction formulae, also yields several new formulae. The applications of the main results are exhibited by considering some examples in the concluding section.

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Related Measures and L-orthogonal Polynomials

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Let ψ be a bounded, non decreasing function on $[a, b]$, $0 \leq a < b \leq \infty$, with infinitely many points of increase in $[a, b]$ and such that all the moments $\mu_n = \int_a^b t^n d\psi(t)$, $n = 0, \pm 1, \pm 2, \dots$, exist. We consider the sequence of monic polynomials $\{Q_n\}_{n=0}^\infty$ defined by $\int_a^b t^{-n+s} Q_n(t) d\psi(t) = 0$, $s = 0, 1, \dots, n-1$.

The sequence $\{t^{-\lfloor (n+1)/2 \rfloor} Q_n(t)\}$ forms a sequence of orthogonal Laurent polynomials with respect to the measure ψ . Thus for convenience, $\{Q_n\}$ is referred here as a sequence of L-orthogonal polynomials. It is known that

$$Q_{n+1}(z) = (z - \beta_{n+1})Q_n(z) - \alpha_{n+1}zQ_{n-1}(z), \quad n \geq 1,$$

with $Q_0(z) = 1$ and $Q_1(z) = z - \beta_1$, where $\beta_n > 0$ and $\alpha_{n+1} > 0$, $n \geq 1$.

Such L-orthogonal polynomials were first considered in [2] in order to solve the strong Stieltjes moment problem. Gaussian type quadrature rules involving these polynomials were treated, for example, in [1] and [5]. Studies of polynomials satisfying recurrence relations of the above type have appeared prior to [2] in the theory of continued fractions and two-point Padé approximants (see [3] and [4]).

Let ψ_0 and ψ_1 be two strong positive measures supported within $[a, b]$ and connected to each other by $(t - \kappa)d\psi_1(t) = \gamma d\psi_0(t)$. For $i = 0, 1$, let $\{Q_n^{(i)}\}$ be

the sequence of monic L-orthogonal polynomials with respect to the measure ψ_i .

Considering the connection formulas that exist between the two sequences of polynomials $\{Q_n^{(0)}\}$ and $\{Q_n^{(1)}\}$, some useful relations that exist between the coefficients of the respective three term recurrence relations are obtained. As an application of these relations, some monotonicity properties of the zeros of certain L-orthogonal polynomials are derived.

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Analysis and Applications of Some Modified BESSEL Functions

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The properties of kernels of KONTOROVITCH–LEBEDEV integral transforms–modified BESSEL functions of the second kind with pure imaginary order $K_{i\beta}(x)$ and with complex order $K_{1/2+i\beta}(x)$ are elaborated. Some new representations of these functions and transforms are justified. The approximation and computation of kernels of KONTOROVITCH–LEBEDEV integral transforms–modified BESSEL functions of the second kind with pure imaginary order $K_{i\beta}(x)$ and with complex order $K_{1/2+i\beta}(x)$ are elaborated on the basis of several approaches [1-3].

The inequalities which give estimations for their kernels - the real and imaginary parts of the modified Bessel functions of the second kind $ReK_{1/2+i\beta}(x)$ and $ImK_{1/2+i\beta}(x)$ for all values of the variables x and β are obtained. A proof of

inversion formulas and PARSEVAL equations for modified KONTOROVITCH-LEBEDEV integral transforms is developed [3].

The hypergeometric type differential equations of the second order with polynomial coefficients are considered. The computational scheme of Tau method is extended for the systems of hypergeometric type differential equations [4].

The effective applications of the modified BESSEL functions for the numerical solution of some mixed boundary value problems in wedge domains are given. The KONTOROVITCH-LEBEDEV integral transforms and dual integral equations are used. The analysis of using of these functions and transforms is elaborated in detail [5].

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A Common Fixed Point Theorem for Strict Contractive Condition

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In this paper we establish a common fixed point theorem for a quadruple of non-continuous mappings by using a strict contractive condition and property (E.A.) under occasionally weakly compatible maps. Our result generalize and extend the result of Singh-Pant [1] and others.

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Radon Transforms on the Heisenberg Group and Transversal Radon Transforms

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Let $\mathbf{H}_n = \mathbf{C}^n \times \mathbf{R}$ be the Heisenberg group with multiplication $(z, t) \circ (\zeta, \tau) = (z + \zeta, t + \tau - \text{Im}(z \cdot \bar{\zeta})/2)$. We study the Heisenberg-Radon transform

$$(R_H f)(z, t) = \int_{\mathbf{C}^n} f((z, t) \circ (\zeta, 0)) d\zeta, \quad (z, t) \in \mathbf{H}_n, \quad (4)$$

which was introduced by Strichartz [2], who obtained a weak L^2 -type inversion formula for R_H and a mixed norm estimate on L^p functions with $1 \leq p \leq p_0$, $p_0 = 1 + 1/(2n + 1)$. We also consider the transversal Radon transform

$$(R_T F)(a, b) = \int_{\mathbf{R}^{m-1}} F(x', a \cdot x' + b) dx', \quad (a, b) \in \mathbf{R}^{m-1} \times \mathbf{R} = \mathbf{R}^m, \quad (5)$$

which integrates a function F on \mathbf{R}^m over hyperplanes $x_m = a \cdot x' + b$, transversal to the last coordinate axis.

If $m = 2n + 1$, then, changing variables $a = (-\text{Im } z, \text{Re } z)/2$, $b = t$, one can write R_H in the form (5). Using this connection, we prove that for $f \in L^p(\mathbf{H}_n)$ the Radon transform (4) is finite a.e. provided that $1 \leq p < 1 + 1/2n$. This bound is sharp and improves the upper bound p_0 in [2].

We obtain new boundedness results and explicit inversion formulas for R_H and R_T (for any $m \geq 2$, not only odd) on L^p functions in the full range of the parameter p . We also show that these transforms are isomorphisms of the corresponding Semyanisty-Lizorkin spaces of smooth functions. In the framework of these spaces we obtain inversion formulas which are pointwise counterparts of the corresponding weak-type formulas in [2].

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On a Class of Harmonic Univalent Functions Defined by a Linear Operator

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Let S_H denote the class of functions $f = h + \bar{g}$, which are harmonic, univalent and sense preserving in the unit disc Δ , where

$$h(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad g(z) = \sum_{k=1}^{\infty} b_k z^k, \quad z \in \Delta.$$

We define a new subclass $SHL(\alpha, \beta)$ of S_H by using a linear operator of harmonic univalent functions

$$\mathcal{L}(a, c)f(z) = z + \sum_{k=2}^{\infty} \frac{(a)_k}{(c)_k} z^k, \quad a \in \mathbb{R}, \quad c \in \mathbb{R} - \mathbb{Z}_0^- = \{0, -1, -2, \dots\}, \quad z \in \Delta.$$

In this paper, coefficient bounds, distortion bounds and extreme points are obtained for the class $SHL(\alpha, \beta)$.

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Some Difference Double Sequence Spaces Defined by Orlicz Function

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In this article we introduce some vector valued difference double sequence spaces defined by Orlicz function.

An Orlicz function M is a mapping $M : [0, \infty) \rightarrow [0, \infty)$ such that it is *continuous, non-decreasing* and *convex* with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \rightarrow \infty$, as $x \rightarrow \infty$.

For a seminorm q , we introduce the following difference double sequence spaces.

$${}_{2}\ell_{\infty}(M, \Delta, q) = \{ \langle a_{nk} \rangle \in {}_{2}w(q) : \sup_{n,k} M \left(q \left(\frac{\Delta a_{nk}}{\rho} \right) \right) < \infty, \text{ for some } \rho > 0 \}$$

$${}_{2}c(M, \Delta, q) = \{ \langle a_{nk} \rangle \in {}_{2}w(q) : \lim_{n,k} M \left(q \left(\frac{\Delta a_{nk} - L}{\rho} \right) \right) = 0, \text{ for some } \rho > 0 \}$$

Also $\langle a_{nk} \rangle \in {}_{2}c^R(M, \Delta, q)$ i.e. regularly convergent if $\langle a_{nk} \rangle \in {}_{2}c(M, \Delta, q)$ and the following limits hold:

There exists $L_k \in X$, such that $M \left(q \left(\frac{\Delta a_{nk} - L_k}{\rho} \right) \right) \rightarrow 0$, as $n \rightarrow \infty$, for some $\rho > 0$ and all $k \in N$.

There exists $J_n \in X$, such that $M \left(q \left(\frac{\Delta a_{nk} - J_n}{\rho} \right) \right) \rightarrow 0$, as $k \rightarrow \infty$, for some $\rho > 0$ and all $n \in N$.

We study some of their properties like solidness, symmetricity, completeness etc. and some inclusion results will be proved.

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On Some Metrical and Algebraic Questions for General Nonholonomic Spaces

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We study tangent cone questions for quasimetric spaces with dilations, in particular sub-Riemannian manifolds in a general setting [4].

On a (quasi)metric space, dilations can be defined as continuous one-parameter families of contractive homeomorphisms given in a neighborhood of each point. Most important examples of such spaces are sub-Riemannian manifolds which model nonholonomic processes and naturally arise in many applications.

We prove the existence of the tangent cone, provided dilations accord with the quasimetric in a certain way. The notion of the tangent cone to a quasimetric space was introduced in [5].

If, additionally, the limit of a certain combination of dilations exists, we prove that the tangent cone to the given quasimetric space is a graded nilpotent Lie group (this is a joint result with S. K. Vodopyanov [6]). The proof of this fact uses algebraic tools, in particular Mal'cev's theorem on local and global topological groups which helps to overcome difficulties concerned with investigation of a local version of the H5 Problem.

The motivation of our investigation can be found in [2, 4, 5, 6], see also [1, 3] on related topics. We consider not metrics but quasimetrics (for which the triangle inequality holds only in a generalized sense: $d(u, v) \leq Q(d(u, w) + d(w, v))$, $1 \leq Q < \infty$), according to a general situation [4] when the intrinsic Carnot-Carathéodory metric on a sub-Riemannian manifold might not exist.

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Some Inequalities for Harmonic Univalent Maps Involving Wright Generalized Hypergeometric (Wgh) Functions

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A class S_H of sense preserving harmonic univalent maps is introduced by Cunie and Sheil-Small [2] in 1984. Sufficient coefficient conditions for a map $f \in S_H$ to be in classes S_H^* and K_H are obtained by Jahangiri [3,4]. In this paper, a harmonic univalent map W generated by Wright's generalized hypergeometric (Wgh) functions [5] is considered. Under certain convergence conditions, derived with the help of Gauss's multiplication theorem, some Wgh inequalities ensuring sense preserving nature and belongingness to the classes S_H^* and K_H of that harmonic map are examined and proved. In addition an integral representation of W is also discussed in terms of its belongingness to these classes. Further a convolution of two harmonic maps is studied in terms of its class preserving properties for the subclasses $S_{H^0}^*$ and K_{H^0} with the help of conjectures proved by Cunie and Sheil-Small. Some special cases [1] of our results are also mentioned.

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On Escaping Sets of Entire Functions

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Let f be a transcendental entire function and $f^n, n \in \mathcal{N}$ denote the n^{th} iterate of f . Eremenko [2] defined an escaping set as

$$I(f) := \{z \in \mathcal{C} : f^n(z) \rightarrow \infty \text{ as } n \rightarrow \infty\}.$$

A subset of $I(f)$ in which the iterates of a transcendental entire function tend to infinity arbitrarily fast was considered by Bergweiler and Hinkkanen [1] who defined the set

$$A(f) := \{z \in \mathcal{C} : \text{there exists } L \in \mathcal{N} \text{ such that } |f^n(z)| > M(R, f^{n-L}) \text{ for } n > L\}$$

where $M(R, f) = \max_{|z|=R} |f(z)|$, R is any value such that $R > \min_{z \in J(f)} |z|$, and $J(f)$ is the Julia set of f .

An alternate definition of $A(f)$ was given by Rippon and Stallard [3] who defined the set

$$B(f) := \{z \in \mathcal{C} : \text{there exists } L \in \mathcal{N} \text{ such that } f^{n+L}(z) \notin \widetilde{f^n(D)}, n \in \mathcal{N}\}$$

where D is an open disk meeting Julia set of f and \widetilde{U} denotes the union of U and its bounded complementary components.

For transcendental entire functions f and g , in this paper we have found some relations between $B(f \circ g)$ with $B(g \circ f)$ and also between $B(f \circ g)$ with regards to $B(f)$ and $B(g)$. We have also studied the escaping sets of permutable entire functions.

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Fixed Points of φ -weak Contractions

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Kamran [6] generalized the notion of common property (E.A) by introducing tangential property and improved some results of Y. Liu et. al.[7]. Motivated by Zhang and Song[13], we extend the notion of generalized φ -weak contractions for a pair of single valued self mappings to two hybrid pairs of single and multivalued mappings and improve some results of Kamran [6], Y. Liu et. al.[7] and Zhang and Song[13].

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Fixed Point Theorems Under a Generalized Contractive Condition

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In this note, we aim to generalize the contractive conditions employed in the main results of Cho et. al. [5], Arandelovic et. al. [3] and Aliouche [2]. Here, two common fixed point theorems for two pairs of weakly compatible maps in symmetric spaces are also proved. These results will be helpful in extending and unifying numerous related results in the literature.

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Some Fixed Point Theorems For a Family of Hybrid Pairs of Mappings in Metrically Convex Spaces

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In the present paper we prove some coincidence common fixed point theorems for a family of hybrid pairs of mappings in metrically convex spaces by using the notion of compatibility of mappings. Our results generalize and unify the results due to Imdad and Khan [4], Khan [6], Itoh [5], Ahmad and Imdad [[1], [2]], Ahmad and Khan [3] and several others.

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Involutions in Algebras Arising from Groups and Hypergroups

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Algebra involutions in the context of the second dual B with Arens (1951) products of a Banach algebra A with involution have been studied by various mathematicians. The case of $A =$ group algebra $L^1(G)$ of a locally compact group G continues to draw a lot of attention. This includes study of the spectrum, say, Ω of $A^* = L^\infty(G)$, which inherits some sort of an algebraic structure as well. For G discrete, the study of Arens products was related to that of invariant means by Day (1957). For G discrete, Ω is the Stone-Ćech compactification βG and Civin and Yood (1961) proved that it is a semigroup; we may utilise Comfort and Ross (1966) to see that it is a topological group if and only if G is finite. Dales, Filali, Ghahramani, Hindman, Lau, Neufang, Pym, Runde, Strauss *et al* have more recent significant inputs.

Civin and Yood also noted that the second adjoint of an anti-homomorphism on A is an anti-homomorphism on B with any one Arens product to B with the other Arens product and thus the question of extending an involution T by a suitably modified second adjoint is related to that of Arens regularity of A ; we may note that the question of extending T to an involution on B (with any one Arens product) is related to that of a bijective homomorphism on this B to B (with the other Arens product). Michael Grosser (1984) proved that if $B = L^1(G)^{**}$ admits an involution, then G is discrete. Paul L. Patterson (1994) determined all isometric involutions on $L^1(G)$ and Farhadi and Ghahramani (2007) show that if G has an infinite amenable subgroup, then there is no involution on B that extends the natural involution on $A = L^1(G)$; we obtain non-existence of special involutions extending other involutions on A . A.T.Lau, Medghalchi and Pym (1993) show that, for non-compact non-discrete G , Ω is not a semigroup by displaying μ, ν in Ω whose product is not in Ω . Farhadi

and Ghahramani also demonstrate interesting situations involving existence of involutions when $A^* = L^\infty(G)$ is replaced by certain subalgebras, say, C and, thus, $B = A^{**}$ by certain subalgebras or quotient algebras of B . Ronald G. Douglas (1966) indicates that these products are special probability measures; and for an infinite discrete abelian G gives several subalgebras of B isomorphic to $L^2(bG)$ (which has an involution of its own), with bG , the Bohr compactification of G . We may use these techniques to make the spectrum of certain C into an abstract convolution space (also known as hypergroups developed by Dunkl, Jewett and Spector in 1970's or hypercomplex systems developed by Berezansky et al in 1950's and then others). We also utilize this development, particularly Bloom and Walter (1992) to carry over some of the results to the situation when G is replaced by a hypergroup K .



Matrix Versions of Some Classical Inequalities

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 15A48, 15A18, 15A45

We prove matrix versions of the Chebyshev and Kantotovich inequalities involving the Hadamard product i.e. entrywise product of the matrices. More specifically, we prove that

$$\left(\sum_{i=1}^n w_i A_i \right) \circ \left(\sum_{i=1}^n w_i B_i \right) \leq \left(\sum_{i=1}^n w_i \right) \left(\sum_{i=1}^n w_i (A_i \circ B_i) \right)$$

for $n \times n$ positive semidefinite matrices $A_i, B_i, i = 1, \dots, n$, such that $A_1 \geq \dots \geq A_n$, $B_1 \geq \dots \geq B_n$ where $w_i \geq 0$, $i = 1, \dots, n$, are weights and by $X \geq Y$ ($X > Y$) we means that $X - Y$ is positive semidefinite (positive definite). If $0 < aI_m \leq A_i \leq bI_m$ (here I_m denote the $m \times m$ identity matrix), W_i , $i = 1, \dots, n$ are $n \times n$ positive semidefinite weight matrices and a, b are real numbers, then

$$\left(\sum_{i=1}^n W_i^{1/2} A_i W_i^{1/2} \right) \circ \left(\sum_{i=1}^n W_i^{1/2} A_i^{-1} W_i^{1/2} \right) \leq \frac{a^2 + b^2}{2ab} \left(\sum_{i=1}^n W_i \right) \circ \left(\sum_{i=1}^n W_i \right).$$

Some related inequalities have been discussed. As a consequence we deduce a number of known results.

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Higher Order Approximation by iterates of Modified Beta Operators

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Gupta and Ahmed [2] proposed modified beta operators so as to approximate Lebesgue integrable functions on $[0, \infty)$. It turns out that the order of approximation by these operators is at best $O(n^{-1})$, howsoever smooth the function may be. In order to speed up the rate of convergence by these operators, the technique of linear combination introduced by May [6] and Rathore [8] has been used ([3], [4]). There is yet another approach to improve the order of approximation by linear positive operators, introduced by Micchelli [7] by considering the iterative combination of Bernstein polynomials. Gupta and Vasishtha [5] claimed that the iterative combinations can be applied only for those operators for which t maps exactly into x . Agrawal et al [1] have shown that the iterative combinations can be applied for other operators also which do not reproduce linear functions either.

The present paper deals with an error estimate in terms of higher order modulus of continuity in simultaneous approximation by iterative combination of modified beta operators.

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Certain Generalized Classes of p -valent Analytic Functions Involving Convolution

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 30C45, 30C50, 30C60

In this paper we use, convolution of two p -valent analytic functions, in defining two generalized classes $S^*(p, g, \alpha, m)$ and $S_\lambda^*(p, g, m)$. This convolution generalizes several convolution operators such as Dziok-Srivastava operator [3] which contain well known operators as Hohlov [5], Carlson and Shaffer [2] and Ruschweyh derivative [6]. Further, the convolution reduces to the generalized Salagean operator [1] and to a Salagean operator [7]. We study several coefficient conditions for these classes. Results obtained generalizes several results obtained earlier and generate some new results for special classes of p -valently

starlike, convex and close-to-convex functions of order α . Some of consequent results such as convolution condition and inclusion relation for these classes are also discussed. Further, we obtain inequalities involving partial sums for functions belonging to these classes.

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On Littlewood–Paley Type Inequalities for Subharmonic Functions on Domains in R^n ($n \geq 2$)

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For the unit disc D in C , the classical Littlewood-Paley Inequalities (1936) are as follows: Let h be harmonic on D . There exists a positive constant C , independent of h , such that for all p , $2 \leq p < \infty$,

$$\int_D (1 - |z|)^{p-1} |\nabla h(z)|^p dA(z) \leq C \sup_{0 < r < 1} \int_0^{2\pi} |h(re^{i\theta})|^p d\theta,$$

with the reverse inequality valid for all p , $0 < p \leq 2$.

In the talk we will consider various extensions of the Littlewood-Paley inequalities to subharmonic functions on domains in R^n , $n \geq 2$. Specifically we

will prove that if f is a non-negative C^2 subharmonic function on a bounded domain Ω in R^n with $C^{1,1}$ boundary for which Δf is subharmonic or has subharmonic behavior, then for $1 \leq p < \infty$, there exists a constant C independent of f , such that

$$\int_{\Omega} \delta(x)^{2p-1} (\Delta f(x))^p dx \leq C \sup_{0 < r < r_0} \int_{\partial\Omega} f^p(t - rn_t) ds(t), \quad (6)$$

where $\delta(x)$ is the distance from x to $\partial\Omega$ and n_t is the unit outward normal at $t \in \partial\Omega$. We will also present the analogue of (1) for the case $0 < p < 1$. Taking $f = h^2$, where h is harmonic on Ω , gives the usual Littlewood-Paley inequalities for harmonic functions. We will also consider analogues of the Littlewood-Paley inequalities for non-negative subharmonic functions f for which $|\nabla f|$ is subharmonic or has subharmonic behavior.



Like-Hyperbolic Bloch-Bergman Classes

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Keywords. bounded analytic functions, Bloch-Bergman classes.

In this paper we introduce the like-hyperbolic Bloch-Bergman classes of bounded analytic functions in the open unit disk. We obtain for them, several integral and series characterizations. Likewise we present some metric properties and their relationships with some other well known classes.

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A Note on a Lauricella-saran Triple Hypergeometric Function of Complex Matrix Arguments

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20K: Primary: 33 C 70, 33 C 99. Secondary: 62 H, 44 A 05.

In this paper I have defined the Lauricella-Saran triple hypergeometric function \tilde{F}_K of complex matrix arguments and have established an integral representation for it using the Mathai's matrix transform technique [6].

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Riesz Potentials, Bessel Potentials and Fractional Derivatives on Functions spaces for the Gaussian Measure

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 42C10

In this work we study the boundedness properties of Riesz Potentials, Bessel potentials and Fractional Derivatives on Gaussian Besov-Lipschitz spaces $B_{p,q}^\alpha(\gamma_d)$ and on Triebel-Lizorkin spaces $F_{p,q}^\alpha(\gamma_d)$. In [3] Gaussian Lipschitz spaces $Lip_\alpha(\gamma_d)$ were considered and the boundedness of Fractional Integrals and Fractional Derivatives on them was studied in detail, we are going to extend those results for Gaussian Besov-Lipschitz and Triebel-Lizorkin spaces. Also these results can be extended to the case of Laguerre or Jacobi expansions and even further to the general framework of diffusions semigroups

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L_p -Spaces of Differentiable Forms and Mappings With Controlled Distortion

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We investigate necessary and sufficient conditions on approximately differentiable mappings $f : \mathbf{M} \rightarrow \mathbf{M}'$ of Riemannian manifolds to induce a bounded (with respect to Lebesgue norms) pull-back operator of differential forms. As a consequence, we obtain, in particular, that a homeomorphism $f : \mathbf{M} \rightarrow \mathbf{M}'$ of the class $\text{ACL}(\mathbf{M})$, for which the pull-back operator of spaces of differential forms with finite \mathcal{L}_p -norm is an isomorphism, is either quasiconformal or quasiisometric [1].

An approximately differentiable mapping $f : \mathbf{M} \rightarrow \mathbf{M}'$ has k -finite distortion if $\mathcal{H}^n(Z) = 0$ at $k = 0$ and $\text{rank app } Df(x) < k$ for \mathcal{H}^n -almost all $x \in Z$ at $1 \leq k \leq n$ where $Z = \{x \in \mathbf{M} : \det \text{app } Df(x) = 0\}$. We say that an approximately differentiable mapping $f : \mathbf{M} \rightarrow \mathbf{M}'$ has (q, p) -bounded distortion (in symbols $f \in \mathcal{CD}_{q,p}^k(\mathbf{M}; \mathbf{M}')$) if f has k -finite distortion and

$$\mathbf{M}' \ni y \mapsto H_{k,q}(y) = \left(\sum_{x \in f^{-1}(y) \setminus (\Sigma \cup Z)} \frac{|\Lambda^k f(x)|^q}{|J(x,f)|} \right)^{\frac{1}{q}} \in L_\kappa(\mathbf{M}')$$

where $\frac{1}{\kappa} = \frac{1}{q} - \frac{1}{p}$, $\kappa = \infty$ ($\kappa = q$) when $q = p$ ($p = \infty$). (Here $\Lambda^k f(x)$ is the pull-back operator of differential forms of degree k which is well-defined at the points of approximate differentiability.)

The main result is the following

Theorem [1]. *Let $f : \mathbf{M} \rightarrow \mathbf{M}'$ be an approximately differentiable mapping. The pull-back operator $\tilde{f}^* : \mathcal{L}_p(\mathbf{M}', \Lambda^k) \rightarrow \mathcal{L}_q(\mathbf{M}, \Lambda^k)$, $1 \leq q \leq p \leq \infty$, defined by $\mathcal{L}_p(\mathbf{M}', \Lambda^k) \ni \omega \mapsto \tilde{f}^*\omega(x) = f^*\omega(x)$ (0) if $x \in \mathbf{M} \setminus (Z \cup \Sigma)$ (otherwise), is bounded iff $f \in \mathcal{CD}_{q,p}^k(\mathbf{M}; \mathbf{M}')$. Moreover, the norm of the operator $\tilde{f}^* \sim \|H_{k,q}(\cdot) \| L_\kappa(\mathbf{M}')\|$.*

Some applications to the theory of $\mathcal{L}_{q,p}$ -cogomologies of Riemannian spaces are also given.

The proof of the Theorem is based on methods of [2].

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On the Absolute Convergence of Fourier Series of Functions of $\Lambda BV^{(p)}$ and $\varphi\Lambda BV$

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Let f be a 2π periodic function in $L^1[0; 2\pi]$ and $\hat{f}(n)$, $n \in \mathbb{Z}$, be its Fourier coefficients. Extending the classical result of Zygmund [3], Schramm and Waterman [2] obtained the sufficiency conditions for the absolute convergence of Fourier series of functions of $\Lambda BV^{(p)}$ and $\varphi\Lambda BV$. Here we have generalized these results ([1], [4] and [5] for non-lacunary Fourier series and [2]) by obtaining certain sufficiency conditions for the convergence of the series $\sum_{k \in \mathbb{Z}} |\hat{f}(n_k)|^\beta$ ($0 < \beta \leq 2$), where $\{n_k\}_{k=1}^\infty$ is a strictly increasing sequence of natural numbers and $n_{-k} = n_k$, for all k , for such functions.

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On the Translates of Powers of a Continuous Periodic Function

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Let $C(T)$ denote the Banach space of 2π -periodic, real valued continuous functions on \mathbb{R} where T denotes the unit circle.

For $f \in C(T)$, let $V(f)$ denote the subspace generated by all translates of integer powers of f . One notices that $V(\cos(\theta))$ is an algebra under pointwise multiplication and due to Stone-Weierstrass Theorem it is dense in $C(T)$.

The purpose of this note is to characterize the set of functions in $C(T)$ which share with $\cos(\theta)$ this property. It turns out that the underlying reason for $V(\cos(\theta))$ to be dense is the fact that $\cos(\theta)$ obtains its maximum value at a single point in $[0, 2\pi)$. That it any function $f \in C(T)$ which takes a given value at only one point (which is necessarily its maximum or minimum) has $\overline{V(f)} = C(T)$.



On q -Mellin Transforms of Certain Basic Hypergeometric Functions

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In the present paper, three theorems involving the q -Mellin transforms of certain basic hypergeometric functions have been derived. Applications of these theorems in terms of the q -Mellin transforms of various basic hypergeometric functions, q -polynomials and, their products have also been investigated.

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Section 9

Functional Analysis and Applications

Weakly Continuous Hilbert Bundles over Stonean Spaces and their C^* -algebras

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If Δ is a Stonean space and if $(\Delta, \{H_s\}_{s \in \Delta}, \Omega)$ is a continuous Hilbert bundle over Δ , then there is an associated set Ω_{wk} of vector fields that satisfy continuity properties relative to the weak topology of the Hilbert space fibres H_s . We prove that this set of vector fields carries the structure of a Kaplansky–Hilbert module over $C(\Delta)$ [5]—a special type of C^* -module—and that the algebra $B(\Omega_{\text{wk}})$ of bounded (adjointable) endomorphisms of Ω_{wk} is a type I AW^* -algebra. Further, we show that $B(\Omega_{\text{wk}})$ is the injective envelope and second order local multiplier algebra [1] of the C^* -algebra $K(\Omega)$ of compact endomorphisms of the Hilbert C^* -module Ω . In fact, we show that $B(\Omega_{\text{wk}}) = M_{\text{loc}}(M_{\text{loc}}(A)) = I(A)$ for the

spatial continuous trace C^* -algebra A [4] induced by the continuous Hilbert bundle $(\Delta, \{H_s\}_{s \in \Delta}, \Omega)$, generalizing a situation considered in [2] and [3]. Regarding the relation between the AW^* -algebra $B(\Omega_{\text{wk}})$ and the underlying bundle structure, we show that endomorphisms $b \in B(\Omega_{\text{wk}})$ are representable, up to meagre subsets of Δ , by operator fields on Δ satisfying a weak continuity property.

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Convex Functions and Matrix Inequalities

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Inequalities for convex and concave functions have been generalized to the case of matrices by several authors over a period of time. These leads to some interesting inequalities for matrices, which in some cases coincide with, and in other cases are at variance with, the corresponding inequalities for real numbers. In this short talk, I shall discuss some of these inequalities and their further consequences.

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Ball Remotality in Banach Spaces

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We call a subspace Y of a Banach space X a DBR subspace if its unit ball B_Y admits farthest points from a dense set of points of X . We study the problem for subspaces of classical sequence spaces c_0 , c , ℓ_1 and ℓ_∞ ; some function spaces and also ball remotality of a Banach space in its bidual.



Inclusion Systems and Amalgamated Products of Product Systems

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One parameter semigroups of contractive completely positive maps on a C^* -algebra, known as quantum dynamical semigroups, can be dilated to semigroups of endomorphisms (E -semigroups) of a larger algebra ([2], [3]). When the C^* -algebra is actually the von Neumann algebra of all bounded operators on a Hilbert space, E -semigroups can be classified by looking at their tensor product systems of Hilbert spaces [1].

In the present work, we introduce the notion of ‘inclusion systems’ which are exactly like tensor product systems but linking unitary maps are replaced by isometries. We show that a simple inductive procedure gives us product

systems from inclusion systems. We use this technique to understand amalgamated products of product systems, where the amalgamation is done through contractive morphisms. This is a joint work with Mithun Mukherjee [4].

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Differential Structures in C^* -algebras

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 46L80, 46L85, 46L87, 46L89

Given a dense $*$ -subalgebra \mathcal{U} of a C^* -algebra \mathcal{A} , various classes of smooth subalgebras of \mathcal{A} generated by \mathcal{U} using differential norms will be discussed. These include differential Fréchet algebras, smooth algebras, C^k -algebras, C^∞ -algebras, analytic algebras and entire analytic algebras. Their smoothness properties like spectral invariance, K -theory isomorphisms and closure under functional calculi will be exhibited. Several examples of smooth algebras will be reviewed.

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On n -normed Linear Space Valued Strongly ∇_r -Cesàro and Strongly ∇_r -lacunary Summable Sequences

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In this article we introduce the spaces $|\sigma_1|(X, \nabla_r)$ and $N_\theta(X, \nabla_r)$ of X -valued strongly ∇_r -Cesàro summable and strongly ∇_r -lacunary summable sequences respectively, where X , a real linear n -normed space and ∇_r is a new difference operator, where r is a non-negative integer. This article extends the notion of strongly Cesàro summable and strongly lacunary summable sequences to n -normed linear space valued (n -nls valued) difference sequences. We study these spaces for existence of norm as well as for completeness. Further we investigate the relationship between these spaces.

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**Metric Projections onto Finite Dimensional Subspaces of Spaces of Continuous Functions**

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Let T be a locally compact Hausdorff space, and $C_0(T)$ the space of real continuous functions on T that vanish at infinity, equipped with the uniform norm. Let G denote a finite dimensional linear subspace of $C_0(T)$. Then $P_G : C_0(T) \rightarrow \mathcal{P}(G)$ denotes the set-valued metric projection of $C_0(T)$ onto G , that is, for each $f \in C_0(T)$, the set $P_G(f)$ is the set of best uniform approximations to f from G . Metric projections of the form P_G have been studied for 50 years but the story is incomplete. We are concerned with the following possible properties of the metric projections.

- (1) G is Chebyshev, that is, $\text{card } P_G(f) = 1$ for all $f \in C_0(T)$. Such G are characterised by the Haar condition (Haar 1918).
- (2) P_G is lower semi-continuous. Such G were characterised by Wu Li (1989).

- (3) P_G admits a continuous selection, that is a continuous function $s : C_0(T) \rightarrow G$ such that $s(f) \in P_G(f)$ for all $f \in C_0(T)$. Such G were characterised by G. Nürnberger and M. Sommer (Sommer 1980) for the case $T = [0, 1]$, and by Wu Li (1991) in the general case (see also A. L. Brown 2006).
- (4) P_G admits a unique continuous selection. Such G were characterised by J. Blatter (1990, 1991).

Note that (1) \implies (2) \implies (3) \iff (4) \iff (1). The known characterising conditions for (2), (3) and (4) are not easily exploited and the relations between them are unclear. A new, calculable condition, which will be called a *Generalised Haar Condition (GHC)*, will be described. If the space T has the property that each one point component of T is an isolated point of T then the (GHC) characterises those $G \subseteq C_0(T)$ which have a lower semi-continuous metric projection. The condition allows the construction of examples: if $T = [0, 1] \times \{1, \dots, k\}$ where $2 \leq k \in \mathbb{N}$ then there exist many non-Chebyshev G such that P_G is lower semi-continuous; if $T = T_1 \sqcup S^1$ and $G \subseteq C_0(T)$ is such that $\dim G|_{S^1} \geq 2$ and P_G is lower semi-continuous then $G = G|_{T_1} \oplus G|_{S^1}$. Much less is known for (3) and (4).



Reconstruction of Analytic Signals with Prescribed Symmetries

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Recently one can observe a growing interest in Dunkl operators, or differential-difference operators associated to specific finite reflection groups. These operators are particularly useful for the study of structures with predefined symmetries common in electromagnetism, fluid dynamics or quantum mechanics. Also, such structures are appearing in texture analysis of crystallography. Using this as motivation, we construct spherical Dunkl wavelets based on approximate identities and we give some practical examples (see [1]).

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Lucas' Theorem and Numerical Range

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The Lucas' theorem states that if p is a polynomial with complex coefficients then all zeros of its derivative p' lie in the convex hull of the set of zeros of p . Let A be an $n \times n$ matrix. The numerical range of A is the set of complex numbers $W(A) = \{x^*Ax : x \in \mathbf{C}^n, |x| = 1\}$.

The Lucas' theorem is generalized to the numerical ranges of 3×3 companion matrices. Let $p(t) = t^3 + a_2 t^2 + a_1 t + a_0$ be a real polynomial, and C and C' be respectively the companion matrices of $p(t)$ and $p'(t)/3$. We determine conditions of the generalization $W(C') \subset W(C)$. Examples are provided to show a negative answer to the question raised by Zemanek.

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Multipliers on Weighted Semigroups

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Given a weighted discrete abelian semigroup (S, ω) , the semigroup $M_\omega(S)$ of ω -bounded multipliers as well as the Rees quotient semigroup $M_\omega(S)/S$ together with respective weights $\tilde{\omega}$ and $\tilde{\omega}_q$ induced by ω are discussed; their associated Beurling Banach algebras $\ell^1(S, \omega)$, $\ell^1(M_\omega(S, \omega), \tilde{\omega})$ and $\ell^1(M_\omega(S)/S, \tilde{\omega}_q)$ will be exhibited; and their Banach algebra structure involving semisimplicity, Gel'fand spaces, uniqueness of uniform norm and regularity will be exhibited revealing their dependence on the weight ω . The involutive analogues of these are also considered. A number of examples will be discussed.

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Large Time Behavior of Solutions to Some Classes of Second Order Evolution Equations and Difference Equations

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2010 MATHEMATICS SUBJECT CLASSIFICATION. Primary 47J35, 34G25; 39A12; 39A11; Secondary 47H05, 47H04.

We consider the following second order nonhomogeneous evolution equation

$$\begin{cases} u''(t) - cu'(t) \in Au(t) + f(t) & \text{a.e. } t \in (0, +\infty) \\ u(0) = u_0, \quad \sup_{t \geq 0} |u(t)| < +\infty \end{cases}$$

where A is a monotone operator in a real Hilbert space H , c is a real number, and $f : \mathbb{R}^+ \rightarrow H$ is a given function, as well as its discrete analogue corresponding to the following second order difference equation

$$\begin{cases} u_{n+1} - 2u_n + u_{n-1} \in c_n Au_n; & n \geq 1 \\ u_0 \in H, \quad \sup_{n \geq 0} |u_n| < +\infty \end{cases}$$

where $\{c_n\}$ is a positive real sequence. We prove ergodic theorems, as well as weak and strong convergence theorems for solutions to these equations, converging to an element of $A^{-1}(0)$, implying in particular that solutions exist if and only if $A^{-1}(0) \neq \emptyset$. Our results extend and give simpler proofs to previous results by several authors who studied special cases of similar problems by assuming that $A^{-1}(0) \neq \emptyset$, and have many applications in approximation and optimization theory.



Recent Development in Metric Fixed Point Theory and its Applications

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 54H25, 47H10

In this talk we will discuss some recent developments in metric fixed point theory and some application of fixed point theorems to obtain existence theorems

for nonlinear differential and integral equations. Our treatment includes some standard well-known results as well as some recent ones.



On Convergence of Regularized Modified Newton's Method for Nonlinear Ill-posed Problems

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In this paper we consider regularized modified Newton's method for approximately solving the nonlinear ill-posed problem $F(x) = y$, where the right hand side is replaced by noisy data $y^\delta \in Y$ with $\|y - y^\delta\| \leq \delta$ and $F : D(F) \subset X \rightarrow Y$ is a nonlinear operator between Hilbert spaces X and Y . Under the assumption that Fréchet derivative F' of F is Lipschitz continuous, a choice of the regularization parameter and a stopping rule based on a majorizing sequence are presented. We prove that under a general source condition on $x_0 - \hat{x}$, the error $\|\hat{x} - x_{k,\alpha}^\delta\|$ between the regularized approximation $x_{k,\alpha}^\delta$ ($x_0 := x_{0,\alpha}^\delta$) and the solution \hat{x} is of optimal order.



Automatic Continuity of n -homomorphisms between Fréchet Algebras and Topological Algebras

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A linear map $\theta : A \rightarrow B$ between algebras A and B is an n -homomorphism if $\theta(a_1 a_2 \cdots a_n) = \theta(a_1) \theta(a_2) \cdots \theta(a_n)$ for all elements $a_1, a_2, \dots, a_n \in A$. If $(A, (p_m))$

is a commutative regular Fréchet algebra, then $A/\ker p_m$ is a Fréchet Q -algebra with respect to the quotient topology, whenever $\pi_m^{-1}(\text{rad}A_m) \subseteq \ker p_m$, where A_m is the completion of $A/\ker p_m$ with respect to the norm $p_m(x + \ker p_m) = p_m(x)$, $x \in A$. In particular, if A_m is semisimple then $A/\ker p_m$ is a Fréchet Q -algebra. This implies that if θ is an n -homomorphism on certain Fréchet algebras $(A, (p_m))$ into semisimple commutative Fréchet algebras $(B, (q_m))$ such that $\theta(\ker p_m) \subseteq \ker q_m$ for large enough m , then θ is continuous.

Let A be a Fréchet Q -algebra, B be a semisimple Fréchet algebra, $\theta : A \rightarrow B$ be a dense range n -homomorphism such that $\theta(A)$ is factorizable, and the spectral radius ν_B is continuous on the separating space of θ . Then θ is automatically continuous.

Following Ransford's method we show that if A is an *lmc* Q -algebra and B is a factorizable, advertibly complete, *lmc* semisimple algebra, then every surjective n -homomorphism $\theta : A \rightarrow B$ has closed graph. We then obtain extensions of Johnson's theorem for surjective n -homomorphisms and a theorem, due to C. E. Rickart, for dense range n -homomorphisms, on certain topological algebras.

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Discrete Monogenic Signals

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 30G35

In 2004 Sommer and Felsberg introduced the so-called monogenic signal [1], a concept of an analytic signal in higher dimensions which is nowadays widely applied in texture analysis of images. Recently, this concept was extended to the so-called conformal monogenic signal [2]. From a mathematical point of view monogenic signals are boundary values of monogenic functions, i.e. null-solutions of the Dirac operator. In this way the theory behind is still a continuous theory while the signals under considerations are given as directly sampled signals. In this talk we propose a completely discrete version of a monogenic signal, based on the notion of a boundary value of null-solution of a corresponding discrete Dirac operator (see, for instance, [3]) and study some of its properties, in particular the construction of the related discrete Hilbert transform. In the end we will show its practical applicability.

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Reverse Brunn-Minkowski and Reverse Entropy Power Inequalities for Convex Measures

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The reverse Brunn-Minkowski inequality of V. D. Milman [1] has been influential in Convex Geometry and the Asymptotic Theory of Normed Spaces. It states that, given two convex bodies A and B in \mathcal{R}^n , one can find linear volume preserving maps $u_i : \mathcal{R}^n \rightarrow \mathcal{R}^n$ ($i = 1, 2$) such that with some absolute constant C

$$|\tilde{A} + \tilde{B}|^{1/n} \leq C \left(|A|^{1/n} + |B|^{1/n} \right), \quad (1)$$

where $\tilde{A} = u_1(A)$, $\tilde{B} = u_2(B)$, $A + B$ is the Minkowski sum, and $|A|$ is the n -dimensional volume of A . Note that $|\tilde{A} + \tilde{B}|^{1/n} \geq |A|^{1/n} + |B|^{1/n}$, holds true for any such u_i by the usual Brunn-Minkowski inequality.

We develop an entropic generalization of (1) for arbitrary log-concave probability distributions. Given a random vector X in \mathcal{R}^n with density $f(x)$, consider the entropy functional $h(X) = -\int_{\mathcal{R}^n} f(x) \log f(x) dx$, and the entropy power $H(X) = e^{2h(X)/n}$. The Shannon-Stam entropy power inequality asserts that $H(X+Y) \geq H(X) + H(Y)$, for any two independent random vectors X and Y in \mathcal{R}^n for which the entropy is defined. It is closely related, though not directly equivalent, to the Brunn-Minkowski inequality.

We show that if X and Y are independent and have log-concave densities, then for some linear volume preserving maps $u_i : \mathcal{R}^n \rightarrow \mathcal{R}^n$, $H(\tilde{X} + \tilde{Y}) \leq C(H(X) + H(Y))$, where $\tilde{X} = u_1(X)$, $\tilde{Y} = u_2(Y)$, and C is an absolute constant.

Since the entropy is invariant under linear volume preserving transformations, this is indeed a reverse entropy power inequality, in the same sense that (1) is a reverse of the Brunn-Minkowski inequality. By taking X, Y uniformly distributed in convex bodies $A, B \subset \mathcal{R}^n$, and noting that $h(X) = \log |A|$, we also recover (1). We also show an extended reverse inequality that holds for the larger class of convex or hyperbolic measures on \mathcal{R}^n in the sense of Borell. The proof relies on Milman's notion of M -ellipsoids as well as on various information-theoretic inequalities.

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Continuity of Bessel Wavelet Transform on some Distribution Spaces

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In this paper continuity of the Bessel wavelet transform of a suitable function ϕ in terms of an appropriate mother wavelet ψ is investigated on certain Distribution spaces.

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One more Pathology of C^* -algebraic Tensor Products

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There are several known pathologies for tensor products of C^* -algebras. Here we reveal one more pathology: C^* -tensor products need not be associative.

Let D be a C^* -algebra. Given two (separable) C^* -algebras, A and B , consider asymptotic homomorphisms $\varphi = (\varphi_t)_{t \in [0, \infty)} : A \rightarrow D$ and $\psi = (\psi_t)_{t \in [0, \infty)} : B \rightarrow D$. They give rise to an asymptotic homomorphism $\varphi \otimes \psi : A \odot B \rightarrow D \otimes_{\min} D$, where \odot denotes the algebraic tensor product and \otimes_{\min} is the minimal tensor product of C^* -algebras.

For $c = \sum a_i \otimes b_i \in A \odot B$, set $\|c\|_{D,0} = \sup_{\varphi, \psi} \|(\varphi \otimes \psi)(c)\|$, where the supremum is over all asymptotic homomorphisms, and $\|c\|_D = \min(\|c\|_{D,0}, \|c\|_{\min})$. For $D = B(H)$, this norm was introduced in [1]. The completion of $A \odot B$ with respect to the norm $\|\cdot\|_D$ is a C^* -algebra denoted by $A \otimes_D B$.

Theorem ([2]). Let $D = \prod K$ be the product of countably many copies of the C^* -algebra K of compact operators. Then the tensor product \otimes_D is not associative. Namely, there exist C^* -algebras A, B, C such that the canonical associativity isomorphism of $A \odot B \odot C$ doesn't extend to an isomorphism of the C^* -algebras $A \otimes_D (B \otimes_D C)$ and $(A \otimes_D B) \otimes_D C$.

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Bhattacharyya Divergence-Based Mean of Symmetric Positive-Definite Matrices

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The notion of divergence function was introduced in 1930 by Mahalanobis to measure the discrimination between two correlated normal multivariate distributions [4]. Since then, closeness between two probability distributions on an event space is usually measured by a divergence function (also called dissimilarity measure or relative entropy). Among these we mention here the Kullback-Leibler divergence and the Bhattacharyya divergence [2].

The set of Hermitian positive-definite matrices plays fundamental roles in many disciplines such as mathematics, numerical analysis, probability and statistics, engineering, and biological and social sciences. In the last few years, there has been a renewable interest in developing the theory of means for elements in this set [1, 3]. This is due to theoretical and practical implications. In this work we present a divergence function on the space of symmetric positive-definite matrices which coincides with the Bhattacharyya divergence in the case of multivariate Gaussian distributions of zero means. We then study the invariance properties of this divergence function as well as the matrix means based on it. We present a fixed-point algorithm for computing the Bhattacharyya divergence-based mean. A convergence result of this algorithm is provided.

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Abelian and Tauberian Theorems for the Laplace Transformations On Duals of Ordered Topological Vector Spaces

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Theorems connecting the asymptotic behaviour of a generalized function in a neighborhood of zero with the asymptotic behaviour of its integral transform at infinity are called Tauberian. The theorems which are inverse to Tauberian are called Abelian. In this paper we study distributions which are bounded on the sides of a wedge W in \mathbb{R}^n , tempered distributions having their support in a wedge W in \mathbb{R}^n and holomorphic generalized functions defined on the tube region T^v . The notion of distributions having asymptotic, strong asymptotic of order α is defined and the compatibility of these notions with the lattice properties in $D'(W)$, $S'(W)$ respectively is proved. Those functions which are holomorphic in T^v form a convolution algebra $H(W)$ which is isomorphic to $S'(W)$ via the Laplace transformation. We define an order relation on $H(W)$ by identifying a cone in $H(W)$ and assign a topology to $H(W)$ with respect to which the above cone is normal. The notion of elements in $H(W)$ having asymptotic is defined and is observed to be compatible with lattice properties in $H(W)$. The Tauberian and Abelian theorems in this new background for the Laplace transform

are proved. Two corollaries extending the results of the theorem to monotone nets are also stated. A special case of the Tauberian theorem applied to the one-dimensional case is also stated.

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Common Fixed Point Theorem for Uniformly C_q -commuting Mappings Satisfying a Generalized Asymptotically Nonexpansive Condition

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The aim of this paper is to establish a common fixed point theorem for newly introduced α -generalized asymptotically \mathcal{S} -nonexpansive mappings under the notion of uniformly C_q -commuting mappings. Best approximation results have also been determined as its application. Our work improves, extends and generalizes the corresponding results of Al-Thagafi [1], Al-Thagafi and Shahzad [2], Beg et al. [3], Hussain et al. [5] and Vijayaraju and Hemavathy [11].

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Common Fixed Points for a Rational Inequality under Weakly Compatible Maps in Cone Metric Spaces

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Huang and Zhang [1] recently introduced the cone metric spaces by replacing real numbers with ordered Banach spaces and Olaleru [5] later generalized the concept by replacing ordered Banach spaces with ordered topological vector spaces. The interest in cone metric spaces is informed by their recently discovered applications in optimization theory. For example, see [3]. Several fixed

point theorems are recently proved for different contractive operators on cone metric spaces. For example, see [5] and [2].

In this paper, we present common fixed point results for a rational inequality of weakly compatible maps in cone metric spaces.

The results are generalizations and extensions of several fixed point results in cone metric spaces and consequently in metric spaces including the recent results in [4].

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Fixed Point Theorems for Generalized Asymptotic Contractions

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In 2003, Kirk introduced the notion of *asymptotic contractions* on a metric space and obtained a fixed point theorem for the same. Many subsequent extensions and generalizations of Kirk's theorem appeared (cf. [1]–[11] and references thereof). In this paper, we present a brief development of numerous extensions and generalizations, which have come during a short span of five years. Further, following Suzuki's asymptotic contraction of Meir-Keeler type (ACMK), we obtain a coincidence theorem for a generalized ACMK for a pair of maps and derive some general fixed point theorems on metric spaces.

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Composition Operators on Holomorphic Sobolev Spaces in B_n

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We study the composition operator C_Φ on holomorphic Sobolev spaces induced by an analytic self-map Φ of B_n in \mathbf{C}^n that extends to be smooth on $\overline{B_n}$. We characterize the boundedness and the compactness of C_Φ on $A_{\alpha,s}^p$, and prove the jump phenomenon of C_Φ on $A_{\alpha,s}^p$. Moreover, we show an interesting result that the boundedness of C_Φ on $A_{\alpha,s}^p$ is equivalent to the compactness of $C_\Phi : A_{\alpha,s}^p \rightarrow A_{\beta,t}^q$ for appropriate $A_{\beta,t}^q$, for example $A_{\beta,t}^q = A_{\alpha+1/4,s}^p$. We provide examples to show that our results are sharp.

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On Two Classes of Operators

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A bounded linear operator on a Hilbert space H is n -normal if $T^n T^* = T^* T^n$; and is n -generalized skew projection if $T^n = -T^*$ ($n \in N$). These two classes

of operators will be discussed with examples. Continuity of spectral parts of n -normal operators will be established. The spectrum of an n -generalized skew projection is calculated. This gives analogue of results of GroB and Trenkler [2]. Conditions are developed for a linear combination of generalized skew projections to be a generalized skew projection; providing an analogue of a result of Baksalary and Baksalary [1].

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Weak Compactness in the Dual Space of a JB*-triple is Commutatively Determined

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The following criterium of weak compactness in the dual of a JB*-triple is obtained in [1] and [2]: a bounded set K in the dual of a JB*-triple E is not relatively weakly compact if and only if there exist a sequence of pairwise orthogonal elements (a_n) in the closed unit ball of E , a sequence (φ_n) in K , and $\vartheta > 0$ satisfying that $|\varphi_n(a_n)| > \vartheta$ for all $n \in \mathbb{N}$. Consequently, a bounded subset in the dual space of a JB*-triple, E , is relatively weakly compact whenever its restriction to any abelian subtriple of E is.

This result generalizes the characterization of weak compactness in the dual of a C*-algebra obtained by H. Pfitzner in [3].

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Existence of Common Fixed Points for a Pair of Generalized Weakly Contractive Maps

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We prove the existence of a point of coincidence for a pair of selfmaps (f, T) of a metric space (X, d) in which T is a generalized weakly contractive map with respect to f , through the convergence of the Picard iteration. Also, we deduce the existence of common fixed points for occasionally weakly compatible maps.

Further, we prove the existence of common fixed points for a pair of occasionally weakly compatible selfmaps satisfying generalized weakly contractive condition and property (E.A).



Banach Spaces without Approximation Properties of Type p

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 46B28. Spaces of operators; tensor products; approximation properties

The main purpose of this talk is to show that the question posed in the paper of Sinha D.P. and Karn A.K. [1] (see the very end of that paper) has a negative answer, and that the answer was known, essentially, in 1985 after the papers [2] and [3] by Reinov O.I. have appeared in 1982 and in 1985 respectively (for a translation of [3], see arXiv:1002.3902v1 [math.FA]). In [1] it was shown that for each $p > 2$ there is a Banach space without the AP of type p (the notion introduced in [1]). The open question from [1] was: are there such spaces for $1 \leq p < 2$? We show that there are (for a proof, see arXiv:1003.0085v1 [math.FA]).

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Mathematical Justification of Viscoelastic Beam Models by Asymptotic Methods

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 74K10, 35C20, 74A05, 74A10, 74D05, 41A60

In solid mechanics, the obtention of classical models for beams and plates is based on *a priori* hypotheses on the displacement and/or stress fields, which upon substitution in the equilibrium and constitutive equations of three-dimensional elasticity, leads to useful simplifications. Nevertheless, there is a need to justify the validity of most of the models obtained this way.

In the past decades many models have been derived and justified by the use of the asymptotic expansion method, whose foundations can be studied in Lions [1]. Earlier works were due to Ciarlet and Destuynder [2] in order to justify the linearized theory of plate bending. The asymptotic method was successfully used by Bermúdez and Viaño [3] to justify the Bernoulli-Navier model for bending-stretching of elastic thin rods.

Nevertheless, elasticity models cannot describe important mechanical phenomena such as hardening, memory or relaxation of the materials involved.

Therefore, we devoted this work to derive and justify bending-stretching models for viscoelastic beams (see [4]). The justification is based on the introduction of a change of variable and a scaling of unknowns (displacements and stresses) of the three-dimensional viscoelasticity problem posed in the volume Ω^ε occupied by the rod (ε gives the size of the diameter of the transversal section), with unknowns \mathbf{u}^ε and $\boldsymbol{\sigma}^\varepsilon$ – displacements and stresses. This way, the

problem is reduced to other equivalent, posed in a reference domain Ω , with unknowns $\mathbf{u}(\varepsilon)$ and $\boldsymbol{\sigma}(\varepsilon)$ – scaled displacements and stresses. The mathematical justification of the Bernoulli-Navier model is supported by a convergence result of the form $\mathbf{u}(\varepsilon) \rightarrow \mathbf{u}^0$ in $[H^1(\Omega)]^3$, for all time, where $\mathbf{u}^0 = (u_i^0)$ is the classical Bernoulli-Navier displacement.

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Related Fixed Point Theorems for Two Set Valued Mappings on Two Uniform Spaces

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 54H25, 47H10

In this paper, we obtain some related fixed point theorems for two set valued mappings on two complete and compact uniform spaces. Our results generalize the results of Namdeo, Tiwari, Fisher and Kenan [12].

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Banach-Stone Theorems for Spaces of Vector-valued Functions

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 46B40

Let X be a compact Hausdorff space and let E be a Banach space. Let $C(X, E)$ denote the space of E -valued continuous functions equipped with the supremum norm. For Banach spaces E, F , let $Iso(E, F)$ denote the set of surjective isometries, equipped with the strong operator topology. A well-known Banach-Stone theorem that describes the structure of a surjective isometry $\Phi : C(X, E) \rightarrow C(Y, F)$ states that when E, F have trivial centralizers, $\Phi(f)(y) = \tau(y)(f \circ \phi)(y)$, for a surjective homeomorphism $\phi : Y \rightarrow X$ and for a continuous map $\tau : Y \rightarrow Iso(E, F)$ ([1]). Thus in particular we have that X is homeomorphic to Y and that E is isometric to F . An interesting variation on this theme of Banach-Stone theorems, is to put 'local' conditions on the isometry Φ and ask if the same conclusions can be obtained, including a possible description of Φ . Recently there has been a lot work in this direction when E and F have additional structures like being an abstract M -space or a Banach lattice ([2], [3]). In these cases of course one considers Riesz isomorphisms rather than isometries. In this talk we formulate and prove an order unit Banach space version of the Banach-Stone theorem.

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Refinements of Operator Jensen's Inequality

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2010 MATHEMATICS SUBJECT CLASSIFICATION. Primary 47A63; secondary 46L08, 46L05, 47B10, 47A30

The Davis–Choi–Jensen inequality states that if f is an operator convex function on an interval J , then for every self-adjoint operator A acting on a Hilbert space \mathcal{H} with spectra in J and each unital positive linear map Φ on $\mathcal{B}(\mathcal{H})$,

$$f(\Phi(A)) \leq \Phi(f(A)) \quad (2)$$

holds. In particular,

$$f\left(\sum_{i=1}^n A_i^* X_i A_i\right) \leq \sum_{i=1}^n A_i^* f(X_i) A_i \quad (3)$$

for every n -tuple (X_1, \dots, X_n) of elements of $\mathcal{B}(\mathcal{H})$ with spectra in J and every n -tuple (A_1, \dots, A_n) of operators in $\mathcal{B}(\mathcal{H})$ with $\sum_{i=1}^n A_i^* A_i = I$. Also inequality (3) is true if $0 \in J$, $f(0) \leq 0$ and $\sum_{i=1}^n A_i^* A_i \leq I$. It is known that (2) is equivalent to the operator convexity of f . In this talk, we discuss some results concerning with Jensen's inequality, some equivalent conditions to the operator convexity, inequalities involving eigenvalues and present some refinements of the Choi–Davis–Jensen inequality for strictly positive maps.



On Weierstrass Transform of Tempered Boehmians

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 46F10, 46F12, 46F99, 44A15.

Tempered Boehmians are introduced as a natural extension of tempered distribution. In this paper we have attempted for an extension of Weierstrass tranform, which is, further studied for the tempered Boehmians.

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p-Adic wavelets

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 42C40, 11E95

First p -adic wavelet basis was found by S.V. Kozyrev [1] in 2002. For $p = 2$, his basis is an analog of the classical Haar basis. It appears that these wavelets are very useful to solve p -adic pseudo-differential equations.

The development of the real wavelet theory is based on the notion of multiresolution analysis (MRA). Following this idea, the notion of p -adic MRA was introduced in [2], and a general scheme for its construction was described. Also, this scheme was realized to construct the p -adic Haar MRA with using

$$\phi(x) = \sum_{r=0}^{p-1} \phi\left(\frac{1}{p}x - \frac{r}{p}\right) \quad (4)$$

as a generating refinement equation. Note that (4) reflects a *natural* “self-similarity” of the space \mathbf{Q}_p : the unit disc $B_0(0) = \{x : |x|_p \leq 1\}$ is represented as the union of p mutually *disjoint* discs $B_{-1}(r) = \{x : |x - r|_p \leq p^{-1}\}$, $r = 0, \dots, p - 1$, and so the characteristic function of $B_0(0)$ is a solution of (4) (scaling function). In contrast to the real setting, there exists infinitely many different orthonormal wavelet bases in the same Haar MRA. A wide class of orthogonal scaling functions generating a MRA was constructed in [3]. However, it was proved in [4] that all of these functions lead to the same Haar MRA and that there exist no other orthogonal test scaling functions generating a MRA. A more general definition of MRA (with non-orthogonal scaling functions) were introduced in [3], and a complete characterisation of test functions generating a MRA was given. Also, methods for the construction of MRA-based wavelet frames and Riesz bases have been developed.

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Some Common Fixed Point Theorems of Integral Type in Menger PM Spaces

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Primary 54H25, Secondary 47H10.

Keywords. Menger spaces, common property (E.A), weakly compatible mappings, and t-norm.

In this paper, we propose the integral type version of fixed point theorems in Menger spaces satisfying common property (E.A). Also using common property (E.A), some common fixed point theorems are proved for self mappings satisfying quasi-contraction and ϕ -type contraction in Menger PM spaces. Our results generalize several known results in Menger as well as metric spaces. Some related results are also derived besides furnishing an illustrative example.

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Coincidence and Fixed Point Theorem Satisfying Integral Type Implicit Relations in Symmetric Spaces

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 54H25, 47H10.

The aim of this paper is to obtain coincidence and fixed point theorem in symmetric spaces satisfying integral type implicit relations due to their unifying power besides admitting new contraction condition. Our main result is a generalized and improved form of several known results.



The Characterization of a Class of Quantum Markov Semigroups and the Associated Dirichlet Forms Based on Hilbert C^* -modules

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 46L, 47H

In joint work with Guo Maozheng, first, we characterize that the generator of A -linear and strict continuous and symmetric contraction semigroup $\{T_t\}_{t \in R^+} \subset L(H \otimes A)$ is a non-positive definite self-adjoint regular module operator on $H \otimes A$, where H is a separable Hilbert space and A is a finite dimensional C^* -algebra, $L(H \otimes A)$ is the C^* -algebra of all adjointable modular maps on $H \otimes A$.

Next, we give a one to one correspondence between the set of non-positive definite self-adjoint regular modular operators on $H \otimes A$ and the set of non-negative densely defined A -valued quadratic forms.

In the end, we obtain that a strict continuous symmetric semigroup $\{T_t\}_{t \in R^+} \subset L(H \otimes A)$ being Markovian if and only if the associated densely defined A -valued quadratic form is a Dirichlet form.

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Discrete Algebraic Equations and Discrete Operator Equations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 47S99, 26B40

We give constructive results of Hilbert's problem 13th for discrete functions. By them we give formula solution expressed by a superposition of functions of

one variable to equations constructed by discrete functions and equations with parameterized discrete operations. Further more we give formula solution expressed by a superposition of operators of one variable to equations constructed by discrete operators and equations with parameterized discrete operators.

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Section 10

Dynamical Systems and Ordinary Differential Equations

Golden Tilings

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 37E30, 37C15

In this talk we present the definition of a golden sequence $\{r_i\}_{i \in \mathbb{N}}$. These golden sequences have the property of being Fibonacci quasi-periodic and determine a tiling in the real line. We prove a one-to-one correspondence between:

- (i) affine classes of golden tilings;
- (ii) smooth conjugacy classes of Anosov diffeomorphisms, with an invariant measure absolutely continuous with respect to the Lebesgue measure, that are topologically conjugate to the Anosov automorphism

$$G_A(x, y) = (x + y, x)$$

- (iii) solenoid functions.

A. Pinto and D. Sullivan developed a theory relating 2-adic sequences (Pinto-Sullivan tilings in the real line) with smooth conjugacy classes of doubling expanding circle maps. The solenoid functions give a parametrization of the infinite dimensional space consisting of the mathematical objects described in the above equivalences.

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Rigidity and Flexibility of some Group Actions Related to Real-rank One Lie Groups

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 37C85

By \mathbb{H}^n , we denote the n -dimensional hyperbolic space. Let $\text{Isom}_+(\mathbb{H}^n)$ be the group of orientation preserving isometries of \mathbb{H}^n and B be its Borel subgroup (e.g., the subgroup of elements that fix $\infty \in \partial\mathbb{H}^n \simeq \mathbb{R}^{n-1} \cup \{\infty\}$). For any given cocompact lattice Γ of $\text{Isom}_+(\mathbb{H}^n)$, the group B naturally acts on $\Gamma \backslash \text{Isom}_+(\mathbb{H}^n)$ from right. We denote the action by ρ_Γ .

In this talk, we will discuss about the deformation of ρ_Γ . More precisely, we will give the complete family of the deformation of the above action for $n = 2$ and show the local rigidity for $n \geq 3$.

Theorem A. *Suppose that $n = 2$. Let n_o be the dimension of the Teichmüller space of $\Gamma \backslash \mathbb{H}^2$ and n_p be the first Betti number of $\Gamma \backslash \text{Isom}_+(\mathbb{H}^2)$. Then, there exists a C^∞ family $\{\rho_t\}_{t \in \mathbb{R}^{n_o+n_p}}$ of actions of B on $\Gamma \backslash \text{Isom}_+(\mathbb{H}^2)$ such that*

$\rho_0 = \rho_\Gamma$ and any locally free action of B on $\Gamma \backslash \text{Isom}_+(\mathbb{H}^2)$ is C^∞ conjugate to ρ_t for some t .

Theorem B. *Suppose that $n \geq 3$. Then, ρ_Γ is C^∞ locally rigid.*

In the proof of Theorem A, the deformation theory of low dimensional Anosov systems plays an important role. In the proof of Theorem B, the rigidity theorem of conformal Anosov systems does.

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On Qualitative Behavior of Solutions to Nonlinear Ordinary Differential Equations of Higher Order

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 34C15, 34C10

Qualitative behavior of solutions to quasi-linear ordinary differential equations of the higher order is described. In particular, to the equation

$$y^{(n)} + \sum_{j=0}^{n-1} a_j(x) y^{(j)} + p(x) |y|^k \text{sgn} y = 0$$

with $n \geq 1$, real (not necessary natural) $k > 1$, and continuous functions $p(x)$ and $a_j(x)$, uniform estimates for solutions with the same domain ([4]), sufficient conditions for existence of non-oscillatory solutions, a criterion for existence of non-oscillatory solutions with non-zero limit at infinity, sufficient conditions for existence of solutions equivalent to those of the related linear differential equation (cf.[2]) are formulated. In the case of even order and positive potential $p(x)$, a criterion is obtained for all solutions to be oscillatory ([3], cf.[1]).

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An Impulsive Two-prey One-predator System with Seasonal Effects

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 34A37, 34D23, 34H05, 92D25.

The dynamical relationships between predator and prey can be represented by the functional response which refers to the change in the density of prey attached per unit time per predator as the prey density changes. One of well-known functional responses is the Beddington-DeAngelis functional response introduced by Beddington and DeAngelis et al, independently. The main difference of this functional response from a classical Holling type ones is that this one contains an extra term presenting mutual interference by predators. There are a lot of factors to be considered in the environment to describe more realistic relationships between predators and preys. One of important factors is seasonality, which is a kind of periodic fluctuation varying with changing seasons. Also, the seasonality has an effect on various parameters in the ecological systems. For this reason, it is valuable to carry out research on systems with periodic ecological parameters which might be quite naturally exposed such as those due to seasonal effects of weather or food supply etc [2, 4]. There are several ways to reflect the effects caused by the seasonality on ecological systems [1, 3, 5]. In this talk we consider the intrinsic growth rate a of the prey population as periodically varying function of time due to seasonal variations. In other words we adopt $a_0 = a(1 + \epsilon \sin(\omega t))$ as the intrinsic growth rate of the prey. Here the parameter ϵ represents the degree of seasonality, $a\epsilon$ the magnitude of the perturbation in a_0 and ω the angular frequency of the fluctuation caused by seasonality.

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Singular Solution of Reduced ODEs of Rotating Stratified Boussinesq Equations

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In this paper we investigate the complete integrability of the system of six coupled nonlinear ODEs, which arises in the ODE reduction of rotating stratified Boussinesq equations in the context of theory of basin scale dynamics; the details are given in the paper of Leo R. M. Maas [1]. Also, Desale [2] has discussed the complete integrability of this system via concept of first integrals. But we use Painlevé test approach to investigate the complete integrability of the system. And we conclude that the system is completely integrable only if the Rayleigh number $Ra = 0$. The singular solution of the system admits the movable pole type singularity in complex domain.

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Digraphs of Unimodal Cycles

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Let θ be a unimodal cyclic permutation. Define the RL -pattern for θ as the element $S_1 S_2 \dots S_n \in \{R, L\}^n$ satisfying

$$S_i = \begin{cases} R, & \text{if } \theta^i(1) > \theta^{i-1}(1), \\ L, & \text{if } \theta^i(1) < \theta^{i-1}(1). \end{cases}$$

For example, the RL -pattern for $\theta = (12435)$ is $RRLRL$. Let \mathfrak{C} denote the class of unimodal cycles whose RL -pattern does not contain two consecutive R 's. It is well known that the maximal cycle of order n ($n \geq 4$) in \mathfrak{C} with respect to the forcing relation is

$$\bar{\theta}_n = (1, k+1, k, k+2, k-1, k+3, k-2, \dots, n-1, 2, n), \quad \text{where } k = \frac{n}{2}.$$

This research aims to investigate the nature of the coefficients found in the characteristic polynomial of the digraph representation of $\bar{\theta}_n$.



Stability of Solutions of Linear Differential Systems of Neutral Type with Constant Coefficients

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We deal with stability of linear systems neutral differential equations

$$\dot{x}(t) = D\dot{x}(t - \tau) + Ax(t) + Bx(t - \tau) \quad (1)$$

where $t \geq 0$ is an independent variable, $\tau > 0$ is a constant delay, A, B and D are $n \times n$ constant matrices and $x: [-\tau, \infty) \rightarrow R^n$ is a column vector-solution. We use Lyapunov-Krasovskii functionals of a quadratic type depending on running coordinates as well as on their derivatives

$$V_0[x(t), t] = x^T(t)Hx(t) + \int_{t-\tau}^t e^{-\beta(t-s)} [x^T(s)G_1x(s) + \dot{x}^T(s)G_2\dot{x}(s)] ds$$

and $V[x(t), t] = e^{pt}V_0[x(t), t]$ where x is a solution of (1), β and p are real parameters, $n \times n$ matrices H, G_1 and G_2 are positively definite, and $t > 0$. Although many approaches in the literature are used to judge the stability, our approach, except others, not only determines whether the system (1) is exponentially stable but also gives delay dependent estimation of solutions in terms of norms for both $\|x(t)\|$ and $\|\dot{x}(t)\|$ even in the case of instability.

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The Riemann-Hilbert Problem on a Compact Riemann Surface

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 34M45,34M50,34M56

In the talk I am going to touch two aspects dealing with the The Riemann-Hilbert problem on a compact Riemann surface.

Firstly, I shall discuss the solvability of the Riemann-Hilbert problem on a compact Riemann surface. Also I shall give an estimate of the number of apparent singularities.

The problem is the following. Let M be a Riemann surface of positive genus, let us fix some finite set $S = \{a_1, \dots, a_n\}$. We shall consider the system of p linear differential equations with fuchsian singularities in S . It is known that in the case of positive genus the dimension of the space of monodromies is bigger than the dimension of the space of such systems. So, typically, so Riemann-Hilbert problem cannot be solved. But it can we solved if we allow the system to have additional singularities with trivial monodromy. It will be shown, that $2pg - g + 1$ additional singularities are enough.

Also the conditions on the monodromy representation that are necessary (and typically sufficient) for the solvability of the problem (without apparent singularities) will be discussed.

Secondly, I am going to discuss isomonodromic deformations of pairs (E, ∇) (E is a holomorphic bundle and ∇ is a connections) on surfaces. Although those deformations were considered by a quite large number of authors we shall present one more description of isomonodromic deformations on surfaces. Our description will be geometrical and it is baced on the representation of a surface as a factor of a disc. In fact this approach generalizes the ordinary and elliptic Shlesinger systems. In our description it turns out that the deformations are described by an ordinary Shlesinger system on a Riemann sphere and some linear equation.



Slow-fast Bogdanov-Takens Bifurcations

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The talk deals with perturbations from planar vector fields having a line of zeros and representing a singular limit of Bogdanov-Takens (BT) bifurcations. We introduce, among other precise definitions, the notion of slow-fast BT-bifurcation and we provide a complete study of the bifurcation diagram and the related phase portraits. Based on geometric singular perturbation theory, including blow-up, we get results that are valid on a uniform neighbourhood both in parameter space and in the phase plane. The talk is based on joint work with Peter De Maesschalck.

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Determination of Periodic Orbits with Bifurcation Values, Strange Attractor, Lyapunov Exponents and Various Fractal Dimensions on Two-Dimensional Discrete Systems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 37G15, 37G35

This paper on ‘Chaos and Fractal’ highlights four objectives on a two dimensional discrete dynamical system: $h(x, y) = (y, \mu x + \lambda y - y^3)$, where μ and λ are adjustable parameters. Firstly, by adopting suitable computer programmes we evaluate period doubling bifurcation values of the parameter ‘ λ ’ for the periodic orbits of periods $2^0, 2^1, 2^2, 2^3, \dots$ and obtain the Feigenbaum universal constant $\delta = 4.66920161\dots$, a route from order to chaos and the accumulation point $\alpha = 3.24069766596\dots$ beyond which chaotic region occurs. Secondly, as the crucial feature of periodic orbits for their long time behaviour, the strange attractor of the system is achieved. Thirdly, the notion of exponential divergence of nearby trajectories and the existence of chaos are confirmed by determining the Lyapunov exponents. Fourthly, Box Dimension, Information Dimension, Correlation Dimension and Hausdorff Dimension are studied, and some illuminating results are obtained as the measure of chaos.

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Limit Cycle Problems

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We develop the global bifurcation theory of planar polynomial dynamical systems and suggest a new geometric approach to solving Hilbert’s Sixteenth Problem on the maximum number and relative position of their limit cycles in two special cases of such systems. First, using geometric properties of four field rotation parameters of a new canonical system, we present the proof of our earlier conjecture that the maximum number of limit cycles in a quadratic system is equal to four and their only possible distribution is $(3 : 1)$. Then, by means of the same geometric approach, we solve the Problem for Liénard’s polynomial system (in this special case, it is called Smale’s Thirteenth Problem). Besides, generalizing the obtained results, we present the solution of Hilbert’s Sixteenth Problem on the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and, applying the Wintner–Perko termination principle for multiple limit cycles, we develop an alternative approach to solving the Problem. By means of this approach we complete also the global qualitative analysis of a generalized Liénard cubic system, a neural network cubic system, a Liénard-type piecewise linear system and a quartic dynamical system which models the population dynamics in biomedical and ecological systems [1]–[6].

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Dynamical Systems of Fractional order

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Fractional Calculus dates back to correspondence between LHospital and Leibniz towards the end of 17th century. The pioneering works of Euler, Lagrange, Abel, Liouville, Riemann, Grunwald and Letnikov has led to formulation of fractional integrals and derivatives with subsequent development of fractional calculus. Formal mathematics of fractional calculus existed in the literature, for a long time, however utility and applicability of fractional calculus to various branches of Science and Engineering have been realised rather recently, [1, 2].

Study of dynamical systems of fractional order is receiving increasing attention in the recent years. Financial systems in economics displaying fractional order dynamics are known. Furthermore Lorenz, Chen, L, Rossler systems of fractional order have been studied widely in the literature. Effect of delay on chaotic solutions in fractional order dynamical system is investigated by the present author [3, 4, 5].

In this presentation we explore the interrelation between the order of fractional system and chaos, synchronisation and delay.

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Approximate Analytical Solution of Fractional Lotka-Volterra Equations

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The article presents the solutions of Lotka-Volterra equations of fractional order time derivatives with the help of analytical method of nonlinear problem called the Homotopy perturbation method. By using initial values, the explicit solutions of predator and prey populations for different particular cases have been derived. The numerical solutions show that only a few iterations are needed to obtain accurate approximate solutions. The method performs extremely well in terms of efficiency and simplicity to solve this historical biological model.



On a Perturbed Quadratic Fractional Integral Equation of Abel Type

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 45G10, 45M99, 47H09

We present an existence theorem for monotonic solutions of a perturbed quadratic fractional integral equation in $C[0, 1]$. Our equation contains the famous Chandrasekhar's integral equation as a special case. The concept of measure of noncompactness related to monotonicity, introduced by J. Banaś and L. Olszowy, and a fixed point theorem due to Darbo are the main tools in carrying out our proof. Moreover, we give an example for indicating the natural realizations of our abstract result presented in this work.

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Nonlinear Stability in the Generalized Photogravitational Restricted Three Body Problem with Poynting-Robertson Drag

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 70F15

Keywords. Nonlinear Stability/Triangular Points/Generalized Photogravitational/RTBP/P-R Drag.

The Nonlinear stability of triangular equilibrium points has been discussed in the generalized photogravitational restricted three body problem with Poynting-Robertson drag. The problem is generalized in the sense that smaller primary is supposed to be an oblate spheroid. The bigger primary is considered as radiating. We have performed first and second order normalization of the Hamiltonian of the problem. We have applied KAM theorem to examine the condition of non-linear stability. We have found three critical mass ratios. Finally we conclude that triangular points are stable in the nonlinear sense except three critical mass ratios at which KAM theorem fails.



Iterative Methods for Solving Fractional Differential Equations

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Fractional differential equations, both partial and ordinary ones, have received more attention in recent years. Various phenomena in physics, like diffusion in a disordered or fractal medium, or in image analysis, or in risk management have been modeled by means of fractional equations.

Fractional diffusion-wave equation has important applications to mathematical physics. Iterative methods to solve fractional and ordinary differential equation are receiving increasing attention in recent years, for example the Adomian Decomposition Method (ADM) [3, 4], Homotopy Analysis Method (HAM) [5, 6], Homotopy Perturbation Method (HPM) [2, 7], Variational Iterative Method [2, 8].

In this work we have used HAM, HPM, VIM, ADM and DJ method [1] for solving linear and nonlinear fractional differential equations.

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On an Abstract Nonlinear Functional Integro-differential Equation with Nonlocal Condition

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Primary 45J05; Secondary 45N05, 34G20, 35K55

Let X be a Banach space with norm $\|\cdot\|$. Let $C = C([-r, 0], X)$, $0 < r < \infty$, denotes the Banach space of all continuous functions $\psi : [-r, 0] \rightarrow X$ endowed with supremum norm

$$\|\psi\|_c = \text{Sup}\{\|\psi(\theta)\| : -r \leq \theta \leq 0\}$$

If x is a continuous function from $[-r, T]$, $T > 0$, to X and $t \in [0, T]$ then x_t denotes the element of C given by $x_t(\theta) = x(t + \theta)$, for $\theta \in [-r, 0]$. Consider the nonlinear functional integro-differential equation with non local condition

$$x'(t) = f\left(t, x_t, \int_0^t k(t, s)h(s, x_s)ds\right), \quad t \in [0, T] \quad (2)$$

$$x(t) + (g(x_{t_1}, \dots, x_{t_p}))(t) = \phi(t), \quad t \in [-r, 0] \quad (3)$$

where $x : [-r, T] \rightarrow X$, the functions $k : [0, T] \times [0, T] \rightarrow \mathbb{R}$, $h : [0, T] \times C \rightarrow X$ and $f : [0, T] \times C \times X \rightarrow X$ are continuous functions, g is given function satisfying some assumptions and ϕ is given element of C .

We investigate the global existence of solutions of the above system by using topological transversality theorem known as Leray-Schander alternative.

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On some Estimates for the First Eigenvalue of the Sturm—Liouville Problem with Third-type Boundary Conditions

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Consider the Sturm-Liouville problem:

$$y''(x) - q(x)y(x) + \lambda y(x) = 0, \quad \begin{cases} y'(0) - k^2 y(0) = 0, \\ y'(1) + k^2 y(1) = 0, \end{cases}$$

where $q(x)$ is a non-negative bounded summable function on $[0, 1]$ such that

$$\int_0^1 q^\gamma(x)dx = 1, \quad \gamma \neq 0.$$

By A_γ denote the set of all such functions. We estimate the first eigenvalue $\lambda_1(q)$ of this problem for different values of γ and k . For $M_\gamma = \sup_{q \in A_\gamma} \lambda_1(q)$ and $m_\gamma = \inf_{q \in A_\gamma} \lambda_1(q)$ some estimates were obtained. In particular, we have

- Theorem 1.** 1. If $\gamma > 1$ and $k = 0$, then $m_\gamma = 0$;
 2. if $\gamma \leq 1$ and $k = 0$, then $m_\gamma \geq 1/4$.
 3. If $\gamma \in (-\infty, 0) \cup (0, 1)$, then $M_\gamma = +\infty$;
 4. if $\gamma \geq 1$, then $M_\gamma \leq \pi^2 + 2$;
 5. if $\gamma \geq 1$ and $k = 0$, then $M_\gamma = 1$;
 6. if $\gamma = 1$ and $k \neq 0$, then $M_1 = \xi_*$, where ξ_* is the solution to the equation $\arctan \frac{k^2}{\sqrt{\xi}} = \frac{\xi-1}{2\sqrt{\xi}}$.

Remark. The problem for the equation $y'' + \lambda q(x)y = 0$, $q(x) \in A_\gamma$, and conditions $y(0) = y(1) = 0$ was considered in [1].

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On Existence Results of Impulsive Integrodifferential Inclusions

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 34A37, 34A60, 34G20, 35R10, 45J05, 47H20

We consider the class of first order impulsive integrodifferential inclusions of the type:

$$x'(t) - Ax(t) \in Bx(t) + \int_0^t k(t, s)F(s, x(s))ds, \quad a.e. \quad t \in J \quad (4)$$

$$J = [0, b], \quad t \neq t_k, \quad k = 1, \dots, m,$$

$$\Delta x|_{t=t_k} = I_k(x(t_k^-)), \quad k = 1, \dots, m, \quad (5)$$

$$x(0) = x_0, \quad (6)$$

where $F : J \times X \rightarrow P(X)$ is a multivalued map, X a real separable Banach space with norm $\|\cdot\|$, $P(X)$ is the family of all nonempty subsets of X , A is the infinitesimal generator of a family of semigroup $\{T(t) : t \geq 0\}$ in X , $k : D \rightarrow R$, $D = \{(t, s) \in J \times J : s \leq t\}$, B is a bounded linear operator from

X into X , $x_0 \in X$, $0 < t_1 < t_2 < \dots < t_m < t_{m+1} = b$, $I_k \in C(X, X)$ ($k = 1, \dots, m$), $\Delta x|_{t=t_k} = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{h \rightarrow 0^+} x(t_k + h)$ and $x(t_k^-) = \lim_{h \rightarrow 0^-} x(t_k - h)$ represent the right and left limits of $x(t)$ at $t = t_k$.

The purpose of the present paper is to study the class of first order impulsive integrodifferential inclusions in a real separable Banach spaces by using semigroup theory and suitable fixed point theorems when the multivalued map has convex and nonconvex values.

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Some Topological Properties of Julia Components of Transcendental Entire Functions

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Let f be a transcendental entire function and f^n denote the n -th iterate of f . Recall that the *Fatou set* $F(f)$ and the *Julia set* $J(f)$ of f are defined as follows:

$$F(f) := \{z \in \mathbf{C} \mid \{f^n\}_{n=1}^{\infty} \text{ is normal in a neighborhood of } z\}, \quad J(f) := \mathbf{C} \setminus F(f).$$

In what follows, we consider the case where f has a multiply-connected wandering domain. Then $J(f)$ is disconnected for such an f and we investigate some

topological properties of connected components of the Julia set, which we call *Julia components*. The results are the following:

Theorem A. Let f be a transcendental entire function which has a multiply-connected wandering domain and let C be a Julia component with bounded orbit.

- (1) There exists a polynomial g such that C is homeomorphic to a Julia component of the Julia set $J(g)$.
- (2) If C is full (i.e., the complement C^c in the complex plane \mathbf{C} is connected), then C is a buried component.
- (3) If C is a wandering Julia component (i.e., $f^m(C) \cap f^n(C) = \emptyset$, $\forall m \neq n$), then C is a buried singleton component.

Corollary B. For every repelling periodic point p , let $C(p)$ be the Julia component containing p .

- (1) If $C(p)$ is full, then $C(p)$ is a buried component.
- (2) If $C(p)$ is not full, then the bounded components of $C(p)^c$ consist of immediate attractive basins, immediate parabolic basins and Siegel disks and their preimages.
- (3) The point p is a buried point unless it is on the boundary of an immediate attractive basin, an immediate parabolic basin or a Siegel disk.

We also show some results on Julia components under some additional conditions on the behavior of singular values of f .

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Chaos near Non-hyperbolic Equilibria or Non-transversal Homoclinic Points

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Let $V \in 0$ be a bounded domain in the Euclidean space \mathbb{R}^n . Consider the set X of C^1 diffeomorphisms $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ such that $F(V) \subset V$. Here \bar{V} is the

closure of the set V . Define the C^1 – distance at the space X by the formula

$$d_1(F, F') = \max_{x \in K} (|F(x) - F'(x)|) + \max_{x \in K} |DF(x) - DF'(x)|.$$

We denote by the symbol $|\cdot|$ all Euclidian norms of vectors and corresponding matrix norms and by DF and DF' the Jacobi matrices of the corresponding mappings. We shall identify all the mappings which coincide on \bar{V} . Fix a mapping $F \in X$ and assume that the point O is the equilibrium of F . Let $\lambda_1, \dots, \lambda_n$ be the eigenvalues of the matrix $A = DF(0)$ (some of them may be equal) and $m \in \{1, 2, \dots, n_1\}$ be such that

$$|\lambda_1| \leq |\lambda_2| \leq \dots \leq |\lambda_m| < 1 \leq |\lambda_{m+1}| \leq |\lambda_n|. \quad (1.1)$$

The equilibrium 0 of the mapping F is *strongly conditionally unstable* if

1. there exist a neighborhood $U_0 \ni 0$ and such a continuous mapping $V : U_0 \rightarrow [0, +\infty)$ that $V(x) = 0$ if and only if $x \in W_{loc}^s$ and $V(F(x)) \geq V(x)$ for all $x \in U_0 \cap F^{-1}(U_0)$;
2. the equilibrium 0 of the mapping $F^{-1}|_{W_{loc}^{uc}}$ is asymptotically stable.

We say that the equilibrium 0 of the mapping F is *strongly conditionally stable* if it is *strongly conditionally unstable* by the respect of the mapping F^{-1} .

Condition 1. The point 0 of the mapping F is strongly conditionally stable. The manifolds W^{sc} and W^u intersect transversally in a point $p \neq 0$.

Condition 2. The point 0 of the mapping $F|_{W_{loc}^{uc}}(0)$ is strongly conditionally unstable. The manifolds W^{sc} and W^u intersect transversally in a point $p \neq 0$.

Theorem 1. Suppose the coordinate origin is an equilibrium of the diffeomorphism $F \in X$, and either conditions 1 or conditions 2 are satisfied. Then for any neighborhood $U \ni 0$ there exists such $\delta > 0$ that for any $G \in X$ such that

$$\|F - G\|_X < \delta$$

there is an infinite subset $P_G \in U$ such that

1. all points $q \in P_G$ are periodic to the respect of G ;
2. for any $m \in \mathbb{N}$ there is a point $q \in P_G$, such that the period of q is bigger than m ;
3. $\text{card} \bar{P}_G = \aleph$.

Corollary. Let $F \in X$ be such that $x = 0$ is a hyperbolic equilibrium. Suppose that F can be C^1 linearized in a neighborhood U of 0 . Assume that the corresponding stable W^s and unstable W^u manifolds intersect in a point $p \neq 0$ in such a way that there exists such a disk D and a neighborhood U_1 of the point p with the following properties.

1. $\dim D = \dim W^u$, the manifolds D and W^s intersect transversally at the point p . Denote by w^s and w^u the connected components of intersections of corresponding manifolds with U_1 , containing the point p
2. There exist such a smooth coordinate system $\xi = \text{col}(\eta, \zeta)$ in U_1 that $\xi(x)$ is a local diffeomorphism of x in $U_1 \ni p$, $\eta(x) = \eta(p)$ for any $x \in D$, $\zeta(x) = 0$ for any $x \in w^s$ and the set w^u is a graph of the function $\eta = g(\zeta)$, $|\zeta| < \delta$, such that the mapping g is smooth for all $\zeta : 0 < |\zeta| < \delta$.

Then for any neighborhood $U \subset 0$ there is an infinite subset $P \in U$ such that the same statements 1.–3. held true.

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Global Existence for Volterra-Fredholm Functional Integrodifferential Equations

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Let R^n be the Euclidean n -space with Euclidean norm $|\cdot|$. Let $C = C([-r, 0], R^n)$, $0 < r < \infty$, denotes the Banach space of all continuous functions $\psi : [-r, 0] \rightarrow R^n$ endowed with supremum norm

$$\|\psi\|_C = \sup\{|\psi(\theta)| : -r \leq \theta \leq 0\}.$$

If x is a continuous function from $[-r, T]$, $T > 0$, to R^n and $t \in [0, T]$ then x_t denotes the element of C given by $x_t(\theta) = x(t + \theta)$ for $\theta \in [-r, 0]$. Consider the nonlinear functional mixed Volterra-Fredholm integrodifferential equation of the form

$$x'(t) = f(t, x(t), x_t, \int_0^t a(t, s)g(s, x(s), x_s)ds, \int_0^T b(t, s)h(s, x(s), x_s)ds), \quad t \in [0, T],$$

$$x(t) = \phi(t), \quad -r \leq t \leq 0,$$

where $f : [0, T] \times R^n \times C \times R^n \times R^n \rightarrow R^n$, $x : [-r, T] \rightarrow R^n$, the functions $a, b : [0, T] \times [0, T] \rightarrow R$, $g, h : [0, T] \times R^n \times C \rightarrow R^n$, are continuous functions and ϕ is a given element of C .

We investigate the global existence of solutions of the above system by using topological transversality theorem known as Leray-Schauder alternative.

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Chaotic Dynamics in some Pendulum Type Equations

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We investigate the presence of chaotic-like dynamics for a class of second order ODEs using the concept of “stretching along paths” and the theory of “linked twist maps”. The chaotic dynamics considered is of the coin-tossing type as in the Smale’s Horseshoe. The proof relies on some recent results about chaotic planar maps combined with the study of geometric features which are typical of linked twist maps.

We study the case in which a perturbation is introduced for the conservative scalar equation $\ddot{x} + f(x) = 0$ in the form of a weight function $q(t)$.

The ODEs investigated are of the form

$$\ddot{x} + q(t)f(x) = 0, \quad \left(\dot{x}(t) = \frac{d}{dt}x(t) \right)$$

where $f = f(x) : \mathbb{R} \rightarrow \mathbb{R}$ is locally Lipschitz and $q = q(t) : \mathbb{R} \rightarrow \mathbb{R}$ is a T -periodic weight function which belongs to $L^1([0, T])$.

We consider two different scenarios for $f : \mathbb{R} \rightarrow \mathbb{R}$, depending on the assumptions for $q(t)$.

(Case:1) $q(t)$ changes its sign and $f(x)$ is periodic but not necessarily odd. In this case we assume that for some constant $L > 0$, f satisfies:

$$(H_1) : f(x + L) = -f(x), \quad \forall x \in \mathbb{R} \text{ and } f(x) > 0, \quad \forall x \in]0, L[.$$

(Case:2) $q(t)$ is of constant sign and jumps between two different values and $f(x)$ is not necessarily periodic or odd.

In the second case we assume that f satisfies:

(H₂) : There exist a, b with $a < 0 < b$ such that $f(a) = f(0) = f(b) = 0$, and

$$f(x) < 0 \quad \forall x \in]a, 0[, \quad f(x) > 0 \quad \forall x \in]0, b[, \quad \int_a^b f(s) ds = 0.$$



Stability Analysis of Non-autonomous Reaction-diffusion Systems: The Effects of Growing Domains

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By using asymptotic theory, we generalise the Turing diffusively-driven instability conditions for reaction-diffusion systems with slow, isotropic domain growth. There are two fundamental biological differences between the Turing conditions on fixed and growing domains, namely: (i) we need not enforce cross nor pure kinetic conditions and (ii) the restriction to *activator-inhibitor* kinetics to induce pattern formation on a growing biological system is no longer a requirement. Our theoretical findings are confirmed and reinforced by numerical simulations for the special cases of isotropic linear, exponential and logistic growth profiles. In particular we illustrate an example of a reaction-diffusion system which cannot exhibit a diffusively-driven instability on a fixed domain but is unstable in the presence of slow growth.



On Integrable Codimension One Anosov Actions of \mathbb{R}^k

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In this work, we consider the problem of topological classification of codimension one Anosov actions of \mathbb{R}^k , $k \geq 1$, on closed connected orientable manifolds of dimension $n + k$ with $n \geq 3$. This is a natural continuation of [1]. We show that the fundamental group of the ambient manifold is solvable if and only if the weak foliation of codimension one is transversely affine. We also study the situation where one 1-parameter subgroup of \mathbb{R}^k admits a cross-section, and compare this to the case where the whole action is transverse to a fibration over a manifold of dimension n . As a byproduct, generalizing a Theorem by Ghys [2] in the case $k = 1$, we show that, under some assumptions about the smoothness of the sub-bundle $E^{ss} \oplus E^{uu}$, and in the case where the action preserves the volume, it is topologically equivalent to a suspension of a linear Anosov action of \mathbb{Z}^k on \mathbb{T}^n .

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Asymptotic Stability of Solutions to Delay Differential Equations with Periodic Coefficients

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We consider the systems of delay differential equations of the form

$$\frac{d}{dt}y(t) = A(t)y(t) + B(t)y(t - \tau) + G(t, y(t), y(t - \tau)), \quad t > \tau > 0, \quad (1)$$

where $A(t)$ and $B(t)$ are matrices with continuous T -periodic entries; i.e.,

$$A(t + T) \equiv A(t), \quad B(t + T) \equiv B(t), \quad T > \tau,$$

$G(t, u, v)$ is a real-valued vector-function satisfying the Lipschitz condition and such that

$$\|G(t, u, v)\| \leq q_1 \|u\|^{1+\omega_1} + q_2 \|v\|^{1+\omega_2}, \quad q_1, q_2, \omega_1, \omega_2 \geq 0 \text{ are constant.}$$

We study the asymptotic stability of the zero solution to (1). Using an approach developed in [1, 2], we obtain estimates for solutions to (1) that characterize the decay rates as $t \rightarrow \infty$ and find attraction domains of the zero solution to (1) without finding roots of characteristic quasipolynomials. This approach is based on the Riccati type matrix differential inequality

$$\begin{aligned} \frac{d}{dt}H(t) + H(t)A(t) + A^*(t)H(t) + H(t)B(t)K^{-1}(\tau)B^*(t)H(t) < -K(0), \quad t \in [0, T], \\ K(s) = K^*(s) > 0, \quad \frac{d}{ds}K(s) < 0, \quad s \in [0, \tau]. \end{aligned}$$

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Dynamics of Axiom A Polynomial Skew Products on \mathbb{C}^2

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In this talk, we study polynomial skew products $f : \mathbb{C}^2 \rightarrow \mathbb{C}^2$ of the form $f(z, w) = (p(z), q(z, w))$, where $p(z)$ and $q(z, w)$ are polynomials of degree $d \geq 2$. We consider the dynamics of regular Axiom A polynomial skew products on $J_p \times \mathbb{C}$. Especially, we are interested in their postcritical sets. Put $q_z(w) = q(z, w)$ and let $C_{J_p} = \{(z, w) \in \mathbb{C}^2; z \in J_p, q'_z(w) = 0\}$ be the critical set over J_p . For any subset X in \mathbb{C}^2 , its accumulation set is defined by

$$A(X) = \bigcap_{N \geq 0} \overline{\bigcup_{n \geq N} f^n(X)}.$$

DeMarco & Hruska [DH1] defined the *pointwise* and *component-wise* accumulation sets of C_{J_p} respectively by

$$A_{pt}(C_{J_p}) = \overline{\bigcup_{x \in C_{J_p}} A(x)} \quad \text{and} \quad A_{cc}(C_{J_p}) = \overline{\bigcup_{C \in \mathcal{C}(C_{J_p})} A(C)},$$

where $\mathcal{C}(C_{J_p})$ denotes the collection of connected components of C_{J_p} . We will give characterizations of the equalities $A_{pt}(C_{J_p}) = A_{cc}(C_{J_p})$ and $A_{pt}(C_{J_p}) = A(C_{J_p})$ in terms of saddle basic sets on $J_p \times \mathbb{C}$. We also give a characterization of $A_{cc}(C_{J_p}) = A(C_{J_p})$, which was posed in [DH1].

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Omitted Values and Dynamics of Meromorphic Functions

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Let M be the class of all transcendental meromorphic functions $f : \mathbb{C} \rightarrow \mathbb{C} \cup \{\infty\}$ with at least two poles or one pole that is not an omitted value, and $M_o = \{f \in M : f \text{ has at least one omitted value}\}$. Some dynamical issues of the functions in M_o are addressed in this article. A complete classification of all the multiply connected Fatou components is made. As a corollary, it follows that the Julia set is not totally disconnected unless all the omitted values are contained in a single Fatou component. Non-existence of both Baker wandering domains and invariant Herman rings are proved. Eventual connectivity of each wandering domain is proved to exist. For functions with exactly one pole, we show that Herman rings of period two also do not exist. A necessary and sufficient condition for the existence of a dense subset of singleton buried components in the Julia set is established for functions with two omitted values. The conjecture that a meromorphic function has at most two completely invariant Fatou components is confirmed for all $f \in M_o$ except in the case when f has a single omitted value, no critical value and is of infinite order. Some relevant examples are discussed.



Invisible Parts of Attractors

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This paper deals with attractors of generic dynamical systems, and to what extent they can be “invisible”. We introduce the notion of *rate of invisibility*, which measures how rarely parts of the attractor are actually visited by generic orbits.

One would expect areas around the attractor to be visited quite often, but this is not so. For any $n \gg 1$, we present an example of a dynamical system whose defining parameters are all of order n and whose distance from structurally unstable dynamical systems (in the C^1 -norm) is expected to be $O(n^{-4})$, and in the space of skew products it *is* proved to be at least $O(n^{-2})$. However, a very large part of the attractor is visited by generic orbits with frequency $\leq 2^{-n}$, and therefore we conclude that the rate of invisibility of this dynamical system is $\leq 2^{-n}$.

The number 2^{-n} is so tiny compared with the defining parameters of the dynamical system (no smaller than order n^{-1}), that it’s hard to imagine any experiment that would actually see orbits reach the “invisible” part of the attractor. This prompts the question: should that part be considered to lie in the attractor at all? From a qualitative viewpoint, we prove that it respects a number of generally accepted definitions of attractor. But from a quantitative viewpoint, it is visited so rarely that it has no practical relevance to the asymptotic behavior of orbits.

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On Weak Product Recurrence

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A point x in a topological dynamical system (X, T) is said to be (weakly) product recurrent if for every topological dynamical system (Y, S) and for every (resp. uniformly) recurrent point $y \in Y$, the pair (x, y) is recurrent under the product action (T, S) .

It was proved many years ago that product recurrence is equivalent to distality, however full characterization of weak product recurrence is still an open problem (it was recently proved by Haddad and Ott that the class of weakly product recurrent points is essentially larger than the class of product recurrent points).

In this talk we will discuss results known from the literature and present some new sufficient conditions for weak product recurrence.

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Explicit Estimates on Certain Dynamic Inequalities in Two Variables on Time Scales

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In the present paper we establish explicit estimates on some fundamental dynamic integral inequalities in two variables on time scales which can be used as tools in the study of certain dynamic equations on time scales. Applications of one of our result are also given.

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A Model of a Plankton-nutrient Interaction with Instantaneous Nutrient Recycling

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Keywords. Nutrient, phytoplankton, zooplankton, coexistence, persistence, bloom, control.

We have considered a three-component model consisting of dissolved limiting nutrients (N) supplied at constant rate and partially recycled after the death of plankton by bacterial decomposition, Phytoplankton (P) and Zooplankton (Z). For a Realistic representation of the open marine plankton ecosystem, we have incorporated various natural phenomena such as nutrient recycling, inter-species competition and grazing at a higher level. Nutrient-phytoplankton-zooplankton interactions are observed to be very complex and situation specific.

For the model with constant nutrient input and different constant washout rates, conditions for boundedness of the solutions, existence and stability of non-negative equilibria, as well as persistence are given. Different exciting results, ranging from stable situation to cyclic blooms, may occur under different favorable conditions, which may give some insights for predictive management. Local stability of the equilibria is analyzed. It is shown that the positive equilibrium loses its stability when the nutrient input concentration passes through a critical value and the Hopf-bifurcation occurs that induces oscillations of the populations.



Symmetry and Integrability Aspects of a Generalised Damped Nonlinear Oscillators and Systems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 34C14, 22E60, 34G20, 37K10

Identifying integrable nonlinear differential equations and exploring their underlying solutions is one of the challenging problems in nonlinear dynamical systems. Different methods have been proposed in order to identify new integrable cases and to understand the underlying dynamics associated with the finite dimensional nonlinear dynamical systems. The most widely used methods include Painlevé analysis, Lie symmetry analysis, Noether's theorem and direct linearization etc. In this paper, we consider a general damped second-order nonlinear ordinary differential equation of the form $\ddot{x} + (k_1x^q + k_2)\dot{x} + k_3x^{2q+1} + k_4x^{q+1} + \lambda x = 0$, where over dot denotes differentiation with respect to t , and k_i , s , $i = 1, 2, 3, 4$, λ and q are arbitrary parameters. For $q = 1$, we carry out the Painlevé analysis, obtained the symmetry and then integrability. We repeat the analysis for $q = 2$ and finally for $q =$ arbitrary. It is interesting to see that the above equation includes a large number of physically important nonlinear oscillators and systems such as the anharmonic oscillator, force-free Helmholtz oscillator, force-free Duffing oscillator, force-free Duffing - van der Pol oscillator, modified Emden type equation and its hierarchy. Our results show several new equations which have signature of integrability.

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Controlling the Spread of Malaria Using Bacteria: A Mathematical Approach

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In this paper, we propose and analyze a non-linear mathematical model to control malaria by rearing biocontrol agents such as bacteria which eats mosquito larvae, in malaria prevalent areas along with vaccination of susceptible human population. In the modeling process, it is assumed that the bacteria and

mosquito population grows logistically in nature. The analysis of the model shows that as the magnitude of control parameters i.e., rate at which bacteria eats mosquito larvae and the rate of vaccination in system under consideration increases, the spread of malaria decreases and it may be eliminated completely if these parameters are very large.



The N -body Problem in Spaces of Negative Curvature

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In this work we generalize the Newtonian N -body problem to spaces of constant curvature κ , we derive the new equations of motion and study the 2-dimensional case. For $\kappa < 0$, we use the Weierstrass hyperbolic model of hyperbolic geometry and we will compare it with other models. We will illustrate several results concerning central configurations, relative equilibria, i.e. solutions where the distances between any two particles are constant during the motion and homographic solutions, i.e. orbits for which the configuration of the system remains similar with itself for all time. Part of the material presented in this talk is contained in [3].

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Stability Analysis of Delayed Neural Networks with Polytopic Uncertainties

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In this paper, we give novel criteria for the exponential stability of a more general class of delayed cellular neural networks. The delay neural networks in consideration are time-varying with polytopic uncertainties and various activation functions. Based on augmented parameter-dependent Lyapunov-Krasovskii functionals [1], new delay-dependent conditions for the global exponential stability are obtained for two cases of time-varying delays; that is, delays which are differentiable and have an upper bound of the delay-derivatives, and delays which are bounded but not necessary to be differentiable. Extending the results of [2, 3], the conditions are presented in terms of linear matrix inequalities, which allow to compute simultaneously two bounds that characterize the exponential stability rate of the solution.

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An Asymptotic Universal Focal Decomposition for a Family of Mechanical Systems

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Galileo, in the XVII century, observed that the small oscillations of a pendulum seem to have constant period. In fact, the Taylor expansion of the period map of the pendulum is constant up to second order in the initial angular velocity around the stable equilibrium. It is well known that, for small oscillations of the pendulum and small intervals of time, the dynamics of the pendulum can be approximated by the dynamics of the harmonic oscillator. We study the dynamics of a family of mechanical systems that includes the pendulum at small neighbourhoods of the equilibrium but after long intervals of time so that the second order term of the period map can no longer be neglected. We characterize such dynamical behaviour through a renormalization scheme acting on the dynamics of this family of mechanical systems. The main theorem states that the asymptotic limit of this renormalization scheme is universal: it is the same for all the elements in the considered class of mechanical systems. As a consequence we obtain an universal asymptotic focal decomposition for this family of mechanical systems. Furthermore, we obtain that the asymptotic trajectories have a Hamiltonian character and compute the action of each element in this family of trajectories. We conclude with a description of the utility that the asymptotic universal focal decomposition may have in the computation of propagators in semiclassical physics.

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Soret and Dufour Effects on Mixed Convection in a Micropolar Fluid Saturated Darcy Porous Medium

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A mathematical model for the steady, mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a micropolar fluid saturated Darcy porous medium in the presence of Soret and Dufour effects is presented. The nonlinear governing equations and their associated boundary conditions are initially cast into dimensionless forms using similarity transformations. The resulting system of equations is then solved numerically using the implicit finite difference scheme known as Keller-box method. The higher values of the coupling number N (i.e., the effect of microstructure becomes significant) result in lower velocity distribution and but higher wall temperature, wall concentration distributions in the boundary layer compared to the Newtonian fluid case. The numerical results indicate that the skin friction coefficient as well as rate of heat and mass transfers in the micropolar fluid are lower compared to that of the Newtonian fluid. The opposite nature can be found in the case of Darcy number. The present analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects.

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Mañé-Bochi Theorem in Dimension 3

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In 1982, Mañé announced that generically among conservative diffeomorphisms in dimension 2, the Oseledec's splitting was dominated [4], a complete proof of this fact was later presented by Bochi [2]. This means that generically among conservative diffeomorphisms of dimension 2, either all Lyapunov exponents vanish almost everywhere, or else the system is Anosov.

Further generalizations of this fact were made for instance, by Bochi-Viana, where they prove that generically among conservative systems the Oseledec's splitting of each orbit is dominated [3]. Note that, however, different behaviors could coexist, such as vanishing of all Lyapunov exponents and non-uniform hyperbolicity.

Here we show that generically among conservative systems in dimension 3, the Oseledets splitting is globally dominated, and that either all Lyapunov exponents vanish almost everywhere, or the system is non-uniformly hyperbolic and ergodic. This uses recent results by Avila-Bochi [1], and proves a conjecture therein.

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Dynamical Complexities in a Food Chain Model with Holling Type-II and Crowley-Martin Functional Responses

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We study how predator behavior influences community dynamics of predator-prey systems. It turns out that predator behavior plays a dominant role in community dynamics. The model system studied in this paper reveals that period-doubling and period-doubling reversals can generate short-term recurrent chaos which mimics chaotic dynamics observed in natural populations. Short-term recurrent chaos manifests itself when deterministic changes in a system parameter interrupt chaotic behavior at unpredictable intervals. The result reinforces an earlier suggestion that period-doubling reversals could control chaotic dynamics in ecological models.

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From Simple Dynamics to Chaos Through Nonmonotone Delayed Feedback

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Some seemingly simple nonlinear delay differential equations still pose massive problems to the understanding of their global dynamics, even after many

decades of intensive research. In the talk an overview of some recent results will be presented for two celebrated model equations with unimodal feedback: the Nicholson blowflies equation arisen in population dynamics, and the Mackey-Glass equation which has been proposed to model blood cell production and haematological diseases, and well known for its chaotic behavior. In particular, we give conditions that ensure that all solutions eventually enter the domain where the feedback is monotone, thus chaotic behavior can be excluded. We give sharp (in certain sense the sharpest) bounds for the global attractor and construct heteroclinic orbits from the trivial equilibrium to a slowly oscillating periodic orbit around the positive equilibrium. We discuss the coexistence of rapidly oscillating periodic solutions, and provide many numerical examples for different scenarios.

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A Numerical Analysis of Chaos in the Double Pendulum by Using the Multiple Scales Method

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The double pendulum is a simple physical system that exhibits rich dynamic behavior where the motion is governed by a set of coupled ordinary differential equations. Using the multiple scales methods; our original non-autonomous system is reduced to a third-order approximate autonomous system.

For certain energies the motion of double pendulum undergoes chaotic motion. Chaos theory is an area of inquiry in mathematics and physics, which studies the behavior of certain dynamical systems that are highly sensitive to

initial conditions. Mathematically, chaos means deterministic behavior which is very sensitive to its initial conditions; that is, tiny differences in the input of chaotic systems can be quickly amplified to create overwhelming differences in the output. This is the so-called butterfly effect.

Poincaré sections and bifurcation diagrams are constructed for certain, characteristic values of energy. The largest Lyapunov characteristic exponents are also calculated, where the Lyapunov exponents (also known as characteristic exponents) associated with a trajectory are essentially a measure of the average rates of expansion and contraction of trajectories surrounding it. They are asymptotic quantities, defined locally in state space, and describe the exponential rate at which a perturbation to a trajectory of a system grows or decays with time at a certain location in the state space. All three methods confirm the passing of the system from the regular low-energy limit into chaos as energy is increased.

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Asymmetric Nonlinear Oscillators

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We consider oscillators described by the equation

$$x'' = -\lambda f(x^+) + \mu g(x^-), \quad (7)$$

where $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$ and generally nonlinear functions $f(z)$ and $g(z)$ are continuous, positive valued for $z > 0$ and satisfy the condition $f(0) = g(0) = 0$. This means that the restoring forces act nonlinearly and they

are different for $x > 0$ and $x < 0$. The restoring forces are controlled by the nonnegative parameters λ and μ . If $f(z) = g(z) = z$ then equation (7) reduces to the Fučík equation ([1])

$$x'' = -\lambda x^+ + \mu x^-. \quad (8)$$

A set of all (λ, μ) such that the problem (8),

$$x(a) = 0, \quad x(b) = 0 \quad (9)$$

has a nontrivial solution is called Fučík spectrum. It is known to be a union of hyperbola looking branches and each branch corresponds to solutions with definite nodal structure.

We study the problem (7), (9) under the normalization condition $|x'(a)| = \alpha$. The respective spectrum is called α -normalized spectrum. We obtain ([2]) the description of α -normalized spectra in terms of the time-map functions (the first zero functions) corresponding to the Cauchy problems $u'' = -f(u)$, $u(a) = 0$, $u'(a) = \gamma$ and $v'' = -g(v)$, $v(a) = 0$, $v'(a) = \gamma$.

The properties of α -normalized spectra are described in a series of statements. It is to be mentioned that some of these properties seem to be unusual and closely relate to strange behavior ([3]) of technical structures subject to asymmetrical forcing.

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The Dynamics of Holomorphic Germs Tangent to the Identity Near a Smooth Curve of Fixed Points

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Let $f \in \text{End}(\mathbb{C}^2, O)$ be tangent to the identity and with order $\nu(f) \geq 2$. We study the dynamics of f near the set of fixed points $\text{Fix}(f)$. Using results of Abate [1], we prove that if the set of fixed points of f , $\text{Fix}(f)$, is smooth at the origin, f is tangential to this set, and the origin is not singular, then there are no parabolic curves for f at the origin. After that and using some techniques and results of Hakim [5], [6], we prove that $\text{Fix}(f)$ is smooth at the origin and this last one is a singular point of f , with the pure order of f , $\nu_0(f) = 1$, then there exist $\nu(f) - 1$ parabolic curves for f at the origin. Finally and using always the same results of Hakim [5], [6], we prove that if O is dicritical, then there exist infinitely many parabolic curves[9].

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Comparative Study on the Dynamics of Certain Families of Transcendental Meromorphic Functions

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Keywords. Bifurcation, chaos, iteration, Julia set, Fatou set, singular value

In the iteration theory of analytic functions, the dynamics of transcendental meromorphic functions has been explored by many researchers in recent years. Devaney and coworkers studied the dynamics of meromorphic functions with rational Schwarzian derivative. Baker and coworkers proved many results on the dynamics of meromorphic functions. We studied the dynamics of certain critically finite meromorphic functions and certain non-critically finite meromorphic functions. In this paper, we attempt a comparative study on the dynamics of one parameter families of functions $\lambda \frac{(z+\mu_0)}{(z+\mu_0+4)}e^z$, $\lambda \frac{\sinh^2 z}{z^4}$, $\lambda \tanh(e^z)$, $\lambda \tan z$, etc.



Complexity in a Prey-predator Delay Model of Leslie-Gower Type

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The complexity in a two-dimensional dynamical model is analyzed for prey predator system with Leslie-Gower type interaction incorporating discrete delay involved in growth of two species. The existence of periodic solutions via Hopf-bifurcation with respect to delay parameters are established. Numerical simulation substantiate the analytical results and also shows the appearance of chaos.

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Differentiation of SBR Measures for Topological Slow Recurrent Unimodal Maps

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Consider *deformations* of unimodal maps, that is, smooth families of unimodal maps f_t which are inside a topological class. Assuming that this topological class satisfies the so-called topological slow recurrence condition, a (topological) condition that implies the more familiar Benedicks-Carleson and Collet-Eckmann conditions, we study the differentiability of the SBR (Sinai-Bowen-Ruelle) measures of f_t with respect to the parameter t . In one of the main steps of this study we show that the family of conjugacies h_t satisfying $h_t(f_0(x)) = f_t(h_t(x))$ is differentiable with respect to the parameter t , and its derivative is continuous with respect to x , at least on the orbit of the critical point and the set of periodic points.

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Singular Cauchy Initial Value Problem for Certain Classes of Integro-differential Equations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 45J05

The existence and uniqueness of solutions and asymptotic estimates of solution formulas are studied for certain classes of integrodifferential equations in a neighbourhood of a singular point [1]–[4]. Solutions are located in a domain homeomorphic to a cone having vertex coinciding with the initial point. The proofs are based on a combination of the topological method of T. Ważewski and the Schauder's fixed point theorem or on the Banach contraction principle, respectively.

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Modeling and Analysis of HIV Infection and Drug Therapy

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 92B05, 37N25, 34D20

Here we consider a model of HIV and CD 4+ T Cells dynamics via system of ordinary differential equations. We shall consider the effect of Protease Inhibitor and RT Inhibitor drugs on dynamics of HIV. It is noted that due to inefficacy of RT Inhibitor a fraction of CD4+ T cells reverts back to uninfected cell population. So we model this phenomenon and analytically study the model. Further we shall consider the emergence of drug resistance strain of HIV in the presence of both the Protease Inhibitor and RT Inhibitor. The results found analytically, are supported and analyzed numerically.



Diffusive Driven Instabilities and Spatio-temporal Patterns in a Predator-prey System

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35K57

Predator - prey communities are building blocks of an ecosystem. Feeding rates reflect interference between predators in several situations, e.g., when predators form a dense colony or perform collective motion in a school, encounter prey in a region of limited size, etc. We perform spatio-temporal dynamics and pattern formation in a model aquatic system in both homogeneous and heterogeneous environments. Zooplanktons are predated by fishes and interfere with individuals of their own community. Numerical simulations are carried out to explore Turing and non-Turing spatial patterns. We also examine the effect of spatial heterogeneity on the spatio-temporal dynamics of the phytoplankton-zooplankton system. The phytoplankton specific growth rate is assumed to be a linear function of the depth of the water body.

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On Approximate Solutions of Integrodifferential Equation with Nonlocal Condition

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2000 MATHEMATICS SUBJECT CLASSIFICATION. Primary 45N05; Secondary 47B38, 47H10

We consider the following nonlinear mixed Volterra-Fredholm integrodifferential equation of the form:

$$x'(t) = Ax(t) + f(t, x(t), \int_0^t k(s, x(s))ds, \int_0^b h(s, x(s))ds), \quad t \in J = [0, b], \quad (1)$$

$$x(0) + g(t_1, t_2, \dots, t_p, x(\cdot)) = x_0, \quad (2)$$

where A is an infinitesimal generator of C_0 - semigroup of $T(t)$ on a Banach space X , $t \geq 0$, $0 \leq t_1 < t_2 < \dots < t_p \leq b$, f, k, h, g are given functions. We assume that $f \in C(J \times X \times X \times X, X)$, $k, h \in C(J \times X, X)$, $g(t_1, t_2, \dots, t_p, \cdot) : X \rightarrow X$ and x_0 is a given element of X .

In this paper, we investigate the approximate solutions, uniqueness and other properties of solutions of (1) – (2). The method of approximations to the solutions is a very powerful tool which provides valuable information, without the need to know in advance the solutions explicitly of various dynamic equations. We apply the method of approximations to the solutions of the initial value problem (1) – (2) and investigate new estimates on the difference

between the two approximate solutions of equation (1) and convergence properties of solutions of approximate problems. The main tool employed in the analysis is based on the application of a variant of a certain integral inequality with explicit estimate due to B. G. Pachpatte, the method of approximations and the theory of semigroups.

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Spatio-temporal Dynamics and Pattern Formation in Aquatic Predator-prey Systems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 92-08

Predator-prey communities are building blocks of an ecosystem. Feeding rates reflect interference between predators in several situations, e.g., when predators form a dense colony or perform collective motion in a school, encounter prey in a region of limited size, etc. We perform spatio-temporal dynamics and pattern formation in a model aquatic system in both homogeneous and heterogeneous environments. Zooplanktons are predated by fishes and interfere with individuals of their own community. Numerical simulations are carried out to explore Turing and non-Turing spatial patterns. We also examine the effect of spatial heterogeneity on the spatio-temporal dynamics of the phytoplankton-zooplankton system. The phytoplankton specific growth rate is assumed to be a linear function of the depth of the water body. It is found that the spatiotemporal dynamics of an aquatic system is governed by three important factors: (i) intensity of interference between the zooplankton, (ii) rate of fish predation and (iii) the spatial heterogeneity. In homogeneous environment, the temporal dynamics of prey and predator species are drastically different. While prey

species density evolves chaotically, predator densities execute a regular motion irrespective of the intensity of fish predation. When the spatial heterogeneity is included, the two species oscillate in unison. It has been found that the instability observed in the model aquatic system is diffusion driven and fish predation acts as a regularising factor. We also observed that spatial heterogeneity stabilizes the system. The idea contained in the paper provides a better understanding of the pattern formation in aquatic systems.



Singularities as Seeds of Complex Dynamics

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Since singularities of vector fields are easily detectable objects and, in many cases, organizing centers of complex behaviours, they become essential elements to explain dynamics. For instance, in order to prove the existence of chaos in a given family of vector fields, it was proved in [4, 5] that it is enough to find certain singularities that are unfolded by the family as *seeds of strange attractors*. The singularities with the lowest codimension for which an analytical proof is given are the nilpotent singularities of codimension 3 on \mathbb{R}^3 .

Motivated by answering whether it is possible to create chaos by means of coupling dynamics, where the dynamics and the mechanisms of coupling are chosen to be as simple as possible, we proved in [1] that the family consisting of two Brusselators linearly coupled by diffusion has a nilpotent singularity on \mathbb{R}^4 which is an organizing center of codimension 4. After proving that any generic unfolding of an n -dimensional nilpotent singularity of codimension n contains generic unfoldings of $(n - 1)$ -dimensional nilpotent singularities of codimension

$(n-1)$, we conclude that the family also unfolds generically nilpotent singularities of codimension 3 and therefore strange attractors, as we showed numerically in [2].

On the other hand, a linearly coupled system by diffusion is a natural framework for the arising of Hopf-pitchfork singularities due to the symmetries of those systems. These singularities can become the organizing centers of chaotic dynamics as well as synchronization/desynchronization processes in coupled systems. In [3] we illustrated this in the above-mentioned family.

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The Existence of Super-eight Solution for the 4-body Problem: a Golden-section-assisted Proof

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 70F10, 39A10

The research described in this paper aims to prove the existence of Gervert's super-eight solution for the 4-body problem with equal masses, which is one of the open problems on N-body problem, announced ever on many occasions. Normally, symmetric constraints are introduced to N-body system, for the discovery of the periodic solutions with different shapes, and for the exclusion of any collision.

We start from the above an exploration of the super-eight orbit, which is performed with not only the traditional symmetric constraint, but a dynamical geometric constraint. We employ two smooth curves with non-self-intersection that joint when $t = a$, $a \in [0, 1/4]$, at the only common point of the two curves, in one quarter of the orbit. Then the piecewise orbit, formed by 8 jointed curves,

provides the functional one class of orbits with continuous deformation. Within the mixing constraints, we develop a numerical variational method. We then prove that the rounding error is limited by the infinitesimal of the same order as the discrete step. In this case, the minimizing of the functional transforms into an optimization problem.

Afterwards, in the process of minimizing the functional with Newtonian potential for 4 equal masses, we immediately discover the bifurcation on a : (i) when a approaches to 0, the functional attains its minimum, 19.841, at a clockwise circle; (ii) when a approaches to $1/4$, the functional attains the same, at an anticlockwise circle; while, (iii) when a approaches to the golden section, the functional attains its minimum at the super-eight of the value 26.289. In this bifurcation, one of the four bodies in the initial configuration, located on the positive part of x -axis, will pass the self-intersection of the orbit for the first time when $t = 0.121$. Furthermore, the mixing constraints guarantee that there will not exist any collision in super-eight whenever the collisions on self-intersection are excluded.

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Section 11

Partial Differential Equations

On Some Nonclassical Problems for Non-stationary Equations of Mathematical Physics

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35K20, 35K90, 35L20, 35L90, 35Q40,
11J68

Interest in the subject of partial differential equations with nonclassical initial and boundary conditions has been growing fast in the last years since they are often used to construct mathematical models of various non-stationary processes of physics, biology and ecology ([1]–[3]). Nonlocal initial-boundary value problems are nonclassical problems, where instead of classical boundary or initial conditions a relationship between the boundary or initial values of the unknown function and its values at internal points of the domain or at later times are given. Discrete spatially nonlocal problem for Laplace equation first was systematically investigated in [4] and later various generalizations of the problem posed in [4] were studied for elliptic, parabolic and hyperbolic equations. Nonlocal in time problems for parabolic equation, which involve nonlocal initial condition, were studied in [5]. Note that nonlocal in time problems are generalizations of periodical problems and can be also considered as problems of controllability by initial conditions.

The present paper deals with nonlocal in time problems for parabolic, hyperbolic and Schrödinger type equations and systems. Nonclassical problems for first and second order evolution and Schrödinger equations in abstract spaces with various nonlocal initial conditions are studied and iterative algorithms of approximation of such type problems by classical ones are constructed. For

some nonlocal in time problems for hyperbolic and Schrödinger equations the dependence of existence and uniqueness of solution on algebraic properties of expressions containing time moments and geometric characteristics of the space domain is studied.

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Some Mathematical Results for the Fowler Equation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 47J35, 35G25, 76B25

We study the existence of travelling-waves and local well-posedness in a suitable function space for a nonlinear nonlocal evolution equation recently proposed by Andrew C. Fowler to describe nonlinear dune formation.

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Critical Points of Solutions to Elliptic Problems in Planar Domains

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Given a planar domain Ω , and an analytic function f , we describe the set of critical points for the solution u of the semilinear elliptic problem $\Delta u = f(u)$ in Ω , $u = 0$ on $\partial\Omega$. For simply connected domains we establish that the set of critical points is finite while for non-simply connected domains we show that this set is made up of finitely many isolated points and finitely many analytic Jordan curves. Further results are given in the case that Ω is an annular domain whose border has nonzero curvature. We also present some explicit calculations that help to illustrate the theory as well as some open questions regarding the cardinality of the critical set of the solutions.

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The Comparison Principle in Generalizing the Solvability of a Nonlinear Parabolic Equation

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In 2002, Arceo et al [2] considered radially symmetric solutions to the nonlinear parabolic equation

$$u_t = \Delta_p u + \frac{|u|^{q-2}u}{(1-|x|)^\alpha}$$

with Dirichlet boundary conditions over the unit ball in R^N , $N \geq 3$. In 2008, Arceo et al [1] established generalizations, which will be discussed in this talk along with related updates. The generalizations include modifications in the requirements for the initial data and the special exponents p and q with respect to dimension N . The proving techniques mainly involve the Comparison Principle.

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Analytic Solution for a Miscible Displacement Model in Heterogeneous Porous Media

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The standard model for incompressible miscible displacement for a mixture of oil and a solvent with concentration c in a gravity-free environment, with mixed boundary conditions and initial data c_0 , is given by the system:

$$\begin{cases} \Phi \frac{\partial c}{\partial t} + \nabla \cdot (uc - D(u)\nabla c) = \tilde{c}q, & (x, t) \in \Omega \times [0, T], \text{ and } c(x, 0) = c_0, x \in \Omega. \\ \operatorname{div}(u) = q; \quad u = -\frac{K(x)}{\mu(c)} \nabla p, \\ p = 0 \text{ in } \Gamma_D \times [0, T], \quad (D\nabla c) \cdot n = u \cdot n = 0, \text{ in } \Gamma_N \times [0, T], \end{cases}$$

where $T > 0$, $\Omega \subset \mathbb{R}^2$ is bounded, with a Lipschitz continuous boundary $\partial\Omega$. Furthermore, for the Dirichlet and Neumann boundary conditions, we have $\partial\Omega = \Gamma_D \cup \Gamma_N$ and $\Gamma_D \cap \Gamma_N = \emptyset$. The variables involved are K , the rock permeability, μ the viscosity of the fluid, which depends on the solvent concentration; p the pressure of the fluid, Φ the porosity of the medium, q the volumetric external flow rate per unit volume; \tilde{c} the specified concentration of solvent in the injection well ($q > 0$) and the resident concentration in the producer ($q < 0$). Finally, D is the diffusion-dispersion tensor.

We divided our study in two parts: when the viscosity is constant which yield in a linear problem. For this part we have used the classic theory of monotone operators [1], combined with variational techniques (non convex type as in [2]) for the evolution equation, and Hilbert Spaces Methods for the elliptic equation. In the second part of this work, we consider the non linear coupled system, using as approximation for a solution, a family of solutions of the linear uncoupled system. The approximation method uses the Zorn's Lemma. To complete the

work, we used mixed finite elements [3] and the standard Modified Method of Characteristics (MMOC) [4] to find a numerical solution.

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Γ -convergence of Power-law Functionals with Variable Exponents and Related PDEs

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Motivated by the analysis of various models related to polycrystal plasticity, the asymptotic behavior of several classes of power-law functionals acting on fields belonging to variable exponent Lebesgue spaces and which are subject to constant rank differential constraints is studied via Γ -convergence [3], [4]. We extend, in several directions, the corresponding Γ -convergence results in [2] and [5].

A number of highly degenerate nonlinear partial differential equations arise as Aronsson equations associated to variational principles for the limiting functionals, including the ∞ -Laplace equation (an excellent survey on this equation and related variational problems can be found in [1]) and its generalization to the variable exponent case, as well as new systems of PDEs that are relevant for applications which require considering differential constraints other than $\operatorname{curl} v = 0$.

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Nontrivial Solutions for the Nonlinear Hyperbolic System

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We investigate the uniqueness and multiple nontrivial solutions $u(x, t)$ for perturbations $\mu[(u + v + 1)^+ - 1]$, $\nu[(u + v + 1)^+ - 1]$ of the hyperbolic system with Dirichlet boundary condition

$$u_{tt} - u_{xx} = g_1((u + v + 1)^+ - 1) \quad \text{in} \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \mathbf{R},$$

$$v_{tt} - v_{xx} = g_2((u + v + 1)^+ - 1) \quad \text{in} \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \mathbf{R},$$

where $u^+ = \max\{u, 0\}$, μ, ν are nonzero constants and the nonlinearity $(\mu + \nu)[(u + v + 1)^+ - 1]$ crosses the eigenvalues of the wave operator.

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Decay and Nonexistence in a Nonlinear Evolution Equation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35L70, 35B35

In Esquivel-Avila [1], we studied the quasilinear riser equation

$$\begin{aligned}
 &u_{tt} + \alpha u_t + 2\beta u_{xxxx} - 2[(ax + b)u_x]_x + \frac{\beta}{3}(u_x^3)_{xxx} \\
 &- [(ax + b)u_x^3]_x - \beta(u_{xx}^2 u_x)_x = f(u), \quad (x, t) \in (0, 1) \times (0, T),
 \end{aligned}$$

with homogeneous boundary conditions

$$u(0, t) = u(1, t) = u_{xx}(0, t) = u_{xx}(1, t) = 0, \quad t \in (0, T),$$

and initial conditions

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = v_0(x), \quad x \in (0, 1),$$

where $\alpha \geq 0$, and $a > 0, b > 0, \beta > 0$, and

$$|f(s)| \leq \mu |s|^{r-1}, \quad \forall s \in \mathcal{R}, \quad \mu > 0, \quad r > 2.$$

We proved blow up and exponential decay for initial energies $E_0 < d =$ depth of the potential well. In [2], we showed blow up and global existence for initial energies $E_0 \geq d$. In this talk we want to present these results and list a set of open problems.

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Global Existence and Uniqueness of Solutions to a Model of Price Formation

María del Mar González Noguera

We study a model, due to J.M. Lasry and P.L. Lions, describing the evolution of a scalar price which is realized as a free boundary in a one dimensional diffusion equation with dynamically evolving, non-standard sources. We establish global existence and uniqueness. This is joint work with L. Chayes, M. Gualdani and I. Kim.



Quasielliptic Operators in \mathbb{R}^n and Sobolev Type Equations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35H30, 47F05

We consider the class of matrix quasielliptic operators $\mathcal{L}(D_x)$ in \mathbb{R}^n . This class belongs to that of quasielliptic operators introduced by L.R. Volevich [1] and includes homogeneous elliptic operators, elliptic and parabolic operators in the sense of Petrovskii, elliptic operators in the sense of Douglis–Nirenberg, homogeneous quasielliptic operators. The main results are isomorphism theorems for $\mathcal{L}(D_x)$ in special scales of weighted Sobolev spaces $W_{p,\sigma}^l(\mathbb{R}^n)$. These results imply some well-known isomorphism theorems for elliptic operators and a number of new isomorphism theorems for elliptic and parabolic operators in \mathbb{R}^n . Some of the mentioned results were established by the author in [2, 3, 4]. Isomorphism theorems have numerous applications in the theory of Sobolev type equations [5].

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Global Rough Solutions to the Critical Generalized KdV Equation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q53

Using the I -method, created by Colliander, Keel, Staffilani, Takaoka and Tao [1]–[4], we prove that the initial value problem (IVP) for the critical generalized KdV equation $u_t + u_{xxx} + (u^5)_x = 0$ on the real line is globally well-posed in $H^s(\mathbb{R})$, $s > \frac{3}{5}$, with the appropriate smallness assumption on the initial data. This improves the previous result by Fonseca, Linares and Ponce [5].

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Nonlinearized Fourier Approach and Coherence. Applications to Shock-turbulence Interaction

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35L65, 35L99, 35Q35, 76N10, 76N15.

The context of the considered shock-turbulence interaction assumes a *minimal* nonlinearity – in the form of a nonlinear subconscious [in the sense of P.D. Lax and A. Majda]. The interaction solution is constructed as an admissible solution.

The present analysis has essentially two objectives: (a) finding an *explicit, closed, and optimal* form for the interaction solution, and (b) offering an *exhaustively classifying characterization* of this mentioned solution.

Realizing the objective (a) is connected with: (a₁) considering a *singular limit* of the interaction solution, (a₂) considering a *hierarchy of (natural) partitions* of the singular limit, (a₃) inserting some (natural) *gasdynamic factorizations* at a certain level of the mentioned hierarchy and (a₄) noticing a *compatibility* of these factorizations (indicating a gasdynamic *inner coherence*), (a₅) *predicting some exact details* of the interaction solution, (a₆) indicating some parasite singularities [= strictly depending on the method] to be compensated [= pseudosingularities], (a₇) *re-weighting* the singular limit of the interaction solution.

Realizing the objective (b) is connected with finding some *Lorentz arguments of criticality*. The interaction solution appears essentially to (exhaustively) include a *subcritical* and respectively a *supercritical* contribution distinguished by differences of a “relativistic” nature. Precisely: in the singular limit of the interaction solution the emergent sound is *singular* in the subcritical contribution and it is *regular* in the supercritical contribution. This “relativistic” discontinuity in the nature of the emergent sound, corresponding to the singular limit of the interaction solution, appears to be dissembled (hidden) in the re-weighted interaction solution.

The structure of the present interaction solution is associated first, from a classifying prospect, to M.J. Lighthill’s fundamental representation of the shock-turbulence interaction. • It is noticed that the present interaction solution parallels and extends, from an analytical prospect, H.S. Ribner’s representation and computational approach corresponding to the interaction between a shock discontinuity and a planar vortex whose axis is parallel to this discontinuity. • The details of the “relativistic” separation between a subcritical character and a supercritical character are finally compared with the criticality arguments considered in the recent fundamental numerical studies on the shock-turbulence interaction due to S.K. Lele or K. Mahesh, S.K. Lele and P. Moin.

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Existence of Seven Solutions for an Asymptotically Linear Dirichlet Problem

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In this talk we show that an asymptotically linear elliptic boundary value problem has at least seven solutions. We consider the nonlinear Dirichlet problem

$$\begin{cases} \Delta u + \lambda f(u) = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where $\Omega \subset \mathbb{R}^N$ ($N \geq 2$) is a smooth bounded region, Δ is the Laplacian operator, and $f : \mathbb{R} \rightarrow \mathbb{R}$ is a nonlinear function of class C^1 . We assume that $f(0) = 0$, $f'(0) = 0$, $\lim_{|t| \rightarrow \infty} f'(t) = 1$, and $tf''(t) \geq 0$ for all $t \in \mathbb{R}$. We will denote by $0 < \lambda_1 < \lambda_2 \leq \dots \leq \lambda_k \leq \dots$ the sequence of eigenvalues of $-\Delta$ with zero Dirichlet boundary condition in Ω . Our main result is:

Theorem 1. If $k \geq 2$, $\lambda_k < \lambda_{k+1}$ then there exists $\epsilon > 0$ such that if $\lambda \in (\lambda_k, \lambda_k + \epsilon)$ then (1) has at least seven solutions.

We prove Theorem 1 by using the mountain pass theorem, Lyapunov-Schmidt arguments, existence of solutions that change sign exactly once, and bifurcation properties.

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Principal Eigenvalues for some non Selfadjoint Elliptic Problems and Applications

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35J20, 35P15

We wish to present a minimax formula for the principal eigenvalues of a (generally non selfadjoint) elliptic problem of the form

$$\begin{cases} -\operatorname{div}(A(x)\nabla u) + \langle a(x), \nabla u \rangle + a_0(x)u = \lambda m(x)u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $m(x)$ is a weight function which may be indefinite. Several applications can be considered, which concern for instance the antimaximum principle or some inverse problems. In this talk we will concentrate on the use of that formula to study the asymptotic behavior of the principal eigenvalues when the first order coefficient (drift term) $a(x)$ becomes larger and larger. Such a study is partly motivated by some questions from nonlinear propagation. (Joint work with T. Godoy and S. Paczka from Córdoba, Argentina).

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Scattering Asymptotics for Maxwell-Lorentz System

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q51

These are results of the papers coauthored with A. Komech, H. Spohn, and B. Vainberg, see [1] and citations therein. We consider some systems which describe an interaction of a field in \mathbf{R}^3 (for Klein-Gordon, wave, and Maxwell fields) with a relativistic charged particle of position $q(t)$ and momentum $p(t)$ at a time t . For the Maxwell field the system, in hamiltonian variables, reads

$$\begin{aligned} \dot{E}(x, t) &= -\Delta A(x, t) - \Pi_s(\rho(x - q(t))\dot{q}(t)), \quad \dot{A}(x, t) = -E(x, t), \\ \dot{q}(t) &= \frac{P(t) - A_\rho(q(t), t)}{\left[1 + (P(t) - A_\rho(q(t), t))^2\right]^{1/2}}, \quad \dot{P}(t) = \nabla(\dot{q}(t) \cdot A_\rho(q(t), t)) \\ \nabla \cdot E(x, t) &= 0, \quad \nabla \cdot A(x, t) = 0. \end{aligned}$$

Here $\hat{\Pi}_s$ is the projection onto the space of divergence-free vector fields and

$$A_\rho(x, t) = \int A(y, t)\rho(x - y)dy.$$

The system is translation-invariant and admits soliton-type solutions

$$Y_{a,v}(t) = (E_v(x - vt - a), A_v(x - vt - a), vt + a, P_v), \quad P_v = v/\sqrt{1 - v^2} + \langle \rho, A_v \rangle$$

The states $S_{a,v} = Y_{a,v}(0)$ form the soliton manifold $\mathcal{S} = \{S_{a,v} : a, v \in \mathbf{R}^3, |v| < 1\}$.

Our main result is the asymptotics of the type

$$(E(x, t), A(x, t)) \sim (E_{v_\pm}(x - v_\pm t - a_\pm), A_{v_\pm}(x - v_\pm t - a_\pm)) + W_0(t)\Psi_\pm, \quad t \rightarrow \pm\infty. \tag{1}$$

Here $W_0(t)$ is the group of the free wave equation, Ψ_\pm are scattering states.

Theorem Let i) ρ satisfies certain regularity conditions and the Wiener condition $\hat{\rho}(k) = \int e^{ikx}\rho(x)dx \neq 0 \forall k \in \mathbf{R}^3 \setminus \{0\}$;

ii) $\hat{\rho}$ has the fourth order zero at the point $k = 0$;

iii) the initial data are close to the solitary manifold in a weighted Sobolev space with sufficiently large weight.

Then the asymptotics (1) holds in global energy norm with a remainder tending to zero at a power rate in t .

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Riemann-Hilbert Approach to Scattering Problems in Elastic Media

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q15, 35Q72

We are developing Riemann-Hilbert (RH) approach to scattering problems in elastic media. The approach is based on a version of RH method introduced in nineties by A. Fokas [1] for studying boundary problems for linear and integrable nonlinear PDEs. The suitable Lax pair formulation of the elastodynamic equation is obtained. The integral representations obtained from this vector Lax pair are applied to Rayleigh wave propagation in an elastic quarter space and half space. This reduces the problem to the analysis of certain underdetermined matrix RH problem on a torus. We showed that the problem can be in fact re-formulated as a well-posed RH problem with a shift. Some results of the described analysis will be discussed. Part of this work is done jointly with J. Kaplunov.

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Weighted and Non-local Boundary Value Problems for a Degenerate Forth Order Elliptic Equation and Applications to Cusped Plates

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35J70; 74K20

The paper is devoted to weighted and non-local boundary value problems in a half-plane for a fourth order elliptic equation with the order degeneration on a part of the boundary. Weighted boundary operators are second and third order differential operators. To these mathematical problems lead two following problems for s.c. cusped Kirchhoff-Love plates: (i) on the edge of the plate the bending moment and generalized force (concentrated along the plate edge in the case of a cusped plate) are prescribed; (ii) on the edge of the plate the concentrated at point [at the origin of the coordinate system; in the case of a non-cusped plate concentrated along the vertical segment passing through the point $(0, 0)$] bending moment and generalized shearing force are prescribed. Both the problems are solved in the explicit form. About cusped Kirchhoff-Love plates see [1].

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Some Results on the Global Existence to Navier-Stokes Equations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q30

We consider the global existence of strong solutions of the 3D incompressible Navier-Stokes equations. We in particular show that a set of all initial data in $D(A^{\frac{1}{2}})$ with which global strong solutions exist with some “growth property” is open and locally δ -convex.

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On a Boundary Value Problems for Spectrally Loaded Heat Operator

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In the report we consider the boundary value problems (BVP) for a loaded heat conduction operator (one-dimensional in the space variable) in a quarter-plane which relates to the class of functional-differential operators and has the form $Lu - \lambda Bu$, where L is the differential part and B is the loaded part.

The operator in question is peculiar since the spectral parameter λ is the coefficient of the loaded summand, the order of the derivative in the loaded summand is equal to that of the differential part of the operator, and of the load point defined by the function $x(t)$ moves with a variable velocity, i.e., the derivative $x'(t)$ is not always constant.

Moreover, the load operator B in the generalized spectral problem $Lu = \lambda Bu$ of this report is not invertible. Such operator $L - \lambda B$ is called spectrally loaded.

In the domain $Q = \{x, t | x > 0, t > 0\}$, we consider the BVP

$$(L - \lambda B)u = \begin{cases} u_t - u_{xx} + \lambda u_{xx}|_{x=t^\omega} = f, \\ u(x, 0) = u(0, t) = 0, \end{cases} \quad (1)$$

$$(L^* - \bar{\lambda} B^*)v = \begin{cases} -v_t - v_{xx} + \bar{\lambda} \delta''(x - t^\omega) \otimes \int_0^\infty v(\xi, t) d\xi = g, \\ v(x, \infty) = v(0, t) = v(\infty, t) = v_x(\infty, t) = 0, \end{cases} \quad (2)$$

where $\lambda \in C$ is spectral parameter, $\omega \in (-\infty, \infty)$.

The BVP (2) is adjoint to problem (1).

Here the loaded summand $\lambda u_{xx}|_{x=t\omega}$ in the equation (1) is not a weak perturbation of the differential part $u_t - u_{xx}$ and the loaded differential operator reveals some new properties not enjoyed by the operators with weak perturbation.

We demonstrate that the BVP (1) under consideration is Noetherian and has finite index for some strictly described values of the spectral parameter λ in the complex plane C [1], [2] and the real parameter ω .

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The Periodic Solutions of the Nonlinear Hamiltonian System

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q72

Let $G : R^{2n} \rightarrow R$ be a differentiable function with $G(0, \dots, 0) = 0$ and G' be its gradient. We show the existence of at least m nontrivial periodic solutions of the nonlinear Hamiltonian system

$$\dot{z} = J(G'(z)), \tag{1.1}$$

where $z : R \rightarrow R^{2n}$, $\dot{z} = \frac{dz}{dt}$, $J = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix}$, I_n is the n - dimensional identity matrix. Let $a \cdot b$ and $|\cdot|$ denote the usual inner product and norm on R^{2n} . Let $z = (p, q)$, $p = (z_1, \dots, z_n)$, $q = (z_{n+1}, \dots, z_{2n}) \in R^n$. We assume that G satisfies the following conditions:

- (G1) $G : R^{2n} \rightarrow R$ is C^1 with $G(0, \dots, 0) = 0$.
- (G2) There exists $h \in N$ such that

$$h < \liminf_{|z| \rightarrow \infty} \frac{G'(z) \cdot z}{|z|^2} < h + 1.$$

- (G3) There exists $m \in N$ such that

$$h + 2m < \liminf_{|z| \rightarrow 0} \frac{G'(z) \cdot z}{|z|^2} < h + 2m + 1,$$

or

$$h - 2m - 1 < \limsup_{|z| \rightarrow 0} \frac{G'(z) \cdot z}{|z|^2} < h - 2m.$$

(G4) There exists an integer Γ such that $\Gamma \leq \frac{G'(z) \cdot z}{|z|^2} \leq \Gamma + 1$.

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On the Parametric Interest of the Black-Scholes Equation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 91B28, 35K05

We have discovered some parametrics λ in the Black-Scholes equation which depend on the interest rate r and the volatility σ and later named the parametric interest. On studying the parametric interest λ , we find that such λ gives the sufficient condition for the existence of solutions of the Black-Scholes equation which is either weak or strong solutions. Related topics could be seen in [1], [2], and [3].

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Convergence of Interior Source Methods for Scattering Problems

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We consider interior source methods for solving acoustic or electromagnetic scattering problems, which are usually posed in an exterior domain. Interior source methods replace the differential equations by an integral equation, which, in contrast to boundary element methods are set on a closed curve or a closed surface inside the scattering body. This gives integral equations of type

$$\int_{\gamma} K(x, y)u(y)dS_y = f(x), \quad x \in \Gamma,$$

where γ and Γ (the boundary of the scatterer) are some closed disjoint curves or surfaces. If Γ and γ are analytic, then this is an integral equation of the first kind with an analytic kernel. Results about existence and uniqueness of the solution can often be obtained only in spaces of linear analytic functionals, and in general case, only density of the range of the integral operator can be proved.

We look for approximate solutions of the integral equation as linear combinations of Dirac's δ -functions with supports on γ , because of simplicity of the corresponding solution of the differential equation. The coefficients of the linear combination can be determined either by collocation or by minimizing some convex functional of the residual. This method is also known as discrete source method [1], but no convergence rates have been obtained.

In two-dimensional case, if Γ is analytic it is possible to choose γ and the grid so that the convergence rates of the approximate solutions of both the integral and the original partial differential equations are exponential in the number of variables. If Γ has corners, the collocation method still converges, if one chooses the interior curve, the supports of the δ -functions and the collocation points carefully, but the convergence rate deteriorates to algebraic (see [2]).

In three-dimensional case the L^2 -minimization method always converges, and in certain cases the convergence is exponential. For the collocation method the convergence is still an open question.

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Numerical Simulation of Surface Wave Group Propagation over Slowly Varying Bottom

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The propagation of surface wave group is modeled mathematically by the nonlinear Schrödinger (NLS) equation (Benney and Newell, 1967; Whitham, 1974; Yuen and Lake, 1982). The equation has applications not only in hydrodynamics, but also in nonlinear optics, nonlinear acoustics, plasma physics and so on. It has been studied extensively both theoretically and numerically. In this presentation, we implement the NLS equation with non constant dispersive and nonlinear coefficients to describe the wave propagation over slowly varying bottom, as has been derived by Djordjevic and Redekopp (1978). A numerical scheme using compact finite difference method from Xie et al (2009) is adopted, modified and developed to obtain a better understanding of the wave propagation over slowly varying bottom on a wave tank. A comparison with approximate solution obtained by Benilov and Howlin (2006) is also discussed.

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Exact Solutions of Zakharov-Kuznetsov Equation with Power Law Nonlinearity in (1+3) Dimensions

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q51, 35Q53, 37K10, 78A60.

In this talk we present exact solutions of Zakharov-Kuznetsov's equation in (1+3) dimensions with an arbitrary power law nonlinearity. The method of Lie symmetry analysis will be used to carry out the integration of Zakharov-Kuznetsov's equation. The solutions obtained are cnoidal waves, periodic solutions, singular periodic solutions and solitary wave solutions. Subsequently, the extended tanh function method and the G'/G method will also be used to integrate the Zakharov-Kuznetsov's equation. Finally, the non-topological soliton solution will be obtained by the aid of ansatz method. Numerical simulations throughout the paper will be given to support the analytical development. For some of the work done on Zakharov-Kuznetsov's equation see [1, 2, 3].

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Periodic Solutions for Nonlinear Evolution Equations at Resonance

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We present a method of finding T -periodic solutions for the following problem

$$\dot{u}(t) = -Au(t) + \lambda u(t) + F(t, u(t)), \quad t > 0 \quad (2)$$

where $T > 0$, $\lambda > 0$, a linear operator $A: D(A) \rightarrow X$ is such that $-A$ generates a compact C_0 semigroup $\{e^{-tA}\}_{t \geq 0}$ on $X := L^2(\Omega)$ and $F: [0, +\infty) \times X \rightarrow X$ is the Nemytskii operator for a time T -periodic bounded mapping $f: [0, +\infty) \times \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ where $\Omega \subset \mathbb{R}^n$ is open and bounded. Motivated by [1], [3], we consider the case at resonance i.e. $N := \text{Ker}(\lambda I - A) \neq \{0\}$.

The idea of translations along trajectories is used, that is, periodic solutions of (2) are found as fixed points of $\Phi_T: X \rightarrow X$ being the translation operator associated with (2). Define $g: N \rightarrow N$ by $g(u) := \int_0^T PF(s, u) ds$ for $u \in N$, where $P: X \rightarrow X$ is an orthogonal projection onto N . We show that if $\text{Ker}(\lambda I - A) = \text{Ker}(\lambda I - A^*) = \text{Ker}(I - e^{T(\lambda I - A)})$, then under some Landesman–Lazer type condition and standard assumptions on F

$$\text{deg}_{\text{LS}}(I - \Phi_T, W) = (-1)^{\mu + \dim N} \text{deg}_{\text{B}}(g, W \cap N) \quad (3)$$

provided $W \subset X$ is an open bounded neighborhood of the origin such that $g(u) \neq 0$ for $u \in \partial_N(W \cap N)$, where deg_{LS} and deg_{B} stand for the Leray–Schauder and Brouwer degree, respectively, and μ is the sum of the algebraic multiplicities of eigenvalues of $e^{T(\lambda I - A)}$, lying in $(1, +\infty)$. The formula (3) together with a guiding function argument provides an effective criterion for the existence of T -periodic solutions for (2), which may be applied to many classes of partial differential equations.

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Regularity for 3D Navier–Stokes Equations with Large Data

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We consider the global existence of strong solutions of the 3D Navier-Stokes equations.

1. We first show by a simple argument that a strong solution exists globally when the product of L^2 norms of the initial velocity and the gradient of the initial velocity are small enough.
2. We next consider a thin domain and we present some improvement on the global existence. Generalizing this framework, we introduce a new approximation and investigate its global existence.
3. We finally investigate a global stability of solutions.

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Gradient Bounds for Elliptic Problems Singular at the Boundary and Application to a Stochastic Control Problem with State Constraint

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Let Ω be a bounded smooth domain in R^N , $N \geq 2$, and let us denote by $d(x) = \text{dist}(x, \partial\Omega)$. We deal with a priori estimates, existence and regularity for solutions of nonlinear elliptic equations that are singular at the boundary. The model problem is the following:

$$-\alpha\Delta u + u - \sigma \frac{\nabla u \cdot \nabla d}{d(x)} + d^\beta(x)|\nabla u|^2 = f(x) \quad \text{in } \Omega, \quad (4)$$

where f is $W_{\text{loc}}^{1,\infty}(\Omega)$ function, and $\alpha, \beta, \sigma > 0$.

The main goal is to prove, under suitable assumptions, Lipschitz estimates for solutions of (4) and, furthermore, to study the stability of such estimates as α vanishes, i.e. for the associated first order equation.

The interest in such a class of equations arises from the application to a stochastic control problem with state constraint. Actually, this model has been introduced by J.M. Lasry and P.L. Lions in [1] and that been already studied by the authors in a previous paper (see [2]).

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On Solutions of a Semi-linear Wave Equation with Space-time Dependent Coefficients and a Memory Boundary-like Antiperiodic Condition

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This talk is about the solvability of an initial-boundary value problem of a semi-linear wave equation with space-time dependent coefficients and the so-called memory boundary-like antiperiodic condition. The main tool is the contraction-Galerkin method: the Faedo-Galerkin method is applied for the solvability of a linear problem correspondent to the given problem, then the existence the solution of the given problem is dealt with by a contraction. This study is a generalization of [1, 2, 3].

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System of Chemotaxis with Nonlinear Diffusion

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We study the problem of Cauchy for system of equations

$$\begin{cases} u_t = \nabla \cdot (k(u)\nabla u) - \chi \nabla \cdot (u\nabla v), t > 0, x \in R^n, \\ \Delta v = -u, t > 0, x \in R^n, \end{cases} \quad (5)$$

$$u(0, x) = u_0(x) \geq 0, x \in R^n, \quad (6)$$

where $k \geq 0$, χ – coefficient of chemotaxis. The main question is to define conditions on k, χ and initial function $u_0(x)$, under which unbounded solutions of (5),(6) are not localized. Method of investigation is based on one-parametrical family $\{U, V\}$ stationary solutions of system of equations (5):

$$\begin{cases} \nabla \cdot (k(U)\nabla U) - \chi \nabla \cdot (U\nabla V) = 0, \\ \Delta V + U = 0. \end{cases} \quad (7)$$

It is proved, that spatial structure of stationary solutions of system (7) classifies the evolutional property of solutions of problem (5),(6). We note, that in case $k(u) \equiv 1$ system (5) is Keller-Segel model [1]. Method of stationary states are proposed on monograph [2]. Some other approaches to analysis of system of chemotaxis performed in [3].

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Modelling of Surface Air Temperature and Pricing of Weather Derivatives

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This study attempts to formulate a pricing model for the weather derivatives, whose payoffs depend on surface air temperature. Daily temperature data for the last thirty years is closely analyzed for four cities in U.K. to model a temperature process which captures the daily temperature fluctuations including the seasonal patterns and the year-on-year up-ward trend behaviour of the temperature. This work further evaluates an arbitrage-free option pricing using a Gaussian Ornstein-Uhlenbeck model. Keeping in mind that temperature, the underlying variable of the weather derivative, is non-tradable we consider a risk premium estimator to find the price of a weather derivatives contract. Finally, the study provides results based on these models as well as based on Monte Carlo Simulations.

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A Fourier Type Expansion Formula for Problems in Hydroelasticity

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A large class of problems in the field of Hydroelasticity involve higher order boundary conditions associated with Laplace equation as the governing equation and the eigenfunctions associated with these problems are not orthogonal in the usual sense. In the present paper, a generalized Fourier type integral theorem along with the corresponding orthogonal mode-coupling relations associated with three dimensional Laplace equation are derived to deal with wave structure interaction problems in infinite water depth. The present expansion formula is a generalization of the expansion formula developed by Manam et al. (2006) to deal with two dimensional Laplace equation satisfying higher order boundary conditions. Further, it has been proved that the eigenfunctions associated with the boundary value problems are linearly dependent. Several identities associated with the expansion formulae are derived in a straightforward manner. The utility of the expansion formula is demonstrated by deriving the expansion formula for flexural gravity wave maker problems in three dimensions in the case of infinite water depth. As an application of the expansion formula for flexural gravity waves, the reflection of flexural gravity waves by a rigid wall is analyzed. The present results can easily be applied to study scattering of wave propagation by flexible structures in the field of Acoustic, Electromagnetic Theory, Elasticity, Ocean Engineering, Polar Sciences and Engineering.

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Properties of Solutions of a Class of Degenerate Differential Equations

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Let L be the closure in the norm of the space $L_p(R)$ ($R = (-\infty, +\infty)$), $1 \leq p < \infty$) of differential expression

$$ly = -y'' + qy' + ry, \quad (1)$$

defined on the set $C_0^\infty(R)$ of arbitrarily differentiable finite functions. Functions q and r are assumed continuously differentiable and continuous, respectively. In the case when $q = 0$, the questions of existence, uniqueness, coercive estimates and smoothness of solutions of the equation $Ly = f$ have been studied in [1–4]. If the summand qy' in the expression (1) is a small perturbation of the sum of two others, the results [1–4] can be extended to the general case. It is interesting to investigate the case when qy' does not satisfy these conditions of subordination, in particular, when q is growing faster than r near infinity. This work is devoted to this question.

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Elliptic Problems with a Hardy Potential and Critical Growth in the Gradient

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35D05, 35D10, 35J20, 35J25, 35J70, 46E30, 46E35.

We analyze existence and nonexistence of positive solutions to problem

$$(P_\pm) \quad -\Delta u \pm |\nabla u|^2 = \lambda \frac{u}{|x|^2} + f \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where Ω is a bounded domain containing the origin.

The main results are the following:

- i)* If the quadratic term in the gradient appears in the equation as a reaction term ($-|\nabla u|^2$) and $\lambda > 0$, then there is no solution to problem (P_-) (even in a very weak sense).

- ii*) If the quadratic term in the gradient appears in the equation as an absorption term ($+|\nabla u|^2$), then there exists a positive solution to (P_+) for all $\lambda > 0$ and $f \in L^1(\Omega)$.

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Finite Difference Analysis of Couple Stress Fluid Past an Infinite Vertical Cylinder

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A Numerical analysis is performed to study the transient free convective boundary layer flow of couple stress fluid past an infinite vertical cylinder, in the absence of body forces and body couples. The transformed dimensionless governing nonlinear set of equations for the flow and heat transfer characteristics are derived and solved by using the Crank-Nicolson type of implicit finite difference method. The results concerning the velocity and temperature profiles of both couple stress and Newtonian fluids across the boundary layer are illustrated graphically and discussed for different values of Prandtl number. Transient effects of velocity and temperature are analyzed and compared with those of the Newtonian fluids. The heat transfer characteristics are analyzed with the help of average skin-friction and Nusselt number and are shown graphically. It is observed that in couple stress fluids the deviation of transient velocity and temperature profiles from the hot wall is much more than that of the Newtonian fluids.

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Global Structure Stability of Riemann Solutions for Linearly Degenerate Hyperbolic Conservation Laws under small BV Perturbations of the Initial Data

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In this paper, we study the global structure stability of the Riemann solution $u = U(\frac{x}{t})$ for general $n \times n$ quasilinear hyperbolic systems of conservation laws under a small BV perturbation of the Riemann initial data. We prove the global existence and uniqueness of piecewise C^1 solution containing only n contact discontinuities to a class of the generalized Riemann problem, which can be regarded as a small BV perturbation of the corresponding Riemann problem, for general $n \times n$ linearly degenerate quasilinear hyperbolic system of conservation laws; moreover, this solution has a global structure similar to the one of the self-similar solution $u = U(\frac{x}{t})$ to the corresponding Riemann problem. Our result indicates that this kind of Riemann solution $u = U(\frac{x}{t})$ mentioned above for general $n \times n$ quasilinear hyperbolic systems of conservation laws possesses a global nonlinear structure stability under a small BV perturbation of the Riemann initial data. As an application, we apply the result to the system of the planar motion of an elastic string.

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The Riemann–Hilbert Problem Approach to the Camassa–Holm Equation and the Long-time Asymptotics

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q35

We study the initial value (IV) problem ($-\infty < x < \infty$) and the initial-boundary value (IBV) problem ($0 < x < \infty$) for the Camassa–Holm equation [1]

$$u_t - u_{txx} + 2\kappa u_x + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \quad t > 0$$

with $\kappa > 0$, which is a model equation describing the unidirectional propagation of waves in shallow water over a flat bottom. Our approach is based on expressing the solution in terms of the solution of a matrix Riemann–Hilbert problem formulated in the complex plane of a spectral parameter appearing in the Lax pair representation [1], [2] of the Camassa–Holm equation. The jump conditions for the corresponding RH problem are expressed in terms of the spectral functions, which in turn are expressed in terms of either initial data $u(x, 0)$ (for the IV problem) or initial and boundary (at $x = 0$) values $u(0, t)$, $u_x(0, t)$, and $u_{xx}(0, t)$ (for the IBV problem). In the latter case, the compatibility of initial and boundary values is characterized in terms of algebraic relations among the corresponding spectral functions. We develop the nonlinear steepest descent method [3] and compute the long-time asymptotics of the solutions, which appear to be qualitatively different in different domains of the phase space.

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Wave Propagation in a Thermally Conducting Mixture of an Elastic Solid and a Newtonian Fluid

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The governing equations for generalized thermoelasticity of a mixture of an elastic solid and a Newtonian fluid are formulated in the context of Lord-Shulman and Green-Lindsay theories of generalized thermoelasticity [1-3]. These equations are solved to show the existence of three coupled longitudinal waves and two coupled transverse waves, which attenuate and are dispersive in nature. Reflection from thermally insulated stress free surface is considered for incidence of both coupled longitudinal wave and coupled transverse wave. Reflection coefficients of reflected waves are computed numerically with the angle of incidence for a particular example of the present model.

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Self-similar Shocks in non-ideal Gas

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 74J30, 74J40, 35L67, 35L45, 76M60

In this paper, the method of Lie group invariance is used to obtain a class of self-similar solutions to the problem of shocks in an non-ideal gaseous medium and to characterize analytically the state-dependent form of the medium ahead for which the problem is invariant and admits self-similar solutions. For a particular case of power law, shock path is recovered as special case depending on the arbitrary constants occurring in the expression for the generators of the transformation. Numerical calculations have been performed to obtain the similarity exponents and the profiles of the flow variables, and comparison is made with the known results.

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Solution to Magnetogasdynamics

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35L67, 58J45, 76N15

The Riemann problem for a quasilinear hyperbolic system of equations governing the one dimensional unsteady simple wave flow of an inviscid and perfectly conducting compressible fluid, subjected to a transverse magnetic field, is solved approximately. This class of equations includes as a special case the Euler equations of gasdynamics. It is noticed that in contrast to the gasdynamic case, the pressure is varying across the contact discontinuity. The iterative procedure is used to find densities, between left acoustic wave and right contact discontinuity, and between right contact discontinuity and right acoustic wave, respectively. All other quantities follow directly throughout the (x, t) -plane, except within rarefaction waves, where an extra iterative procedure is used along with Gaussian quadrature rule to find particle velocity; indeed, the determination of the particle velocity involves numerical integration when the magneto-acoustic wave is a rarefaction wave. Lastly, we discuss numerical examples and study the solution influenced by the magnetic field.

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On the Existence of Extreme Waves and the Stokes Conjecture with Vorticity

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35R35, 76B15

We present some recent results on singular solutions of the problem of travelling gravity surface water waves on flows with vorticity. It has been known since the work of Constantin and Strauss [1] that there exist spatially periodic waves of large amplitude for any vorticity distribution. Building on earlier results [2], we show [3] that, for any nonpositive vorticity distribution, a sequence of large-amplitude regular waves converges in a weak sense to an extreme wave with stagnation points at its crests. The proof is based on new a priori estimates, obtained by means of the maximum principle, for the fluid velocity and the wave height along the family of regular waves whose existence was proved in [1]. We also show [2, 3, 4] that this extreme wave has corners of 120° at its crests, as conjectured by Stokes in 1880. Further extensions of the Stokes conjecture, obtained by new geometric methods [4], will also be presented.

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Mathematical Problems of the Q-tensor Theory

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q74, 76A05

The complexity of nematic liquid crystals is described, in Landau-de Gennes theory, through functions defined on two or three-dimensional domains and

taking values into the set of Q -tensors, that is three-by-three symmetric traceless matrices.

The main mathematical challenges are caused by the necessity to finely manipulate “high dimensional” objects. This large dimensionality of the domain and target space allows for specific features inaccessible in lower dimensions (for instance one needs at least a 2D domain to have a real analytic matrix-valued function with discontinuous eigenvectors). Also, in 2D domains, in the so-called “constrained theory”, one can express lifting questions in terms of familiar complex analysis problems, but in 3D one needs to construct the appropriate analogue of the complex analytic language in order to effectively deal with the lifting problem, and this is yet to be done.

I will present some natural physical questions and recent advances in their mathematical treatment, advances that involve a non-standard combination of diverse tools from analysis, algebraic topology, Riemannian geometry, regularity theory and qualitative properties of elliptic and parabolic systems.



Section 12

Mathematical Physics

The Ultra Relativistic, Maxwell and Proca Equation

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We consider the Proca equation which is the Maxwell equation of electromagnetism with mass in the ultra relativistic limit using Snyder-Sidharth Hamiltonian. There is now an extra term involving an extra parity non conserving term and we investigate the consequence both for Proca equation and Maxwell equation and also for understanding of Kaon's decay.

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N-Dimensional Bianchi Type V Universe in Creation-field Cosmology

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 83F05

We have studied the [1], [2] and [3] C-field cosmology with Bianchi type-V space time in N-dimensions. Using methods of [4], the solutions have been studied when the creation field C is a function of time t only. The geometrical and physical aspects for model are also studied.

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Derivation of the “Two-source” Jet Noise Paradigm Using the SO(3) Lie Group Representation for the Reynolds Stress Auto-covariance Tensor

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76Q05

Experiments on jet flows indicate that the overall sound pressure level peaks at small observation angles to the jet axis [1]. Recent work using the generalized acoustic analogy formalism [2] has focused on the kinematic theory of the two-point, time delayed Reynolds stress auto-covariance tensor [3]. Under this approach the starting point is an exact result that expresses the far field acoustic spectrum as the convolution product of a propagator and the (generalized) Reynolds stress auto-covariance tensor, which is a tensor of rank four with one vector dependence. The problem then becomes one of modeling this tensor in order to reduce its number of independent components and then to

determine the consequences for the acoustic spectrum. In this paper, we show the $SO(3)$ Lie group representation [4] can be used to decompose the Reynolds stress auto-covariance tensor into a form that depends upon 6 independent components. The result is fairly general and indicates a “paradigm” of two independent groups of acoustic source terms exist that both contribute to the overall sound pressure level, and can explain the observation from experiments.

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The β -function Over Curved Space-time Under ζ -function Regularization

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This paper generalizes the Connes-Marcolli renormalization bundle to scalar field theories over a curved space-time background, specifically looking at ζ -function regularization. It further extends the idea of renormalization mass scale from a scalar change of metric to a conformal change of metric. In this context, it becomes useful to think of the renormalization mass scale as a complex 1-density over the background manifold.



MHD Transient Flow Past an Impulsively Started Horizontal Porous Plate in a Rotating Fluid with Hall Current

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An exact solution to the problem of an unsteady three dimensional MHD flow of an incompressible viscous electrically conducting fluid past an impulsively started horizontal porous plate taking into account the Hall current is presented. It is assumed that the fluid rotates with a constant angular velocity about the normal to the plate and a uniform magnetic field is applied along the normal and directed into the fluid region. The non-dimensional equations governing the flow are solved by Laplace Transform Technique. The primary velocity, the secondary velocity, the skin friction at the plate due to the primary motion and the skin friction at the plate corresponding to the secondary motion are demonstrated graphically. The effects of the physical parameters involved are discussed graphically and physically interpreted.



Viscous Contributions to the Pressure for Potential Flow Analysis of Electrohydrodynamic Kelvin-Helmholtz Instability

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76E25

In the present paper, we have studied the viscous contributions to the pressure for the potential flow analysis of Kelvin-Helmholtz instability of two viscous fluids in the presence of electric field acting in the direction of streaming. We have introduced the viscous pressure in the normal stress balance along with irrotational pressure and it is assumed that the viscous contributions to the pressure will resolve the discontinuities between the tangential stresses and tangential velocities at the interface. It has been observed that the viscous contributions to the pressure for potential flow solution is more stable than viscous potential flow solution.

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The Effect of Shear Flow on Thermomagnetic Convection in Ferrofluids

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Thermal convection in fluids is one of the most studied instability problems [1]. Plane couette flow between two moving horizontal boundaries is another problem that has attracted scientists. Plane couette flow is linearly stable for all Reynold numbers, but heated plane couette flow becomes unstable after the temperature difference between the boundaries reaches certain critical value. Thermal convection plays a central role in transporting heat and material in vertical direction. Convection in a horizontal layer of fluid, heated from below and subjected to shear velocities at the horizontal boundaries has many applications in industry, in cooling of electronic instruments. Ferrofluids act as a model fluid to study thermomagnetic convection. Magnetic field affects the onset of convection in ferrofluids and this leads to many technological applications [2].

In this paper thermomagnetic convection in horizontal layer of a ferrofluids, in the presence of shear of the base flow and with external magnetic field in the normal direction is investigated [3]. The numerical technique called the Chebyshev tau method is used to study the stability problem. The stability of the flow is determined in terms of Rayleigh number, which is a measure of temperature gradient across the fluid layer. The effect of shear flow is observed through variation of Reynold number.

It is observed that the onset of the longitudinal rolls does not depend upon the shear of the base flow and they always dominate. In the absence of the shear flow, the rolls in any direction appear at the same value of temperature gradient. The onset of transverse rolls depends upon the shear velocity determined by Reynold number. The onset of longitudinal rolls is also independent of the Prandtl number. But increase in Prandtl number increases the required value of the temperature gradient for the onset of transverse rolls. Magnetic field also induces instability. In the absence of gravity, instability is induced at much higher value of magnetic parameter \mathcal{M} for the layer of ferrofluid with moving boundaries.

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Applications of the Method of Stereographic Parametrization

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The idea how to generalize a stereographic projection to an arbitrary semisimple Lie group is expounded in [1]. It is based on comparing Iwasawa and Gauss-Bruhat decompositions of the group. As a result one obtains a projection from a dual space onto an adjoint or coadjoint orbit. In the case of group $SU(2)$ it turns into the well-known stereographic projection onto the extended complex plane.

At the same time, this Lie group construction gives a uniform complex parametrization for orbits. The parametrization is performed in terms of canonical coordinates in a nilpotent subgroup of the complexified semisimple Lie group [2]. A Kählerian potential for an orbit and the corresponding Kählerian metrics and two-form are easily expressed in terms of these complex parameters.

Moreover, a lot of physical applications arise. A phase space for an integrable Hamiltonian system usually has an orbit structure. In particular, a mean field of magnetization in a spin lattice lives and evolves on coadjoint orbits of a Lie group over the corresponding Lie algebra. Dealing with spin s lattice and taking into account higher powers of exchange interaction, it is possible to represent a complete associative algebra of spin operators as $isu(2s+1)$. An evolution of the mean field is governed by a Landau-Lifshitz-like equation, proper for each orbit [3]. Some topological excitations can be represented by holomorphic functions assigned to the complex parameters [4].

Another application was performed for the theory of controllability. The method of stereographic parametrization allows to reconstruct a Hamiltonian for a unitary evolution of a system under control. Note that a unitary evolution is localized on an adjoint orbit of some unitary group over the corresponding algebra [5].

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Scattering of Oblique Water Waves in a Two-layer Fluid Flowing Through a Channel with Bottom Deformation

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The problem of oblique surface wave propagation over a small deformation in a channel flow consisting of two layers is considered. The upper fluid is bounded by a fixed wall, which is an approximation for the free surface, and the lower fluid is bounded by the bottom surface having a small deformation. A simplified perturbation analysis is employed to calculate the first-order corrections to the velocity potentials in two fluids by using the Green's integral theorem in a suitable manner, with the introduction of appropriate Green's functions, and also to calculate the reflection and transmission coefficients in terms of integrals involving the shape function $c(x)$ representing the bottom deformation. Two-dimensional linear water wave theory is utilized for formulating the related boundary value problem. Three special examples of bottom deformation are considered to validate the results. While considering a patch of sinusoidal ripples (having the same wave number), it is observed that the reflection coefficient is an oscillatory function in the ratio of twice the component of the wave number along x -axis and the ripple wave number. When this ratio approaches one, the theory predicts a resonant interaction between the bed and the interface, and the reflection coefficient becomes a multiple of the number of ripples. Similar results are observed for a patch of sinusoidal ripples having different wave numbers. These theoretical observations are supported by graphical results.

This work is motivated mainly by the works done in [1], [2] and [3].

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Integrability Properties of the $\text{AdS}_4 \times \text{CP}^3$ String Sigma Model

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 81T30

The aim of the work is to shed some light on the structure of the duality [1] (usually called AdS/CFT) between quantum gauge theories in Minkowski space-time and theories of strings propagating in Anti-deSitter (AdS) manifolds. In particular, we study the supersymmetric sigma model with target space $\text{AdS}_4 \times \text{CP}^3$, first introduced in [2]. When quantized in the background of a certain classical solution, the worldsheet theory exhibits an interesting spectrum of particle-like states, and there exists a limit, when some of the particles become massless. We find a Lagrangian governing the dynamics of these massless modes — a sigma model with target space CP^3 and a Dirac fermion. We discuss, whether the S-matrix of this theory factorizes, that is if this theory is completely integrable. The latter is important for the integrability of the ambient $\text{AdS}_4 \times \text{CP}^3$ model — a question, which has recently attracted a lot of attention.

We provide the necessary definitions and explain the origin of the conjectures to make the exposition accessible to a person unfamiliar with the subject. In this way we hope to attract the attention of the mathematical community to the problem of finding justifications (or even proofs) for the AdS/CFT duality other than the original one [1]. This might well play a role in a future solution of the “Yang-Mills mass gap problem”.

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Bethe Ansatz Solution of an Integrable, Non-Abelian Anyon Chain with $D(D_3)$ Symmetry

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 81R12

The exact solution for the energy spectrum of a one-dimensional Hamiltonian with local two-site interactions and periodic boundary conditions is determined [1]. The two-site Hamiltonians commute with the symmetry algebra given by the Drinfeld double $D(D_3)$ of the dihedral group D_3 [2]. As such the model describes local interactions between non-Abelian anyons, with fusion rules given by the tensor product decompositions of the irreducible representations of $D(D_3)$. The Bethe ansatz equations which characterise the exact solution are found through the use of functional relations satisfied by a set of mutually commuting transfer matrices, following techniques developed in [3, 4, 5]. The energy eigenvalues of the non-Abelian anyonic chain are found to be generically functions of two sets of roots of a single set of Bethe ansatz equations.

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Correspondence between Ricci and Other Dark Energies

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An exotic form of negative pressure matter called dark energy is used to explain the acceleration of the universe inferred from the observations of distant type Ia supernovae, cosmic microwave background radiation (CMBR), and Sloan Digital Sky Survey (SDSS). Gao et al [1] has discussed how the holographic dark energy model deals with the two fundamental problems of cosmology. This holographic dark energy model and its interacting versions are successful in fitting the current observations [2,3]. Inspired by the holographic dark energy models, Gao et al [1] proposed another possibility, where the density is proportional to the Ricci scalar curvature R and named this dark energy as Ricci dark energy (RDE). In this work we have considered the RDE in presence of dark matter and with suitable choice of the parameters we have seen that the equation of state evolves as quintessence. Also we have considered the correspondence between RDE and other dark energy candidates, namely, tachyonic field [4], DBI-essence [5] and new age-graphic dark energy [6]. We have reconstructed the corresponding scalar fields and potentials accordingly.

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LDP for Periodic States of Quantum Spin Systems

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Let \mathcal{B} denote the Banach space of relatively short range \mathbb{Z}^d -invariant interactions, and for each $\phi \in \mathcal{B}$ let A_ϕ and $P(\phi)$ denote the mean energy and the pressure of ϕ , respectively. Let \mathcal{S} (resp. $\mathcal{S}^{\mathbb{Z}^d}$) denote the set of all (resp. \mathbb{Z}^d -invariant) states on the spin algebra, and let s denote the mean entropy map on $\mathcal{S}^{\mathbb{Z}^d}$. Put $\mathbb{Z}^d_{>} = \{a \in \mathbb{Z}^d : a_i > 0, 1 \leq i \leq d\}$ and $\Lambda(a) = \{x \in \mathbb{Z}^d : |x_i| < a_i, 1 \leq i \leq d\}$ for all $a \in \mathbb{Z}^d_{>}$. For each $a \in \mathbb{Z}^d_{>}$ and each $\omega \in \mathcal{S}^{\mathbb{Z}^d}$, let $U_a(\omega)$ denote the a -periodic state obtained from ω by a standard procedure ([1], Example 4.3.26), and let $V_a \circ U_a(\omega) \in \mathcal{S}^{\mathbb{Z}^d}$ obtained averaging $U_a(\omega)$ along $\Lambda(a)$. Our main result is the following.

Theorem 1. *Let $\phi \in \mathcal{B}$, let $\{\varphi_n : n \in \mathbb{N}\}$ be a countable dense subset of \mathcal{B} and for each $n \in \mathbb{N}$, let ω_{φ_n} be an equilibrium state for φ_n . For each $a \in \mathbb{Z}^d_{>}$ put $\text{Per}_a = \{U_a(\omega_{\varphi_n}) : 0 \leq n \leq \max_{1 \leq i \leq d} a_i\}$ endowed with the probability measure*

$$p_{a,\phi}(\omega) = \frac{e^{|\Lambda(a)|s(\omega) - \sum_{x \in \Lambda(a)} \omega \circ \tau^x(A_\phi)}}{\sum_{\omega' \in \text{Per}_a} e^{|\Lambda(a)|s(\omega') - \sum_{x \in \Lambda(a)} \omega' \circ \tau^x(A_\phi)}}.$$

Then the net $(\sum_{\omega \in \text{Per}_a} p_{a,\phi}(\omega) \delta_{V_a(\omega)})$ satisfies a large deviation principle in \mathcal{S} with powers $(|\Lambda(a)|^{-1})$ and rate function

$$I^\phi(\omega) = \begin{cases} P(\phi) + \omega(A_\phi) - s(\omega) & \text{if } \omega \in \mathcal{S}^{\mathbb{Z}^d} \\ +\infty & \text{if } \omega \in \mathcal{S} \setminus \mathcal{S}^{\mathbb{Z}^d}. \end{cases}$$

It is known that the mean conditional entropy is smaller than s and coincides with s on all equilibrium states, but not on all invariant states ([1], Remark pp. 289). The following corollary specifies the relation between both quantities.

Corollary 2. *The mean entropy is the upper-regularization of the mean conditional entropy.*

Theorem 1 allows us to recover and specify the well-known property that any \mathbb{Z}^d -invariant state can be approximated weakly* and in entropy by ergodic states.

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Metric Stacks and Gromov-Hausdorff Distance

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After Gromov, there is a notion of a distance between two metric spaces which makes the moduli of compact metric spaces itself into a metric space. In this talk I will describe an attempt to define metrics on stacks and to extend Gromov's distance to so-called metric stacks. Time permitting, I will show how metric stacks relate to Marc Rieffel's quantum metric spaces, which are in some sense noncommutative metric spaces, and I will also talk about the applications in physics which motivate the work.



First order chemical reaction on exponentially accelerated vertical plate with mass diffusion and variable plate temperature

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76R10, 80A20

Exact solution of unsteady flow past an exponentially accelerated infinite vertical plate with variable temperature is analyzed in the presence of homogeneous chemical reaction of first order. The dimensionless governing equations of the fluid flow are solved by Laplace-transform technique for finding the concentration of the species, temperature and velocity field. These are presented graphically for different values of physical parameters like thermal Grashof number (Gr), mass Grashof number (Gc) Schmidt number (Sc), accelerating parameter (α) and time. The numerical values of skin friction are studied graphically. It is

observed that the velocity increases with decreasing chemical reaction parameter K . Also the velocity increase with increasing values of a , G_r and G_c .



Supmech: A Noncommutative Geometry Based Universal Mechanics Accommodating Classical and Quantum Mechanics

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 51P05; 81P99

Adopting a stepwise planned approach to the solution of Hilbert's sixth problem (relating to unified axiomatization of physics and probability theory), a universal scheme of mechanics (accommodating both classical and quantum mechanics), called supmech, is developed which combines elements of noncommutative symplectic geometry and noncommutative probability in an observable-state type algebraic framework. It is basically noncommutative Hamiltonian mechanics incorporating the extra condition that the sets of observables and pure states be mutually separating. Consistent description of interaction between two systems in supmech requires the system algebras to be either both commutative or both noncommutative with a 'quantum symplectic structure' characterized by a *universal* Planck type constant. Systems in the latter class (called quantum systems) are shown to inevitably have Hilbert space based realizations (accommodating rigged Hilbert space based Dirac bra-ket formalism) generally admitting commutative superselection rules; finitely generated system algebras have faithful irreducible representations. [arXiv : 0909.4606 v3; 1002.2061 (math-ph)]



Validity of Thermodynamical Laws in Dark Energy Filled Universe

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 83F05

We have considered the flat FRW model of the universe which is filled with only dark energy [1]. The general descriptions of first and second laws of thermodynamics are investigated on the apparent horizon and event horizon of the universe [2]. We have assumed the equation of state of three different types of dark energy models. We have examined the validity of first and second laws of thermodynamics on apparent and event horizons for these dark energies. For these dark energy models, it has been found that on the apparent horizon, first and second laws are always valid. On the event horizon, the laws are break down for some dark energy models [3]. For some particular model [4], first law cannot be satisfied on the event horizon, but second law may be satisfied at the late stage of the evolution of the universe and so the validity of second law on the event horizon depends on the values of the parameters only.

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Cosmological Models of the Universe with Perfect Fluid Coupled with Massless Scalar Field

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 83F05

FRW models of universe filled with perfect fluid coupled with massless scalar field have been studied. The different models of the universe have been obtained by using a special law of variation for Hubble's parameter that yields a constant

value of deceleration parameter. The physical behavior of the models are also discussed.

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Geometric Prequantization of Various Moduli Spaces

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Various moduli spaces, like the Hitchin system, [1], the moduli spaces of abelian and non-abelian vortices, [2], [3], [4], the moduli space of dimensionally reduced and modified Seiberg-Witten equations, [5], can be geometrically prequantized using modifications of Quillen's determinant line bundle construction. Using this, one can construct line bundles on the moduli spaces whose curvatures are proportional to the symplectic form(s) on the moduli spaces, which puts us in the setting of geometric prequantization. In the case of the Hitchin system, for instance, the hyperKähler structure can be prequantized, i.e. one can construct three prequantum lines bundles whose curvatures are proportional to the three symplectic forms in the hyperKähler structure

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Periodic First Integrals for Hamiltonian Systems of Lie Type

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 37J15, 17B80, 17B81

We consider the problem of existence of first integrals for the class of time dependent Hamiltonian systems which are given as linear combination of Hamiltonian vector fields closing under the Lie bracket into a finite dimensional Lie algebra, with coefficients given by time dependent scalar functions [4]. From a natural ansatz for the form of time dependent first integrals, we relate their existence to periodic solutions of an Euler equation on the Lie algebra associated to the initial Hamiltonian system. Under different criteria, one based on properties for the Killing form and the other on the exponential map of the adjoint group of the Lie algebra, we prove the existence of Poisson algebras of periodic first integrals for a large families of periodic Hamiltonian systems. We include an application to the dynamics of the Milne-Pinney oscillator [3] and present a periodic invariant, analogue to that given by Lewis [4] for time dependent oscillators.

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Geometric Constraint Algorithms for Dirac Manifolds

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The geometrically inspired Gotay-Nester constraint algorithm for presymplectic manifolds [1], was originally motivated by the Dirac theory of constraints [2]. It provides, in particular, a useful way of dealing with implicit differential equations arising in classical mechanics, such as the Euler-Lagrange equations for degenerate Lagrangians. In that case, which is the only one considered by Dirac, it is, in a sense, equivalent to Dirac's theory.

In this paper we first generalize the Gotay-Nester algorithm for the case of general Dirac structures (not necessarily integrable) rather than presymplectic forms. For this, we introduce the notion of a Dirac system, which encompasses important equations such as Euler-Lagrange, Hamilton and Lagrange-d'Alembert equations as well as the Kirchoff equations for L-C circuits. We also generalize the Dirac algorithm, obtaining explicit equations of motion in terms of brackets. Both algorithms, called CA and CAD algorithms, respectively, are closely related and, in a sense, equivalent. They provide, in particular, a unified formalism for dealing with Dirac systems mentioned above, which are often implicit differential equations.

In recent times the significance of Dirac structures (integrable or not) [3], in representing geometrically the fundamental equations in several fields, such as Lagrangian or Hamiltonian mechanics, nonholonomic mechanics, several theories of circuits and interconnected systems, has become clarified, thanks to the work of many researchers, see [4]–[5], and references therein. In this work we are interested in the role of Dirac structures in dealing with some implicit differential equations arising in nonholonomic mechanics, and for that purpose the integrability condition is too restrictive.

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Algebraic Computation of Spin Coefficients in Newman-Penrose Formalism using Mathematica

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 83C60

Newman-Penrose (NP)formalism [6], [2] in general relativity is a tetrad formalism in which various geometrical quantities are projected on a chosen null tetrad basis. Spin coefficients in NP formalism replace connection coefficients in geometry. The computations involved are very lengthy and complicated.

Applications of spin coefficients are discussed by many authors, in diverse fields of general relativity and cosmology, like Ahsan et al [1], Hasmani and Ahsan [3], Hasmani and Katkar [4] and also see Kramer et al [5] for other details. Motivated by such a large number of applications and looking to the complexity involved, we have exploited some features of computer algebra package Mathematica to carry out algebraic computation of spin coefficients in Newman-Penrose formalism for given metric and null tetrad. We wish to present this work.

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On Several Fifth Virial Coefficients for the Hard Core Potential

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As a continuation of [1] I present the integrals of several blocks in the fifth virial coefficient for the hard core potential. Some calculations can be extended to the similar blocks of n -th order.

The cluster expansion of an imperfect gas is expressed by the virial series

$$\frac{pv}{kT} = 1 + \frac{B_2}{v} + \frac{B_3}{v^2} + \frac{B_4}{v^3} + \frac{B_5}{v^4} + \dots = 1 + \sum_{k=1}^{\infty} \frac{B_{k+1}}{v^k}.$$

Take the hard core potential with σ twice the radius of the hard sphere. B_5 has 10 types of different block of fifth order. One of them is the fifth order cycle: $(-2/5)C_5 := (-2/5) \int f_{12}f_{23}f_{34}f_{45}f_{51}d^3r_2d^3r_3d^3r_4d^3r_5 = (4/5)(2\pi)^{11/2}\sigma^{12}c_5$ where $c_5 := \int_0^\infty J_{3/2}(t)^5 t^{-11/2} dt$. My calculation becomes $c_5 = \frac{\sqrt{2}}{\pi\sqrt{\pi}} \frac{317}{75600}$. By an iteration of Barnes integral and Weber-Schafheitlin integral of Bessel function, similar calculations become $(-5/6)C_6 = (-5/6)(2\pi)^7\sigma^{15}c_6$ in B_6 , $c_6 = \int_0^\infty J_{3/2}(t)^6 t^{-7} dt = \frac{1564}{637875\pi^2}$. The integral of cycle of n -th order is, for $n \geq 3$

$$-\frac{n-1}{2n}C_n = (-)^{n+1} \frac{n-1}{n} (2\pi)^{(3n/2)-2} \sigma^{3(n-1)} c_n, \quad c_n = \int_0^\infty J_{3/2}(t)^n t^{2-(3n/2)} dt$$

in B_n , and my calculation becomes

$$c_{2m+1} = \frac{9}{4} \sqrt{\frac{\pi}{2}} \left(\frac{2}{9\pi}\right)^m \left\{ \frac{5}{24} - \frac{3(m-1)}{20} + \frac{3(m-1)}{175} + \binom{m-1}{2} \frac{1}{25} \right\}, \quad m \geq 1$$

$$c_{2m} = \frac{9}{5} \left(\frac{2}{9\pi}\right)^{m-1} \left\{ \frac{34}{567} - \frac{4(m-2)}{105} + \frac{m-2}{175} + \binom{m-2}{2} \frac{1}{75} \right\}, \quad m \geq 2.$$

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Hyperbolic Geometry and Invariant Ratio of Quark Masses

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In the context of the unsolved problem of the ‘Existence of generations of Quarks’, the paper attempts to analyse the following important observations:

- (a) The square of mean mass of second generation approximately equals the product of the mean masses of first and third generations.
- (b) The invariant ratio very closely approximates the dimensionless physical constant - the fine structure constant.

This paper attempts to derive the consequences of representing each mean value as a point along the imaginary axis (in the upper half-plane model) of the hyperbolic plane, leading to the model the energy realm as hyperbolic space, with the generations as horocyclic foliation. Moreover, when this cross-ratio is identified with the Weierstrass elliptic lambda-function we arrive at an interpretation of the generations in terms of the fundamental modular fibration of the hyperbolic plane.

Interesting extensions leads to the idea of energy-space duality manifesting as black-hole horizon and conformal boundary in field theories. The gauge symmetries in the standard model of elementary particles are now the the isotropy groups of the special imaginary quadratic fields as the boundary becomes Complex, Quaternionic and Octonionic.

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Higher Dimensional Cosmological Model for Λ -dark Energy

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 83F05

In this paper we have obtained set of solutions for a kinematical Λ , viz., $\Lambda \propto (\frac{\dot{R}}{R})^2$ by assuming the barotropic equation of state in the context of Kaluza-Klein type theory of gravitation. Some results of cosmic density Ω [1], [2] and [3] and deceleration parameter q , have been obtained with consideration of two-fluid structure instead of usual uni-fluid cosmological model.

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Interior Black-Hole Solution With Anisotropic Fluid

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The result of gravitational collapse of a compact body is believed to be a singularity hidden beyond its Schwarzschild radius known as a black hole. In the

interior black hole region, a remarkable change occurs in the nature of the space-time, namely the external spatial radial and temporal coordinates exchange their characters. So, the interior black hole solution is represented by a non-static space-time that is with the time-dependent metric coefficient. With the assumption that the material content of the spherical body is an anisotropic fluid, we have carried out a brief study of the space time metric in the annular region between the physical radius and Schwarzschild radius of a spherical star. The case of isotropic pressures is discussed as a particular case and a regular solution has been obtained.

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Kelvin-Helmholtz Instability of Two Superposed Oldrodian Viscoelastic Fluid Layers in a Horizontal Magnetic Field

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76S05; 76W05.

The Kelvin-Helmholtz instability of the plane interface separating two superposed viscous electrically conducting streaming Oldroydian fluids permeated with surface tension and magnetic field in a porous medium is considered. The stability motion is also assumed to have uniform two dimensional streaming velocity. The stability analysis has been carried out for two highly viscous fluids. By applying the normal mode technique to the linearized perturbation equations, the dispersion relation has been derived. As in the case of superposed Newtonian fluids, the system is stable in the potentially stable case and unstable in the potentially unstable case, that holds also for the present case. The behavior of growth rate with respect to kinematic viscosity, elasticity, permeability of porous medium, surface tension and streaming velocity are examined numerically and discussed in detail in section 5.



On Global Attractors of Nonlinear Hyperbolic PDEs

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35B40, 35B41, 35L10, 35P25, 37K40, 37N20

We consider Klein-Gordon and Dirac equations coupled to U(1)-invariant nonlinear oscillators. The solitary waves of the coupled nonlinear system form two-dimensional submanifold in the Hilbert phase space of finite energy solutions. Our main results read as follows:

Theorem Let all the oscillators be strictly nonlinear. Then any finite energy solution converges, in the long time limit, to the solitary manifold in the local energy seminorms.

The investigation is inspired by Bohr's postulates on transitions to quantum stationary states.

The results are obtained for: a) 1D Klein-Gordon eqn coupled to one oscillator [1, 2, 3] and to finite number of oscillators [4], and b) nD Klein-Gordon and Dirac eqns coupled to one oscillator via mean field interaction [5, 6].

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On the Analytical Solutions for Some Problems of Diagnostic Calculations of Wind-induced Flow

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76D99

Method of solution of hydrodynamical problem with given density field presents the theoretical and practical interest [1, 2, 3].

The system of Ekman's type equations for wind-induced flow of non-homogeneous liquid is investigated. The boundary conditions for the horizontal component of the velocity vector on the undisturbed surface take into account the wind stress. On the bottom of a water body, is imposed the sliding condition. The density of water is a linear function of known temperature. The analytical solution for this problem was found. The obtained solution can be used to determinate the ecological regime of the water body [4].

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**Asymptotic Stability of Kinks for Relativistic Ginzburg-Landau Equation**

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35L70

We prove the asymptotic stability of the moving kinks for the nonlinear relativistic wave equations of the Ginzburg-Landau type in one space dimension: starting in a small neighborhood of the kink, the solution, asymptotically in time, is the sum of a uniformly moving kink and dispersive part described by the free Klein-Gordon equation. The remainder decays in a global energy norm. Crucial role in the proofs play our recent results on the weighted energy decay [3] for the Klein-Gordon equations.

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Small Exotic Smooth \mathbb{R}^4 and String Theory

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Recent work on exotic smooth \mathbb{R}^4 's shows the connection of 4-exotics with the codimension-1 foliations of S^3 , $SU(2)$ WZW models and twisted K-theory $K_H(S^3)$, $H \in H^3(S^3, \mathbb{Z})$ [5, 4]. These results and [1, 5] made possible to explicate some physical effects of exotic 4-smoothness. Based on [3, 6] we show that small exotic smooth \mathbb{R}^4 's can be considered as fundamental structures which underly superstring theory similarly as D-branes do. The cases of D-branes in $SU(2)$ WZW models in finite k stringy regime and in the limiting geometry of the stack of NS5-branes of type II superstring theory, are discussed. The correlation of some configurations of D-branes in various string backgrounds with 4-smoothness of the transversal or ambient spaces, is presented. Moreover, we are able to show that certain quantum D-branes, represented by noncommutative C^* algebras in noncommutative spacetimes, correspond to the net of small exotic \mathbb{R}^4 's embedded in a small exotic \mathbb{R}^4 . Thus exotic smoothness in 4-dimensions captures some higher dimensional effects of superstring theory D-branes, also quantum. This unexpected result shed light on compactification in string theory, and, in fact, serves as a possible counterpart for the compactification. Based on the embeddings of homology 3-spheres in S^6 , or 4-manifolds (Seifert 4-surfaces) in S^7 [2], we can give 4-dimensional and topological origins of D6-brane charges.

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A Simulation-Based Model For Price Prediction

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A simple Ising spin model is proposed to study the price formation mechanism in financial markets. The complex investment behavior of trend followers and fundamentalists ascertaining accurate their final decision when trading in Colombo Stock Exchange (CSE). Compared to other agent-based models, the influence does not flow inward from the surrounding neighbors to the center site, but spreads outward from the center to the neighbors. The model thus describes the spread of opinions among traders. We use Monticarlo Simulations to study the process. The results show that the model is appropriate to describe the behavior of Sri Lankan financial market. Thus we may conclude that this simple model is a good approximation of a number of real financial markets. Most of investors are decision making in Sri Lankan financial market based on the traditional techniques such as following the trend, this model thus provides mathematical and statistical basis for effective share trading by participants.



On the Propagation of SH-Type Waves in Elastic Isotropic and Homogeneous Media Sandwiched by Elastic Inhomogeneous Medium

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 74J15

This paper focuses on the propagation of SH-type waves in three different types of layered homogeneous and inhomogeneous isotropic media. Dispersion equation is derived. Variation of phase velocity with wave number are considered numerically and natures are shown graphically in homogeneous and inhomogeneous cases with the help of Matlab.

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Linear Flow in Three Layers of Fluid Over an Arbitrary Topography

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Steady, two-dimensional, two-layer flow over an arbitrary topography is considered by Belward and L. K. Forbes [1] for its nonlinear solution. Martha and Chakrabarti [2] considered a two-dimensional problem involving irrotational fluid flow in an infinite channel over an arbitrary topography. A linear theory is presented for three layers of fluids where the upper fluid layer is bounded by a rigid lid.

In this paper, a two-dimensional problem involving irrotational fluid flow in three layers of fluid in an infinite channel over an arbitrary topography is considered. The fluid is assumed to be inviscid and incompressible. A linear theory is presented for three layers of fluids where the top surface of the upper layer is a free surface. Fourier analysis is applied to obtain an interface profile with an oscillatory nature downstream of the obstacle. Special examples of bottom topography are examined in detail.

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Inhomogeneous Plane Symmetric Cosmological Model in Scale Invariant Theory

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 83D05

Field equations of Wesson's scale invariant theory are obtained, with the aid of an inhomogeneous plane symmetric metric in the presence of perfect fluid distribution. Model corresponding to stiff fluid is constructed and discussed.

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A Mathematical Model for an Observer of a Set and Its Application in Physics

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 54A05, 37B40, 54A40, 57N99, 83E99, 81Q70

In this talk an observer of a set as a mathematical object will be studied. The notion of topology from the viewpoint of an observer will be considered. Relative manifolds as a mathematical solution for the “*space meaning*” by using of high dimensional observer will be presented. By using of the relative vector fields on relative manifolds a new version of the problem of unity as a realistic approach to the problem of unity will be sketched. We will show that by using of the mathematical notion of the observer in the geometrical structures we can solve the new version of the problem of unity.

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On Phase Transition for Countable State p -adic Potts model on the Cayley tree

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 46S10, 82B26, 12J12, 39A70, 47H10, 60K35

Interest in the physics of non-Archimedean quantum models [3] is based on the idea that the structure of space-time for very short distances might conveniently be described in terms of non-Archimedean numbers. One of the ways to describe this violation of the Archimedean axiom, is the using p -adic analysis. It is known that a number of p -adic models in physics cannot be described using ordinary Kolmogorov's probability theory. New probability models - p -adic probability models were investigated in [1]. This gives a possibility to develop the theory of statistical mechanics in the context of the p -adic theory, since it lies on the base of the theory of probability and stochastic processes.

In this work we develop p -adic probability theory approaches to study of nearest-neighbor countable state Potts models on a Cayley tree in the field of p -adic numbers, which provides more natural concrete examples of p -adic Markov processes. In [2] a construction of p -adic Gibbs measures which depends on weights is given. We found certain conditions to weights for the existence and uniqueness of p -adic Gibbs measures for such a model. In the present work we find other conditions to the weights which provide the existence of the phase transition for the model. Here the phase transition means existence of two different p -adic Gibbs measures. Note that one of such measures is bounded, another one is unbounded.

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**The Stokes Flow Past Shear Free Spheroid**

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76D07

The slow streaming flows of a incompressible viscous fluid past shear stress free oblate and prolate spheroids are considered; where the streams are in the negative direction of the axis of symmetry. Physical properties of interest such as stokes stream function, drag and torque exerted by the fluid on the spheroids are established.

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MHD Two-Phase Flow and Heat Transfer Between Two Parallel Porous Walls in a Rotating System

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76S05, 76T99

Closed form solutions are given for the flow and heat transfer aspects of a magnetohydrodynamic two-phase steady flow between two parallel porous walls under the action of a uniform transverse magnetic field applied in a direction normal to the plane of flow, assuming that the magnetic Reynolds number is small, when both the fluids and walls are in a state of rigid rotation with uniform angular velocity about an axis perpendicular to the plane of flow. It is assumed that the fluids in the two regions are incompressible, immiscible and electrically conducting, having different viscosities, thermal and electrical conductivities. Further, assumed that the transport properties of the two fluids are constant having constant bounding wall temperatures. Numerical calculations for the velocity and temperature distributions for various sets of values of the governing parameters involved are obtained to represent them graphically and are discussed. It is observed that, as the suction number increases there is a significant change in the primary velocity at the upper region but is insignificant in the lower region. While as this number increases the secondary velocity also increases. Also, as suction number increases, there is an increase of the temperature in the two regions.

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Effects of Radiation and Rotation on MHD Free Convection Flow Past an Impulsively Started Plate Embedded in a Porous Medium with Ramped Wall Temperature

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Effects of radiation and rotation on unsteady hydromagnetic free convection flow of a viscous incompressible electrically conducting fluid past an impulsively moving vertical plate embedded in a porous medium is studied. Temperature of the plate has a temporarily ramped profile. Rosseland approximation is used to describe the radiative heat flux in the energy equation. Exact solution of the governing equations, in non-dimensional form, is obtained by Laplace transform technique for both ramped temperature and isothermal plates. Expressions for the Nusselt number and shear stress at the plate are also derived in both the cases. Mathematical formulation of the problem, in non-dimensional form, contains six pertinent flow parameters viz. M (magnetic parameter), K^2 (rotation parameter), K_1 (porosity parameter), G_r (Grashof number), P_r (Prandtl number) and N (radiation parameter). Numerical values of the velocity are depicted graphically for various values of M , K^2 , G_r , K_1 , N and time t while fluid temperature profiles are drawn for different values of P_r , N and t for both ramped temperature and isothermal plates. Numerical values of the shear stress at the plate are presented in tabular form for various values of M , K^2 , G_r , K_1 , N and t whereas that of Nusselt number are given in tables for different values of P_r , N and t for both ramped temperature and isothermal plates.

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Self-similar Flow of a Rotating Dusty Gas Behind the Shock Wave with Increasing Energy, Conduction and Radiation Heat Flux

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A self-similar solution are obtained for one dimensional adiabatic flow behind a cylindrical shock wave propagating in a rotating dusty gas in the presence of heat conduction and radiation heat flux. The dusty gas is assumed to be a mixture of non-ideal (or perfect) gas and small solid particles, in which solid particles are continuously distributed. It is assumed that the equilibrium flow-condition is maintained and variable energy input is continuously supplied by the piston (or inner expanding surface). The heat conduction is express in terms of Fourier's law and the radiation is considered to be of the diffusion type for an optically thick grey gas model. The thermal conductivity and the absorption coefficient are assumed to vary with temperature only. In order to obtain the similarity solutions the initial density of the ambient medium is assume to be constant and the angular velocity of the ambient medium is assume to be decreasing as the distance from the axis increases. The effects of the variation of the heat transfer parameters and non-idealness of the gas in the mixture are investigated. The effects of an increase in (i) the mass concentration of solid particles in the mixture and (ii) the ratio of the density of solid particles to the initial density of the gas on the flow variables are also investigated.



On the Conceptual Interpretation of Brownian Motion Driven Stochastic Differential Equations

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 82C31

There are many problems in theoretical physics which consider the evolution of a state variable $X(t)$ in an uncertain environment. While Brownian motion driven SDEs, $dX(t)/dt = a(t, X) + b(t, X)dB(t)/dt$, where $B(t)$ is a Brownian motion, are a popular approach to such situations, the interpretation of the

term $b(t, X)dB(t)/dt$ as ‘noise’ is often physically unsatisfying. Heuristically, if we take $X(t)$ as the position of a particle, then this noise might be interpreted as successive perturbations on the particle’s trajectory, each occurring over a time period too short to be resolved experimentally. In this presentation, we put this concept on rigorous footing by considering SDEs driven by a stochastic term $b(t, X)dQ(t)/dt$. The process $dQ(t)/dt$ follows different deterministic behaviours over finitely-long time intervals, and can easily be interpreted in terms of successive perturbations on a particle’s trajectory. It will be shown that in the limit of these intervals going to zero, these equations weakly converge in solution to those of a Brownian motion driven SDE. This result strongly supports the heuristic interpretation of the term given above. Moreover, in this limit it can be inferred that the size of these perturbations is finite, supplementing the ‘noise’ in a Brownian motion driven SDE with a sense of size and scale.



Computable Extensions of Generalized Fractional Kinetic Equations in Astrophysics

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 33C20, 33C90

Fractional calculus and special functions have contributed a lot to mathematical physics and its various branches. The great use of mathematical physics in distinguished astrophysical problems has attracted astronomers and physicists to pay more attention to available mathematical tools that can be widely used in solving several problems of astrophysics/physics. In view of the great importance and usefulness of kinetic equations in certain astrophysical problems, the authors derive a generalized fractional kinetic equation involving the Lorenzo-Hartley function, a generalized function for fractional calculus. The fractional kinetic equation discussed here can be used to investigate a wide class of known (and possibly also new) fractional kinetic equations, hitherto scattered in the literature. A compact and easily computable solution is established in terms of the Lorenzo-Hartley function. Special cases, involving the generalized Mittag-Leffler function and the R-function, are considered. The obtained results imply the known results more precisely.

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Quantum Interferometry, Euler angles, Unitary Representations of SU(2)

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 81R99

The Hilbert space of our physical system (the interferometer), \mathbf{H} , is a direct sum of two isometric closed subspaces, \mathbf{H}' and \mathbf{H}'' ; J is a (fixed) Hermitian isometry from \mathbf{H}' to \mathbf{H}'' , I is the identity operator of \mathbf{H} , $J.J = I$.

The unitary operator:

$$S(\alpha) = \cos(\alpha).I + i.\sin(\alpha).J$$

is the scattering operator of a *beam splitter* (a semitransparent mirror, a perfect crystal); if $\cos(\alpha) = 0$ (and $\sin(\alpha) = \pm 1$) we have a true mirror. A splitter because, if the ongoing unit vector f' belong to \mathbf{H}' , the outgoing vector $\cos(\alpha).f' + i.\sin(\alpha).f''$, $f'' = J.f'$, is split between \mathbf{H}' and \mathbf{H}'' . For a physical splitter $\cos(\alpha)$ is always > 0 , but we can obtain a mathematical splitter combining a physical one and a mirror (as $S(\alpha).S(\beta) = S(\alpha + \beta)$).

If E' and E'' are the orthogonal projectors on \mathbf{H}' and \mathbf{H}'' , the unitary operator:

$$T(\theta) = \exp(+i.\theta/2).E' + \exp(-i.\theta/2).E''$$

represents a *phase shifter* (the shift is generally a consequence of strong, electromagnetic or gravitational interactions); the difference of phase is θ ; $J.T(\theta).J = T(-\theta)$.

An *one particle*, Mach-Zehnder interferometer is described by the unitary scattering operator $U(\alpha, \beta, \theta) = S(\alpha).T(\theta).S(\beta)$: splitter, shifter, splitter (plus mirror). Then U is an *unitary representation* of $SU(2)$ in \mathbf{H} ; α, β, θ are the corresponding *Euler angles*.

Now, if the unit vectors f' and f'' belong to \mathbf{H}' and \mathbf{H}'' , $f'' = J.f'$, the MZ interferometer transition probabilities are (an elementary calculation):

$$Prob(f', f') = A(\alpha, \beta) - 2.C(\alpha, \beta). \cos(\theta)$$

$$Prob(f', f'') = B(\alpha, \beta) + 2.C(\alpha, \beta). \cos(\theta)$$

Here A, B, C are polynomials of the sin and cos of α and β , $A + B = 1$. The $\cos(\theta)$ term, coming from the phase shifter, is clearly responsible of the interference effects.

In a similar way it is possible to describe a *two particles* interferometer, for example the EPR (Einstein, Podolski and Rosen) one. Now the Hilbert space of the system is the tensor product of two copies of the same Hilbert space (the one particle space); for every particle there is a phase shifter and a beam splitter; then, for a suitable choice of the ongoing unit vector, we obtain the well know interference and correlation effects.



Euler Characteristic in Mathematical Physics: For Statistical Physics and Field Theory

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The Euler characteristic now forms a very versatile index for geometric and topological invariants. The original idea of Leonhard Euler to define a invariant for Platonic solids, extended to polyhedra, gave $V - E + F = 2$; for V number of vertices, E number of edges and F number of faces.

The recent definitions for the Euler characteristic encompass several branches of mathematics. They show surprising connections between Betti numbers and cohomology, singularities of vector fields on manifolds, moduli spaces and gauge fiber bundles, Gauss Bonet and other geometric invariant forms, Riemann spaces and genus numbers and so on.

The applications of the Euler characteristic index number in mathematical physics are being increasingly discovered and range from statistical physics partition functions to quantised unified fields. Topological phase transitions for

clusters and spin systems in statistical physics as well as topological transitions in compactifications in unified field theory are done.

This paper draws on the author's work [Ref arxiv] in which the Euler characteristic played a key role.



Divergent Multiple Sums and Integrals with Constraints

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 11M06, 52B99, 47G30

We investigate the meromorphic behaviour of

- multiple discrete sums of tensor products of symbols with conical constraints,
- multiple integrals of tensor products of symbols with linear constraints,

and compare their pole structures. We discuss how to extract reasonable finite parts at poles leading to “renormalised” multiple discrete sums and integrals which factorise on tensor products. The first generalise “renormalised” multiple zeta values at non positive integers, whereas the second relate to “renormalised” Feynman integrals in physics.

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On a Plane Steady Exterior Navier–Stokes Problem

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35Q30, 76D03, 76D05

We study the following problem for the steady plane Navier–Stokes equations

$$\begin{aligned} -\nu\Delta\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \nabla p &= \mathbf{f} & \text{in } \Omega, \\ \operatorname{div}\mathbf{v} &= 0 & \text{in } \Omega, \\ \mathbf{v} &= \mathbf{h} & \text{on } \Gamma, \end{aligned} \quad (1)$$

$$\lim_{|x| \rightarrow \infty} \mathbf{v} = 0 \quad (2)$$

in an exterior domain $\Omega \subset \mathbf{R}^2$ ($\Omega = \mathbf{R}^2 \setminus \mathcal{B}$, where \mathcal{B} is a compact connected set with a boundary Γ). The first mathematical investigation of this problem is due to J. Leray, who proved in 1933 that problem (1) admits at least one solution with the finite Dirichlet integral $\int_{\Omega} |\nabla\mathbf{v}|^2 dx < \infty$, provided the boundary datum \mathbf{h} has zero total flux through the boundary:

$$\mathcal{F} = \int_{\Gamma} \mathbf{h} \cdot \mathbf{n} d\Gamma = 0, \quad (3)$$

where \mathbf{n} is the outward (with respect to Ω) unit normal to Γ . However, the question whether or not the Leray's solution satisfies condition (2) remained open in the two-dimensional case (in the three-dimensional case the answer is positive). The asymptotic behavior of solutions to problem (1) with finite Dirichlet integral was investigated by many authors during the last 70 years. However, for problem (1), (2) no existence results were available other than in the case when \mathcal{B} has two orthogonal axes of symmetry, \mathbf{f} and \mathbf{h} satisfies suitable parity conditions and the the boundary datum \mathbf{h} has zero total flux (see (3)).

In this paper we prove the existence of a solution (\mathbf{v}, p) of problem (1), (2) assuming that \mathcal{B} has two orthogonal axes of symmetry and that data satisfies parity conditions. However, the flux \mathcal{F} is arbitrary in our case. To the best of our knowledge this is the first existence result for the two-dimensional stationary exterior problem without restrictions on the value of \mathcal{F} .



Analytical Solutions for Three-dimensional Wind Induced Motion of Viscous Homogeneous Liquid

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The equations of non-stationary three-dimensional wind induced motion of a viscous nonhomogeneous liquid with the corresponding boundary conditions are used for concrete reservoirs [1, 2, 3]. However, analytical solutions for such problems are known only in special cases neither in three-dimensional, nor in a two-dimensional case. It is pointed out that analytical solutions for the Ekman's type model (model without taking into account horizontal viscosity) in case of stationary current are known for constant and variable values of the vertical turbulent exchange [4, 5] and are widely used for the analysis of the solution and computation of specific problems. In the present paper these results has been extended on the model with taking into account horizontal viscosity. Namely, we have found the analytical solution of stationary model of wind induced motion of a viscous homogeneous liquid in the closed reservoir of rectangular form. It is supposed that the value of the vertical turbulent exchange is constant. The obtained solution is compared with the solution for the Ekman's type model, so it makes possible to determine the application area for this simpler model. One can apply obtained solution for testing the computational algorithms of the wind-induced motion of liquid [6, 7].

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Elastic - Plastic Analysis of Thin Rotating Disc Having Variable Thickness and Variable Density with Edge Loading

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Elastic-plastic stresses have been derived for a disc having variable thickness and variable density with edge loading by using Seth's transition theory. The transition theory utilizes the concept of generalized principal strain measure and asymptotic solution at critical points or turning points of the differential system. In this work, it is our main aim to eliminate the need for assuming semi-empirical laws, yield conditions like those of Tresca's, von-Mises. The effect of edge loading has been discussed numerically and depicted graphically. It is concluded that a rotating disc having variable thickness and variable density with edge loading requires higher percentage increase in angular speed to become fully plastic from its initial yielding as compared to a rotating disc having variable thickness under density variation without edge loading.

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Plane Symmetric Cosmic Strings Coupled with Maxwell Fields in Bimetric Theory

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Plane symmetric model is studied with source cosmic cloud strings coupled with electromagnetic field in Rosen's bimetric theory of relativity and vacuum models are established.

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Black Holes in Non-Commutative Geometry

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We study black hole solutions of Einstein's field equations in non-commutative geometry in three- and four-dimensional cases. We are particularly interested in their thermodynamical properties and compare these properties with those of the corresponding solutions in commutative geometry.



Spectral Expansion on the Entire Real Line of the Green Function for a Three-Layer Medium in Fundamental Functions of an Adjoint Sturm-Liouville Operator

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We give a new representation of the Green function in the space R^2 for the Helmholtz equation with coefficient that is a real-valued piecewise constant function depending on a single variable and taking three values. This representation has the form of an expansion in fundamental functions, i.e., bounded (on the entire line R^1) solutions of the Sturm-Liouville equation with a real coefficient [1].

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Thermal Convection of Walters B' Dusty Compressible Viscoelastic Fluid in Porous Medium with Hall Currents

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The thermal convection of compressible Walters B' viscoelastic fluid in porous medium is considered to include the effects of Hall currents and suspended particles. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For the case of stationary convection, Hall currents and suspended particles are found to have destabilizing effects whereas compressibility and magnetic field have stabilizing effects on the system. The medium permeability, however, has stabilizing and destabilizing effects on thermal instability in contrast to its destabilizing effect in the absence of magnetic field. The magnetic field, Hall currents and viscoelasticity parameter are found to introduce oscillatory modes in the system.



An Alternative Well-posedness Property and Static Spacetimes with Naked Singularities

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In the first part of this work, we show that the Cauchy problem for wave propagation in some static spacetimes presenting a singular time-like boundary is well posed, if we only require the waves to have finite energy, although no boundary condition is required. This feature does not come from essential self-adjointness, which is false in these cases, but from a different phenomenon that we call the **alternative well-posedness property**, whose origin is due to the degeneracy of the metric components near the boundary.

Beyond these examples, in the second part, we characterize the type of degeneracy which leads to this phenomenon.

This work was carried out jointly with Ricardo Gamboa Saraví and Philippe Tchamitchian.



A Note on the Disturbance of SH-type of Waves Due to Shearing-stress Dicontinuity in a Visco-elastic Layered Half-space

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The disturbance of SH-type of waves due to shearing stress discontinuity in a visco-elastic layered half space has been considered in this paper. With The help of Laplace and Fourier transform the displacement is obtained in exact form. The numerical calculations are performed for two cases of shearing stress discontinuity. The numerical results are shown graphically. Some special cases are discussed.

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Physics of Rotating and Expanding Black Hole Universe

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Throughout its journey universe follows strong gravity. By unifying general theory of relativity and quantum mechanics a simple derivation is given for rotating black hole's temperature. It is shown that when the rotation speed approaches light speed temperature approaches Hawking's black hole temperature. Applying this idea to the cosmic black hole it is noticed that there is "no cosmic temperature" if there is "no cosmic rotation". Starting from the planck scale it is assumed that- universe is a rotating and expanding black hole. Another key assumption is that at any time cosmic black hole rotates with light speed. For this cosmic sphere as a whole while in light speed rotation "rate of decrease" in temperature or "rate of increase" in cosmic red shift is a measure of "rate of cosmic expansion". Since 1992, measured CMBR data indicates that, present CMB is same in all directions equal to 2.726°K , smooth to 1 part in 100000 and there is no continuous decrease! This directly indicates that, at present rate of decrease in temperature is practically zero and rate of expansion is practically zero. Universe is isotropic and hence static and is rotating as a rigid sphere with light speed. At present galaxies are revolving with speeds proportional to their distances from the cosmic axis of rotation. If present CMBR temperature is 2.726°K , present value of obtained angular velocity is $2.17 \times 10^{-18} \frac{\text{rad}}{\text{sec}} \cong 67 \frac{\text{Km}}{\text{sec.Mpc}}$. Present cosmic mass density and cosmic time are fitted with a $\ln(\text{volume ratio})$ parameter. Finally it can be suggested that dark matter and dark energy are ad-hoc and misleading concepts.

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Exact Solution of Special Classes of Flows in Rotating Fluids

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Analytical fluid dynamics presents a host of problems involving system of non-linear partial differential equations and we know that there are no general methods of solutions to solve these system. Some problems like the ones considered in this paper make use of transformation to render the problem analytically tractable. The dynamics of rotating fluids is a fertile area to scout for general analytical solutions using transformations.

The present study concerns the streamline motion of an incompressible viscous liquid in a rotating frame of reference. The transformation from physical plane to hodograph plane, introduction of Legendre transformation in it, solutions obtained to the flow variables for five special classes of flows and the geometry of streamlines in each case are discussed. The variation of pressure with angular velocity is also analyzed.

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Eigenvalue Bounds for Micropolar Shear Flows

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Linear stability for general viscous 2D micropolar shear flows[3]

$$\mathbf{U} = (U(y), 0, 0), \quad \mathbf{W} = (0, 0, W(y)), \quad y \in (0, 1),$$

is determined by the (dimensionless) equations[2]

$$\begin{aligned} i\alpha [(U - c)(D^2 - \alpha^2) - U''] \tilde{\psi} &= \left(\frac{1}{R_\mu} + \frac{1}{2R_k} \right) (D^2 - \alpha^2)^2 \tilde{\psi} - \frac{R_0}{R_k} (D^2 - \alpha^2) \tilde{w}, \\ i\alpha [(U - c)\tilde{w} - W' \tilde{\psi}] &= \frac{1}{R_\gamma} (D^2 - \alpha^2) \tilde{w} - \frac{2R_0}{R_\nu} \tilde{w} + \frac{1}{R_\nu} (D^2 - \alpha^2) \tilde{\psi}, \end{aligned} \quad (1)$$

where R_γ , R_μ , R_ν , R_k , and R_0 are dimensionless parameters and $D := \frac{d}{dy}$. Let $c = c_r + ic_i$ be any eigenvalue of system (1). We show the following bounds for c_i , which are analogous to the classical result of [1] for flows governed by the Navier-Stokes equations: If $\max\{\frac{R_\mu}{2}, R_k\} < \min\{R_\nu, \frac{R_k}{R_0}\}$, and $\max\{\frac{R_\nu}{2R_0}, R_\gamma\} < \min\{\frac{R_\nu}{2}, \frac{R_k}{2R_0}\}$, then

$$c_i \leq \frac{q_1 + q_2}{2\alpha} - \frac{\pi^2 + \alpha^2}{\alpha R},$$

where $\frac{1}{R} := \min\{\frac{1}{R_1} - \frac{1}{R_2}, \frac{1}{R_3} - \frac{2}{R_2}\}$, $q_1 := \max_{y \in [0,1]} |U'(y)|$, $q_2 := \max_{y \in [0,1]} |W'(y)|$. Moreover, there are no amplified disturbances if

$$\begin{cases} \alpha R q_1 < \frac{(4, 73)^2 \pi}{2} + 2^{\frac{3}{2}} \alpha^3, \\ \text{and} \\ \alpha R q_2 < \sqrt{2(\pi^2 + \alpha^2)} (4, 73)^2, \end{cases} \quad \text{or} \quad \begin{cases} \alpha R q_1 < (4, 73)^2 \pi + 2\alpha^2 \pi, \\ \text{and} \\ \alpha R q_2 < 2\alpha^2 \sqrt{\pi^2 + \alpha^2}. \end{cases}$$

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Cosmological Models Interacting with Massive Scalar Field in Lyra's Manifold

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FRW models of universe interacting with massive scalar mesonic field are investigated in the cosmological theory based on Lyra's Manifold. By considering the well known Hubble's principle and source energy in the wave equation to be absent, exact solutions have been obtained from which different forms of model of the universe are derived. Also some physical and geometrical aspects of the models are also investigated.

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Optimal Bounds on Dispersion Coefficient in Periodic Media

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In this talk, we consider a periodic media and we study the dependence of the dispersion tensor in terms of the microstructure (for the definition of this tensor, we refer the reader to [1] and [2]). We treat one-dimensional and laminated structures, and also we give some perspectives on other cases in higher dimension.

Considering the one dimensional and laminated periodic medium, we completely describe the set in which the dispersion coefficient lies, as the microstructure varies preserving the volume proportion (see [3]). In higher dimension, we study properties on the dispersion tensor for Hashin structures and we characterize the bounds of this tensor in terms of some geometric properties on the reference cell.

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Two-dimensional Instabilities in Fluidised Beds

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The connection between the instability of one-dimensional waves to transverse disturbances and the formation of bubbles in fluidised beds is still unclear. Moreover, recent theoretical and experimental studies have observed the formation of bubbles in gas-fluidised beds, but not in liquid-fluidised beds and, despite a detailed characterisation of the structures is already available, the physical mechanism leading to this differentiation is still unknown [1, 2, 3]. In this work, we focus on the study of the instability of the one-dimensional concentration waves to transverse disturbances, leading to gravitational overturning. We propose an extension of models available in the literature to describe the gravitational instability of unbounded stratified flows [4] to account for the slip velocity between the particles and the fluid, and also to include the inertia of the particles. The rheology of the particulate phase is simplified to retain only the relevant mechanisms in the model [2]. A linear stability analysis is performed in order to determine the dispersion relation of the transverse modes. In addition, a numerical simulation of the full governing equations is carried out and is checked against the theoretical results of the linear stability. The influence of the physical parameters and constitutive relations of the particulate phase rheology in the stability of the waves is evaluated. It is found that fluidised beds are gravitationally unstable to long wavelength transversal modes. However, the results obtained here indicate that the enhanced drag of the fluidising flow on heavy concentrated regions tends to lift them, reducing the growth rates of long waves and even stabilising short waves.

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Effect of g-jitter on the Onset of Thermosolutal Viscoelastic Convection in the Absence of Local Thermal Equilibrium

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The effect of g-jitter on the buoyancy driven convection in a binary viscoelastic fluid saturated porous layer has been studied using linear stability analysis. The Rivlin-Ericksen model [1] has been employed to characterize the viscoelasticity of the fluid, whereas, the porous domain is considered to lack the local thermal equilibrium. The governing equations were converted into a system of ordinary differential equation using the Galerkin method. The stability of the periodic system is investigated using Floquet’s analysis [2] and the obtained stability results are presented in detail. It was found that g-jitter has significant effect on the onset of thermosolutal natural convection.

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Thermal Radiation with Soret and Dufour Effects on MHD Mixed Convection From a Vertical Surface in Darcian Porous Media

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Two dimensional steady , laminar heat and mass transfer by mixed convection from a semi infinite , isothermal and isosolutal vertical plate embedded in a Darcian porous medium in the presence of transverse magnetic field, thermal radiation and Soret and Dufour effects has been studied. The Rosseland approximation for the radiative heat flux is used in the energy equation. It is found that the similarity solution exists in the present case. The resulting set of coupled non-linear ordinary differential equations is solved numerically using shooting technique. Dimensionless velocity, temperature and concentration profiles are presented graphically against η for various values of the mixed convection parameter RP . The numerical values of local Nusselt number and local Sherwood number have been tabulated for various values of involved parameters and discussed in detail.



Convective Instability in a Vibrating Porous Layer Using a Thermal Non-equilibrium Model

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An investigation is made out to study the effect of vertical harmonic vibration in a horizontal porous layer heated from below when the solid and fluid phases are not in local thermal equilibrium. The Brinkman model of flow through porous media is used for the momentum equation and a two-field model that represents the fluid and solid phase temperature fields separately is used for the energy equation. The porous layer is subject to vertical vibrations of arbitrary amplitude and frequency. Linear stability analysis is performed using Floquet theory and the continued fraction technique is applied to solve the resulting algebraic system. It is demonstrated that vibrations can produce a stabilizing or destabilizing effect depending on their amplitude and frequency for a porous layer heated from below. It is also found that increasing interphase heat transfer coefficient in the presence of small values of porosity modified conductivity ratio exposes the competition between synchronous and subharmonic modes for a wider range of vibrational frequencies.

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Topological Phases and Superconducting States in Kitaev-like Lattice Models

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Topological field theories can be realized as topological phases in certain integrable models and two-dimensional strongly-correlated condensed matter systems. We particularly focus on the Kitaev honeycomb lattice model that exhibits the Abelian doubled- \mathbb{Z}_2 and the non-Abelian Ising topological phases. Quasiparticle excitations of these phases have attracted considerable attention recently both for fundamental reasons as a system with non-Abelian fractional statistics and for potential applications in topological quantum computation.

We present an exact solution of the Kitaev spin model on the honeycomb lattice. We employ a Jordan-Wigner type fermionization and find that the Hamiltonian takes a Bardeen-Cooper-Schrieffer (BCS) type form, allowing the system to be solved by Bogoliubov transformation. Our fermionization does not employ non-physical auxiliary degrees of freedom and the eigenstates we obtain are completely explicit in terms of the spin variables. The ground-state is obtained as a BCS condensate of fermion pairs over a vacuum state which corresponds to the toric code state with the same vorticity. We show in detail how to calculate all eigenstates and eigenvalues of the model on the torus. In particular, we find that the topological degeneracy on the torus descends directly from that of the toric code, which now supplies four vacua for the fermions, one for each choice of periodic vs. anti-periodic boundary conditions. The reduction of the degeneracy in the non-Abelian phase of the model is seen to be due to the vanishing of one of the corresponding candidate BCS ground-states in that phase. This occurs in particular in the fully periodic vortex-free sector. The true ground-state in this sector is exhibited and shown to be gapped away from the three partially anti-periodic ground-states whenever the non-Abelian topological phase is gapped. The exact solution of the related star lattice chiral spin liquid is also presented.

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On Integrability of Riccati Equation and Its Relation with Some Computational Methods Used to Find Exact Solutions to NLPDE's

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Using the Lie groups theory we show a method for solving a one-parameter family of Riccati equations. We obtain a new case of integration of the general Riccati equation and we use it to solve several Riccati equations.

On the other hand, several computational techniques which use Riccati equations have been implemented in the last two decades with the aim to find exact solutions to NLPDE's. Among these methods are the tanh method [1], the generalized tanh method [2], the extended tanh method [3], the improved tanh-coth method [4],[5], the G'/G -expansion method [6][7] and the General Riccati equation method [8]. The principal objective of this work consists on show that from the point of view of physical applications, all theses methods are equivalents. We illustrate the results solving the one dimensional Burguers', the Tzitzeica–Dodd–Bullough (TDB) and the modified Korteweg-de Vries (MKdV) equations.

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Section 13

Probability and Statistics

On Restricted Fuzzy Linear Regression using Fuzzy Entropy

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Tanaka et. al. [1, 2, 3] introduced the concept of fuzzy regression model with fuzzy regression coefficients. Since a fuzzy number can be uniquely determined through its position and entropy [4], therefore by using the concept of fuzzy entropy, the estimators of the fuzzy regression coefficients may be estimated.

In the present communication, a fuzzy linear regression (FLR) model with some restrictions in the form of prior information is considered. We have obtained the estimators of regression coefficients with the help of fuzzy entropy for the restricted FLR model. Further, a few numerical examples are provided to illustrate the the proposed model and the obtained estimators.

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Weak Convergence for the Stochastic Heat Equation Driven by Gaussian White Noise

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60B12; 60G60; 60H15

We consider a quasi-linear stochastic heat equation on $[0, 1]$, with Dirichlet boundary conditions and controlled by the space-time white noise. We formally replace the random perturbation by a family of noisy inputs depending on a parameter $n \in N$ such that approximate the white noise in some sense. Then, we provide sufficient conditions ensuring that the real-valued *mild* solution of the SPDE perturbed by this family of noises converges in law, in the space $\mathcal{C}([0, T] \times [0, 1])$ of continuous functions, to the solution of the white noise driven SPDE. Making use of a suitable continuous functional of the stochastic convolution term, we show that it suffices to tackle the linear problem. For this, we prove that the corresponding family of laws is tight and we identify the limit law by showing the convergence of the finite dimensional distributions. We have also considered two particular families of noises to that our result applies. The first one involves a Poisson process in the plane and has been motivated by a one-dimensional result of Stroock, which states that the family of processes $n \int_0^t (-1)^{N(n^2 s)} ds$, where N is a standard Poisson process, converges in law to a Brownian motion. The second one is constructed in terms of the kernels associated to the extension of Donsker's theorem to the plane.

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Stochastic Growth Processes based on Random Segments

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60D05, 60J20

PRELIMINARY: Let \mathcal{K}_0 be a fixed interval of length $L > 0$ and $\mathcal{I}_n := [\omega, \omega + d]$, $n \in \mathbb{N}$, randomly placed intervals, i.e. segments, each of length $d > 0$ with ω as uniformly distributed random variables. We examine the length $L(\mathcal{K}_n)$ of growth processes $(\mathcal{K}_n)_{n \in \mathbb{N}}$ generated by $\mathcal{K}_n = \mathcal{K}_{n-1} \cup \mathcal{I}_n$ under the following two conditions **(1)**: $\mathcal{K}_{n-1} \cap \mathcal{I}_n \neq \emptyset$ and **(2)**: $\mathcal{K}_0 \cap \mathcal{I}_n \neq \emptyset$ respectively, both in **continuous** $\mathcal{C} := \mathcal{K} \subset \mathbb{R}$ and **discrete** $\mathcal{D} := \mathcal{K} \subset \mathbb{Z}$ space; so we get the four random processes **(1)**: $(\tilde{X}_n)_{n \in \mathbb{N}} = L(\mathcal{C}_n)$ and $(\hat{X}_n)_{n \in \mathbb{N}} = L(\mathcal{D}_n)$ as well as **(2)**: $(\tilde{Y}_n)_{n \in \mathbb{N}} = L(\mathcal{C}_n)$ and $(\hat{Y}_n)_{n \in \mathbb{N}} = L(\mathcal{D}_n)$.

RESULTS: **(1)**: Due to the unbounded growth of the random variables X_n we investigate the order of the growth, i.e. of the expected value $\mathbb{E}(X_n)$:

Cont.: We proof $O(\sqrt{n}) \leq \mathbb{E}(\tilde{X}_n) \leq O(n)$ due to Jensen's inequality and discuss a nice sequence based on $\mathbb{E}(\tilde{X}_{n+1}) = \mathbb{E}(\mathbb{E}(\tilde{X}_{n+1} | \tilde{X}_n)) = \mathbb{E}(H(\tilde{X}_n)) \approx H(\mathbb{E}(\tilde{X}_n))$ with $H(x) := (x^2 + x \cdot d + d^2)/(x + d)$ which motivates that $\mathbb{E}(\tilde{X}_n) = O(\sqrt{n})$ holds.

Disc.: $(\hat{X}_n)_{n \in \mathbb{N}}$ can be modeled as a Markov process and in the simplest case $d = 1$ we compute via Chapman-Kolmogorov equation and inverse z-transform the density of the distribution $\mathbb{P}(\hat{X}_n = i) = \frac{i+2}{i!} \cdot \sum_{k=1}^i (-1)^{i-k} \binom{i}{k} (k+2)^{i-1} \left(\frac{k}{k+2}\right)^n$.

(2): The random variables Y_n are bounded and we accomplish exact expressions:

Cont.: $\mathbb{E}(\tilde{Y}_n) = L + 2d - 2\frac{d+L}{n+1} [1 - (\frac{L}{d+L})^{n+1}]$, $d + L > 0$ and $n \in \mathbb{N}$ as well as

Disc.: $\mathbb{E}(\hat{Y}_n) = L + 2d - 2 \sum_{k=1}^d (1 - \frac{k}{L+d+1})^n$, $d + L > 0$ and $n \in \mathbb{N}$.

A comparison derives: $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N (1 - \frac{k/N}{1+\lambda+1/N})^n = \frac{1+\lambda}{n+1} \cdot [1 - (\frac{\lambda}{1+\lambda})^{n+1}]$. In both spaces we have a closer look to the density functions of the random variables Y_n . In the conclusion we will give an outlook of growth processes in the plane where the intervals \mathcal{I}_n are segments creating a convex hull by their vertices.

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On Ageing Properties of Semi-markov System and Total Time on Test Transforms

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First passage times of appropriate stochastic process have often been used to represent times to failure of devices or systems which are subject to shocks and wear, random repair time and random interruptions during their operations: [2]. The life distribution properties of these processes have therefore been widely investigated in reliability and maintenance literature. Use of total time on test transform in identification of failure rate models (Increasing

failure rate/Decreasing failure rate/ Bathtub shaped/constant) in the binary system case is discussed in literature: [3]. Later, [4] presented some relationship between the total time on test transform and some other ageing properties of random variable. In this paper, ageing properties of semi-Markov performance process of a multistate system are considered: [1]. We give the method of identification of the failure rate model of first passage time distribution of the semi-Markov system based on the transition probability functions. An illustrative example is given.

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Asymptotic Expansion with Double Layers of Singularly Perturbed Diffusions

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For the diffusion process X_t^ϵ on R^3 , $t \leq T$ with a fixed initial distribution $p(0, x)$, $x \in R^3$ and generator $L^\epsilon = L_1 + 1/\epsilon L_2 + 1/\epsilon^2 L_3$ where

$$L_1 = 1/2 \sum_{i,j=1}^3 \sigma_{i,j}^{(1)}(t, x) \partial_{i,j} + \sum_{i=1}^3 b_i^{(1)}(t, x) \partial_i,$$

$$L_2 = 1/2 \sum_{i,j=1}^2 \sigma_{i,j}^{(2)}(t, x) \partial_{i,j} + \sum_{i=1}^2 b_i^{(2)}(t, x) \partial_i$$

and

$$L_3 = 1/2 \sigma_3^3(t, x) \partial_{33}(t, x) + b_3^3(t, x) \partial_3,$$

we shall develop the asymptotic expansion (with double layers) of its density functions $p(t, x)$. If all the coefficients $\sigma_{i,j}^k(t, x)$ and $b_i^k(t, x)$ are in $C^{(n, 2n+1)}$, we show that there exist functions $u_i(t, x)$, $v_i(t/\epsilon, x)$ and $w_i(t/\epsilon^2, x)$ such that $\sup_{t \in [0, T], x \in R^3} |p(t, x) - \sum_{i=0}^n \epsilon^i u_i(t, x) - \sum_{i=0}^n \epsilon^i v_i(t/\epsilon, x) - \sum_{i=0}^n \epsilon^i w_i(t/\epsilon^2, x)| =$

$O(\epsilon^{n+1})$. Our results generalize that in [1] where there are only two time scales and only one boundary layer. The proof is based on the associated Fokker-Planck equation.

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Intermittency in a Hyperbolic Anderson Problem

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60H15

We study the asymptotics of the even moments of solutions to a stochastic wave equation in spatial dimension 3 with linear multiplicative noise. Our main theorem states that these moments grow more quickly than one might expect. This phenomenon is well-known for parabolic stochastic partial differential equations, under the name of intermittency. Our results seem to be the first example of this phenomenon for hyperbolic equations. For comparison, we also derive bounds on moments of the solution to the stochastic heat equation with linear multiplicative noise. This is joint work with Carl Mueller [1]. It makes strong use of a Feynman-Kac type formula for moments of this stochastic wave equation developed in joint work with Carl Mueller and Roger Tribe [2].

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Mathematical Modeling of Mortality for Countries with Limited and Defective Data

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 62Nxx

The survivorship function plays a vital role in life tables. Inefficient registration of vital events and geographically remote areas in many developing countries hinder the regular availability of accurate data. Hence mathematical modeling of such data is of utmost importance because of their role as development indicators. In this paper, survivorship function of a developing country particularly Nepal is regressed on age and time variables. A parsimonious regression model with a very few regression coefficients, specially suited to such countries, keeping the constraints of limited and defective data is developed. The developed form satisfies all the criteria of a good model not only for developing countries like Nepal and India, but also for developed countries like Germany. A comparison of results is made between the data of countries with abundant and good quality data on one hand and limited and defective data on the other hand.

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Saddlepoint-Based Inferences for Nonlinear Regression Models

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20K

A novel method for making small sample inference on the nonlinear parameter in a conditionally linear nonlinear regression model is proposed. It is based on Saddlepoint approximations to the distribution of the estimating equation whose unique root is the parameter's maximum likelihood estimator (MLE) are obtained, while substituting conditional MLE's for the remaining (nuisance) parameters. A key result of Daniels [1] enables to relate these approximations to those for the estimator of interest. Standard methods either rely on large samples or fail to provide guidance in this context. The method's performance relies on a model reparametrization that orthogonalizes the nonlinear parameter with the nuisance parameters, thereby also validating the substitution of conditional MLE's in for the latter. The methodology is shown to be applicable for inference on ratios of regression parameters in ordinary linear models, calibration and many others. Simulations results for the proposed method yield confidence intervals with lengths and coverage probabilities that compare favorably with those from several competing methods.

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The EM Algorithm and Optimal Designs for Mixtures of Distributions

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Maximum Likelihood Estimates (MLE) for a model with mixture of distributions is usually an unaffordable task from a computational point of view, even for simple cases when the number of distributions is known. The EM algorithm is frequently used in this context to approximate the MLE. Louis (1982) in a celebrated paper [1] provides the information matrix for the EM (“pure”) estimates. The EM algorithm provides approximate MLE, thus the information matrix to be used must be the Fisher information matrix for the marginal log-likelihood of the observations. Pure EM estimates are computed and compared to the MLE. Some comparisons of the two information matrices are also performed. Finally, optimal designs are computed for a mixture of normal distributions with modeled means throughout an explanatory variable.

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Estimation of Regression Coefficients in a Replicated Measurement Error Model under Restrictions

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 62J05, 62H12

A multiple linear regression model with replicated observations is adopted. It is considered that all the variables in the model are observed with additive measurement errors. It is also assumed that some prior information on regression coefficients is available in the form of exact linear restrictions. Under such a setup, the usual estimators are either inconsistent or do not satisfy the given exact linear restrictions on regression coefficients. We obtain such estimators of regression coefficients which are consistent as well as satisfy the given restrictions. Asymptotic properties of the estimators are analyzed. A simulation study is also conducted to study the small sample properties of the estimators. It is common to assume the normal distribution of measurement errors. In the present work, no such assumption is made.

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On Construction Of New Circular Models

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 62H11

In many diverse fields, the measurements are directions- A biologist may be interested in the direction of flight of the bird or orientation of an animal while geologist may be measuring the direction of earth's magnetic field. Such directions may be in two or three dimensions. A set of such observations on the directions is referred to as 'directional data'. In particular analysis pertaining to two dimensional directional data falls under the topic 'CIRCULAR STATISTICS'. For such data, several Statistical models were constructed and

inference procedures were studied. Most of the existing circular models were constructed using the method of wrapping the corresponding linear models. In order to study the characteristics of a new circular model the convenient form of the density function is in terms of trigonometric moments which is basically derived using the characteristic function of the corresponding linear model. In this note an attempt is made to derive wrapped versions of Logistic distribution, Extreme Value distribution, Lognormal distribution, Weibull distribution, Half Logistic distribution and Binormal distribution. Also it was pointed out that there is one - one correspondence between Toeplitz Hermitian Positive Definite (THPD) matrix obtained using positive definite sequence from the characteristic function and a circular model. Therefore, this process can be treated as a method of construction of a new circular model. In addition to this construction of circular model based on the Rising Sun function is also included.

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An Application of the Response Surface Methodology (RSM) to the Production of Pectynolitic Enzymes

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 62K20

This work presents an application of the Response Surface Methodology (RSM) for optimizing the value of the factors influencing the production of pectynolitic enzymes (pectinases) in a process for the growth of yeast using grape skin as a substrate.

The context of the research focuses on the industrial production of enzymes using grape skin. In the process of yeast growth, those pectinases used in the stage of wine fermentation are synthesized because they help release wine color and bouquet. For this growth, grape skin were used as a substrate since their use in industrial production involves not only an economic advantage, but also an environmental one for the grape and wine-growing areas, leading to the exploitation of this fermentation sub-product. This study consists of designing a model with which we can obtain the ideal conditions for the growth of certain type of yeast so that it synthesizes the largest amount of pectinases. Thus, we applied the sequential character of RSM until we obtained a second-order model adjusting it with a compound central design (CCD).

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A Study of Boundedness in PN spaces

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It was shown in [3] that uniform boundedness in a Šerstnev PN space (V, ν, τ, τ^*) , (named *boundedness* in the present setting) of a subset $A \subset V$ with respect to the strong topology is equivalent to the fact that the probabilistic radius R_A of A is an element of \mathcal{D}^+ . Here we extend the equivalence just mentioned to a larger class of PN spaces, namely those PN spaces that are topological vector spaces (briefly TV spaces), but are not Šerstnev PN spaces.

Section 2 presents a characterization of those PN spaces, whether they are TV spaces or not, in which the equivalence holds. In Section 3, a characterization of the Archimedeaness of triangle functions τ^* of the type $\tau_{T,L}$ is given. This work is a partial solution to a problem of comparing the concepts of distributional boundedness (\mathcal{D} -bounded in short) and that of boundedness in the sense of associated strong topology.

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On Some Reliability Properties of Mean Inactivity Time Order under Weighing

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 62E15, 62N05

Jain et.al [1] and Misra et. al [2] studied various properties of reliability measures of weighted distributions of life distributions. Reliability properties regarding the weighted distributions of random variables (and vectors) in univariate and bivariate cases have been discussed by Nanda and Jain [3]. In the present communication, we study the preservation of mean inactivity time under some weight function. Some reliability properties of weighted distributions for mean inactivity time order has also been discussed.

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An Application of the Segal-Bargmann Transform to the Characterization of Lévy White Noise Measures

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60B05, 60H40, 46F25

Being inspired by the observation that the Stein's identity is closely connected to the quantum decomposition of probability measures [3] and the Segal-Bargmann transform [2], we are able to characterize the Lévy white noise measures on the space \mathcal{S}' of tempered distributions associated with a Lévy spectrum having finite second moment. The results not only extends the Stein [4] and Chen's lemma [1] for Gaussian and Poisson distributions to infinite dimensions but also to many other infinitely divisible distributions such as Gamma and Pascal distributions and corresponding Lévy white noise measures on \mathcal{S}' .

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Asymptotic and Martingale Method for a Delay Financial Model

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60G44

In this paper, we extend a delayed geometric Brownian model by adding a stochastic volatility term, which is assumed to have fast mean reversion, to the delayed model. Combining a martingale approach and an asymptotic method, we develop a theory for option pricing under this hybrid model. Core result obtained by our work is a proof that a discounted approximate option price can be decomposed as a martingale part plus a (ignorable) small term. We demonstrate a correction effect driven by the option price under our new model.



Small Value Probabilities

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2010 MATHEMATICS SUBJECT CLASSIFICATION. Primary 60E15; Secondary 60G15.

Small value (deviation) probability studies the asymptotic rate of approaching zero for rare events that positive random variables take smaller values. In the literature, small value probabilities of various types are studied and applied to many problems of interest under different names such as small deviation/ball

probabilities, lower tail behaviors, boundary crossing probabilities, asymptotic evaluation of Laplace transform for large time, etc. We will provide an overview on recent progress and future prospects in the area.

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Mathematical Modelling for LQ45 Index

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LQ45 Index consists of 45 stocks with high liquidity and market capitalization in Indonesia Stock Exchange (IDX). Every 6 months, there is a review process for determination of stocks that can be included in the LQ45 Index. It serves as a benchmark of stocks in IDX and describes the current market condition. The classical model for stock price dynamics is the Geometric Brownian Motion. Using such a model we can successfully apply the Black-Scholes model for option price on stock. However there is a drawback of the GBM model when applied to LQ45 Index data. In a long term period, constant volatility assumed by the GBM model is violated. LQ45 Index data also present multiple mean levels and its return presents fatter tails than a normal distribution. In our paper we attempt to model the dynamics of LQ45 Index data from January 2004 to December 2009 using two stochastic models, viz., mean-reversion model and potential diffusion model, besides the GBM model. We will use some statistical tests to determine which model is the most suitable model for the LQ45 Index.

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Statistically, Failure of Inferential Statistics at Critical Point of Time

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60G25

Inferential statistics has been pivotal tool for the analysis of stochastic processes. Author has described the futility of inferential statistics in determining the down fall and stagnation. Author has tried to cover as many field as possible to depict the failure of inferential statistics at critical point of time. The critical point of time here means the time period in which statistical analysis was supposed to be proven productive, but failed to avoid calamity. The goal of the paper is not to criticize the denouement of inferential statistics; it is rather a critique on inferential statistical analysis. The frequency of failure of inferential statistical analysis may be very small, but failing at critical point of time is devastating. Author has provided statistical surveys and analysis of Great Depression of 1929 in United States of America, astrology, statistical signal processing and a natural disaster in order to support the argument of failure of inferential statistics. All these events were not supposed to transpire as per probability theory, but endured. The proposed alternative for inferential statistics is game theory. The analysis is done using gretl freeware version 1.8.6 [1].

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Stochastic Chains: Matrix Power Series Equations: Algebraic Geometry: Quantity Theory

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60J22

Structured matrix power series equation of the following form

$$\sum X^i D_i = 0 \quad (1)$$

Where D_i 's the sub-matrices of the generator matrix arise in the equilibrium analysis of structured (G/M/1 type) Markov chains. The author questioned the possibility of finding a closed form expression (Jordan Canonical Form) for R.

In this research paper, arbitrary matrix power series of the form in (1) is considered. The following Lemma provides a method of determining the eigenvalues of all possible solutions of (1).

Lemma1: Consider a matrix Y which satisfies (1) and let $H(\lambda) = \sum_{j=0}^{\infty} \lambda^j D_j$

Then $H(\lambda)$ has the following representation $H(\lambda) = (\lambda I - Y)(\sum_{j=0}^{\infty} \lambda^j N_j)$

Theorem1: Consider a matrix power series equation of the form in (1). Let the dimension of \mathbf{X} be 'n'. Let there be "finitely" many, say 'm' ($m > n$) roots of the transcendental function $\text{Det} (H(\lambda) = \sum_{j=0}^{\infty} \lambda^j D_j)$. Then all possible solutions of (1) are divided into atmost $\binom{m}{n}$ equivalence classes and solution in each class is determined as the solution of a linear system of equations.

The results are generalized to multi-matrix/tensor variate polynomial equations.

- It is reasoned that ARBITRARY multi-variate polynomial/power series equations can be imbedded in PROPERLY CHOSEN TENSOR VARIATE POLYNOMIAL/POWER SERIES EQUATIONS (central goal of algebraic geometry).
- Using Lemma 1 and a localization theorem, the author succeeded in finding a Jordan canonical form for the rate matrix R of a structured matrix power series equation (arising in the case of G/M/1-Type Markov chains).
- **TheoryofChances: QuantityTheory:** Also, in this paper, the author proposes the idea of allowing the chance (like probability) to assume negative and complex values. Novel stochastic chains based on real/complex

valued chances are formally discussed. Basic ideas of “quantity theory” (like measure theory) and “unification” (like integration) are formally discussed.



Convergence Rate of Wavelet Expansions of Random Processes

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60G10, 42C40

In various statistical, data compression, signal processing applications and simulation, it could be used to convert the problem of analyzing a continuous-time random process to that of analyzing a random sequence, which is much simpler. Multiresolution analysis provides an efficient framework for the decomposition of random processes.

We consider stationary Gaussian random processes $\mathbf{X}(t)$ and their approximations by sums of wavelet functions, reading as follows:

$$\mathbf{X}_{n, \mathbf{k}_n}(t) := \sum_{|k| \leq k'_0} \xi_{0k} \phi_{0k}(t) + \sum_{j=0}^{n-1} \sum_{|k| \leq k_j} \eta_{jk} \psi_{jk}(t), \quad (2)$$

where $\mathbf{k}_n := (k'_0, k_0, \dots, k_{n-1})$.

On the contrary to many theoretical results with infinite series form of $\mathbf{X}_{n, \mathbf{k}_n}(t)$, in direct numerical implementations we always consider truncated series like (2), where the number of terms in the sums is finite by application reasons. However, there are almost no stochastic results on uniform convergence of finite wavelet expansions to $\mathbf{X}(t)$.

We have obtained the exponential rapidity of convergence of a wide class of wavelet expansions. Namely,

$$P \left\{ \sup_{t \in [0, T]} |\mathbf{X}(t) - \mathbf{X}_{n, \mathbf{k}_n}(t)| > u \right\} \leq 2 \exp \left\{ - \frac{(u - \sqrt{8u\delta(\varepsilon_{\mathbf{k}_n})})^2}{2\varepsilon_{\mathbf{k}_n}^2} \right\},$$

where $u > 8\delta(\varepsilon_{\mathbf{k}_n})$, $\varepsilon_{\mathbf{k}_n}$ are constants which depend only on the covariance function of $\mathbf{X}(t)$ and the wavelet basis.

This work was partly supported by *La Trobe University Research Grant "Stochastic Approximation in Finance and Signal Processing."*

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A New Measure of Relationship among Qualitative Variables

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Pearson's Correlation coefficient is frequently used to study the relationship between two variables. In situations where qualitative variables such as nominal variables, ordinal variables are involved, alternative measures such as Spearman's Rank correlation coefficient, Kendall's Tau and other similar measures are used. Even these measures are originally derived when the underlying variables are continuous, and modified later by accounting for ties (see e.g. [1]), which exist in discrete data, and these do not perform well for qualitative data as proved in this paper using simulation study. In this paper, we propose a Gini-type statistic to quantify the relationship between ordinal variables. We derive an estimate of the measure, and derive a test for independence among ordinal variables. Using a Monte-Carlo simulation study, we show that the measure performs well for qualitative variables as compared to other existing measures. We also discuss some important applications of this new measure.

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Fitting Several Data Sets to Levy and Generalized-t Distributions

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Symmetric and skew Levy and other similar heavy tail distributions are studied by Rathie et al. (See, for example, [1]–[4]) and applied to stock market and currency data sets.

A generalized t distribution function is defined and k-th moments are derived. The results estimating parameters by the method of moments and maximum likelihood method are obtained. This distribution is applied to fit nicely two data sets involving daily change in Petrobras (PETR4.SP PN; 1948 data points) stock market value and the daily change in exchange rates between Brazilian Real and US dollar (3491 data points). The three parameters involved are estimated by maximum likelihood method for these data sets and maximum error is obtained in each case.

The distribution function for the symmetric Levy distribution is derived and used to fit nicely the following six data sets involving: (a) mean monthly sun spots, (b) daily maximum temperature of a Brazilian ecological reserve, (c) daily fluctuations in exchange rates in US dollars of three world currencies (Brazilian Real, Euro, and Swiss franco), and stock market value of J. P. Morgan. Maximum and mean errors are obtained in each case showing a nice fit in each case.

Graphical representations are also given in all the cases.

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On Entropy Convergence of Normalized Partial Maxima of Iid Random Variables

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60F05

Limit distributions of partial maxima of independent and identically distributed (iid) random variables under linear normalization are the well known extreme value laws, also called as l -max stable laws, l denoting that the normalization is linear. If the normalization employed is power in place of linear, then the limit laws have been called as p -max stable laws. Under some conditions on the underlying distribution of the iid random variables, entropy convergence of the density of the normalized partial maxima to the corresponding entropy of the limit law is discussed in this article.



Solutions of the Navier–Stokes and Burgers Equations via Forward-backward SDEs

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65C30, 35Q30, 35Q53

We establish a connection between the strong solution to the spatially periodic Navier–Stokes equations and a solution to a system of forward-backward stochastic differential equations (FBSDEs) on the group of volume-preserving diffeomorphisms of a flat torus. Assuming the existence of a solution to the Navier–Stokes equations with the initial data in the Sobolev space H^s for sufficiently large s , we construct a solution of the associated system of FBSDEs. Conversely, if we assume that a solution of the system of FBSDEs exists, then the solution of the Navier–Stokes equations can be obtained from the solution of the FBSDEs. In fact, the constructed FBSDEs on the group of volume-preserving diffeomorphisms can be regarded as an alternative characterization to the Navier–Stokes equations for studying the properties of the latter. On the

other hand, we describe a probabilistic construction of H^s -regular solutions to the spatially periodic Burgers equation by proving the existence and uniqueness theorem for the associated forward-backward stochastic system.

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Phase Transitions for Dilute Particle Systems with Lennard-Jones Potential

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 82B21, 60F10, 60K35, 82B31, 82B05, 82B26

We consider a classical dilute particle system in a large container with pair interaction given by a Lennard-Jones-type potential. The inverse temperature

is picked proportionally to the logarithm of the number of particles. We identify the free energy per particle in terms of a variational formula and show that this formula exhibits a cascade of phase transitions as the temperature parameter ranges from zero to infinity. Loosely speaking, the lower the temperature, the larger the relevant crystal structures that give the main contribution to the free energy. The phases are characterised by the size of the relevant configurations. Our main tool is a new large deviation principle for sparse point configurations.



Improvements of Some Moment Inequalities

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60E15

For an arbitrary probability law with support on $(0, \infty)$, define

$$\lambda(s) = \lambda_s(X) := \begin{cases} (EX^s - (EX)^s)/s(s-1), & s \neq 0, 1; \\ \log(EX) - E(\log X), & s = 0; \\ E(X \log X) - (EX) \log(EX), & s = 1. \end{cases}$$

It is known [1] that $\lambda(s)$ is log-convex (hence convex) for $s \in \mathbf{R}$, that is,

$$\xi(s, t) := \lambda(s)\lambda(t) - \lambda^2\left(\frac{s+t}{2}\right) \geq 0; s, t \in \mathbf{R},$$

providing that the corresponding moments exist.

Some refinements of the above are

$$\begin{aligned} & \left[\lambda(s) - 2\lambda\left(\frac{s+t}{2}\right) + \lambda(t) \right] \left[\lambda(u) - 2\lambda\left(\frac{u+v}{2}\right) + \lambda(v) \right] \\ & \geq \left[\lambda\left(\frac{s+u}{2}\right) - \lambda\left(\frac{s+v}{2}\right) + \lambda\left(\frac{t+v}{2}\right) - \lambda\left(\frac{t+u}{2}\right) \right]^2; \end{aligned}$$

or

$$\xi(s, t)\xi(s, v) \geq \left[\xi\left(s, \frac{t+v}{2}\right) - \xi\left(\frac{s+t}{2}, \frac{s+v}{2}\right) \right]^2.$$

Note that for $s, t, u, v \in \mathbf{2N}$, the above moment inequalities are valid for arbitrary probability distributions with support on $(-\infty, +\infty)$, [2].

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A Stochastic Model for the Expected Time to Recruitment in a Single Graded Manpower System with Two Thresholds following SCBZ Property and Correlated Inter-decision Times

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In this paper, an organization subjected to random exit of personnel due to the policy decisions taken by the organization is considered. There is an associated loss of manhours if a person quits. As the exit of personnel is unpredictable, a new recruitment policy involving two thresholds—one is optional and the other is mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach, a mathematical model is constructed involving an appropriate univariate recruitment policy. The mean and variance of the time to recruitment are obtained when (i) the loss of manhours process forms a sequence of independent and identically distributed continuous random variables (ii) the inter-decision times are exchangeable and constantly correlated exponential random variables and (iii) the optional threshold level as well as the mandatory threshold level follow SCBZ property. The present results extend those in [1] for correlated case. The analytical results are numerically illustrated and analysed by assuming specific distributions.

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Length Biased Weighted Residual Inaccuracy Measure

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The concept of weighted distribution introduced by Rao [5] is widely used. These distributions arise when the observations generated from a stochastic process are recorded with some weight function. Let X be a non-negative continuous random variable (r.v.) with p.d.f. $f(x)$ and X^w be a weighted r.v. corresponding to X with weight function $w(x)$. When $w(x) = x$, X^w is said to be a length biased random variable.

Consider X and Y two non-negative r.v.'s representing time to failure of two systems with p.d.f. $f(x)$ and $g(x)$ respectively, and let $F(x)$ and $G(y)$ be failure distributions. Then the measure of differential entropy associated with the r.v. X , measure of discrimination of X about Y and measure of inaccuracy are given by Shannon [6], Kullback and Leibler [4] and Kerridge [3] respectively. Using the information theoretic approach to measure the uncertainty of a system which has survived up to time t , the corresponding dynamic measure of uncertainty, discrimination and dynamic inaccuracy are given by Ebrahimi [1], Ebrahimi and Kirmani [2] and Taneja et al. [7] respectively. Extending the concept of residual inaccuracy, in the present communication we introduce a length biased weighted residual inaccuracy measure between two residual lifetime distributions over the interval (t, ∞) . Based on proportional hazard model (PHM), a characterization problem for this inaccuracy measure has been studied and a lower bound has been derived.

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Valuation of American Basket Options by a Simple Binomial Tree

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In this article we address the problem of valuing and hedging American options on baskets and spreads, i.e., on portfolios consisting of both long and short positions. We adopt the main ideas of the Generalized Lognormal (GLN) approach introduced in Borovkova et al. (2007) and extend them to the case of American options. We approximate the basket price process by a suitable Geometric Brownian motion, shifted by an arbitrary parameter and possibly reflected over the x -axis. These adjustments to the GBM are necessary for dealing with negative basket values and possible negative skewness of basket increments distribution. We construct a simple binomial tree for an arbitrary basket, by matching the baskets volatility, and evaluate our approach by comparing the binomial tree option prices to those obtained by other methods, whenever possible. Moreover, we evaluate the delta-hedging performance of our method and show that it performs remarkably well, in terms of both option pricing and delta hedging. The main advantages of our method is that it is simple, computationally extremely fast and efficient, while providing accurate option prices and deltas.

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Comparative Study of Normal Plots for Analyzing Unreplicated Factorial Designs

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Unreplicated two-level factorial designs are very useful in many applications, especially in industrial experimentation [4]. One of the reasons of their wide spread use is the cost savings associated with such designs [3]. However, without replication, a direct estimation of error variance is not possible [2]. One common method to assess the significance of effects is to use normal or half-normal probability plots [1]. Some experimenters use the normal plot, while others prefer the half-normal plot [5]. The choice between these two plots seems to be subjective. In this paper we present a study carried out to compare these two graphical techniques.

We intend to verify in what situations one plot could be better than the other. We use simulation and case studies to evaluate the abilities of both graphical techniques to identify significant effects, to detect outliers, and to identify inadvertent split-plotting in unreplicated two-level factorial designs. We show that these simulations can provide potentially useful insights to practitioners when interpreting results from an experiment. We also discuss the advantages and limitations of each procedure.

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Section 14

Combinatorics

Balanced Tournaments for Games

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We consider a game played on a graph with n vertices, or countries, and b edges, or borders. A tournament consists of n games where no player plays the same country twice, represented by n labellings of the graph where no label is given to the same vertex twice. A tournament is balanced if the number of times each pair of players shares a common border is strictly within 1 of $n \cdot \frac{b}{\binom{n}{2}}$.

We determine all pairs (n, b) with $n \leq 7$ for which there is a graph having no balanced tournament. We also consider the problem of finding an algorithm which will always produce an “almost balanced” tournament.



The Graph Equation $Sz(G) = W(G) + k$ and an Application in Nanoscience

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Let G be a connected graph and $\eta(G) = Sz(G) - W(G)$, where $W(G)$ denotes the Wiener index and $Sz(G)$ denotes the Szeged index of G . A well-known result of Klavžar, Rajapakse and Gutman [3] states that $\eta(G) \geq 0$ and by a result of Dobrynin and Gutman $\eta(G) = 0$ if and only if each block of G is complete [1, 2]. In this paper an edge-path matrix for the graph G is presented by which it is possible to present a new characterization for the graphs in which the Wiener and Szeged are the same. It is also shown that $\eta(G) \neq 1, 3$ and a classification of all graphs with $\eta(G) = 2, 4, 5$ are presented. Finally, it is proved that for a given positive integer $k, k \neq 1, 3$, there exists a graph G with $\eta(G) = k$.

We apply our result to compute the Wiener index of some molecular graphs applicable in nanoscience.

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Square 2-designs on a New Family of Binary Codes

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In this paper, we develop a family of new codes and investigate whether this family of codes holds a square design or not[1]. Let F_q^n denote the linear space of all n -tuples over the finite field $F_q = GF(q)$. An (n, M) code C over F_q is a subset of F_q^n of size M . If C is a k -dimensional subspace of F_q^n , C is called an $[n, k]$ linear code over F_q . The field F_2 is very special in coding theory, and codes over F_2 are called binary codes and similarly, codes over F_3 are called ternary codes and so on. Thus codes over F_q are called q -ary codes [2, 3].

C is an $(n, M, d)_q$ code over F_q of length n with M codewords and minimum distance d . The code C can be either linear or nonlinear. A t - (v, k, λ) design D is a set X of v points together with a collection of k -subsets of X (called block) such that every t -subset of X is contained in exactly λ blocks [1, 4]. Here, a new family of binary codes $(\frac{3^n-1}{2}, \frac{3^n-1}{2}, 2 \cdot 3^{n-2})$, $n \geq 3$ is developed and some of their properties are studied. It is also shown that this family of codes holds 2 - $(\frac{3^n-1}{2}, 3^{n-1}, 2 \cdot 3^{n-2})$ design.

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On Chainos Total-ctree Graph of a Graph

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A nontrivial connected graph G is called a chain if every block is incident with at most two cutpoints of G and every cutpoint is incident with exactly two blocks. In other words a graph is a chain if its block-cutpoint graph is a path. The total-ctree of a graph is the graph whose points can be put in one-to-one correspondence with the set of blocks and cutpoints of a graph in such a way that two points of total-ctree graph are adjacent if and only if the corresponding members of a graph are adjacent, co-adjacent or incident.

In this paper, the concept of chainos total-ctree graph of a graph is introduced. We present a characterization of those graphs whose chainos are eulerian, hamiltonian, planarity, outerplanarity and minimally nonouterplanarity. Also, the necessary and sufficient conditions for chainos total-ctree graph to have crossing number one or two are established. Further covering invariants and chromatic number are studied.

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On Ramsey $(2K_2, P_n)$ -minimal Graphs

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For any given graphs G and H , the notation $F \rightarrow (G, H)$ means that any red-blue coloring of the edges of F forces F to contain a red subgraph G or a blue subgraph H . Graph F is a *Ramsey* (G, H) -*minimal* graph if $F \rightarrow (G, H)$ but $F^* \not\rightarrow (G, H)$ for any proper subgraph $F^* \subset F$. The class of all (G, H) -minimal graphs is denoted by $\mathcal{R}(G, H)$. The pair (G, H) is called *Ramsey-finite* or *Ramsey-infinite* depending upon whether $\mathcal{R}(G, H)$ is finite or infinite. Some results related to the finite class $\mathcal{R}(2K_2, H)$ for some H have been obtained as follows.

Burr, Erdős and Lovász [3] showed that $\mathcal{R}(2K_2, 2K_2) = \{3K_2, C_5\}$. Then Burr et al. [2] determined all graphs in $\mathcal{R}(2K_2, C_3)$. Mengersen and Oeckermann [5] determined the members of $\mathcal{R}(2K_2, tK_2)$ for $t \leq 5$ and characterizing its members for $t \geq 5$. Another results are the determination of all graphs belonging to $\mathcal{R}(2K_2, K_{1,n})$ for $n \leq 3$ and the characterization of its members for $n \geq 3$ in [4]. Recently, Yulianti et al. [6] determined all graphs in $\mathcal{R}(2K_2, C_4)$ and gave some necessary conditions for graphs in $\mathcal{R}(2K_2, C_n)$ for $n \geq 3$. It has been shown in [1] (without proof) and [4] that $\mathcal{R}(2K_2, P_3) = \{2P_3, C_4, C_5\}$. In this paper we will characterize the graphs belonging to $\mathcal{R}(2K_2, P_n)$ for $n \geq 3$.

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Strong (Weak) Edge-Edge Domination Number of a Graph

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Keywords. Edge-Edge Dominating sets (EED sets), Strong Edge-Edge Dominating sets (SEED sets), Weak Edge-Edge dominating sets (WEED sets).

For any edge $x = uv$ of an isolate free graph $G(V, X)$, $V_x = \{w \in V | w \text{ is adjacent to } u \text{ or } v\}$. Then $\langle N[x] \rangle$ is the subgraph induced by the set V_x . The edge degree of $x = uv$ is defined as $d_e(x) = d(u) + d(v) - 2$. We say that an edge x , e-dominates an edge y if $y \in \langle N[x] \rangle$. An edge x strongly e-dominates an edge y if $y \in \langle N[x] \rangle$ and $d_e(x) \geq d_e(y)$. A set $L \subseteq X$ is an *Edge-Edge Dominating Set* (EED-set) if every edge in $X - L$ is e-dominated by an edge in L . And L is said to be a *Strong Edge-Edge Dominating set* (SEED-set) if every edge in $X - L$ is strongly e-dominated by an edge in L . The *edge-edge domination number* $\gamma_{ee}(G)$ (*strong edge-edge domination number* $\gamma_{see}(G)$) is the minimum cardinality of an EED-set (SEED-set). In this paper, we find the relationship between the new parameters and some known graph parameters. Further the *Edge-Edge degree* of an edge is defined and bounds for γ_{ee} in terms of maximum Edge-Edge degree is established.

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Regulation of Uncertain Nonlinear Systems: A Differential Equation Approach to Robust Control Design with Applications to Biological Systems

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The development of effective prevention and control strategies requires taking into account the uncertainties inherent in any realistic dynamical process, because they often induce complex behaviors (oscillations, instability, poor performance, etc). Problems with uncertainty are the most ambitious and most difficult of control theory, but their analysis is necessary and of great importance for real-world applications. The basis of robust control theory is to consider these uncertain behaviors, and analyze how the control system can cope with this problem. The uncertainty can be of two types: errors or defects from the model and unmeasured noises and disturbances that affect the dynamical systems. These terms of uncertainty often lead to great instability. The aim of robust control theory is to control these instabilities, either by acting on certain parameters to maintain the system in a desired state, or by calculating the limit of these parameters before the system becomes unstable (“*observe, measure and provide for effective actions*”).

The fundamental idea of our approach is the connection between the game theory approach and the problem of stabilizing uncertain nonlinear distributed parameter systems, described by nonlinear partial differential equations (NPDE's). This is motivated, by the fact that: first, it is well now that the most appropriate mathematical models for the real dynamical systems are the *full nonlinear* ones, and second, the robust control theory can be represented as a differential game between an engineer seeking the best control which stabilizes the system with limited control efforts, and simultaneously plant or unexpected events seeking the maximally malevolent disturbance which destabilizes the system with limited disturbance magnitude. This area concerns investigation of the minimax control, stability and adjoint control optimization of infinite-dimensional dynamical systems (which are, in general, systems of coupled and time-varying NPDE's). Details of our approach and several physical and biological applications are reported in [1]. In this review, we provide an overview of our approach, by presenting the basic ideas behind the main theories as well as the reasons why such an approach is a good alternative for the robust regulation of full nonlinear dynamical systems, and by showing how to apply them in a practical way. Finally, in order to explain our theoretical proposals on practical cases, we give a biological application.

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On Connected Splitting Matroid

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In this paper, we prove that if M is a binary connected and vertically 3-connected matroid of cogirth at least 4 and girth at least 3, then the matroid obtained by splitting away any pair of elements of M is connected.

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On Some P -partitions and Partitions into Four Cubes

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The theory of ordinary P -partitions which is a common generalization of the theory of partitions and the theory of compositions was introduced by Richard Stanley in 1972 [3]. I must recall that an ordinary P -partition is an order-preserving map from a partially ordered set P to a chain with special rules specifying where equal values may occur ([1], [3]).

In this talk, I will describe how we can use the generating function for some P -partitions and the ordinary graph induced by the covering relation of P in order to obtain a formula for the number of partitions of a positive integer n into four cubes with two of them equal. In particular, these results allow us to provide advances to the question *Can every natural number k be put in the form $x^3 + y^3 + 2z^3$, where x, y, z are integers?* mentioned by Richard Guy in [2].

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Transit Functions of Higher Arity

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Transit functions of arity 2 and associated convexities is a well studied topic since 1951, see [3], [2] and [1]. We generalize the transit function from binary to n -ary, $n > 2$. Let V be a non-empty set. Then a function $R: V \times V \times \dots \times V \rightarrow 2^V$ is a *transit function of arity n* (or *n -ary transit function*) on V if R satisfies the following axioms.

- (t1) $u_i \in R(u_1, u_2, \dots, u_n)$ for all $u_i \in V, i = 1, \dots, n$;
- (t2) $R(u_1, u_2, \dots, u_n) = R(\pi(u_1, u_2, \dots, u_n))$ for all $u_i \in V$, where $\pi(u_1, u_2, \dots, u_n)$ is any permutation of (u_1, u_2, \dots, u_n) ;
- (t3) $R(u, u, \dots, u) = \{u\}$ for all $u \in V$.

If R is a transit function on V , a subset W of V is R -convex if $R(u_1, u_2, \dots, u_n) \subseteq W$ for any $u_1, u_2, \dots, u_n \in W$.

A convexity \mathcal{C} on V is of *arity $\leq n$* if $\mathcal{C} = \{C \subseteq V \mid F \subseteq C, |F| \leq n \Rightarrow \langle F \rangle_{\mathcal{C}} \subseteq C\}$. We have that: \mathcal{C} is an S_1 -convexity on V of arity $\leq n$ if and only if \mathcal{C} is an R -convexity for some n -ary transit function R on V . We extend the betweenness properties of binary transit functions to n -ary functions and analyze their implications. The following betweenness axioms can be considered for an n -ary transit function R . For any $u_1, u_2, \dots, u_n, x, x_1, x_2, \dots, x_n \in V$, define

- (b1) $x \in R(u_1, u_2, \dots, u_n), x \neq u_k \Rightarrow u_k \notin R(y_1, y_2, \dots, y_n)$, where $y_i = u_i$ if $y_i \neq u_k$ else $y_i = x$ for $i = 1, 2, \dots, n$.
- (b2) $x \in R(u_1, u_2, \dots, u_n) \Rightarrow R(x, u_2, \dots, u_n) \subseteq R(u_1, u_2, \dots, u_n)$.
- (m) $\forall x_1, x_2, \dots, x_n \in R(u_1, u_2, \dots, u_n) \Rightarrow R(x_1, x_2, \dots, x_n) \subseteq R(u_1, u_2, \dots, u_n)$.

Also discuss some examples of n -ary transit functions on simple graphs and study the underlying hypergraphs of n -ary transit functions.

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Domination Parameters of Circulant Graphs

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Let Γ be a finite group with e as the identity. A generating set of the group Γ is a subset A such that every element of Γ can be expressed as the product of finitely many elements of A . Assume that $e \notin A$ and $a \in A$ implies $a^{-1} \in A$. The Cayley graph $G = (V, E)$, where $V(G) = \Gamma$ and $E(G) = \{(x, y)_a | x, y \in V(G), \text{ there exists } a \in A \text{ such that } y = xa\}$ and it is denoted by $\text{Cay}(\Gamma, A)$. A Cayley graph on a finite cyclic group is called a circulant graph. Suppose G is a graph, the open neighbourhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of vertices adjacent to v . The closed neighbourhood of v is $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood $N(S)$ is defined to be $\cup_{v \in S} N(v)$ and the closed neighbourhood of S is $N[S] = N(S) \cup S$. A set $S \subseteq V$ of vertices in a graph $G = (V, E)$ is called a *dominating set* if every vertex $v \in V$ is either an element of S or adjacent to an element of S [1]. A dominating set S is a *minimal dominating set* if no proper subset is a dominating set. The *domination number* $\gamma(G)$ of a graph G is the minimum cardinality of all dominating sets in G [1] and the corresponding dominating set is called a γ -set. One can refer [1] for definitions of other domination parameters. Let n is a fixed positive integer, $Z_n = \{0, 1, 2, \dots, n-1\}$ and $G = \text{Cay}(Z_n, A)$, where $A = \{1, n-1, 2, n-2, \dots, k, n-k\}$ where $1 \leq k \leq \frac{n-1}{2}$. In this paper, we obtain the value of domination number, total and connected domination numbers of $G = \text{Cay}(Z_n, A)$. Also we describe under what conditions subgroups of Z_n become efficient dominating sets in of $G = \text{Cay}(Z_n, A)$.

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Integral Trees of Arbitrarily Large Diameter

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In this poster we give the construction of trees having only integer eigenvalues with arbitrarily large diameters. In fact, we show that for every finite set S of positive integers there exists a tree whose positive eigenvalues are exactly the elements of S . If the set S is different from the set $\{1\}$ then the constructed tree will have diameter $2|S|$.



Forbidden-Minor Characterization for the Class of Graphic Element Splitting Matroids

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This paper is based on the element splitting operation for binary matroids that was introduced by Azadi as a natural generalization of the corresponding operation in graphs. In this paper, we consider the problem of determining precisely which graphic matroids M have the property that the element splitting operation, by every pair of elements on M yields a graphic matroid. This problem is solved by proving that there is exactly one minor-minimal matroid that does not have this property.

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Dynamics of Spanning Tree Graph Operator

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 05C05, 05C76, 37E15, 37E25, 94C15

Given a connected graph G not necessarily finite, denote by V the family of all the spanning trees of G . Define an adjacency relation in V as follows. The spanning trees T_1, T_2 are said to be adjacent if T_1, T_2 differ by a single edge. The resultant graph is called the **Spanning Tree Graph** of G and is denoted by $T(G)$ (see[1]). The operator T is called the spanning tree graph operator. The purpose of this paper is to study the dynamics of this operator such as T -Convergence, T -Divergence, T -Depth and T -Root using the T -Operator.

We prove that

1. For a connected graph G not necessarily finite, $T(G)$ is connected if and only if any two spanning trees of G differ by at most a finite number of edges.
2. A connected graph G is T -convergent if and only if G has no cycle or G has exactly one cycle and its length is 3.
3. The T -depth of any finite connected graph is finite and no infinite connected graph has a T -root.
4. For any graph G which has a root there are infinitely many roots under the T -operator.

The T -Root problem is still not completely solved but we obtain some partial results on that also.

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Forbidden Configuration Characterization for Interval Digraphs/Bigraphs

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Interval digraph $D(V,E)$ is a directed graph for which to each vertex v , we can assign an ordered pair (S_v, T_v) of interval such that uv is an edge iff S_u intersects T_v . *Interval bigraph* is a bipartite graph where to each vertex we can assign an interval so that vertices in opposite partite sets intersect. These two concepts are equivalent [2,4]. *Ferrers digraphs* are digraphs satisfying any of the conditions.

- i) Successor sets (or, predecessor sets) are linearly ordered by inclusion.
- ii) The rows and columns can be permuted (independently) so that the 1s cluster in the upper right (or, lower left) as Ferrers diagram.
- iii) The adjacency matrix has no 2-by-2 permutation matrix as a submatrix.

The following theorem characterizes interval digraphs. Theorem (Sen, Das, Roy and West [3]) The following conditions are equivalent

- i) D is an interval digraph.
- ii) The rows and columns of $A(D)$ can be permuted (independently) so that each zero can be replaced by one of $\{R, C\}$ in such a way that every R has only R 's to its right and every C has only C below it.
- iii) D is the intersection of two Ferrers digraph whose union is complete.

We discuss interval (di/bi)graphs wholly in terms of binary adjacency matrices, and call such a matrix an interval matrix. Das and Sen [1] obtained partial results on forbidden configuration for interval matrices. Our aim in this paper is characterization of interval matrix in terms of forbidden configurations.

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On the Coloring of Steiner Triple System

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(Joint work with S. Akbari, R. Manaviyat and M.J. Nikmehr)

A *Steiner triple system* of order v , $\text{STS}(v)$, is an ordered pair $S = (V, T)$, where V is a set of size v and T is a collection of triples of V such that every pair of V is contained in exactly one triple of T . A k -coloring of (V, T) is a function $c : V \rightarrow \{1, \dots, k\}$ such that $|\{c(x), c(y), c(z)\}| \geq 2$, for every triple $\{x, y, z\}$ of T . A k -coloring of a $\text{STS}(v)$ is called a *balanced coloring* if the size of color classes are the same. A Steiner triple system, S , is called k -chromatic if it admits a k -coloring but not a $(k - 1)$ -coloring. In this talk, we show that if there exists a balanced k -chromatic $\text{STS}(v)$, then for every natural number w , $w \equiv 1$ or $3 \pmod{6}$, there exists a balanced k -chromatic $\text{STS}(vw)$. Moreover, it is shown that if there is a k -chromatic $\text{STS}(v)$ and v is divisible by a prime number less than $k + 1$, then there exists a k -chromatic $\text{STS}(2v + 1)$.

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Graphs with Many ± 1 or $\pm\sqrt{2}$ Eigenvalues

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A pseudo (v, k, λ) -design is a pair (X, \mathcal{B}) where X is a v -set and $\mathcal{B} = \{B_1, \dots, B_{v-1}\}$ is a collection of k -subsets (blocks) of X such that each two distinct B_i, B_j intersect in λ elements; and $0 < \lambda < k < v - 1$. We use the notion of pseudo designs to characterize graphs of order n whose spectrum contains either ± 1 or $\pm\sqrt{2}$ with multiplicity $(n - 2)/2$ or $(n - 3)/2$. It turns out that the subdivision of the star $K_{1,k}$ is determined by its spectrum if $k \notin \{\ell^2 - 1 \mid \ell \in \mathbb{N}\} \cup \{\ell^2 - \ell \mid \ell \in \mathbb{N}\}$. Meanwhile, partial results confirming a conjecture of O. Marrero on characterization of pseudo (v, k, λ) -designs are obtained.

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Adjacency Matrices of Probe Interval Graphs

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We obtain several characterizations of the adjacency matrix of a probe interval graph. In course of this study we describe an easy method of obtaining interval representation of an interval bigraph from its adjacency matrix. Finally, we note that if we add a loop at every probe vertex of a probe interval graph, then the Ferrers dimension of the corresponding symmetric bipartite graph is at most 3.

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The Ekeland Variational Principle For Set-valued Maps Involving Coderivatives

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The well-known Ekeland Variational Principle ([1]) says roughly that for any lower semicontinuous function f bounded from below on a complete metric space X , there exists an approximate minimizer of f which is an exact minimizer of a perturbed function. When X is a Banach space and f is Gâteaux differentiable, its derivative can be made arbitrarily small. Moreover, if f satisfies the Palais-Smale condition then it attains a minimum on X .

Our aim is to extend the above results to the case of a set-valued map F which is defined on a Banach space and takes values in a partially ordered Banach space. We obtain ([2] and [3]) several variants of the variational principle for F involving its coderivative in the senses of Ioffe, Clarke and Mordukhovich, and establish sufficient conditions for F to have a weak minimizer, a properly positive minimizer, a Henig proper minimizer and a superminimizer under Palais-Smale type conditions.

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Algorithm for Finding the Coefficients of Rook Polynomials

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In solving problems on finding the number of arrangements of n objects with restriction, it is usually required to find the coefficients of certain polynomials called rook polynomials. See [1], [2], and [3] for topics on rook polynomials. It is not difficult to find these coefficients when n is not large. However, when n becomes larger the calculations becomes laboring and less practical. In this paper we propose a simple algorithm for finding these coefficients. Also, this algorithm can be modified for more general problems on rook polynomials

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On Ordinary Generalized Geometric-arithmetic Index

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Topological indices are numerical parameters of a graph which invariant under graph isomorphism. For a graph G , let $V(G)$ and $E(G)$ be the vertex-set and edge-set of G respectively. Vukicevic and Furtula in [2] introduced the geometric-arithmetic index, GA, of G as follows:

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{(d_u + d_v)},$$

where d_u denotes the degree of vertex $u \in E(G)$. A class of geometricarithmic topological indices is defined [1]

$$GA_{general}(G) = \sum_{uv \in E(G)} \frac{\sqrt{Q_u Q_v}}{\frac{1}{2}(Q_u + Q_v)},$$

where Q_u is some quantity that in a unique manner can be associated with the vertex u of the graph G . Some of this class GA indices have been studied in [1, 3]. It is natural that we consider the ordinary geometric-arithmetic index of G . To this purpose for each positive real number k we define

$$OGA_k = \sum_{uv \in E(G)} \left[\frac{\sqrt{2d_u d_v}}{d_u + d_v} \right]^k = \sum_{uv \in E(G)} \frac{(4d_u d_v)^{\frac{k}{2}}}{(d_u + d_v)^k}.$$

In this paper we obtain some results related to this index, especially we obtain lower and upper bound in terms of other graph invariants and topological indices. Also, we obtain this index for some nanotubes and nanotorus.

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No Equal Length Cycles Problem of Erdos

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Let $f(n)$ be the maximum number of edges in a graph on n vertices in which no two cycles have the same length. In 1975, Erdős raised the problem of determining $f(n)$ (see [1], p.247, Problem 11). Shi [7] proved that $f(n) \geq n + [(\sqrt{8n - 23} + 1)/2]$ for $n \geq 3$. Boros, Caro, Füredi and Yuster [2] proved that $f(n) \leq n + 1.98\sqrt{n}(1 + o(1))$. Lai [6] proved that $f(n) \geq n + \sqrt{2.4}\sqrt{n}(1 - o(1))$

and make the following conjecture: $\lim_{n \rightarrow \infty} \frac{f(n)-n}{\sqrt{n}} = \sqrt{2.4}$. We think it is difficult to prove this conjecture. It would be nice to prove the following sixteen years old conjecture [5]: $\liminf_{n \rightarrow \infty} \frac{f(n)-n}{\sqrt{n}} \leq \sqrt{3}$. Let $f_2(n)$ be the maximum number of edges in a 2-connected graph on n vertices in which no two cycles have the same length. Shi [7] proved that $f_2(n) \leq n + [\frac{1}{2}(\sqrt{8n-15} - 3)]$, for $n \geq 3$. Chen, Lehel, Jacobson, and Shreve [3] proved that $f_2(n) \geq n + \sqrt{n/2} - o(\sqrt{n})$. Boros, Caro, Füredi and Yuster [2] improved this lower bound significantly: $f_2(n) \geq n + \sqrt{n} - O(n^{\frac{9}{20}})$ and make the following conjecture: $\lim \frac{f_2(n)-n}{\sqrt{n}} = 1$. Markström [4] raised the problem of determining the maximum number of edges in a hamiltonian graph on n vertices with no repeated cycle lengths.

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Acyclic List Edge Coloring of Planar Graphs without Short Cycles

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A proper edge coloring of a graph is said to be *acyclic* if any cycle is colored with at least three colors. The *acyclic chromatic index*, denoted $a'(G)$, is the least number of colors required for an acyclic edge coloring of G . An *edge-list* L of a graph G is a mapping that assigns a finite set of positive integers to each edge of G . An acyclic edge coloring ϕ of G such that $\phi(e) \in L(e)$ for any $e \in E(G)$ is called an *acyclic L -edge coloring* of G . A graph G is said to be *acyclically k -edge choosable* if it has an acyclic L -edge coloring for any edge-list L that satisfies $|L(e)| \geq k$ for each edge e . The *acyclic list chromatic index* is the least integer k such that G is acyclically k -edge choosable.

In [1, 2, 3], upper bounds for the acyclic chromatic indexes of several classes of planar graphs without short cycles were obtained. Let the *girth* of a graph be the shortest length of a cycle in that graph. We establish various upper bounds for the acyclic list chromatic indexes of planar graphs with girth at least 4, 5, 6, or 16, respectively.

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On b -perfect Graphs and b -chromatic Number of Regular Graphs

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The b -chromatic number $\varphi(G)$ of a graph G is the largest integer k such that G admits a proper k -coloring in which every color class contains at least one vertex adjacent to some vertex in all the other color classes [3]. A graph G is b -perfect [1] if every induced subgraph H of G satisfies $\varphi(H) = \chi(H)$, where $\chi(H)$ denotes the chromatic number of H . In [2] a collection of twenty two minimally b -imperfect graphs were given and it was conjectured that these are the only ones. In this paper, we characterize b -perfect distance hereditary graphs and observe that the above conjecture is true for this class.

The b -chromatic number of a d -regular graph G lies in the interval $2 \leq \varphi(G) \leq d + 1$. In [5] and [6] classes of d -regular graphs for which $\varphi(G) = d + 1$

are obtained and from these results it follows that there are only a finite number of d -regular graphs with $\varphi(G) \leq d$. In [4], it is proved that except for 4 graphs, $\varphi(G) = 4$ for every cubic graph G . In the last section, we discuss the possible values of $k \leq d$ for which there exists a d -regular graph with $\varphi(G) = k$.

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Controllability of Nonlinear Fractional Integrodifferential Systems

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In this paper we establish a set of sufficient conditions for the controllability of nonlinear fractional integrodifferential systems of the form

$$\begin{aligned} D^q x(t) &= Ax(t) + Bu(t) + f(t, x(t), \int_0^t g(t, s, x(s)) ds, u(t)), \quad t \in J = [0, b], \\ x(0) &= x_0, \end{aligned}$$

where $0 < q < 1$, $x \in R^n$, $u \in R^m$, A is an $n \times n$ matrix, B is an $n \times m$ matrix and $g : J \times J \times R^n \rightarrow R^n$ and $f : J \times R^{2n} \times R^m \rightarrow R^n$ are continuous functions. The results are obtained by using the recent formula for solution representation of system of fractional differential equations [1, 2, 4] and the application of the Schauder fixed point theorem [3]. Examples are provided to illustrate the results.

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Acyclic List Edge Coloring of Graphs

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 05C15

A proper edge coloring of a graph is said to be *acyclic* if any cycle is colored with at least three colors. An *edge-list* L of a graph G is a mapping that assigns a finite set of positive integers to each edge of G . An acyclic edge coloring ϕ of G such that $\phi(e) \in L(e)$ for any $e \in E(G)$ is called an *acyclic L -edge coloring* of G . A graph G is said to be *acyclically k -edge choosable* if it has an acyclic L -edge coloring for any edge-list L that satisfies $|L(e)| \geq k$ for each edge e . The *acyclic list chromatic index* is the least integer k such that G is acyclically k -edge choosable. We develop techniques to obtain bounds for the acyclic list chromatic indexes of outerplanar graphs, subcubic graphs, and subdivisions of Halin graphs.



On a Principal Impossibility to Prove P=NP

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The Cook problem, notably $P=NP?$, was raised years ago. At the same time in graph theory a problem on the duality between vertex and edge graphs exists [1]. A Church-Turing thesis on the principle normalization of any algorithm exists too [2]. In addition we know that a directed graph can serve as a pictorial representation of any algorithm. Unfortunately all attempts to reach the solution of “Graph isomorphism’s Problem” in the statement of searching for the effective algorithm for the proving of the equivalence between the vertex graphs [3] still are fruitless. Can we find the equivalent conversion of the vertex graph into the edge graph which will considerably differ from the exhaustion method? Yes, the problem is solved and published [4]. The justification of the application of this solution to the theory of algorithms is also proved.

Still at the graph’s converting a problem between the strict duality and semi-duality arises. Generally the converting of the vertex graph into the edge graph not always happens to the equivalence of the cyclomatic numbers. But for some mass problems the strict duality is necessary. The analysis of the converting process unequivocally reduces us to the fact that generally the graphs can be divided into three classes. The strict duality exists only for the graphs without the contours and some special intervals (holonomic graphs) [4]. The strict duality does not exist in principal for the graphs with the contours (progressive-heteronomous graphs). The graphs without the contours, but with some special intervals, can be reduced to the form which possesses the property of the strict duality (bounded-heteronomous graphs). It is reasonable to make a suggestion that any mass problem against the task’s graph also could be divided into three classes of tasks, possessing different properties. And the solution of $P=NP?$ problem will be different for the different classes inside the concrete mass problem. But we’ll never succeed in proving $P=NP$.

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A Dual to Erdős-Ko-Rado Theorem(?)

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 05E30

Erdős-Ko-Rado Theorem states the following: Let d, n be positive integers such that $2d < n$. Let X be a set with n elements. The size of any intersecting family of d -subsets in X is $\leq \binom{n-1}{d-1}$ and it is equal to $\binom{n-1}{d-1}$ if and only if the family consists of all d -subsets containing a fixed element.

Our theorem states the following: Let d, n be positive integers such that $2d < n$. Let $X = \{a_1, \dots, a_n\}$ where a_1, \dots, a_n are real numbers and $a_1 + a_2 + \dots + a_n \geq 0$. A d -subset of X is nonnegative if the sum of all the elements in A is ≥ 0 . Then any family consisting of nonnegative d -subsets in X must have size $\geq \binom{n-1}{d-1}$ and it is equal to $\binom{n-1}{d-1}$ if and only if the family consists of all d -subsets containing a fixed element. Is this a dual to E-K-R theorem?



P_h -(super)magic Labelings of C_n^{+p}

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Let P_h be a *path* on h vertices. A graph $G = (V, E)$ admits a P_h -*covering* if every edge in E belongs to a subgraph of G that is isomorphic to P_h . That G is called P_h -*magic* if there is a total labeling $f : V \cup E \rightarrow \{1, 2, \dots, |V| + |E|\}$ such that for each subgraph $H = (V', E')$ of G that is isomorphic to P_h , $\sum_{v \in V'} f(v) + \sum_{e \in E'} f(e)$ is constant. Additionally, G is called P_h -*supermagic* if $f(V) = \{1, 2, \dots, |V|\}$.

The P_h -(super)magic labelings was first studied by Gutiérrez et.al [2] in 2005. They gave P_h -(super)magic labelings of P_n and a cycle C_n . In [3] Maryati

et.al provided P_h -(super)magic labelings of some trees, namely, shrubs and banana trees. For further information of the (super)magic labelings, see [1] and [4].

We study the P_h -(super)magic labelings of some graphs constructed from C_n by adding a number of pendant edges p , denoted by C_n^{+p} . We characterize the P_h -(super)magicness of this graph. We give the necessary condition for a graph which contain a cycle with a pendant being P_h -(super)magic for a fixed h . We also consider the P_h -(super)magic labeling for C_n^{+p} . We obtain the following results.

Theorem 1. If G contains C_n^{+1} , for $n \geq 3$ then G is not P_{n+1} -magic.

Theorem 2. If G contains C_n^{+1} , for $n \geq 4$ then G is not P_{n-1} -magic.

Theorem 3. Let $n \geq 3$ and $3 \leq p \leq n$. For odd n or even n with $p < n$, C_n^{+p} is P_{n+2} -supermagic.

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Principle of Inclusion-Exclusion for Finite Fuzzy Subsets

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The idea of the principle of Inclusion-Exclusion is a counting technique to count the elements of a specified subset of a finite non-empty set X satisfying a finite collection of dichotomous properties which are not necessarily mutually exclusive. see for instance [1].

In this talk [2], we propose several settings of that principle in the case of finite fuzzy subsets of X taking degrees of membership values in the unit interval \mathbf{I} ; that is, a finite set X on which several properties are defined, each

property may or may not be a dichotomous property or in other words, each may be crisp, fuzzy, vague or linguistically defined. The ideas are all based on the α -cuts of fuzzy subsets and some equivalence relations on the fuzzy subsets. We illustrate the ideas with some applications to problems encountered in day-to-day shopping.

We also briefly discuss the associated concept of Möbius functions and the Möbius inversion formula. The results are published in [3].

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On the Clique and Chromatic Number of the Annihilating-Ideal Graph

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 05C15, 05C69, 13E05, 13E10, 16P60.

Let R be a commutative ring and $\mathbb{A}(R)$ be the set of ideals with non-zero annihilator. The annihilating-ideal graph of R is defined as the graph $\mathbb{AG}(R)$ with vertex set $\mathbb{A}(R)^* = \mathbb{A}(R) \setminus \{(0)\}$ and two distinct vertices I and J are adjacent if and only if $IJ = (0)$. In this paper, we study some connections between the graph theoretic properties of this graph and some algebraic properties of the commutative rings. We investigate commutative rings whose annihilating-ideal graphs are a complete graph. Also, we present some results on the clique number and the chromatic number of the annihilating-ideal graph of a commutative ring.

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The Total Vertex Irregularity Strength of an Amalgamation of Stars

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For a simple graph G with the vertex set V and the edge set E , a labeling $\lambda : V \cup E \rightarrow \{1, 2, \dots, k\}$ is called a vertex irregular total k -labeling of G if for any two different vertices x and y in V we have $wt(x) \neq wt(y)$ where $wt(x) = \lambda(x) + \sum_{xz \in E} \lambda(xz)$. The total vertex irregularity strength of G , denoted by $tvs(G)$, is the smallest positive integer k for which G has a vertex irregular total k -labelling. In this paper, we determined the total vertex irregularity strength of an amalgamation of stars.



A Different Approach to Hadamard's Maximum Determinant Problem: Update

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 05B20

For a positive integer n , consider the finite set D_n given by the set of absolute values of the determinant function over the set of all real n -by- n matrices whose coefficients are either 0 or 1. D_n is called the determinant spectrum for order n (0,1) matrices.

At the 1998 ICM in Berlin, it was suggested [3] that approximating D_n could solve the Hadamard maximum determinant problem. Finding D_n exactly is called the determinant spectrum problem; it was studied in [1] and has appeared in more recent research, cf. [2] and [4]. We report on the progress made on the spectrum problem as well as its application to Hadamard's problem.

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Minimizing Laplacian Spectral Radius of Unicyclic Graphs with Fixed Girth

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Let G be a finite simple graph and $L(G)$ be its Laplacian matrix [2]. The largest eigenvalue of $L(G)$ is called the Laplacian spectral radius of G . Let $\mathcal{U}_{n,g}$ be the class of all unicyclic graphs on n vertices with fixed girth g . In [1], the author has determined the graph which maximizes the Laplacian spectral radius over the class $\mathcal{U}_{n,g}$. In this work, we have studied the minimum Laplacian spectral radius over the class $\mathcal{U}_{n,g}$. Consider a cycle on g vertices and append a pendent vertex of the path on $n - g$ vertices to one of the vertices of the cycle. The new graph is a unicyclic graph on n vertices with girth g and we denote it by $C_{n,g}$. We have studied the eigenvectors corresponding to the Laplacian spectral radius of $C_{n,g}$. We show that when $g = 3$ or 4 , $C_{n,g}$ has the minimum Laplacian spectral radius over the class $\mathcal{U}_{n,g}$. We prove that the same result is true when n is large with respect to fixed girth g . However, this need not be true when n is small comparing to large girth g .

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Characterization of Quasi-symmetric Designs in Terms of Residuals of Biplanes

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Let X be a finite set of v elements, called points and β be a finite family of distinct k -subsets of X , called blocks. Then the pair $D = (X, \beta)$ is called a block design (or 2-design) with parameters (v, b, r, k, λ) ($v > k \geq 3$) if $|X| = v$, $|\beta| = b$, each block contains k points, each point occurs in r blocks. each pair of points occurs in λ blocks.

For $0 \leq x < k$, x is called an intersection number of D if there exists $B, B' \in \beta$ such that $|B \cap B'| = x$. A 2-design with two intersection numbers is said to be quasi-symmetric design. We denote these intersection numbers by x and y , and assume $0 \leq x < y < k$. We consider the proper quasi-symmetric designs i.e., both the intersection numbers occur.

Symmetric designs are 2-designs with $b = v$, equivalently $r = k$. It is well known that any two distinct blocks of symmetric design intersects in λ points. Block residual of a symmetric design is a design with parameters $(v - k, v -$

$1, k, k-\lambda, \lambda$) obtained by removing a block B and all points in B from remaining blocks. In view of the parameters of a block residual, 2-design satisfying $r = k + \lambda$ is said to be quasi-residual. Quasi-residual design is called residual or embeddable if it actually is the block residual of a symmetric 2-design. Quasi-residual design with $\lambda = 2$ has parameters $v = k(k+1)/2, b = (k+1)(k+2)/2, r = k+2$ and $x = 1, y = 2$. Hall and Connor proved that a quasi-residual 2-design with $\lambda = 2$ is residual. Symmetric design with $\lambda = 2$ is called biplane. Biplane with parameters $v = (m+3)(m+2)/2 + 1, k = m+3, \lambda = 2$ are known to exist only when $m = 1, 2, 3, 6, 8, 10$ and it is an open question if there are other values of m for which a biplane exists. In view of this the following result is proved.

Let D be a quasi-symmetric design with intersection numbers x, y and $y - x = 1$. Then D is a design with parameters $v = (1+m)(2+m)/2, b = (2+m)(3+m)/2, r = m+3, k = m+1, \lambda = 2, x = 1, y = 2$ and $m = 2, 3, \dots$ or complement of one of these design or D is a design with parameters $v = 5, b = 10, r = 6, k = 3, \lambda = 3$ and $x = 1, y = 2$.

Classification of quasi-symmetric designs with $y - x = 2$ will be discussed.

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Sum-integral Interpolators and the Euler-Maclaurin Formula for Polytopes

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Given a polytope P in a vector space V equipped with a lattice Λ and a function f on V , can one relate the sum of f over the lattice points in P to the integral of f over P ? One would expect a main term, the integral of f over P , as well as correction terms involving the integrals of f over the proper faces of P .

If f is a constant function, one wishes to express the number of lattice points in P in terms of volume of P and the volumes of the faces of P . One hopes for a local formula, meaning that for each face F of P , the formula contains a term that is the volume $\text{vol}(F)$ multiplied by a coefficient $\mu(\text{Supp}(P, F))$ that depends only on the supporting cone $\text{Supp}(P, F)$ to P along F . McMullen [McM] proved the existence of local lattice point counting formulas in a nonconstructive way.

The second author and Thomas [PT] gave an explicit construction of a rational valued function μ , given a fixed *complement map* on V .

Berline and Vergne constructed in [BV] an explicit local Euler-MacLaurin formula for the sum of a polynomial function f over the lattice points $P \cap \Lambda$ of a rational polytope. This construction is local and requires an inner product on the vector space V . The formula results from a relationship between the integral and the sum of an exponential function over a polytope.

In this work, we introduce the concept of an *interpolator* between the families of exponential sums (S) and exponential integrals (I) over rational polytopes in a rational vector space V . Our main result states that a complement map on the vector space V gives rise in a natural way to an effectively computable SI-interpolator on V , and hence a local Euler-Maclaurin formula, an IS-interpolator (and a reverse local Euler-Maclaurin formula) and an IS⁰-interpolator. In particular, an inner product on V or a complete flag in V gives rise to a local Euler-Maclaurin formula. These formulas generalize the work of Berline-Vergne, Pommersheim-Thomas, and Morelli.

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Root Graphs of Anti-Gallai Graphs

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The anti-Gallai graph $\Delta(G)$ of a graph G has the edges of G as its vertices and any two vertices of G are adjacent in $\Delta(G)$ if the corresponding edges of G lie on a triangle in G and hence is a spanning subgraph of the line graph of G .

The study on the structure of the anti-Gallai graph is well motivated in [2] where it is shown that the four color theorem can be equivalently stated in terms of anti-Gallai graphs.

A root graph of an anti-Gallai graph H is a graph G such that $H \cong \Delta(G)$. In [1] it is shown that there are infinitely many anti-Gallai graphs with more than one root graph. In [3] an algorithm is provided to partition the edge set of a line graph to the edges of Gallai and anti-Gallai graphs of its root graph.

In this paper we introduce an algorithm to recognize and hence find all the root graphs of anti-Gallai graph.

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Semi-complete Graph and some of its Properties

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A semi-complete graph, a weaker graph than a complete graph is introduced and its properties are studied. These graphs are very useful in safety problems of defence and banking.

Definition: A graph is said to be semi-complete iff it is simple and for any two of its vertices there exists a third vertex with which these two vertices are adjacent (in the graph).

To avoid trivialities, we consider a non-trivial graph with at least three vertices.

Observations: (i) Any semi-complete graph is a non empty graph (i.e it has edges) (ii) Any complete graph is semi-complete but the converse is false. (iii) Any semi-complete graph is connected and hence contains a spanning tree. (iv) Any semi-complete graph contains K_3 and hence it is not a tree and not bipartite. Its girth is at least 3 and is 3 colourable.

Main Results:

- (i) The Harary graph $H_{m,n}$ is semi-complete iff either
 - (a) m is even and $4 \leq m < n \leq 2m + 1$ (or)
 - (b) m is odd, $3 \leq m < n \leq 2m$.
- (ii) A simple graph G with atleast three vertices is semi-complete iff any two vertices lie on the same K_3 or on different K_3 having a common vertex in G .
- (iii) If a semi-complete graph is such that its edge set is a union of triangles such that no two triangles have a common edge then all the triangles have a common vertex.

Further concepts like weak semi-complete, strong semi-complete, super strong semi-complete graphs are introduced.

Characterization theorems for semi-complete graph to be (i) weak semi-complete (ii) strong semi-complete (iii) super strong semi-complete are obtained.

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**Acyclic Choosability of Planar Graphs**

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Let $L = \{L(v), v \in V(G)\}$ be a list assignment of a graph G . The graph G is acyclically L -choosable if there is a proper coloring c of the vertices of G such that $c(v) \in L(v)$ for any $v \in V(G)$, and G does not contain any bicolored cycle. A graph G is acyclically k -choosable if it is acyclically L -choosable for any list assignment L with $|L(v)| = k$ for all $v \in V(G)$. In this short communication we will survey recent results on acyclic 4-choosability of planar graphs with forbidden cycles. In particular, we prove [4] that every planar graph without $\{4, i, j\}$ -cycles, for $5 \leq i \leq j \leq 8$, is acyclically 4-choosable.

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Graph Reductions Using the 4-polygon to 4-star Transformation

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A graph G is an ordered pair (V, E) , where V is a some set and E is a set of 2-point subsets of V . The elements of the set V are called vertices of the graph G and the elements of E edges of G .

A type of transformation on a graph, which leaves invariant the number of edges of the graph is known as a ΔY transformation. A $\Delta \rightarrow Y$ transformation, is the deletion of three edges of a triangle with vertices x, y, z , and the addition of a new vertex w together with three new edges wx, wy, wz . A $Y \rightarrow \Delta$ transformation, is the deletion of one vertex w of degree 3 together with its three incident edges wx, wy, wz , and the addition of three edges xy, yz, xz [2, 4].

A graph is ΔY reducible if it can be reduced to only one vertex by means of a sequence of ΔY transformations and series-parallel reductions. It is known that every plane graph is ΔY reducible [1, 2]. In this work we generalize the definition of $\Delta \rightarrow Y$ transformation in a graph to that of n -polygon $\rightarrow n$ -star transformation, and explore the case $n = 4$.

It is said that a graph is 4-polygon reducible if it can be reduced to only one vertex by means of a sequence of 4-polygon \rightarrow 4-star transformations and series-parallel reductions. We present some families of graphs which are 4-polygon reducible, one of them is the family of finite grids. Among the families of graphs with chords which will be shown to be 4-polygon reducible are wheels and complete graphs. The complete graphs K_n with $n \geq 6$ are not ΔY reducible, but they are 4-polygon reducible. In this talk, we also exhibit families of quasi-cubic graphs which are not 4-polygon reducible, and non-polygon reducible cubic graphs that are minimal with respect to the number of vertices.

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On Certain Types of Graphs

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Graphs that play an important role in artificial intelligence are introduced and their useful properties are obtained. The elementary study is being done by Saradhi [2], we follow the terminology and notation given in Bondy & Murthy [1].

Definition: Let m, n be positive integers and s be a non negative integer $\leq (m - 1)$, then the simple graph G with vertex set $V = V(G) = \{1, 2, \dots, n\}$ and the edge set $E = E(G) = \{(u, v) : \text{where } u, v \in V \text{ with } v \neq u \text{ and } u + v \equiv s \pmod{m}\}$ is called a number theoretic graph. (and is denoted by $G_{m,n}(s)$).

The following are the main results:

Theorem (1): For the (simple) graph $G_{m,n}(s)$ $m, n \geq 3$ and $s = 0, 1, 2, \dots, (m - 1)$ with vertex set $V = V(G) = \{1, 2, \dots, m\}$ and with $V_i = \{v \in V : v \equiv i \pmod{m}\}$ ($i = 0, 1, 2, \dots, m - 1$), the number of edges $\varepsilon(G_{m,n}(s))$ of $(G_{m,n}(s))$ is

(i) $\sum_{i=0}^{\frac{s}{2}-1} |V_i| |V_{s-1-i}| + |V_{\frac{s}{2}}| \binom{|V_{\frac{s}{2}}|-1}{2} + \sum_{i=\frac{s}{2}+1}^{\frac{m}{2}-1} |V_{\frac{s}{2}+i}| |V_{\frac{m+s}{2}-i}| + |V_{\frac{m+s}{2}}| \binom{|V_{\frac{m+s}{2}}|-1}{2}$
if both m and s are even (and a similar expressions when (ii) m is even and s is odd (iii) m is odd and s is even and (iv) m and s are odd.)

Theorem (2): (a) $G_{2,n}(0)$ is bipartite iff $n = 2, 3, 4$ and $G_{2,n}(1)$ is bipartite for all $n \geq 2$. (b) For all $n, m \geq 3$ and $s \in 0, 1, \dots, m - 1$, the graph $G_{m,n}(s)$ is bipartite iff any one of the following holds: (i) m is even and s is odd. (ii) m is even, and $s = 0$ $n \leq (\frac{5m-1}{2})$ (iii) m is odd and $s = 0$ $n \leq 3m-1$ (iv) s assumes even integers from 2 to λ where $\lambda = m-2$ or $m-1$ according as m is even or odd $n \leq \frac{4m+s-2}{2}$ (v) both m and s are odd $n \leq \frac{5m+s-2}{2}$.

Theorem (3): For $n, m \geq 3$ and $s \in \{0, 1, \dots, m - 1\}$, & $n = mq + r$ ($r = 0, 1, \dots, m - 1$), the graph $G_{m,n}(s)$ has a perfect matching iff any one of the following holds: (i) m is even and s is odd and $r = 0$. (ii) m is even and $s = 0$, q is even $r = 0$; (iii) m is odd $s = 0$, q is even $r = 0$ or $m - 1$ (iv) m is even and s is even and $s \in \{2, \dots, m - 2\}$, q is even and $r = 0$ (v) m is odd, s is even and $s \in \{1, 2, \dots, m - 1\}$, either q is even, $r = 0$ or q is odd and $r = s - 1$ (vi) m is odd, s is odd, q is even and $r = 0$.

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On the Number of Inversions in Simply Generated Trees

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Consider a rooted tree T labelled with the integers $1, \dots, n$, where n is the number of nodes of T . An *inversion* in T is a pair of nodes (i, j) , such that $i > j$ and node i lies on the (unique) path from the root to node j .

We study the number of inversions in labelled families of simply generated trees, i.e., families \mathcal{T} which can be described by a formal equation of the form $\mathcal{T} = \circ \times \varphi(\mathcal{T})$, where \circ is a node and $\varphi(\mathcal{T})$ a substituted structure, cf. [4]. For some important families of this type exact results on the number of inversions are known, see, e.g., [3] for inversions in Cayley trees and [2] for inversions in ordered trees and cyclic trees. Furthermore, in [1] the distributional behaviour of the number of inversions in Cayley trees has been studied via relations to cost measures in a linear probing hashing algorithm.

We extend the existing work by a study of the limiting distribution behaviour of the number of inversions in arbitrary simply generated tree families. For a randomly chosen tree T of size n , we analyze both the total number X_n of inversions in T and the number $Y_{n,j}$ of inversions *generated by node j* , i.e., the number of inversions of the type (i, j) , $i > j$.

We can show that after proper normalization X_n is asymptotically Airy distributed, whereas $Y_{n,j}$ follows asymptotically a Rayleigh distribution. We can even characterize the behaviour of $Y_{n,j}$ if we let $j = j(n)$ grow with n . In this case we obtain three different limit laws depending on the order of growth of j .

Our considerations are not limited to simply generated trees, generalizations to other combinatorial structures, e.g., to so-called k -trees (or k -dimensional trees), are possible.

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Minimal Prime Ideals and Cycles in Annihilating-Ideal Graphs

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Let R be a commutative ring with identity and $\mathbb{A}(R)$ be the set of ideals with non-zero annihilator. The annihilating-ideal graph of R is defined as the graph $\mathbb{AG}(R)$ with the vertex set $\mathbb{A}(R)^* = \mathbb{A}(R) \setminus \{0\}$ and two distinct vertices I and J are adjacent if and only if $IJ = 0$. In this paper, we study some connections between the graph theoretic properties of this graph and some algebraic properties of a commutative ring. We prove that if $\mathbb{AG}(R)$ is a tree, then either $\mathbb{AG}(R)$ is a star graph or a path of order 4 and in the latter case $R \cong F \times S$, where F is a field and S is a ring with exactly one non-trivial ideal. Moreover; we prove that if R has at least three minimal prime ideals, then $\mathbb{AG}(R)$ is not a tree. It is shown that for every reduced ring R , if R has at least three minimal prime ideals, then $\mathbb{AG}(R)$ contains a triangle. Finally, it is proved that, if $|\text{Min}(R)| = 1$ and $\mathbb{AG}(R)$ is a bipartite graph, then $\mathbb{AG}(R)$ is a star graph.

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Polyhedral Approach to Integer Partitions

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We develop the polyhedral approach to integer partitions [3]. Its main idea is to study the set of partitions of any positive integer n as a polytope $P_n \subset R^n$, which is the convex hull of the set of incidence vectors x of all partitions of n . With the use of representation of P_n as a polytope on a partial algebra, we give subadditive characterization of its non-trivial facets. They correspond to extreme rays of the cone of subadditive functions on $\{1, 2, \dots, n\}$ with some additional requirements.

Studying vertices of P_n is the first attempt to reduce the sets of partitions to their subsets. We show that the vertices of all partition polytopes form a partition ideal of the Andrews partition lattice [1] and propose a lifting method for constructing them. Of special importance is the criterion of whether a given partition x is a convex combination of two others: this is true iff there exist two equal sum collections of parts of x . The criterion yields necessary conditions for vertices, in particular, exact upper bounds on the numbers of all parts $\lceil n/2 \rceil$ and distinct parts $\lfloor \log(n+1) \rfloor$ of a vertex. It also reveals relations of vertices to sum-free sets, Sidon sets [4], and knapsack partitions [2], the latter being just those partitions that cannot be convexly expressed via two others. We prove that the problems of recognizing knapsack partitions, as well as certain multisets embracing the additive structures mentioned above, are co-NP-complete.

We find that there exists a subset of support vertices of P_n , from which all other vertices can be recursively generated using two operations of merging parts of partitions. Starting from any vertex, one can also use these operations to build sequences of vertices generating complete subgraphs of the partition polytope graph. Numerical data testify to the number of support vertices being considerably less than the number of vertices, which, in its turn, is much less than the total number of partitions.

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\mathcal{C} -Consistency of Signed Line Structures

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A *signed graph* (or *sigraph* in short) is an ordered pair $S = (S^u, \sigma)$, where S^u is a graph $G = (V, E)$, called the *underlying graph* of S and $\sigma : E \rightarrow \{+, -\}$ is a function from the edge set E of S^u into the set $\{+, -\}$, called the *signature* of S . For a sigraph S , its *line sigraph* $L(S)$ is a sigraph in which the edges of S are represented as vertices, two of these vertices are defined adjacent whenever the corresponding edges in S have a vertex in common and any such edge ef is defined to be negative whenever both e and f are negative edges in S . The \times -*line sigraph* of S denoted by $L_\times(S)$ is a sigraph defined on the line graph $L(S^u)$ of the graph S^u by assigning to each edge ef of $L(S^u)$, the product of signs of the adjacent edges e and f in S . The *canonical marking* on S is defined as: for each vertex $v \in V(S)$, $\mu(v) = \prod_{e_j \in E_v} \sigma(e_j)$, where E_v is the set of edges e_j incident at v in S . Now, if every vertex of a given sigraph S is canonically marked, then a cycle Z in S is said to be *canonically consistent* (\mathcal{C} -consistent) if it contains an even number of negative vertices and the given sigraph S is said to be \mathcal{C} -consistent if every cycle in it is \mathcal{C} -consistent. In this paper, we obtain a characterization of sigraphs whose line sigraphs and \times -line sigraphs are \mathcal{C} -consistent.

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Gamma - Half Graphs

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We consider only simple graphs. A new domination parameter, namely, λ - domination number $\gamma_\lambda(G)$ of a graph was introduced in [1]. Let λ be such that $0 < \lambda < 1$. Let G be a graph. To each vertex u of G , define a map f_u on $V(G)$ as follows:

$$f_u(v) = \begin{cases} 1 & \text{if } d(u, v) \leq 1 \\ \lambda & \text{if } d(u, v) = 2 \\ 0 & \text{otherwise.} \end{cases}$$

where $d(u, v)$ is the distance between the vertices u and v . A subset D of V is said to be a λ - dominating set of G if for each v in $V(G)$, $\sum_{u \in D} f_u(v) \geq 1$ holds.

The minimum cardinality of a λ - dominating set is called the λ - domination number of G and is denoted by $\gamma_\lambda(G)$. A λ - dominating set with cardinality $\gamma_\lambda(G)$ is said to be a γ_λ - set of G . By taking $\lambda = \frac{1}{2}$, various bounds for $\gamma_{\frac{1}{2}}(G)$ have been obtained in [1]. In this paper also we take $\lambda = \frac{1}{2}$.

Given a graph G , we associate a new graph denoted by $\gamma_{\frac{1}{2}} \cdot G$ and called the $\gamma_{\frac{1}{2}}$ - graph of G . The vertex set of $\gamma_{\frac{1}{2}} \cdot G$ is the set of all $\gamma_{\frac{1}{2}}$ - sets of G . Two $\gamma_{\frac{1}{2}}$ - sets A and B are adjacent in $\gamma_{\frac{1}{2}} \cdot G$ if and only if $|A \cap B| = \gamma_{\frac{1}{2}}(G) - 1$. For various standard graphs G ; $\gamma_{\frac{1}{2}}$ - graphs of G are determined in [2]. A graph G is said to be a $\gamma_{\frac{1}{2}}$ - graph if it is a $\gamma_{\frac{1}{2}}$ - graph of some graph H , in otherwords G is isomorphic to $\gamma_{\frac{1}{2}} \cdot H$ for some graph H . In this paper we initiate a study on $\gamma_{\frac{1}{2}}$ - graphs. We show that not all graphs are $\gamma_{\frac{1}{2}}$ - graphs. Complete bipartite graphs $K_{m,n}$, where $2 \leq m$ and $3 \leq n$ are not $\gamma_{\frac{1}{2}}$ - graphs; Every tree is a

$\gamma_{\frac{1}{2}}$ - graph, every unicyclic graph is a $\gamma_{\frac{1}{2}}$ - graph. We also find various methods to obtain new $\gamma_{\frac{1}{2}}$ - graphs from known $\gamma_{\frac{1}{2}}$ - graphs. We prove that cartesian product of two $\gamma_{\frac{1}{2}}$ - graphs is a $\gamma_{\frac{1}{2}}$ - graph. We establish that every induced subgraph of a $\gamma_{\frac{1}{2}}$ - graph is a $\gamma_{\frac{1}{2}}$ - graph.

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Mod Difference Digraphs

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Mod difference labeling is a compact representation for many digraphs wherein we represent a digraph with n positive integers. A digraph $D = (V, E)$ is a *mod difference digraph* if there exist a labeling L (called *mod difference labeling*) and a positive integer m such that $(x, y) \in E$ if and only if $L(y) - L(x) \equiv L(w) \pmod{m}$ for some $w \in V$. This is an extension of the definition of monographs defined for undirected graphs by Bloom et.al. [1] and difference digraphs defined by S.V. Gervacio [4], made in order to label more classes of digraphs, particularly those with cycles.

In this paper we discuss some properties of mod difference digraphs, labeling of some important classes of digraphs like directed cycles, paths, acyclic directed graphs and tournaments.

In addition to giving a labeling for these classes of digraphs, we characterize the label sets which are often referred to as signatures of digraphs. For example we define a relationship between the power set of a set and signatures of non-isomorphic tournaments.

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An Explicit Acyclic Edge Coloring Algorithm For A Class of Complete Graphs Using Near-One-Factors

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All the graphs considered here are simple and finite. An edge coloring of a graph is proper if no pair of incident edges at any vertex receive the same color. A proper edge coloring of a graph is called acyclic if it does not have any bi-chromatic cycle.

A one-factor of a graph G of even order is a one regular spanning subgraph of G . A one-factorization of a graph G of even order is a partition of the edge set into a set of one-factors. Analogously, a near-one-factor of a graph of order $2n + 1$ is a set of n edges and one vertex (call it isolated vertex) that between them cover all vertices; a near-one-factorization is a set of near-one-factors that contain every edge precisely once. In general, (near-)one-factorization of

a graph may or may not exist. But complete graphs always admit a (near-)one-factorization. It can be easily visualized that the union of two disjoint near-one-factors is always a collection of disjoint cycles of even length and a path connecting the isolated vertices.

In this paper, cycle structure of the union of a pair of near-one-factors is completely determined up to the edge level. These near-one-factors are from the near-one-factorization of a graph of order pq , where p and q are twin primes, produced by the method given in [4]. The knowledge of the cycle structure is then exploited to design an algorithm that uses $pq+2p$ colors. Color assignment, in this algorithm, is carried out by evaluating an expression. This is in contrast to all the existing algorithms in the literature [2, 3] except the one in [1].

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On Characterizations of Strong Posets

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In this paper, we have introduced a concept of strong poset and the forbidden configuration is obtained for strong posets in terms of LU-subsets. We have shown that the concept of strongness, and property being balanced are equivalent in upper semimodular J^* -posets. Also, characterizations of atomistic and dually atomistic posets are obtained. Further, we have shown that for balanced poset of finite length, the length of the subposet of join-irreducible elements equals the length of its subposet of meet-irreducible elements. This result generalizes the results of Ganter and Rival, Reuter and Stern for modular and balanced lattices respectively.

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Zero-Sum Flows in Graphs

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For a graph G , a zero-sum flow is an assignment of non-zero real numbers on the edges of G such that the total sum of the assignments of all edges incident with any vertex of G is zero. A zero-sum k -flow for a graph G is a zero-sum flow with labels from the set $\{\pm 1, \dots, \pm(k-1)\}$. In [1] a necessary and sufficient condition for the existence of zero-sum flow for a graph is given. Also it was

conjectured that if G is a graph with a zero-sum flow, then G has a zero-sum 6-flow. It is shown that the conjecture is true for a 2-edge connected bipartite graph, and also for r -regular graphs with even r . In [2] the authors proved that every $3r$ -regular graph has a zero-sum 5-flow. In this talk we give an affirmative answer to the conjecture except for $r = 5$.

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Section 15

Mathematical Aspects of Computer Science

A Logic Game for classification of Regular Languages

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We present a novel logic game for classification of Regular Languages. We discuss the motivation behind the game and demonstrate how the game can be applied towards a solution to the long-standing generalized star height 2 problem, which can be stated as the following.

Generalized Star Height 2 Problem. Does there exist a Regular language L such that no regular expression of star height ≤ 1 represents L .

There have been similar attempts from Qiqi Yan and Wolfgang Thomas [1]. However, our game is more generalized and the application to the generalized star height 2 problem is more straight forward. In addition, we present a rigid framework for similar logic games to be developed and applied to other fundamental problems related to Regular languages.

It has been a common trend to classify Regular languages by their star height [2]. We see the Star Height 2 Problem as a very basic question about Regular Languages and if we cannot yet answer it, we believe we lack the basic tools for our understanding of Regular Languages at large. Insight into this particular problem is important to solve the even rigorous *star height problem* which deals with the existence of an algorithm to compute the star height of a given regular language.

In this short communication, we introduce our game, demonstrate its applications and introduce a flexible framework on which similar games can be developed in future. We also discuss the close relation of our game to existing

games in logic, such as the FO(TC) game by Erich Grädel [3], for instance. We also discuss the similarities and differences of our game with similar previous attempts [1].

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On Soft Set Relations

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Molodtsov [1] introduced the theory of soft sets as a generalized tool for modeling complex systems involving uncertain or not clearly defined objects. Owing to the fact that many mathematical objects such as fuzzy sets, topological spaces, rough sets [see [1, 2]] can be considered as particular types of soft sets, it is a very general tool for handling objects which are defined in terms of loose or general set of characteristics. A soft set can be considered as an approximate description of an object precisely consists of two parts, namely predicate and approximate value set. Exact solution to the mathematical models constructed are needed in classical mathematics. If the model is so complicated that we cannot set an exact solution, we can go for approximate solution and there are many methods for this. In contrary to this, in soft set theory as the initial description of object itself is of approximate nature, we need not have to introduce the concept of exact solution.

There are many mathematical tools available for modeling complex systems such as probability theory, fuzzy set theory, interval mathematics etc. But there are inherent difficulties associated with each of these techniques. Probability theory is applicable only for a stochastically stable system. Interval mathematics is not sufficiently adaptable for problems with different uncertainties. Setting the membership function value is always been a problem in fuzzy set theory. Moreover all these techniques lack in parametrization of the tools and

hence they could not be applied successfully in tackling problems especially in areas like economic, environmental and social. Soft set theory is standing in a unique way in the sense that it is free from the above difficulties and has a wider scope for many applications in a multidimensional way.

This paper is an attempt to open up the theoretical aspects of soft sets by extending the notions of relations, composition of relations and partitions to the framework of soft sets. Rather than defining relations as subsets of the cartesian product, we also obtain induced relations from universal set and the attribute set. Equivalence relations and partitions on soft sets are defined and a one one relationship between them is also established. Further composition of functions are introduced in soft set context with related results.

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Maximum Flow is Approximable by Deterministic Constant-time Algorithm in Sparse Networks

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We show a deterministic constant-time parallel algorithm for finding an almost maximum flow in multisource-multitarget networks with bounded degrees and bounded edge capacities. As a consequence, we show that the value of the maximum flow over the number of nodes is a testable parameter on these networks.



Various Operators on Rough Fuzzy Groups-An Impact on Information Systems

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In 1982, Z. Pawlak [4] introduced the theory of rough sets and it found wider applications in computing the minimal features in the information systems. In 1988, Dubois and Prade hybridized Rough and Fuzzy approaches [3] which helped the industries to develop the tools on rough approaches in the information systems working under fuzzy environment. Several Researchers including Yao, Slezak, Skowran etc. have been effective in developing various mathematical tools as well as in implementing the concepts into the information systems.

In 2004, G. Ganesan [2] contributed the concept of rough fuzzy groups. In his paper, the construction of the closure axioms are defined in terms of the iterative approaches on fuzzy ordered pairs using max-min and min-max operators. In 2007, G. Ganesan [1] discussed the importance of using max-max, max-min, min-max and min-min operators in the information systems using intuitionistic fuzziness.

In this lecture, the methods of computation of minimal features in the information systems are discussed under the four compound operations mentioned above and the various levels of significance of the constructing rough fuzzy groups using these operations.

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Fourth-order Nonlinearity of Monomial Partial Spread Function on 10-variables

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The r th-order nonlinearity of an n -variable Boolean function f denoted by $nl_r(f)$, is defined as the minimum Hamming distance of f from all n -variable Boolean functions of degrees at most r ($r \geq 1$). In this paper we tighten the lower bounds of fourth-order nonlinearity of monomial Partial Spread Function

on 10-variables. It is also demonstrated that this lower bound is better than the lower bound of inverse function and Dillon bent function obtained by Carlet [1].

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Affine Grassmann Codes

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We consider a new class of linear codes, called affine Grassmann codes. These can be viewed as a variant of generalized Reed-Muller codes and are closely related to Grassmann codes. We determine the length, dimension, and the minimum distance of any affine Grassmann code. Moreover, we show that affine Grassmann codes have a large automorphism group and determine the number of minimum weight codewords. In geometric terms, some of our results could be viewed as a generalization of elementary facts about hyperplanes over finite fields to “determinantal hyperplanes”, that is, hypersurfaces defined by linear combinations of minors (of varying sizes) of a generic matrix. The auxiliary results obtained in the course of proving the main theorems and the techniques employed may also be of some independent interest.

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Application of Non Associative Algebraic Structure: Quasigroup to Cryptography

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The current trend in cryptography is to search for approaches to the cryptographic algorithm design. One such possibility is to use other algebraic structures such as quasigroup rather than the traditional. Even though quasigroups are not in the mainstream of cryptographic research, but there is hardly other single mathematical paradigm to develop cryptographic applications.

Quasigroups are algebraic structures closely related to Latin Squares which have many different applications. This article describes a block cipher based on quasigroup. Almost all ciphers based on quasigroups consider a scenario where the quasigroups(which acts as a part of key) to be used for encryption/decryption are predetermined. The existing algorithm has been modified as the proposed cipher uses two randomly generated quasigroups, a method is illustrated to construct two random Latin Squares which in turn creates a pair of quasigroups of dual operations. The construction of the block cipher is based on quasigroup string transformations. Since quasigroup in general do not have algebraic properties such as being associative, commutative, neutral elements, inverting these functions seems to require exponentially many readings from look up tables that define them in order to check whether the initial conditions are satisfied, thus making them strong candidates for cryptographic functions. According to analysis the method is extremely secure, some theoretical proofs about the cryptographically secure algorithm are presented. Besides that the plaintext and its ciphertext are of same length and the enciphering is of stream nature guarantying a very fast implementation.

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Rough Multiset and its Properties

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Classical set theory implicitly assumes that all mathematical objects occur without repetition in a set. Many fields of modern mathematics have emerged based on this principle. In classical set theory, a set is a well-defined collection of distinct objects. If repeated occurrences of any object is allowed in a set, then a mathematical structure, that is known as multiset (mset, for short), is obtained. In 1986, R R Yager [3] gave a formal definition for the multiset and developed an elementary algebra of multisets. W. D. Blizard [1] provides the relevance and logical implications of multisets in various fields of mathematics and other disciplines.

Rough set theory, proposed by Zdzislaw Pawlak [2] in 1982 can be seen as a mathematical approach to vagueness. Rough set theory is a powerful tool for dealing with the uncertainty, granularity and incompleteness of knowledge in information systems. When there is a huge amount of data, it is very difficult to extract useful information from the information systems. In any information system, some situations may occur, where the respective counts of objects in the universe of discourse are not single. In such situations we have to deal with collections of information in which duplicates are significant. In such cases multisets play an important role in processing the information. The information system dealing with multisets is said to be an information multisystem.

In this paper we provide a new dimension to Pawlak's rough set theory by replacing its universe by multisets. This is called a rough multiset and it is a useful structure in modeling information multisystem. Rough multiset is defined in terms of lower and upper mset approximations, many properties connected to lower mset approximations and uppermset approximations are obtained.

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Tools for Supporting Graphs in Computer Science

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Graphs are used almost everywhere in computer science and are routinely included in many courses in undergraduate and graduate programs. Students, researchers and specialists who need to work with graphs and implement graph algorithms may not have the necessary knowledge to do so. In the paper, we describe the Wiki GRAPP and WEGA systems intended to help in teaching and research in graph theory, graph algorithms and their applications to computer science.

In 1999 our dictionary [1] was published, which covered more than 1500 main graph-related terms from monographs in Russian. It was the first dictionary of graphs in computing and it aroused a great interest of readers. Our new dictionary [2] is an extended dictionary of 1999 and it includes more than 1000 new terms from journal articles whose abstracts were published in Abstract Journal “Mathematics” in section “Graph Theory”, as well as from volumes of annual conferences “Graph-Theoretic Concepts in Computer Science” and book series “Graph Theory Notes of New York”. The Wiki GRAPP system is an on-line “edition” of the dictionaries on the basis of the MediaWiki system. At present it includes definitions of all terms from our first dictionary.

The WEGA system is a Web-Encyclopedia of Graph Algorithms which is based on the book [3]. We use a high-level and language-independent representation of graph algorithms in our book and system. In our view, such an approach allows us to describe algorithms in a form that admits direct analysis

of their correctness and complexity, as well as a simple translation of algorithms to high-level programming languages without disturbance of their correctness and complexity. We also believe that visualization could be very helpful for readers in understanding graph algorithms. Therefore, we give a particular attention to embed capabilities of interactive animation of graph algorithms into the WEGA system.

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Homomorphic Encryption Schemes

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Homomorphic encryption [1] is a special class of encryption functions which allows the encrypted data to be operated on directly without requiring knowledge about the decryption function. Let $E_k(\cdot)$ be an encryption function with key k and $D_k(\cdot)$ be the corresponding decryption function. Then $E_k(\cdot)$ is Homomorphic with the operator (\cdot) if there exists an efficient algorithm Alg such that $\text{Alg}(E_k(x), E_k(y)) = E_k(x) \cdot E_k(y)$, where x and y are two different messages.

There are mainly three types of Homomorphic encryption schemes (i) Multiplicative Homomorphic encryption scheme where $E_k(x \cdot y) = E_k(x) \cdot E_k(y)$ (ii) Additive Homomorphic encryption scheme where $E_k(x + y) = E_k(x) + E_k(y)$ and (iii) Scalar Homomorphic encryption scheme where $E_k(tx) = s\text{Multi}(E_k(x), t)$. i.e. $E_k(tx)$ can be found easily from t and $E_k(x)$ without needing to know

what x is. Besides these, there are some special Homomorphic schemes [2] which are both multiplicative as well as additive Homomorphic [3]. Homomorphic encryption schemes have recently gained importance and are widely used in cloud computing, secure packet forwarding in mobile adhoc networks, wireless sensor networks and electronic voting system.

Our work includes efficient implementation of Homomorphic schemes over large integers which depict Multiplicative as well as Additive Homomorphism. We are also designing fast and secure Homomorphic encryption schemes for wireless sensor networks based on simple integer arithmetic [4] and computations on Elliptic Curves. Implementation has been carried out on Linux platform with GMP libraries for large number arithmetic using the C++ programming language.

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Planar Graph Isomorphism is in Log-space

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Graph Isomorphism is the prime example of a computational problem with a wide difference between the best known lower and upper bounds on its complexity. There is a significant gap between extant lower and upper bounds for planar graphs as well. We bridge the gap for this natural and important special case by presenting an upper bound that matches the known log-space hardness [2]. In fact, we show the formally stronger result that planar graph canonization is in log-space. This improves the previously known upper bound of AC^1 [4].

Our algorithm first constructs the biconnected component tree of a connected planar graph and then refines each biconnected component into a triconnected component tree. The next step is to log-space reduce the biconnected planar graph isomorphism and canonization problems to those for 3-connected planar graphs, which are known to be in log-space by [1]. This is achieved by using the above decomposition, and by making significant modifications to Lindell's algorithm for tree canonization [3], along with changes in the space complexity analysis. The reduction from the connected case to the biconnected case requires further new ideas, including a non-trivial case analysis and a group theoretic lemma to bound the number of automorphisms of a colored 3-connected planar graph. This lemma is crucial for the reduction to work in log-space.

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The Conditional Covering Problem on Unweighted Interval Graphs with Nonuniform Coverage Radius

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Let $G = (V, E)$ be an interval graph with n vertices and m edges. A positive integer $R(x)$ is associated with every vertex $x \in V$. In the conditional covering problem, a vertex $x \in V$ will cover a vertex $y \in V$ ($x \neq y$) if $d(x, y) \leq R(x)$ where $d(x, y)$ is the shortest distance between the vertices x and y . The conditional covering problem (CCP) asks to find a minimum cardinality vertex set C in V so as to cover all the vertices of the graphs and every vertex in C to be covered by another vertex of C . This problem is NP-complete for general graphs. In this paper, we propose an efficient algorithm to solve the CCP with nonuniform coverage radius in $O(n^2)$ time when G is an interval graph containing n vertices.

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$L(2, 1)$ –Labelling of Cactus Graphs

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An $L(2, 1)$ –labelling of a graph G is an assignment of nonnegative integers to the vertices of G such that the difference between the labels assigned to any two adjacent vertices is at least two and the difference between the labels assigned to any two vertices which are at distance two is at least one. The span of an $L(2, 1)$ –labelling is the maximum label number assigned to any vertex of G . The $L(2, 1)$ –labelling number of a graph G , denoted by $\lambda(G)$, is the least integer k such that G has an $L(2, 1)$ –labelling of span k .

A cactus graph is a connected graph in which every block is either an edge or a cycle. In this paper, we label the vertices of a cactus graph by $L(2, 1)$ –labelling.

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Quantum Computations in the Bulk Ensemble NMR Quantum Computer

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NMR quantum computation system executes parallel bulk ensemble computation, in which an Avogadro's number of molecules are used as individual quantum computers. As pointed out by Collins([1]) and Nishino([2]), since measurements in NMR quantum computer are given by the expectation value quantum computation, the Grover's algorithm will be faster than a single quantum computer.

In [3], we discussed some of the implications on the Deutsch-Jozsa algorithm in a bulk quantum computing, and presented some new approach to realize experimentally.

According to Nishino's result, this speed-up effects in the bulk ensemble computation will appear in 4 qubits or more.

So, our main objective in this reserch is to discuss experimental realizations of quantum algorithms with Lie group techniques.

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L-valued Automata and Associated Topology

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 18B20, 54A05, 03E72.

The study is to show a nice interplay among L -fuzzy approximation operator (L is a complete orthomodular lattice) on an ' L -fuzzy approximation space', L -valued topology and L -valued automaton. We begin by noting that each L -fuzzy approximation space is associated with a L -fuzzy approximation operator, which turns out to be Kuratowski L -fuzzy closure operator, if the L -fuzzy relation associated with approximation space is reflexive and transitive, which in turn give rise to a L -valued fuzzy topology. Also, if the lattice is distributive, we can get a dual L -valued fuzzy topology. It is shown that many properties of L -valued fuzzy automata can be conveniently described in terms of these L -valued fuzzy topologies.

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A Note on Ambiguous Probabilistic Deterministic Finite state automata

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 68Q10 and 68Q45

In this paper we define various forms of a probabilistic finite state automata, and probability of strings generated by a probabilistic finite state automata is discussed in terms of extended transition functions. On the same line, the ambiguity of probabilistic deterministic finite state automata is discussed.

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Proving Easy Programming Languages by Denotational Semantics

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Logic of any programming languages that can be written in the form of mathematical notations can be verified by their syntactic and semantic properties. This paper presents how to verify the logic by denotational semantics or dynamic semantics. In order to verify the accuracy of programming language logic, the syntax and semantics must be defined in the form of "Lambda calculus".

The theory of Lambda calculus is one of many useful tools for dealing with higher-order functions in denotational semantics. The verification of a simple but representative programming language including two issues; i.e., substitution and types, is presented as an example.

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Intuitionistic Fuzzy Multisets

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In conventional fuzzy set theory, a membership function assigns to each element of the universe of discourse, a number from the unit interval to indicate the degree of belongingness to the set under consideration. Since Zadeh introduced fuzzy sets in 1965, many new approaches and theories treating imprecision and uncertainty have been proposed. In 1986, Krassimir.T.Atanassov [1] introduced the concept of Intuitionistic fuzzy set (IFS) as a theory developed in (a kind of) intuitionistic logic. Intuitionistic fuzzy set is characterized by two functions expressing the degree of belongingness and the degree of nonbelongingness, respectively. The name Intuitionistic fuzzy set is due to George Gargove, with the motivation that their fuzzification denies the law of excluded middle-one of the main ideas of intuitionism. This idea, which is a natural generalization of a standard fuzzy set, seems to be useful in modelling many real life situations, like negotiation processes, psychological investigations, reasoning etc.

Many fields of modern mathematics have been emerged by violating a basic principle of a given theory only because useful structures could be defined this way. While sets permit us to have at most one occurrence of each element, multisets or bags, permit us to have multiple occurrences of the elements. A complete account of the development of multiset theory can be seen in [2]. As a generalization of multiset, Yager [3] introduced the concept of fuzzy multiset (FMS). An element of a fuzzy multiset can occur more than once with possibly the same or different membership values.

In this paper an attempt is made to consider all the above concepts together by introducing a new concept named as Intuitionistic fuzzy multiset (IFMS). We discuss operations on Intuitionistic fuzzy multisets such as union, intersection, addition, multiplication etc. We introduce $\alpha\beta$ - cut of an Intuitionistic fuzzy multiset, Cartesian product of Intuitionistic fuzzy multisets and discuss their various properties. Also we define two operators over the set of all Intuitionistic fuzzy multisets which will transform every Intuitionistic fuzzy multisets into fuzzy multisets and discuss various properties connecting all these.

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The Generalized Transportation Model of “Bottleneck” type

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Many of the economical decision problems lead to multiple criteria transportation model of fractional type with identical denominators, where the “bottleneck” criterion appears as a “minmax” time constraining. In [1] is proposed a method for solving the single-criteria transportation problem of fractional type. We studied the transportation problem of “bottleneck” type

with multiple fractional criteria [3], that is defined as follows:

$$\begin{aligned} \min z_1 &= \frac{\sum_{i=1}^m \sum_{j=1}^n c_{ij}^1 x_{ij}}{\max_{i,j} \{t_{i,j}/x_{i,j} > 0\}} & \min z_2 &= \frac{\sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 x_{ij}}{\max_{i,j} \{t_{i,j}/x_{i,j} > 0\}} \\ & & \dots & \\ \min z_r &= \frac{\sum_{i=1}^m \sum_{j=1}^n c_{ij}^r x_{ij}}{\max_{i,j} \{t_{i,j}/x_{i,j} > 0\}} & \min z_{r+1} &= \max_{i,j} \{t_{i,j}/x_{i,j} > 0\} \end{aligned} \quad (1)$$

$$\sum_{j=1}^n x_{ij} = a_i, \quad \forall i = \overline{1, m} \quad \sum_{i=1}^m x_{ij} = b_j, \quad \forall j = \overline{1, n} \quad \sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

where c_{ij}^k , $k = 1, \dots, r$, $i = 1, \dots, m$, $j = 1, \dots, n$ correspond to the concrete interpretation of the respective criteria, a_i – availability at source i , b_j – requirement at destination j x_{ij} – amount transported from source i to destination j . In order to solve the model (1), I suggest its reducing to another multicriterial linear model like in [2], after that I propose an iterative procedure to found the all basic efficient solutions of it. The Theorems that prove the equivalence of the both models are given, meaning the common set of their basic efficient solutions.

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A Message Authentication Code Based On Quasigroups

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A quasigroup (Q, o) is a set Q with a binary operation ‘ o ’ (that is a magma), such that for all a, b in Q , there exist unique elements x, y in Q such that

$aox = b$ and $yoa = b$. These quasigroups are not required to be associative and commutative. A cryptographic message authentication code is a short piece of information used to authenticate a message. In [5] Kristen Ann Meyer created a new kind message authentication code whose security based on non associativity of Quasigroups. This paper focuses on a new kind of message authentication code based on quasigroup and its non-associativity and such quasigroup structure is also studied.

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The Algebraic IV Differential Attack AIDA: A Cryptanalytic Tool

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Any cryptographic bit function f like the n -th output bit of a stream cipher can be written – as can every Boolean function – in Disjunctive and Algebraic Normal Forms. Let f depend on the IV bits v_1, \dots, v_{n_1} and the key bits k_1, \dots, k_{n_2} :

$$f = \bigvee_{I \subset \{1, \dots, n_1\}} \left(v_I^\wedge \bar{v}_I^\wedge \wedge \left(\bigoplus_{J \subset \{1, \dots, n_2\}} d_{I,J} k_J^\wedge \bar{k}_J^\wedge \right) \right) = \bigoplus_{I \subset \{1, \dots, n_1\}} \left(v_I^\wedge \wedge \left(\bigoplus_{J \subset \{1, \dots, n_2\}} a_{I,J} k_J^\wedge \right) \right),$$

where \vee, \wedge, \oplus are the logical *or*, *and*, and *exclusive or*, respectively, $v_I^\wedge = \bigwedge_{i \in I} v_i$ and $\bar{k}_J^\wedge = \bigwedge_{j \in \{1, \dots, n_2\} \setminus J} \bar{k}_j$, $k_j = 1 - k_j$. $d_{I,J}, a_{I,J} \in \mathbb{F}_2$ exclude (if 0) or include (if 1) the respective term. Each DNF coefficient $d_{I,J}$ can be obtained by one simulation.

AIDA consists in first observing that $a_{I,J} = \bigoplus_{M \subset I} d_{M,J}$ by the inclusion-exclusion-principle, requiring $2^{|I|}$ simulations, and second searching for index sets I such that $a_{I,J} = 1$ only for one-element sets $J = \{j\}$ that is linear dependence on the key bits. Given n_2 such linear relations, we recover the key by Gaussian elimination.

AIDA [2] was republished a year late(r) by Dinur and Shamir in [1], renamed into “cube attack”.

Results

1. AIDA successfully attacks TRIVIUM with reduced setup length of 792 [4]. However, linear AIDA will not break TRIVIUM with full setup length of 1152 [3].
2. AIDA completely breaks BIVIUM-A and -B in just minutes, see [5].
3. The Fast Reed-Muller Transform speeds up AIDA by a factor of 5000, see [3].
4. The Wavefront Model [3] as linearity test requires $d + 13$ simulations (d the hypercube dimension), while the BLR test [1] needs $3(d + 1) + 1$ simulations.
5. Fast multiplication of ANFs over n variables ($N := 2^n$ entries) is possible in $4N \log N$ steps instead of $O(N^2)$, using the fast RMT [3].

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Answer to Question P/NP is $P \neq NP$

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This paper sets up P -operator defined by two standards: class A - the realistic existence of polynomial algorithm, and class B - the possible existence of polynomial algorithm. Prove:

Lemma 1: $(C \neq \Phi) \rightarrow (C = \bar{A} \cap B)$. (C denotes an obscure class. Φ denotes an empty set.)

Lemma 2: $(NP \subseteq B) = \Phi$. (“ NP is a subset of B ” can not be true).

Lemma 3: $NP = \Phi$. The definition of NP is not correct.

Lemma 4: $(NP \neq \Phi) \rightarrow (NP = \bar{A} \cap B = C)$. If the NP is correct, then $NP = C$.

Theorem 5: $P \neq NP$.

Due to above theories, no matter what standard of P taken, and regardless of the definition of NP right or wrong, will be $P \neq NP$. The answer to question P/NP is $P \neq NP$.

This problem-solving using logical analysis of the new method is proposed criteria for the classification must be consistent with the definition of correctness, feasibility, and distinct requirements, but also pointed out that the existing theories such as “ P is a subset of NP ”, “*Hanoi Tower problem* is an arithmetic problem in \bar{P} ” and “cannot determine the $P = NP$ or $P \neq NP$ ” errors and their causes, from the algorithm, arithmetic problems, structure, function, standards, five levels of analysis of the plight of the definition of NP , it is recommended to abandon this chaos concept.

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Classification of Some Distance-regular Graphs

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 05E30

We give classification of some distance-regular graphs with intersection array $a_1 \geq 2$ and with the second largest eigenvalue of its local graph at most 1.

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Model of Numbers Tree and its applications

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The paper is devoted by questions modeling of Numbers Tree arising at the analysis of the numerical and corresponding text information. It is shown that many questions of mathematical analysis of problems and their applications are reduced to construction of models in extremes regimes and connected with it and construction of Model Numbers Tree. For any positive number N there are natural numbers $p, n, m > 1$ and positive numbers $a_i, a_{ij}, \dots, a_{i,m}$ for which take place $N^p = a_1^n + a_2^n + \dots + a_m^n$, $a_j^p = a_{1j}^n + a_{2j}^n + \dots + a_{mj}^n$, $a_{ij}^n = a_{1ij}^n + a_{2ij}^n + \dots + a_{mij}^n$, $a_{ijk}^p = a_{1ijk}^n + a_{2ijk}^n + \dots + a_{mijk}^n$, ..., $a_{ijk\dots s}^p = a_{1ijk\dots s}^n + a_{2ijk\dots s}^n$ and in last presentation members of the right part can not beat are submitted as the final sum composed $n - th$ degrees of some integers so-called by a basis of a tree[1]. Main results are next: 1). The number N is uniquely represented as Numbers Tree representation: $N^p = \sum k_{j_q} a_{ij_1j_2\dots j_q}^n$, where k_{j_q} are numbers of occurrence

of a basic element $a_{i_1 j_1 \dots j_q}$ in a tree of numbers. 2). Representation for N and corresponding Numbers Tree representation are optimum representation. More over with help of transformation $a_{im} = xa_{i,m-1}, i = \overline{1, m-1}, a_{mm} = y \sqrt[n]{N_{m-1}^p}, N_m = zN_{m-1}$, where $x^n + y^n = z^n$ we have the polinom $N_m^p = (x^{m-1})^n + \sum_{i=2}^m \left(yx^{m-i} z^{\frac{p(i-2)}{n}} \right)^n$ which describes the process of Numbers Tree grows[2].

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Section 16

Numerical Analysis and Scientific Computing

A Parameter Uniform Difference Method for a Singularly Perturbed Three-point Boundary Value Problem

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The purpose of this study is to present a uniform finite difference method for numerical solution of nonlinear singularly perturbed second order convection-diffusion problem with nonlocal and third type boundary conditions. The numerical method presented here comprises a fitted-difference scheme on a piecewise uniform mesh. We have derived this approach on the basis of the method of integral identities using interpolating quadrature rules with the weight and remainder terms in integral form. This results in local truncation errors containing only second-order derivatives of exact solution and hence facilitates examination of the convergence. In the boundary layers, we introduce the special uniform meshes, which are constructed by using the estimates of derivatives of the exact solution and the analysis of the local truncation error. Optimal order error estimates, uniformly in the diffusion parameter, are proven. Some numerical results illustrate in practice the result of convergence proved theoretically.



Numerical Solution of Camassa-Holm Equation by Using Homotopy Analysis Method

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The following Camassa-Holm (CH) equation is used to model the statistical of turbulent fluid flows.

$$u_t + 2ku_x - u_{xxt} + auu_x = 2u_xu_{xx} + uu_{xxx}, \quad u(x, 0) = f(x), \quad u_{xx}(x, 0) = g(x). \quad (1)$$

In recent years, some works have been done in order to find the solution of (1)[1–4]. In this work, we apply the homotopy analysis method (HAM) [5] to solve the Eq. (1) and compare this method with the homotopy perturbation method (HPM) [3]. For this purpose, the uniqueness of the solution and the convergence of the method are proved. Also, an algorithm is proposed in order to compute the approximate solution of the eq. (1) and it has been shown that the computational complexity of the algorithm for HAM is less than HPM. The results of the algorithm show that the approximate solution of Camassa-Holm equation is calculated with smaller error and less number of iterations for HAM in comparison with the HPM. Hence, one can observe that the HAM is more rapid convergence and accurate than the HPM.

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Artificial Diffusion-Convection Problem in One Dimension: A Computational Approach

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Convection is the process in which heat moves through a gas or a liquid as the hotter part rises and the cooler, heavier part sinks, where as in the diffusion a gas or liquid diffuses or is diffused in a substance, it becomes slowly mixed with that substance. In the linear convection-diffusion problem with variable co-efficients, transport mechanism dominates where as diffusion effects are confined to a reasonably small part of the domain. In this paper the asymptotic nature of solution to stationary convection-artificial diffusion problem is considered and a numerical technique to control the oscillatory behavior of the computed solution at the specific value of argument is studied. The co-efficient of diffusion introduced controls the oscillations at the boundary layer. On increasing artificial diffusion by a certain amount the solution pattern matches with the series solution of the same problem.

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Discrete Element Method for Granular Flow and Cracks Propagation

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Granular Materials (GM) are everywhere in nature and are the second-most manipulated material in industry after water, but as once written by Pierre-Gilles de Gennes, their statistical physics is still in its infancy.

In this presentation, after a short overview of the mathematical challenges and the state of the art related to the diverse set of behaviors of GM, I will present new numerical simulations, by using the contemporary Discrete Element Method in order to simulate a wide variety of cases in both rapid and dense granular flow regimes. I will also characterized the industrial relevance of the simulations, the link with the cracks propagation and what can these two active research fields learn from each other.

Granular Matter is often referred to as the fourth state of the matter and depending on the situation, GM can behave as a solid, a liquid, or a gas. When dry sand is poured, it acts as a fluid, while the pile on which it is poured is solid-like. When dry sand is fluidized by blowing air through it or by strong shaking, it behaves gas-like. Leo P. Kadanoff quoted “One might even say that the study of the granular materials give one a chance to reinvent statistical mechanics in a new context”!

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Approximation by Spline Functions Derived from Slopes and Integral Values

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Abstract: Recently, we witness brand new developments on the construction of the spline functions by using slopes or derivative values at the knots or the integral values in each of the subintervals, see for example [1], [2] and [3]. This is totally a new approach and new idea to construct spline functions without using the function values. In this talk, few interesting methods will be presented for cubic and quintic splines. Also, numerical examples and the order of convergence analysis will be discussed. These are shortcut algorithms to build cubic and quintic splines to approximate the original function when the function values are not given.

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Operator Splitting Methods for Maxwell's Equations in Dispersive Media

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The accurate simulation of wave propagation in complex media is an important issue that arises in various fields such as acoustics, electromagnetics and elasticity. We consider here the electromagnetic interrogation of dispersive dielectrics which are of interest in applications including biomedical imaging. Biological tissue interactions with electromagnetic fields are defined by their

complex permittivity, which is a function of the various electric and magnetic polarization mechanisms, and conductivity of the biological media [2]. These electromagnetic properties of dispersive dielectrics are frequency dependent.

Computational simulations of the propagation and scattering of transient electromagnetic waves in dispersive dielectrics can be studied by numerically solving the time-dependent Maxwell's equations coupled to equations that describe the evolution of the induced macroscopic polarization [3]. The latter incorporates the physical dispersion of the medium and its response to the electromagnetic pulse.

We consider Maxwell's equations in dispersive media of Debye type (e.g., biological tissue). In such relaxing dielectric media, the presence of different wave speeds leads to stiffness in the temporal domain [1]. We present an operator splitting scheme that decouples fast and slow moving processes in the problem to develop separate sub-problems. This alleviates the stringent requirements on the time-step which, along with stability conditions, requires small spatial steps and hence excessive computations for long-time integration of Maxwell's equations.

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A High-Order Discontinuous Galerkin Method for Elliptic Interface Problems

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In this work, we implement the high-order local discontinuous Galerkin method [1] with the numerical flux proposed by Guyomarc'h et. al. [3], to solve the elliptic interface problem.

Instead of the triangular elements and the P^k -polynomial space used in [3], we use the quadrilateral elements and the Q^k -polynomial space. To treat the curved interface and boundaries, we use the transfinite blending mappings [2] to construct curved elements. The discretization generates a symmetric system and the system can be solved directly with standard methods or reduced into smaller systems with matrix reduction techniques.

The numerical experiments show the h and p convergence properties of the our high order scheme. Moreover, our numerical experiments show that the high-order method is more efficient than the low-order one. To achieve the same magnitude of accuracy, the degree of freedom of the generated system using the coarsest mesh is much smaller than the one using the finest mesh. As a result the CPU time required to solve the system using the coarsest mesh is much smaller, too.

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Newton's Forward Difference Interpolation Formula - Extended

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There are various interpolation formulae, which are useful when given tabular values correspond to a polynomial function. Problem arises when the tabular values do not correspond to a polynomial function, rather, to a function of the type $y = a^x$. For example, given the tabular values of x and y as (1,3), (2,9), (3,27); where $y(x) = 3^x$. Using Newton's Forward difference interpolation formula we get $y(4) = 57$ and $y(5)=99$. But as the values were derived from the function $y(x) = 3^x$ we have actual values of $y(4)=81$ and $y(5) = 243$. We can interpolate this type of functions, by extending Newton's forward difference

interpolation formula in a generic fashion taking Newtons forward difference table as the base.

We have to take the initial ordinates of each higher order difference as the values of y corresponding to x denoting it by $y^1(x)$ and form another difference table. From this difference table we form another difference table in the same manner and term the values of y corresponding to x as $y^2(x)$ and so on till $y^m(x)$, when the values become constant. This is shown in the table below:

Table 1. Extended Difference Tables

x	y	Δ	Δ^2	x	y^1	Δ	Δ^2	x	y^2	Δ	Δ^2
1	3			1	3			1	3		
2	9	6		2	6	3		2	3	0	
3	27	18	12	3	12	6	3	3	3	0	0

Depending upon the number of difference tables so formed, we extend the Newton's forward difference interpolation formula as follows:

$$y_n(x) = (m+1)y_0 + p(\Delta y_0 + \Delta y_0^1 + \Delta y_0^2 + \dots + \Delta y_0^m) + \frac{1}{2!} p(p-1)(\Delta^2 y_0 + \Delta^2 y_0^1 + \Delta^2 y_0^2 + \dots + \Delta^2 y_0^m) + \dots + \frac{1}{n!} p(p-1)(p-2)\dots(p-n+1)(\Delta^n y_0 + \Delta^n y_0^1 + \Delta^n y_0^2 + \dots + \Delta^n y_0^m)$$

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Polynomial Chaos Approach for Approximating Cole-Cole Dispersive Media

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We consider electromagnetic interrogation of dispersive dielectrics. We are interested in polydisperse materials which exhibit relaxation mechanisms not sufficiently modeled by first order linear (Debye) polarization models. Heuristic

generalizations, including the Cole-Cole model, have been successful in replicating the complex permittivity of such materials. However, these models no longer correspond to ODEs in the time-domain, and therefore are difficult to simulate.

We implement an alternative approach based on using the first order linear ODE Debye model, but with distributions of relaxation times. The need for multiple relaxation times was first discussed by von Schweidler in 1907, long before the Cole brothers published their model in 1941, and even before Debye published *Polar Molecules* in 1929. Since then various efforts have been made both in fitting Cole-Cole parameters and distributions to data. A significant amount of this work is reviewed in the survey paper by Foster and Schwan [1].

However, with regard to time-domain simulations using these models, there have been relatively few attempts to develop numerical methods, and these have been nearly exclusively aimed at the Cole-Cole model (see [2] for an exception). Indeed, most practitioners attempt to replace the Cole-Cole representation with a multi-pole Debye model instead. While significantly faster, this approach is limited to a narrow frequency range of applicability.

We develop a novel approach to the time-domain simulation of polydisperse materials by applying Polynomial Chaos [3] to the Debye polarization model including a distribution of relaxation times. The traditional Yee scheme is employed to discretize Maxwell's equations which are coupled to the mean value of the polarization. We examine the accuracy and efficiency of the resulting method as compared to competing approaches using the Cole-Cole model and multi-pole Debye.

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Exponential Type Integrators for Abstract Quasilinear Parabolic Equations with Variable Domains

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In this talk, I propose an exponential explicit integrator for the time discretization of quasilinear parabolic problems. My numerical scheme is based on Magnus methods. In an abstract formulation, the initial-boundary value problem is written as an initial value problem on a Banach space X

$$u'(t) = A(u(t))u(t) + b(t), \quad 0 < t \leq T, \quad u(0) \text{ given}, \quad (2)$$

involving the sectorial operator $A(v) : D(v) \rightarrow X$ with variable domains $D(v) \subset X$ with regard to $v \in V \subset X$. Under reasonable regularity requirements on the problem, I analyze the stability and the convergence behaviour of the numerical methods.

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A Compact Streamfunction-Velocity Method for Incompressible Viscous Flows

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We recently [1] proposed a new paradigm for solving the steady-state two-dimensional (2D) Navier-Stokes (N-S) equations using a compact streamfunction-velocity (ψ - v) formulation. This formulation has been shown to avoid the difficulties associated with the traditional formulations (primitive variables, and streamfunction- vorticity formulations). The new formulation has been found to be second order accurate, and yields accurate solutions of a number of fluid flow problems.

In this presentation, we describe the ideas behind the development of the streamfunction-velocity (ψ - v) formulation for steady state as well as transient flows, and present results for a variety of fluid flow problems.

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Parametric Septic Splines Approach to the Solution of Sixth-order Two Point Boundary-value Problems

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Keywords. Parametric septic splines; Boundary-value Problems; Spline function approximation.

Parametric splines, which are equivalent to seven-degree polynomial splines, are used to develop a class of numerical methods for solution of sixth-order boundary-value problems are presented. The spline function is used to derive some consistency relations for computing approximations to the solution of sixth-order two point boundary-value problems. Second, fourth, sixth, and eighth order convergence is obtained. It is shown that the present method gives approximations, which are better than those produced by other splines and domain decomposition methods. Two numerical examples are given to illustrate the practical usefulness of the new approach.



Vector-valued Wave Packets

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The concept of wave packet has been studied in a series of papers by Labate, Christensen, Hernandez et al, see for example [3], [4] and [5] and references therein. Vector-valued multiresolution analysis and associated wavelets and wavelet packets have been investigated in [1], [2] and [3]. A concept of vector valued wave packet is introduced and its basic properties are investigated including relationship with vector-valued multiresolution analysis and its variants.

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Multisymplectic Integrators for Hamiltonian Wave Equations

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2010 MATHEMATICS SUBJECT CLASSIFICATION. Primary 65L06, 65L07, 65L20; Secondary 33C20, 34E05.

Hamiltonian ODEs conserve a symplectic form, $\omega_t = 0$, and can be integrated numerically by symplectic integrators that conserve ω . Hamiltonian PDEs have a multisymplectic conservation law such as $\omega_t + \kappa_x = 0$; numerical methods that have a discrete multisymplectic conservation law can be derived using discrete Lagrangian methods or (what is often equivalent) symplectic partitioned Runge–Kutta (SPRK) in space and time applied to a multi-Hamiltonian formulation of the PDE. We survey the following results on the numerical and dynamical behaviour of these methods:

1. Symplectic RK methods (including the popular box scheme) can unconditionally preserve the form of the dispersion relation of any multi-Hamiltonian PDE.
2. The implicit discrete equations of symplectic RK may not have solutions, even for fine grids.
3. We give sufficient conditions on the PDE for SPRK to yield explicit local semidiscretizations.

4. We characterize those SPRK methods that yield stable semidiscretizations and determine their dispersion relation.
5. We express the frequency response of s -stage Lobatto IIIA–IIIB family of SPRK methods (whose simplest member is the central difference approximation of the nonlinear wave equation) explicitly in terms of continued fractions and hence prove that the entire family is stable in this sense.
6. We describe the ability of these methods to capture steady states and travelling wave solutions and compare to non-multisymplectic methods.

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First Step into Pattern-finding DNA Kernel

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In post-Jeffrey [1] period, from 1990 till “Hierarchical multifractal representation of symbolic sequences” [2], “Wavelet-based multifractal analysis of DNA sequences by using chaos-game representation” [3], and interestingly an online chaos game at “<http://www.g-language.org/wiki/cgr>” [4], any convergence a propos general pattern on DNA has not been attained. CGR (Chaos Game Representation) is just another window to represent a string of letters in a DNA word in the form of self-similar fractal image, but the “mini” image, of the unit which is supposed to generate self-similarity in the whole image, is a function of the exact composition of the DNA word, and so clearly the generality resides in just being self-similar, which is rather a direct consequence of implementing CGR algorithm or its fork.

To investigate farther deep into the DNA fractal images in the hope of extracting some pattern, one would, a priori, require knowing some general truths (biological or real contrasted against theoretical) regarding DNA sequences. As to the current status-quo of DNA-CGR analyses, one must know a perfectly “good” word size, regardless of composition.

Because in certain sense a pattern, be it “visual” or “theoretical”, is user-defined hence there could be a multitude of idiosyncratic heuristics to represent or treat a given DNA sequence or a set of sequences to arrive at some putative pattern and idiosyncrasy also exists in using a particular software environment and so there could arise a need of a unified and dedicated DNA software base to put all such idiosyncrasies and also to leave room for future developments and such a software base run on bare-metal-machine would finally give way to analyses and manipulations of everything structured as such into address space in a unified-user-defined and clear-cut manner. It may be noted in passing that the very “object” called pattern is actually unknown, in a sense, though researchers are searching for pattern yet nobody precisely knows the description of that pattern per se. Keeping such issues in the backdrop, it could be worthwhile to design a kernel rather than just writing an ordinary program specifying another idiosyncratically defined pattern finding algorithm in an arbitrarily chosen software environment; and moreover a kernel, which is by definition standalone, and in this case designed solely for putting A-C-G-T-pattern on platform, can easily import all open source codes for the existing symbolic sequence pattern-finding programs, along with, if required, the source codes for DNA sequence analysis programs and the whole of the DNA database on the fly.

GENEUS does exist [5], but started to develop during pre-Jeffrey period, hence the developers somehow missed to get the system optimized for allowing some convergence vis-à-vis fractal-like or some exotic pattern even for locally defined scenarios.

Commercial Simics environment [6] has been used as a full-system virtual model for the present Unix-like Kernel, which is complete and stable, and upon being booted starts running “idle”, “init”, and “shell” without human intervention, and shows no propensity to fall into Simics debugger, at least when no new DNA words are integrated.

A 32-bit application, exemplified with three DNA words (E.coli K-12 chromosome [7]; Mexican Cotton or Upland Cotton, biological nomenclature - *Gossypium hirsutum*, BAC sequences, Phase 1 BACS, Segment ID 124001821, 150 kilobase long, source “<http://www.plantgdb.org>”; and Human chromosome 22, Contig NT 027140.6,) has been written and installed in this present kernel. This application randomly fragmentizes a given pair of DNA sequences into user-defined size of fragments, and then pair-wise compares the generated fragments, extracted from the given DNA sequence-pair, yielding a 0/1 score matrix, a 0 for a mismatch and a 1 for a match; whereupon, the matrix elements get summed up row-wise (all rows) and column-wise (all columns), to yield a row-vector and a column-vector, and then the inner-product of these vectors gets imaged in an inner-product space and finally, the generated image gets distorted following the template of classic barrel distortion. The so called, here, “barrel image” shows some humanly perceptible pattern consistency depending on *whether* the image gets generated using pure fragments (with all same

DNA letters) compared with all fragments (irrespective of pure or hybrid) *or* hybrid fragments (all or some DNA letters/letter not same) compared with all fragments.

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Conjugate Gradient Methods for Nonsymmetric Systems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65F10

Positive definite linear systems $Ax = b$ are solved by Conjugate Gradient methods. For the indefinite systems, polynomial preconditioning techniques [1] are applied. This method cannot be used for nonsymmetric systems. Instead of generating parameters dynamically in the conjugate gradient methods, parameters are generated in advance based only on the smallest and largest eigenvalues; and the size of the nonsymmetric matrix. These parameters are used in the conjugate gradient methods to solve the system.

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**Uniformly Convergent Numerical Method for Singularly Perturbed Second Order Ordinary Delay Differential Equations of Convection-diffusion Type**

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In this paper a numerical method is suggested for singularly perturbed second order ordinary delay differential equations of convection-diffusion type. The numerical method is based on the standard finite difference operator applied on the piece wise uniform mesh (Shishkin type) A parameter uniform error bound for the numerical solution is obtained. Numerical results are provided to illustrate the theoretical results.

**Lagrange Multiplier Method with Penalty for elliptic Interface Problems**

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20K

The basic requirement for the stability of the mortar element method is to construct finite element spaces which satisfy certain criteria known as inf-sup (well known as LBB, i.e., Ladyzhenskaya-Babuška-Brezzi) condition. Many natural and convenient choices of finite element spaces are ruled out as these spaces

may not satisfy the inf-sup condition. In order to alleviate this problem Lagrange multiplier method with penalty is used in this article. The existence and uniqueness results of the discrete problem are discussed without using the discrete LBB condition. The results of numerical experiments support the theoretical results obtained in this article.

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Some Markov-Bernstein type Inequalities and Certain Class of Sobolev Polynomials

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Let (μ_0, μ_1) be a vector of non-negative measures on the real line, with μ_0 not identically zero, finite moments of all orders, compact or non compact supports, and at least one of them having an infinite number of points in its support. We

show that for any linear operator T on the space of polynomials with complex coefficients and any integer $n \geq 0$, there is a constant $\gamma_n(T) \geq 0$, such that

$$\|Tp\|_S \leq \gamma_n(T)\|p\|_S,$$

for any polynomial p of degree $\leq n$, where $\gamma_n(T)$ is independent of p , and

$$\|p\|_S = \left\{ \int |p(x)|^2 d\mu_0(x) + \int |p'(x)|^2 d\mu_1(x) \right\}^{\frac{1}{2}}.$$

We find a formula for the best possible value $\gamma_n(T)$ of $\gamma_n(T)$ and inequalities for $\gamma_n(T)$. Also, we give some examples when T is a differentiation operator and (μ_0, μ_1) is a vector of orthogonalizing measures for classical orthogonal polynomials.

This poster is an abridged version of the paper of the same title [3].

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A Second Generation Wavelet Based Approach for Multiscale Solution of Biharmonic Plate Equation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65T60, 65N30

In this paper we propose a new wavelet based approach for solving biharmonic equation. The wavelets are constructed from cubic Hermite spline, these wavelets are based on recent papers on semi-orthogonal [1] and biorthogonal wavelets [2], the wavelets are adapted over the boundaries of domain. The extension to two dimensions is performed via tensor product [3]. The biharmonic equation is discretized using wavelet Galerkin method. The conditioning of multiresolution stiffness matrices are experimentally verified for various

wavelet constructions, and are found to be asymptotically optimal. Whereas the classical approaches using finite element and hierarchical bases lead to poor conditioning. Our numerical experiments clearly demonstrate the advantages of wavelets in providing a stable, hierarchical, incremental [4] and scalable algorithm for an efficient, adaptive solution of biharmonic plate bending equation.

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A Fully Implicit Numerical Method for Integration of Stiff Stochastic Differential Equations

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Constructing numerical methods for stiff stochastic ordinary differential equations in the Itô sense, where the stiffness in the diffusion coefficients dominates or is at least as significant as the stiffness in the drift coefficient, has been an open problem for a while [1] because of the narrow coverage of drift-diffusion plane by the (mean square) stability regions of the existing implicit schemes [2]. In this contribution we show that it is possible to construct a fully implicit method that can have a stability region that covers almost the whole drift-diffusion plane. The method uses a low pass filter to deal with the stiffness by effective homogenization and the stability is maintained by a modified real

Schur decomposition in the time stepping of the method. An analysis shows that this method is convergent with strong order 1.0 and can have a mean square stability region covering almost all of the drift-diffusion plane for certain choice of its algorithmic parameters. A stiff chemical Langevin equation is used to demonstrate the effectiveness of the method. A fuller account of the method can be found in [3].

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Hydrodynamic Stability of Free Convection from an Inclined Elliptic Cylinder in Couple Stress Fluid

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The steady problem of free convective heat transfer from an isothermal inclined elliptic cylinder and its stability is investigated. The cylinder is inclined at an arbitrary angle with the horizontal and immersed in an unbounded, viscous, incompressible couple stress fluid. It is assumed that the flow is laminar and two dimensional and that the Boussinesq approximation is valid. The full steady Navier-Stokes and thermal energy equations are transformed to elliptical coordinates and an asymptotic analysis is used to find appropriate far-field conditions. A numerical scheme based on finite differences is then used to obtain numerical solutions. Results are found for small to moderate Grashof and Prandtl numbers, and varying ellipse inclinations and aspect ratios. A linear stability analysis is performed to determine the critical Grashof number at which the flow loses stability. Comparisons are made with longtime unsteady solutions.

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Compressive Sampling Techniques and their Application to Data Regularization

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 94A20, 42A05, 15A52, 05A18, 90C05, 90C25

For data that can be sparsely generated, one can obtain good reconstructions from reduced number of measurements - thereby compressing the sensing process rather than the traditionally sensed data. A wealth of recent developments [1][3][4] in applied mathematics, by the name of Compressed Sensing or Compressive Sampling (CS) aim at achieving this objective through l_1 -norm minimization. Recent results on CS indicate that CS has a lot of potential applications in fields such as Image Processing, Siesmology [3], to name a few.

Constrained by the practical and economical aspects, one often uses data sampled irregularly and insufficiently. The use of such data in applications does in deed result in certain artifacts and poor spatial resolution. Therefore, before being used, the measurements are to be interpolated onto a regular grid. One of the methods [2] achieving this objective is based on the Fourier reconstruction, which involves an underdetermined system of equations. The present work applies CS to the Fourier-based interpolation problem. For the signals having sparse Fourier spectra, the algorithm computes the Fourier coefficients on a regular grid from a few samples of the signal measured over irregular locations. The algorithm being deterministic in nature generates error out of irregularity in the measurement coordinates, and then applies CS to achieve its objective. To justify the applicability of our algorithm, we present the empirical performance of it on different sets of measurement coordinates as a function of number of nonzero Fourier coefficients. Our simulation results and the explicit error bounds indicate that CS based technique has promising features for the regularization of data from irregular and incomplete set of measurements.

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Biharmonic Computation of the Flow Past an Impulsively Started Circular Cylinder

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In this paper, we extend a second order temporally and spatially accurate finite difference scheme for biharmonic form of the transient incompressible 2D Navier-Stokes (N-S) equations in rectangular domains to problems on irregular physical geometries that are expressible in terms of conformal mappings. This formulation is used to simulate viscous flow past an impulsively started circular cylinder for Reynolds number (Re) ranging from 10 to 9500. We have studied time evolution of flow structure and compared our computed solutions with the experimental and numerical results available in literature[1, 2]. Excellent comparison has been obtained both qualitatively and quantitatively.

Considering conformal transformation $x = x(\xi, \eta)$, $y = y(\xi, \eta)$ of the physical plane into a rectangular computational plane, the biharmonic form of transient N-S equation in terms of stream function ψ is given as

$$\begin{aligned} \frac{\partial}{\partial t} \nabla^2 \psi &= \frac{1}{JRe} \left[\nabla^4 \psi - (2C + Re\psi_\eta) \frac{\partial}{\partial \xi} \nabla^2 \psi - (2D - Re\psi_\xi) \frac{\partial}{\partial \eta} \nabla^2 \psi \right. \\ &\quad \left. + (E + CRe\psi_\eta - DRe\psi_\xi) \nabla^2 \psi \right] \end{aligned} \quad (3)$$

where $C = J_\xi/J$, $D = J_\eta/J$, $E = 2C^2 + 2D^2 - J_{\eta\eta}/J - J_{\xi\xi}/J$, J being the jacobian of the conformal transformation.

Equation (3) is discretized by the proposed extension that uses values of ψ and its gradients ψ_ξ, ψ_η in the compact square cell. For time marching we have adopted a predictor-corrector approach and also carry out a stability analysis.

The flows for $10 \leq Re \leq 40$ were time marched to the steady state while for $Re = 200$ and 300 periodic flow state was reached. For higher Reynolds numbers $Re = 3000, 5000, 9500$ only early stages of the flow were analyzed.

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Parameter-uniform Numerical Method for Singularly Perturbed Differential Equations with Discontinuous Data

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We propose a parameter-uniform numerical method for singularly perturbed differential equations of reaction-diffusion type with discontinuous data. In addition to presence of boundary layers at both end points, strong interior layers can also be present due to the discontinuities in the source term and/or coefficients. Numerical method based on piecewise uniform Shishkin meshes are constructed and parameter-uniform error bounds for the numerical solution and its derivatives are established. Numerical results are presented to support the theoretical results.



Section 17

Control Theory and Optimization

More Realistic Mathematical Models of Traffic Equilibria

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The traffic equilibrium problem (TEP) has been studied for many decades. Several formulations have been proposed under various assumptions. Earlier TEP formulations made use of assumptions which are found to be unnatural or unrealistic (e.g., that the travel costs are independent of the link flows) in order to obtain TEP models which are easy to analyze. Most of the existing TEP formulations assume that route costs are additive, that is, *the route costs are simply the sum of the arc costs for all the arcs on the route being considered* [3, 4]. Another assumption used in most TEP models is that every traveler has a complete and accurate information about the characteristics of the traffic network and all travelers have the same route cost perception and travel behavior. There are various situations, however, when the route costs are non-additive [5]. Moreover, different individuals may have different travel behavior and such travel behavior may be affected by the different time or weather of the day.

In this paper, we consider a more realistic TEP model which is solvable using existing solution methods. In particular, reformulations of the TEP with nonadditive route costs and the TEP under uncertainty will be discussed.

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Proximal Point Method under Metric Regularity

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The classical proximal point algorithm is basically an approximative method for finding fixed points of *nonexpansive* mappings in Hilbert spaces. This method seems to have been applied for the first time to convex optimization by Martinet [4]. Later it was thoroughly explored in a subsequent path-breaking paper by Rockafellar [6].

We consider the following general version of the proximal point algorithms for solving the inclusion $0 \in T(x)$, where T is a set-valued mapping acting from a Banach space X to a Banach space Y . First, choose any sequence of functions $g_n : X \rightarrow Y$ with $g_n(0) = 0$ that are Lipschitz continuous in a neighborhood of the origin. Then pick an initial guess x_0 and find a sequence x_n by applying the iteration $0 \in g_n(x_{n+1} - x_n) + T(x_{n+1})$ for $n = 0, 1, \dots$. We prove that if the Lipschitz constants of g_n are bounded by half the reciprocal of the modulus of regularity of T , then there exists a neighborhood O of \bar{x} such that for each initial point $x_0 \in O$ one can find a sequence x_n generated by the algorithm which is linearly convergent to \bar{x} . Moreover, if the functions

g_n have their Lipschitz constants convergent to zero, then the convergence is superlinear. Similar convergence results are obtained for the cases when T is strongly subregular and strongly regular. Finally, we show that the convergence of this algorithm is stable under small perturbations whenever the set-valued mapping T is metrically regular at a given solution.

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Partial Outsourcing in a Two Warehouse Supply Chain Production Inventory Model

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In this study, an optimal outsourcing policy for a two warehouse supply chain production inventory model with partial backlogging and deteriorating products under inflation has been developed. Cost minimization technique is used to get the approximate expressions for total cost and other parameters. A numerical example and sensitivity analysis are presented to illustrate the model. when only rented or own warehouse is considered, the present value of the total relevant cost is higher than the case when two warehouse is considered.

From the sensitivity analysis, it can be shown that the total cost of the system is influenced by the deterioration rate, the inflation rate and the backlogging ratio.

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Local Observability of Analytic Systems on Time Scales

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A time scale is a closed subset \mathbb{T} of the set \mathbb{R} of real numbers. It is a model of time, which can be continuous ($\mathbb{T} = \mathbb{R}$), discrete (e.g. $\mathbb{T} = \mathbb{Z}$ - the set of integers, or $\mathbb{T} = \{q^n : n \in \mathbb{N}\}$ - the quantum scale), or mixed (e.g. a union of closed intervals). Calculus on time scales, based on the concept of delta derivative, is a unification of the classical differential calculus and the calculus of finite differences. Nabla differential equations generalize differential and difference equations (see e.g. [4]).

Let \mathbb{T} be an arbitrary time scale. We are interested in local observability of a control system Σ with output: $x^\Delta(t) = f(x(t), u(t))$, $y(t) = h(x(t))$, where $t \in \mathbb{T}$, $x(t) \in \mathbb{R}^n$ (state), $u(t) \in \mathbb{R}^m$ (control or input), $y(t) \in \mathbb{R}^r$ (observation or output) and $x^\Delta(t)$ is the delta derivative of x at time t . Local observability at a point $x_0 \in \mathbb{R}^n$ is defined in the same way as for continuous-time or discrete-time systems.

Similarly as in the continuous-time and discrete-time cases we define the observation algebra $H(\Sigma)$ of the system Σ . It is an algebra of analytic functions on \mathbb{R}^n . Let I_{x_0} denote the ideal of the algebra \mathcal{A}_{x_0} of germs of analytic functions at x_0 that consists of the germs of functions from $H(\Sigma)$ that vanish at x_0 . Then I_{x_0} is contained in the unique maximal ideal m_{x_0} of \mathcal{A}_{x_0} . Let $\sqrt[\mathbb{R}]{I_{x_0}}$ denote the real radical of the ideal I_{x_0} .

Theorem. The system Σ is locally observable at x_0 if and only if $\sqrt[m]{I_{x_0}} = m_{x_0}$.

It is an extension of the characterizations of local observability obtained for continuous-time ([1, 3]) and discrete-time ([2]) systems.

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Some Study on Fuzzy Control Theory: Special Attention on Queuing Control at Air and Rail Traffic Network

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 93C42

Keywords. Fuzzy control, Train status, Late status, Waiting time etc.

The aim of this paper is to develop a fuzzy control system with a smaller rule base for queuing control in traffic system. Here we have considered the control of queue in rail and air traffic as the field of study. A similar fuzzy control system for road traffic has already been developed and sent to Fuzzy Sets and Systems. Also a comparative study is done throughout this paper related to the other rule bases formed for the traffic system.

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GA based Reliability Optimization in Stochastic Domain

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 90B25

The basic objective of a reliability allocation model is to assign reliability to sub-systems so as to arrive at a prefixed reliability goal for the system as a whole, subject to various constraints operating on the system/subsystems. Earlier, heuristic methods, reduced gradient method, dynamic programming method and branch and bound method were used to solve such reliability allocation problems. However, with the advent of genetic algorithm and other numerical optimization methods, researchers have started paying more attention on reliability optimization via numerical methods. In almost all the works, the design parameters involved in the optimization problem have usually been taken to be constants. However, the design parameters are not constant in nature. These parameters can be viewed as estimated values, which in turn follow certain stochastic laws. Unfortunately, constraints involving these estimated values of the optimization problems are usually solved in the deterministic domain and need to be solved in the stochastic domain. Further, distributional parameters may not be of single value. They may be allowed to vary over an interval to take care of the sensitivity of the factor market. Keeping these considerations in the backdrop, the reliability optimization problem can be best described as a problem of chance constraints with distributional parameters assuming interval

values. Study of the system reliability where the component reliabilities are imprecise and/or interval valued has already been initiated by some authors (see [1], [2]). The goal of this work is to examine the redundancy allocation problem under imprecise reliability with constraints, expressed in terms of coefficient matrix and availability vectors, as chance constraints. Even for the random coefficient matrix and availability vector, it is proposed to consider interval valued means and variances so that the optimization problem can be dealt with under a generalized setup and specific solutions can be arrived at by collapsing an interval valued parameter into a point.

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Comparison of two Hemivariational Control Problems and Convergence of their Galerkin Approximation

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In this paper we shall consider two nonlinear and nonmonotone optimal control problems governed by parabolic hemivariational inclusions in $W(0, T) = \{y \in L^2(0, t; V) : y' \in L(0, T; V)\}$, where V is a real reflexive Banach space,

$$\begin{cases} y'(t) + A(t)y(t) + \chi(t) = f(t) & a.e. t \in (0, T) \\ y(0) = y_0 \\ \chi(x, t) \in \tilde{\beta}(x, t, u(x, t), y(x, t)) & a.e. (x, t) \in Q \end{cases}$$

and

$$\begin{cases} y'(t) + A(t)y(t) + \chi(t) = (Bu)(t) & a.e. t \in (0, T) \\ y(0) = y_0 \\ \chi(x, t) \in \hat{\beta}(x, t, y(x, t)) & a.e. (x, t) \in Q \end{cases}$$

where the time derivative is understood in the sense of distribution and u is the control variable. The multivalued functions $\tilde{\beta}$ and $\hat{\beta}$ are nonmonotone and

include the vertical jumps. The operator A is assumed to be monotone and satisfy certain coerciveness and boundeness hypotheses. These problems arise in many important real-life models of control.

The optimal control problem formulation is to find an optimal pair (u_0, y_0) which minimizes a continuous convex and coercive functional J .

After giving some results on the existence of an optimal control, we treat the optimization problem by Galerkin approximation. Then we prove the convergence of optimal values for approximated optimization problems to the ones for the original problem. Finally we compare the results received in that two cases of control.

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Optimal Economic Production Quantity Policy for an Imperfect Production System of Ameliorating Items

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It has been observed that most of the classical EPQ models assumed perfect production. But it is not necessary that all the units of an item produced are of perfect quality. There may be the mixture of perfect and imperfect quality. This type of production falls in the category of imperfect production process. In the present paper, we develop the model for imperfect production/inventory system of ameliorating items with time varying demand. Here, ameliorating term is used in the reference of the items whose value or utility or quantity increases with time. The unit production cost is taken to be a convex function of the production rate which is also a variable. The mathematical expression for the expected profit function is derived and the effects of amelioration on

the inventory replenishment policies are studied with the help of numerical examples.

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Approximate Optimality Conditions for Minimax Programming Problems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 90C46, 90C26, 49K35

The Lagrange multiplier rules and the KarushKuhnTucker (KKT) necessary optimality conditions lie at the heart of non-linear optimization. To derive the necessary optimality conditions of KKT type in which the Lagrange multiplier associated with the objective function is non-zero, one needs to impose some kind of constraint qualification. However, in absence of constraint qualifications, the KKT optimality conditions may fail to hold.

In this work, we consider non-smooth Lipschitz programming problems with set inclusion and abstract constraints. Here we develop approximate optimality conditions for minimax programming problems in absence of any constraint qualification. The optimality conditions are worked out not exactly at the optimal solution but at some points in a neighbourhood of the optimal solution. For this reason, we call the conditions as approximate optimality conditions. One

may wonder as to why we do not aim to achieve the desired scenario of optimality conditions at the exact optimal solution instead of approximate optimality conditions. The reasons are twofold. Firstly (purposefully) we want to avoid constraint qualification and secondly the approximate optimality rules seem to be more advantageous in the sense that they may allow us to work with the gradients of the functions rather than their subdifferentials. It all sounds very promising, however, even for a locally Lipschitz function the proximal subdifferential may turn out to be an empty set. This bottleneck can be improved by moving to the limiting subdifferential which is non-empty and possess good calculus rules. We extend the results in terms of the limiting subdifferentials in presence of an appropriate constraint qualification thereby leading to the optimality conditions at the exact optimal point.

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Quadratic Order Optimality Conditions for Extremals Completely Singular in Part of Controls

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 49K15

For the control system $\dot{x} = f(t, x, u) + F(t, x)v$, consider the problem of minimizing the cost $J = \varphi_0(x(0), x(T))$ under constraints $\varphi(x(0), x(T)) \leq 0$ and $\eta(x(0), x(T)) = 0$, where $x \in R^n$, $u \in R^{r_u}$, $v \in R^{r_v}$, $\varphi \in R^m$, $\eta \in R^s$.

Take a process $w^0 = (x^0(t), u^0(t), v^0(t))$ satisfying the first order necessary conditions for a weak minimum with a unique collection of multipliers. Let Ω be the second variation of the corresponding Lagrange function, and K be the cone of critical variations $\bar{w} = (\bar{x}, \bar{u}, \bar{v})$, i.e. those satisfying the linearized relations of the problem. Define the quadratic order of minimum

$$\gamma(\bar{x}, \bar{u}, \bar{v}) = |\bar{x}(0)|^2 + \int_0^T (|\bar{u}|^2 + |\bar{y}|^2) dt + |\bar{y}(T)|^2, \quad \dot{\bar{y}} = \bar{v}, \quad \bar{y}(0) = 0.$$

Here \bar{y} is the variation of an additional, artificial state variable. The control variation \bar{v} does not come explicitly in γ .

Theorem 1. *a)* Let w^0 provide a weak minimum with respect to both u and v (weak–weak minimum). Then $\Omega(\bar{w}) \geq 0$ for all $\bar{w} \in K$. In particular, Ω satisfies the Legendre condition with respect to u (i.e. $-H_{uu} \geq 0$), the Goh conditions with respect to v (see [1, 2]).

b) Let Ω satisfy the above Legendre and Goh conditions, and also $\exists a > 0$ such that $\Omega(\bar{w}) \geq a\gamma(\bar{w})$ for all $\bar{w} \in K$. Then w^0 provides a strict weak–weak minimum. Moreover, in some uniform neighborhood of w^0 the increment of the cost is estimated from below by γ .

These conditions are close to each other with no gap between them. We also consider a weak minimum with respect to u and a so-called Pontryagin minimum with respect to v (weak–Pontryagin minimum). Theorem 1 remains valid if the Legendre and Goh conditions are complemented by some condition of equality type on the third variation of the Lagrange function with respect to x , v (see [1, 2]).

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Optimal Life Insurance, Consumption and Investment

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 91G10, 91G80, 93E20, 90C39

We propose an extension to Merton's famous continuous time model of optimal consumption and investment, in the spirit of previous works by Pliska and Ye, to allow for a wage earner to have a random lifetime and to use a portion of the income to purchase life insurance in order to provide for his estate, while investing his savings in a financial market composed by one risk-free security and an arbitrary number of risky securities whose diffusive term is driven by a multi-dimensional Brownian motion. The wage earner's problem is then to find the optimal consumption, investment, and insurance purchase decisions in order to maximize expected utility of consumption, of the size of the estate in the event of premature death, and of the size of the estate at the time of retirement. We use dynamic programming methods to obtain explicit solutions for the case of constant relative risk aversion utility functions. We obtain new theoretical results for this class of utility functions and provide the corresponding economic interpretations with the help of suitable numerical examples.

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Revisiting Optimality Conditions in Convex Programming

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 90C25

The phrase convex optimization refers to the minimization of a convex function over a convex set. However the feasible convex set need not be always described by convex inequalities. In this article we consider a convex feasible set which are described by inequality constraints which are locally Lipschitz and not necessarily convex and need not be smooth. We show that if the Slater's constraint qualification and a simple non-degeneracy condition is satisfied then the Karush-Kuhn-Tucker type optimality condition is both necessary and sufficient.

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Characterizing the Lagrangian Strong Duality in Constrained Nonconvex Optimization

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 90C30, 41A65, 52A07

After establishing a general Gordan-type alternative theorem involving cones with possibly empty interior (generalizing that in [2]), we apply it to characterize the Lagrangian strong duality in nonconvex optimization problems. Thus, we revise those results in [1]. The case with a single constraint is particularly analyzed. If time allows, applications to vector optimization will be presented as well.

This is a joint work ([3]) with Cristián Vera from Universidad Católica de la Santísima Concepción, Concepción.

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Sequential Optimality Conditions for Smooth Constrained Optimization

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 90C30, 49K99, 65K05

Sequential optimality conditions provide adequate theoretical tools to justify stopping criteria for nonlinear programming solvers. We present new sequential optimality conditions related to the Approximate Gradient Projection condition (AGP [4]). When there is an extra set of linear constraints, we define a linear-AGP condition and prove relations with CPLD and KKT conditions. The CPLD [2] is a new constraint qualification strictly weaker than MFCQ and CRCQ. Similar results are obtained when there is an extra set of convex constraints. We define approximate KKT conditions and prove relations to AGP-like conditions. We provide some further generalizations and relations to an inexact restoration algorithm [3].

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Optimal Control Problem of Some Hemivariational Inclusion-Galerkin Approximation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 49J24, 49J40, 49J53

We study the optimal control problem governed by a parabolic hemivariational inclusion. We derive some results on the existence of optimal solutions. Then we introduced Galerkin approximation and prove the convergence of optimal values for approximated control problems to the one for the original problem. Finally, we give a simple example.

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Bottleneck Product Rate Variation Problem with a General Objective

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Mixed-model just-in-time (MMJIT) production system is controlled by setting an evenly distributed sequence of different products. Sequencing of different products with even distribution works when there is a constant rate of usage of all parts. The rate of usage of parts is kept as constant as possible under the consideration of the demand rates of the products. The sequencing problem is called the mixed-model just-in-time sequencing problem (MMJITSP).

The MMJIT production system consists of a hierarchy of a finite and distinct levels. The sequence at the final level is crucial and affects the entire supply chain as all other levels are also inherently fixed because of the pull nature of the system. The MMJITSP only with the final level is the product rate variation problem (PRVP). This is the minimization of the variation in the rate at which different products are produced on the line. This problem with bottleneck objective is the bottleneck PRVP [1].

The bottleneck PRVP with the objective of absolute deviation between the actual and the ideal productions has been solved in pseudo-polynomial time [2]. In this presentation, the bottleneck product rate variation problem with a general objective function is solved and a relation between optimal solutions of the problem with different objective functions is established.

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Inverse Problems for Ill-Posed Variational and Quasi Variational Inequalities

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Numerous mathematical models in applied and industrial mathematics take the form of variational and quasi variational inequalities involving certain coefficients. These coefficients are known and they often are associated to some physical properties of the model. The direct problem in this context is to solve the variational or the quasi-variational inequality. By contrast, an inverse problem asks for the identification of the coefficients when certain measurement of a solution to the variational or quasi variational inequality is available.

This talk will focus on the inverse problem of identification of certain variable parameters in ill-posed variational and quasi variational inequalities. Regularization will be used to handle the data perturbation as well as non-coercive operators. Applications of our results to the identification of variable parameters in partial differential equations will also be discussed. Finite element based numerical examples will be presented. This talk is supported by an AMS/NSF travel grant.



Local Minima, Marginal Functions, and Separating Hyperplanes in Discrete Optimization

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 49K10, 46A22

We prove results in optimization theory of two integer variables which correspond to fundamental results in convex analysis of real variables, viz. that a local minimum of a convex function is global; that the marginal function of a convex function is convex; and that two disjoint convex sets can be separated by a hyperplane. We show by simple examples that none of these fundamental results holds for functions which are restrictions to \mathbf{Z}^2 of convex functions defined on \mathbf{R}^2 . But for a class of functions of two discrete variables called integrally convex functions there are perfect analogues of the three results.

Define a difference operator D_a for $a \in \mathbf{Z}^2$ by $D_a f(x) = f(x+a) - f(x)$, $x \in \mathbf{Z}^2$, $f: \mathbf{Z}^2 \rightarrow \mathbf{R}$.

A function $f: \mathbf{Z}^2 \rightarrow \mathbf{R}$ is said to be *integrally convex* [1, 2] if it satisfies $D_b D_a f \geq 0$ for all $(a, b) \in \mathbf{Z}^2 \times \mathbf{Z}^2$ with $a = (1, 0)$, $b = (1, -1)$, $(1, 0)$, $(1, 1)$ as well as $a = (0, 1)$, $b = (-1, 1)$, $(0, 1)$, $(1, 1)$.

If, given a point $p \in \mathbf{Z}^2$, an integrally convex function satisfies $f(x) \geq f(p)$ for all x such that $\|x-p\|_\infty \leq 1$, then it satisfies $f(x) \geq f(p)$ for all x . Actually sometimes a smaller neighborhood can suffice [2].

For any integrally convex function $f: \mathbf{Z}^2 \rightarrow \mathbf{R}$, its marginal function $h(x) = \inf_{y \in \mathbf{Z}} f(x, y)$, $x \in \mathbf{Z}$, is convex.

Given two integrally convex functions $f, g: \mathbf{Z}^2 \rightarrow \mathbf{R}$, consider the sets

$$A = \{(x, y, z) \in \mathbf{Z}^3; z \geq f(x, y)\}, \quad B = \{(x, y, z) \in \mathbf{Z}^3; -g(x, y) \geq z\}.$$

Then there exists a plane $z = H(x, y)$ separating A and B , i.e., there is an affine function $H: \mathbf{R}^2 \rightarrow \mathbf{R}$ such that $f \geq H|_{\mathbf{Z}^2} \geq -g$, if and only if $f_{\frac{1}{2}} + g_{\frac{1}{2}} \geq 0$, where $f_{\frac{1}{2}}: \mathbf{Z}^2 \cup (\mathbf{Z} + \frac{1}{2})^2 \rightarrow \mathbf{R}$ is defined for $(x, y) \in \mathbf{Z}^2$ by $f_{\frac{1}{2}}(x, y) = f(x, y)$ and

$$f_{\frac{1}{2}}(x + \frac{1}{2}, y + \frac{1}{2}) = \frac{1}{2} \min [f(x, y) + f(x+1, y+1), f(x+1, y) + f(x, y+1)].$$

Work on more than two variables is in progress.

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Controllability of Second Order Volterra Integrodifferential Equations with Nonlocal Conditions

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 93B05

In this paper, we study the controllability of second order volterra integrodifferential systems with nonlocal initial conditions by using Banach fixed point theorem and the theory of strongly continuous cosine family.

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Approximate Controllability of Non-densely Defined First Order Semilinear Integrodifferential Control Systems

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In this paper, we prove some sufficient conditions for the controllability of non-densely defined first order semilinear integrodifferential control systems. Consider the semilinear control system

$$\begin{aligned} x'(t) &= Ax(t) + Bu(t) + \int_0^t f(t, s, x(s))ds; & 0 \leq t \leq T \\ x(0) &= x_0 \end{aligned}$$

where $x : [0, T] \rightarrow V$ is the state function, $u : [0, T] \rightarrow U$ is the control function. Let $Z = L_2[0, T; V]$ and $Y = L_2[0, T; U]$ be function spaces and $B : Y \rightarrow Z$ a bounded linear operator, $f : [0, T] \times [0, T] \times V \rightarrow V$ a nonlinear operator and $A : D(A) \subset V \rightarrow V$ a closed (not necessarily bounded) linear operator whose domain need not be dense in V , that is, $\overline{D(A)} \neq V$.

In this work, using the Schauder fixed point theorem, we establish the controllability for a class of abstract first order semilinear integrodifferential system, where the linear part satisfies the Hille-Yosida condition. An example is provided to illustrate the theory.

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Optimization of Vehicle Routing Problem with Stochastic Demand by Variant of Ant Algorithm

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One of the important issues in this competitive world, for the decision makers of manufacturing firms, has been the efficient methodologies and the selection of best possible delivery routes. There are different types of vehicle routing problems (VRP) prevailing in the literature. Taking into account the real world applications, the vehicle routing problem has been considered with stochastic demand (VRPSD) in which the customer demand has been modeled as a stochastic variable. To solve VRPSD model an efficient metaheuristic procedure based on traditional Ant Colony Optimization (ACO) which is based in the pheromone strategy, inspired in the natural Ants behavior. Ants are inclined to move over the edges with higher pheromone concentration and this behavior is assigned to vehicles. Considering the computational complexity of the problem and to enhance the algorithm performance a neighborhood search embedded Adaptive Ant Algorithm (ns-AAA) is proposed as an improvement of the existing Ant Colony Optimization. ANalysis Of VAriance (ANOVA) is performed to determine the impact of various factors on the objective function value. This algorithm demonstrates better results and proved to be a competitive method related with other metaheuristics. A new method has been proposed to solve the Multi Depot Vehicle Routing Problem (MDVRP) and a modified Ant Algorithm has also been proposed. The effectiveness of Ant Algorithm is compared and the results are proved to be competitive. A comprehensive modeling approach towards stochastic vehicle routing is studied by Bertsimas [1]. In existing

VRPSD models, ACO has empirically shown superior results (Reimann [2]) in resolution of computational complexity. Chao et al.[3] provides a review of the previous heuristics for the multi-depot vehicle routing problem in the operations research literature and introduces a new heuristic, as well.

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Optimal Control Applied to the Ecstasy Model with Peer Pressure

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We apply the optimal control theory to a model of the peer-driven dynamics of ecstasy use. Ecstasy use has continued to be in raves and nightclubs in recent years and the reduction of ecstasy use has become one of the important issues in society. Our goal is to minimize the ecstasy use class and the cost. Optimal control is characterized in terms of the solution of optimality system, which is the state system coupled with the adjoint system and the optimality equations. The numerical simulations show the ideal (optimal) prevention policies of ecstasy use in various scenarios.



Optimal Control of the Inductive Heating Process of Oil-well Casing in Cold, Intermediate and Hot Modes

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Mathematical model of the inductive heating process of oil-well casing is represented as the equation of heat conduction that involved the control action with the boundary conditions and the criteria of heat quality. The performance criterion is energy functional that involves penalty function with penalty parameters governing heat quality during the process [1]. Control action satisfies the constraints according to the physical significance of the process of inductive heating. The optimal control action structure for the internal source heat intensity in cold, intermediate and hot modes is obtained in the paper. It is

performed using the necessary condition of optimality for distributed systems [2] and application of optimization method with internal control actions [3].

The practical problem of optimal inductive heating of oil-well casing in hot mode is solved numerically in this work. Numerical computation is carried out by means of integro-interpolation method using Crank-Nikolson scheme [4]. Numerous experiments and analysis allow tracing the dependence of minimized functional on the penalty parameters and get its three-dimensional plots. The analysis prompts to assign penalty parameters which provide optimal heat mode. According to the results of the carried research recommendations are given how to use them in practice.

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High Order Sufficient Conditions for Tracking Some Mechanical Control Systems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 34H05, 52A27, 93B27, 93C15, 93C95

In control theory there are different problems that can be studied. In this contribution we focus on the tracking problem for mechanical control systems given by an affine connection as described in [2]. The tracking techniques are used either to design suitable feedback control laws or to achieve a particular trajectory within a relatively small error.

In the literature [2] there exist sufficient conditions for tracking affine connection control systems. However, in [3] it was found a control system associated

with a submarine where tracking was feasible, even though those sufficient conditions were not fulfilled. This leads us to generalize the results in [2] that are given in terms of the finite family of the control vector fields using only symmetric products up to degree two.

In this contribution we will describe how to approximate the target trajectories by means of the solutions to the original control system. In order to achieve this we use chronological calculus [1] and some results from average theory [4]. As a result we are able to state new sufficient conditions for tracking. These new conditions are of higher order because symmetric products of higher order of the control vector fields are involved. Moreover these conditions are given in terms of an infinite family of vector fields in contrast to the finite family vector fields known in the literature.

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An Inventory Model for Fair Services of Internet Traffic

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It has been observed that the Internet fair service models use some variant of fair queueing like Deficit Round Robin (DRR) for traffic flow scheduling. Each flow corresponds to a particular differentiated service and its constituent packets get treated accordingly at each network node. It is assumed that per flow Service Level Agreement (SLA) may be achieved using this approach. But there are certain data flows which lose their meaning if they get imperfect service due to any network condition. In this paper, we develop an inventory model for fair services of Internet traffic using DRR service discipline. The different SLA schemes have been analysed for traffic flows for establishing a mathematical model. The analysis proposes a new signalling mechanism for inference of drawbacks in SLA implementation.

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Topological Methods in Controllability Problems for Some Classes of Systems Governed by Differential Inclusions in Banach Spaces

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 93B05, 34A60, 34G25, 34H05, 47H08, 47H11

In the present survey we consider the controllability problem for systems governed by semilinear differential inclusions in a Banach space E of the form

$$y'(t) \in Ay(t) + F(t, y(t)) + Bu(t).$$

Here A is a closed linear operator in E generating the strongly continuous semigroup e^{At} , F is a multivalued nonlinearity, u is a control function and B is a bounded linear operator. We do not assume the compactness of the semigroup e^{At} , but we suppose that F satisfies a regularity condition expressed in the terms of the Hausdorff measure of noncompactness in E .

We present the multivalued operator whose fixed points are solutions of the controllability problem and describe its properties. It allows to apply the topological degree theory for condensing operators and to obtain the controllability results for both upper Carathéodory and almost lower semicontinuous types of nonlinearity under various growth conditions. As application we consider the controllability for a system obeying a perturbed wave equation.

Some extensions are given to the cases of systems governed by degenerate (Sobolev type) inclusions and functional differential inclusions in presence of impulse effects and (finite or infinite) delays.

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Weak Compactness in the Dual Space of a JB*-triple is Commutatively Determined

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The following criterium of weak compactness in the dual of a JB*-triple is obtained in [1] and [2]: a bounded set K in the dual of a JB*-triple E is not relatively weakly compact if and only if there exist a sequence of pairwise orthogonal elements (a_n) in the closed unit ball of E , a sequence (φ_n) in K , and $\vartheta > 0$ satisfying that $|\varphi_n(a_n)| > \vartheta$ for all $n \in \mathbb{N}$. Consequently, a bounded subset in the dual space of a JB*-triple, E , is relatively weakly compact whenever its restriction to any abelian subtriple of E is.

This result generalizes the characterization of weak compactness in the dual of a C*-algebra obtained by H. Pfizner in [3].

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Some Method and Algorithm for Solving D.C Programming

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Global Optimization plays an important role in theory of optimization and applications. Many engineering and economics problems can be formulated as global optimization problems[3]. We consider so called D.C programming problem (difference of two convex functions) which belongs to a class of global optimization. Based on the global optimality conditions by Strekalovsky[4, 5], we derive global optimality conditions for D.C programming. We reduce the problem to concave programming and propose some algorithm and method for solving it. The subproblems of the proposed algorithm are convex optimization problems. Also, we show that indefinite quadratic programming problem can be reduced to D.C programming. Some numerical results are provided.

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Time for Un-Doing! What is the Mantra? Inversion Techniques and Optimization

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The research on forward numerical models, by large, has almost reached a saturation, wherein, the limitations if existed, have to coexist. It is the data assimilation techniques that provide a hope to minimize the numerical limitations using Inversion Techniques.

Optimization is the prescribed medicine from Vedas(Ati Sarvatra Varjayet). It is the need of the hour, which enforces a control of the process for ensuring a kind of balance in any field. Numerical modeling is not an exception.

In this paper, the Inversion and Optimization techniques used in Meteorology and Ocean related fields are highlighted. The deficiencies of various numerical forward models, the effective and efficient way they can be minimized using above techniques are discussed. Improvements that can only be made possible with mathematicians are sought.



An Adaptive Tracking Control of Robot Manipulators in the Task-space under Uncertainties Using Neural Network

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 93C85, 68T40

In this paper, neural network (NN) based controller for the tracking control of robot manipulators in the task-space under uncertainties, is considered. Especially, this controller does not need the prior information of the upper bound of the unstructured uncertainties. By adaptively estimating the upper bound by using feedforward neural network (FNN), effects of unstructured uncertainties can be eliminated and asymptotic error convergence can be obtained for

the closed-loop system. Simulation studies are carried out for a two-link elbow robot manipulator to show the effectiveness of the control scheme.

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The Structured Distance to Non-surjectivity and Its Application to Calculating the Controllability Radius of Linear Systems under Multi-perturbations

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The classical Eckart-Young formula for square matrices [2] identifies the distance to singularity of a matrix. The main purpose of this paper is to get generalizations of this formula. We characterize the distance to non-surjectivity of a linear operator $W \in L(X, Y)$ in finite-dimensional normed spaces X, Y , under the assumption that the operator W is surjective (i.e. $WX = Y$) and subjected to structured perturbations of the form $W \rightarrow W + E\Delta D$. The proof of the main result is based on the theory of multi-valued linear operators [1]. As an application of these results, we derive formulas of the distance $r(A, B)$ from a linear controllable system $\dot{x} = Ax + Bu, t \geq 0, x \in C^n, u \in C^m$ to the nearest uncontrollable system under structured perturbations $[A, B] \rightarrow [A, B] + E\Delta D$. The main result reads

$$r(A, B) = \frac{1}{\sup_{\lambda \in C} \|EW_{\lambda}^{-1}D\|},$$

where $W_{\lambda}^{-1} = [A - \lambda I, B]^{-1} : C^n \times C^m \rightarrow C^n$ is a multi-valued linear operator. The results are extended to the more general case of multi-perturbations

$[A, B] \longrightarrow [A, B] + \sum_{i=1}^N E_i \Delta_i D_i$. Our results unifies and covers some well-known results. In particular, a recent result due to Karow and Kressner [3] is shown to be a consequence of our result.

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On the Optimal Control Problem and Galerkin Approximation for an Extensible Beam Equation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 49J20, 49M15

We discuss the optimal control problem governed by equations

$$\begin{aligned} \frac{\partial^2 y}{\partial t^2} + \Delta^2 y - \left(\alpha + \beta \int_{\Omega} |\nabla y|^2 dx \right) \Delta y &= f + Bu \quad \text{on } S \times \Omega \\ y(t, x) = \frac{\partial y(t, x)}{\partial n} &= 0 \quad \text{on } S \times \Gamma \\ y(0, x) = y_0(x) \text{ and } \frac{\partial y(0, x)}{\partial t} &= y_1(x) \text{ on } \Omega \end{aligned}$$

where $x \in \Omega \subset R^n$, $t \in S = (0, T)$, u is a function representing the control actions, and Γ is boundary of Ω , and n is a normal vector to Γ . The quadratic cost function is classical.

We present the Galerkin approximation of this optimal control problem and we study the approximate family of control problems. The condensation points of a set of solutions of the approxiamte optimization problems are the solutions of the initial optimization problem.

The main result of our paper are the theorems of convergence of optimal values for control problems approximated by the Galerkin method to the one for orginal problem.



On the Optimality of an Initial Data for Delay Differential Equations

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Let $\tau_2 > \tau_1 > 0$ and $t_3 > t_2 > t_1$ be given numbers with $t_3 - t_2 > \tau_2$; suppose that $O \subset R^n$ is an open set and the n -dimensional function $f(t, x, y)$ is continuous on the set $[t_1, t_3] \times O^2$ and continuously differentiable with respect to x and y ; next, let Δ be a set of measurable initial functions $\varphi(t) \in \Phi, t \in [t_1 - \tau_2, t_2]$, where $\Phi \subset O$ is a compact set; let $X_0 \subset O$ be a compact and convex set of initial vectors x_0 .

Under initial data we imply the collection of initial moment $t_0 \in [t_1, t_2]$ and initial vector $x_0 \in X_0$, delay parameter $\tau \in [\tau_1, \tau_2]$ and initial function $\varphi(\cdot) \in \Delta$.

For each initial data $w = (t_0, \tau, x_0, \varphi(\cdot)) \in W = [t_1, t_2] \times [\tau_1, \tau_2] \times X_0 \times \Delta$ we assign the delay differential equation

$$\dot{x}(t) = f(t, x(t), x(t - \tau)), t \in [t_0, t_1]$$

with the initial condition

$$x(t) = \varphi(t), t \in [t_0 - \tau, t_0], x(t_0) = x_0.$$

An initial data $w = (t_0, \tau, x_0, \varphi(\cdot)) \in W$ is said to be admissible if there exists the corresponding solution $x(t) = x(t; w), t \in [t_0, t_3]$ satisfying the conditions $q^i(t_0, \tau, x_0, x(t_3)) = 0, i = \overline{1, l}$. We denote the set of admissible initial data by W_0 .

An initial data $w_0 = (t_{00}, \tau_0, x_{00}, \varphi_0(\cdot)) \in W_0$ is said to be optimal if for arbitrary $w \in W_0$ the following inequality holds

$$q^0(t_{00}, \tau_0, x_{00}, x_0(t_3)) \leq q^0(t_0, \tau, x_0, x(t_3)).$$

Here $x_0(t) = x(t; w_0), x(t) = x(t; w)$ and $q^i(t_0, \tau, x_0, x_1), i = \overline{0, l}$ are continuously differentiable on the set $[t_1, t_2] \times [\tau_1, \tau_2] \times X_0 \times O$.

In this work, the existence theorems of an optimal initial data and necessary optimality conditions for initial data are proved.

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Quantum Theoretical and Computational Optimal Control

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 81Q99

In this communication, quantum control for nucleus is considered in theoretical and computational issues. Particularly, for the quantum dynamics described by Yukawa interaction, we proceed the study in the framework of variational method in Hilbert spaces. Resultant demonstration is well verified the theoretic conclusion with numerical simulation (cf. [1, 2, 3, 4]).

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Asymptotic Analysis of Portfolio Optimization with Stochastic Volatility

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 49L99

In this paper, we study a portfolio optimization problem under CEV model with stochastic volatility. we use an Asymptotic Analysis to extend the constant volatility case of Gao(2009) to obtain the effective volatility result as a leading order term and the correction effect due to the fast mean-reversion of the stochastic volatility.



Section 18

Mathematics in Science and Technology

Effect of Longitudinal Roughness on Magnetic Fluid Based Squeeze Film between Truncated Conical Plates

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An attempt has been made to study and analyze the performance of a magnetic fluid based squeeze film between rough truncated conical plates. The lubricant used here is a magnetic fluid and the external magnetic field is oblique to the lower plate. The bearing surfaces are assumed to be longitudinally rough. The roughness of the bearing surfaces is modeled by a stochastic random variable with nonzero mean, variance and skewness. Efforts have been made to average the associated Reynolds equation with respect to the random roughness parameter. The concerned non-dimensional equation is solved with appropriate boundary conditions in dimensionless form to obtain the pressure distribution. This is then used to get the expression for load carrying capacity, resulting in the calculation of response time. The results are presented graphically. It is observed that the bearing system registers an improved performance as compared to that of a bearing system dealing with a conventional lubricant. The results indicate that the pressure, load carrying capacity and response time increase with increasing magnetization parameter. This investigation reveals that the

standard deviation induces a positive effect. Besides, negatively skewed roughness increases the load carrying capacity and this performance further enhances especially when negative variance is involved. Although, aspect ratio and semi-vertical angle tend to decrease the load carrying capacity, there is a scope for obtaining better performance in the case of negatively skewed roughness.



The Effect of Pore Alignment on Seismic Reflection Amplitudes

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The theory developed by [1] is employed to study the phenomena of reflection and transmission when a plane elastic wave (longitudinal/transverse) becomes incident at the interface between two dissimilar isotropic porous media containing two immiscible fluids. The possible extent of connections between the surface pores of two solids at their common interface is discussed and the effect of connections on different reflected and transmitted waves is studied. Partition of incident energy among various reflected and transmitted waves is also studied. Numerical example calculates the amplitude and energy ratios of reflected and transmitted waves at a plane interface between sandstone containing air-water mixture and a lime stone containing mixture of kerosene-water.

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Mathematical Aspects in Complexity of Biological, Neuropsychological and Psychological Systems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 92B20

We analyse the living system belonging to the human being using (ANs) networks, within the biostructural theory (MBt), through formal mathematical aspects regarding the configuration and functionality of noesistructure (brain-structure). The (ANs) are networks of multidimensional hierarchic evolution, with various ranks. Their complexity varies horizontally, within the same level and vertically, from the lower to the upper level. The biostructural theory (MBt) considers the hierarchic levels of the living matter: 1-coexisting molecular matter; 2-biostructure (the spongy mass); 3-noesistructure (noesismass and coexisting biosic matter (the cortex and the cerebral hemispheres)). The effect of phenomenological functions is quantified by:

$$I_i^{j,j+1(k)} = \int_{\Omega_i^{j,j+1(k)}} ((\partial\varphi_i^{j,j+1(k)} / \partial x) dx + (\partial\varphi_i^{j,j+1(k)} / \partial \tau) d\tau) + C_i^{j,j+1(k)};$$

$$I_{i,i+1}^{(k)} = \int_{\Omega_{i,i+1}^{(k)}} ((\partial\varphi_{i,i+1}^{(k)} / \partial x) dx + (\partial\varphi_{i,i+1}^{(k)} / \partial \tau) d\tau) + C_{i,i+1}^{(k)}$$

x vector of spatial coordinates; τ time. ANs^(k) networks ($k = 1, 2, 3$) associated to three levels of living matter, identified within the biostructural theory (MBt). ANs^(k) complexity: $C_{ANs}^{(k)} = A^{j(k)} + B^{(k)} + A^p(k)$, in which:

$$A^{j(k)} = \sum_{i=0}^3 \sum_{j=0}^{p-1} \left[\int_{S_i^{j(k)}} f_i^{j(k)} dS_i^{j(k)} + \int_{\Omega_i^{j,j+1(k)}} ((\partial\varphi_i^{j,j+1(k)} / \partial x) dx + (\partial\varphi_i^{j,j+1(k)} / \partial \tau) d\tau) + C_i^{j,j+1(k)} \right]$$

$$B^{(k)} = \sum_{i=0}^2 \left[\int_{\Omega_{i,i+1}^{(k)}} (\partial\varphi_{i,i+1}^{(k)} / \partial x) dx + (\partial\varphi_{i,i+1}^{(k)} / \partial \tau) d\tau + C_{i,i+1}^{(k)} \right]$$

$$A^p(k) = \sum_{i=0}^3 \int_{S_i^p(k)} f_i^p(k) dS_i^p(k).$$

Indices k, p, i are shown. The cooperative and hierarchical system of ANs^(k) networks is characterized by the evolution equations: horizontal: $s_i^{j,j+1(k)}(x, \tau) = \varphi_i^{j,j+1(k)}(x, f.., \nabla, \alpha, \tau)$, vertical: $s_{i,i+1}^{(k)}(x, \tau) =$

$\varphi_{i,i+1}^{(k)}(x, f, \dots, \nabla, \alpha, \tau)$, having the solutions: $s_i^{j,j+1(k)}(x, \tau) = S_{0,i}^{j,j+1(k)} + \sum \varepsilon_{st.,i}^{j,j+1(k)}(\tau) \omega_{st.}^{j,j+1(k)}(x) + \sum \varepsilon_{inst.,i}^{j,j+1(k)}(\tau) \omega_{inst.}^{j,j+1(k)}(x)$, respectively: $s_{i,i+1}^{(k)}(x, \tau) = S_{0i,i+1}^{(k)} + \sum \varepsilon_{st.,i,i+1}^{(k)}(\tau) \omega_{st.i,i+1}^{(k)}(x) + \sum \varepsilon_{inst.,i,i+1}^{(k)}(\tau) \omega_{inst.i,i+1}^{(k)}(x)$. Indices k, j, p, i are shown, also, the mathematical aspects in the psychological systems.

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Analytical Solution of Long Waves Generated by Bottom Motion on a Beach with Variable Slope

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2000 MATHEMATICS SUBJECT CLASSIFICATION. (76Bxx)

The problem of tsunami wave generation due to time-dependent arbitrary bottom motion on a beach of uniform slope $y = -qx$ (at equilibrium) is solved by Tuck and Hwang [1] and also by Liu et.al [2]. There seems to have been no attempt so far of finding analytical solution for forced linear long waves on a beach with variable slope. Further the short time analysis of the wave behaviour was not found even on constant slope beaches. In our article an analytical solution is provided for the same problem on a beach of variable slope, $y = -qx^r$, $q > 0$, $r > 0$. To understand the influence of bottom slope on wave elevation and velocity, the problem is studied both at small-time and while at the steady-state assuming a time periodic ground motion $f(x)e^{i\omega t}$. This might be of some importance for the evolution of tsunami waves induced by near-shore earthquakes [3]. Our solution at the steady-state shows a notable feature of no radiation of energy from a finitely distributed time-periodic ground motion for a certain set of values of ω , the circular frequency of the disturbance function. This kind of paradoxical result was first observed by Stoker for steady-state surface waves in infinitely deep water and this peculiar 'resonance' may perhaps be eliminated by assuming small viscosity of the fluid. Although physical settings are different, the generation of long waves by variable atmospheric pressure distribution is analogous to the problem of tsunami formation by bottom displacement [4, 5]. As a result our linear solutions may be used as a benchmark

of various corresponding computational models when one takes into account the non-linear aspect of wave generation not only in steady-state but also in short-time analysis for the first few waves. Finally, we conclude that the characteristics of these forerunners may be useful for tsunami prediction.

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About the Method of Component-based Object Comparison for Objectivity

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2010 MATHEMATICS SUBJECT CLASSIFICATION. Primary: 90B99 Secondary: 68T99.

Keywords. Similarity Analysis, Evaluation, Prediction, Management Decision Making, Linear Programming, Free Datamining Tool.

Our presentation is about the methodology of “COCO” “Component-based Object Comparison for Objectivity” a recently developed Hungarian, Linear Programming based context-free similarity analysis method.

The method investigates the connection between the independent variables X_i , $\underline{X} \in \mathbb{R}^n$ and the depending variable $Y \in \mathbb{R}$ – as regression, but with

a new idea. A certain variable in this method has not got only one constant multiplicative weight in the approximating formula, but the weight is a staircase function of the variable value. The Linear Programming based methodology constructs this staircase functions depending on the approximating formula type (linear, polynomial, multiplicative, mixed, etc.) the error minimization type (linear or nonlinear least squares, etc.) and other parameters (number of the steps in the staircase, etc.).

This datamining method can handle evaluation, benchmarking, forecasting problems from diverse fields [1, 2, 3].

Since there is an available COCO tool on the net [4], we present how to use that tool, we give some examples [5] to show the scope of the method, and we try to specify the theoretical details of it as well [6]. So at our poster we will provide all information about COCO method to the potential users.

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Wave Propagation in a Random Conducting Magneto-generalized-thermo-viscoelastic Medium

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The problem of wave propagation in an interacting random inhomogeneous conducting magneto-generalized-thermo-viscoelastic medium has been studied.

The perturbation technique relevant to stochastic differential equations has been employed to obtain the relation connecting displacement amplitudes of waves propagating in the interacting media. The appropriate Green's tensor essential for discussion has been obtained in course of the analysis. A more general coupled dispersion relation for longitudinal and transverse waves has been deduced to determine the effects of generalized thermal parameters and conductivity on the phase velocity of the coupled waves. The equations have been analyzed for a particular form of thermo-mechanical coupling auto-correlation function to show that the effect (of the order of ε^2 only) of the thermal field is to attenuate the longitudinal type waves and to alter the phase-speed depending upon the values of the viscoelastic parameters and conductivity. Cases of low and high frequencies have also been studied and numerical calculations are being attempted to determine the effects of generalized thermal and viscoelastic parameters, conductivity and thermoelastic coupling on the phase velocity and attenuation coefficients of the waves.



Steven H Strogatz and problem in Nonlinear Dynamics

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It is our tradition to use mathematics in social life and making a better world. I may be a child to say that we can make better world if we can find answer to the question raised by our ancestors (may be mathematicians). I have tried to find the answer to solution of non-linear differential equation, specially in non linear dynamics [1], which is a less transparent example. I found its answers in natural happenings; but how? we can understand it by observing one example: If we drop a stone in still water, ripples will come. If we use differential equation obtained due to this process and try to get computer graphing (trajectory) in abstract space [1] we will get some graph, that graph should match with those ripples in some way. If we follow the reverse path (working back ward) from ripples, the answer is clear. Similarly if bucket full of water is disturbed from bottom, some curves will be formed on the surface of water. The question is that why computer graphing comes like that, it is the miracle of our brain working system and its affiliation with nature. Non linear dynamics and chaos [1], [5] is the important topic in applied mathematics, I have gone through the book written by Steven H. Strogatz [1]. After seeing its one of the graphical work concerning weakly non linear oscillators [1] and comparing with nature's graphing [1], [3], [5] they match each other. I conclude that it is because either earth is oscillating or one of the whole universe is oscillating (depends on time which may be unknown to us) inside which our earth is present. It means that earth

is not only revolving around the sun and spinning around its axis but also some thing more. This backward working can be completely true or completely false, depends on further cooperation with sir Strogatz.

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Intra-cellular Control of Insulin Secretion

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 92C37 (Primary), 35R35 (Secondary)

Patients with diabetes suffer from an absolute or relative lack of the hormone insulin. Insulin is produced by pancreatic β -cells in so-called *regulated exocytosis*. In type 1 diabetes (juvenile diabetes), β -cells are destroyed in auto-immune attacks. In type 2 diabetes and in prediabetic states, we meet a declining β -cell function. Recent advances in observational techniques (ranging from genetic epidemiology and proteomics to multiparameter cell sensing and MRI, ET and nanoparticle-based cell imaging) have brought about the generation of huge new data sets, dealing with ions, DNA, proteins, electrical phenomena, cell membranes, cell organelles, and tissue, in spatial and temporal extreme scale from Ångström to micrometers and from picoseconds to minutes and hours, see [2, 3].

In my talk, I shall briefly describe the common *phenomenological* approach to relate the various data by fancied or statistically more or less well supported ad-hoc assumptions about the regulation. Then I shall advocate for supplementing the phenomenological approach by a *theoretical* approach based

on first principles. As an example, I shall explain how a combination of rigorous geometrical and stochastic methods and electro-dynamical theory naturally draws the attention to fault-tolerant signalling and self-regulation. I consider the making of the fusion pore, anteceding the lipid bilayer membrane vesicle fusion of regulated exocytosis, as a *free boundary problem* and show that one of the applied forces is generated by glucose stimulated intra-cellular Ca^{2+} ions oscillations resulting in a *low-frequent electromagnetic field wave*.

This approach unveils new aspects of the biochemical pathways; provides a new explanation of the basic control of regulated exocytosis; and can direct new model-based measurement in cell analysis. It serves as a *Mathematical Microscope*.

This is joint work with Darya Apushkinskaya (Saarbrücken), Evgeny Apushkinsky (St. Petersburg), and Martin Koch (Copenhagen). For details I refer to [1].

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On a Cyclin Structured Cell Population Model

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 35P05, 92D25

We consider a nonlinear cyclin content structured model of a cell population divided into proliferative and quiescent cells. Under suitable hypotheses, we show existence and uniqueness of a steady state of this model.

We also show, for particular values of the parameters, existence of solutions that do not depend on the cyclin content and hence satisfy an ordinary differential equations system. We analyze the complete asymptotic behavior of this ordinary differential equations system showing that the unique nontrivial steady state (when it exists) is asymptotically stable under some conditions and unstable when the reverse conditions hold. The instability appears through a

Hopf bifurcation which leads to the existence of stable self-sustained oscillations of the populations.

We make numerical simulations for the general case obtaining, for some values of the parameters convergence to the steady state but also oscillations of the population for others.

This is a joint work with Ricardo Borges and Àngel Calsina.

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Stability of Narrow-gap Taylor-Dean Flow with Radial Heating: Stationary Critical Modes

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76Exx, 76E15

A linear stability analysis for Taylor-Dean flow, a viscous flow between concentric cylinders with a pressure gradient acting in the azimuthal direction keeping the cylinders at different temperatures, when the inner cylinder is rotating and outer one is stationary has been implemented. The analysis is made under the assumption that the gap spacing between the cylinders is small compared to the mean radius (small gap approximation).

A parametric study covering wide ranges of β , a parameter characterizing the ratio of representative pumping and rotation velocities and N , the parameter characterizing the direction of temperature gradient ($T_2 - T_1$) is conducted, where T_1 and T_2 are the temperatures of the inner and outer cylinders respectively. The most stable state is always accompanied by keeping the inner cylinder is at higher temperature than the outer one. In the isothermal case ($N = 0$), the flow is most stable near a critical value of $\beta^* = -3.667$, at which the critical wave number (a_c) jumps discontinuously and the discontinuity of a_c corresponds to the fact that the neutral curve consists of two separated

branches occurs precisely at β^* , where there exists an oscillatory axisymmetric mode of approximately equal stability.

Emphasis is given to the occurrence of critical stability for the onset of instability by finding the intersection of the two neutral curves for the inner and outer part in a range of values of the radial temperature gradient $-1.25 < N < 0.25$. We point out the existence of such critical point of stability, where the two neutral curves intersect and disappearance of oscillatory mode when $N = -1.0$.

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Stochastic Growth of Radial Clusters: Weak Convergence to the Asymptotic Profile and Implications for Morphogenesis

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 60G17, 60H15, 82C24, 92C15

The building blocks of mathematical morphogenesis were put several decades ago in the seminal works of Turing [1] and Eden [2]. Their goal was understanding how a macroscopic structure, in particular one breaking the initial homogeneity, could arise out of a multiplicity of simple interactions. While the approach of Turing implied the use of reaction–diffusion equations, Eden concentrated on a probabilistic abstraction of a developing cell colony. In particular, he studied the architecture of a lattice cell colony to which new cells were added following certain probabilistic rules. The objective was studying the

asymptotic colony profile. The original Eden problem can be greatly generalized by means of the use of stochastic partial differential equations. They allow a systematic study of the properties of the colony periphery, particularly of the interface fluctuations. In this work we will summarize our recent progress in this field [3, 4, 5, 6, 7], concentrating on the properties of the realizations of the stochastic growth process. Our goal is unveiling under which conditions the developing radial cluster asymptotically weakly converges to the concentrically propagating spherically symmetric profile.

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Modeling Human Decisions with Game Theory

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 91A35, 91A40, 91E99

We apply Game Theory concepts [6, 7, 8, 9] to the Theory of Planned Behavior [1], that studies the decision-making mechanisms of individuals. We propose the Bayesian-Nash Equilibria as one, of many, possible mechanisms of transforming human intentions in behavior. This process corresponds to the best strategic individual decision taking in account the collective response [2, 4]. We show that saturation, boredom and frustration can lead to splitted strategies, in opposition to no saturation that leads to a constant strategy. Furthermore, we study the role of leaders in individual/group behavior and decision-making [3, 5]. We apply this model to a students success model, describing Nash equilibria, “herding” effects and identifying a hysteresis in the process.

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PDE-constrained Optimization in Application

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 49M, 65K

In this talk efficient numerical method for PDE-constrained optimization problems will be discussed. It is based on simultaneous pseudo-time-stepping in which preconditioned pseudo-unsteady KKT system is integrated in time until a steady state is reached [1, 2, 4]. The preconditioner stems from the reduced SQP methods. Optimization-based multigrid strategy is used for convergence acceleration [5, 3]. The method is applied to problems in aerodynamics.

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MHD Flow due to a Point Sink with Localized Wall Heating (Cooling)

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The process of localized wall heating or cooling (in which a certain portion of the wall is being heated or cooled while, the remaining portion is unchanged) is of practical interest for various scientific and technological applications including thermal protection, energizing the inner protection of the momentum and thermal boundary layer, and in boundary layer control etc. In the present investigation, the effects of localized wall heating (cooling) for the laminar boundary layer flow due to a point sink with an applied magnetic field have been studied. The localized heating or cooling introduces a finite discontinuity in the mathematical formulation of the problem and increases its complexity. In order to overcome this difficulty, a non-uniform distribution of wall temperature is considered at finite sections of the plate. The non linear, coupled partial differential equations governing the flow under boundary layer approximations have been solved numerically by using an implicit finite - difference scheme, along with quasilinearization technique. The effect of the localized wall heating or cooling is found to be very significant on the heat transfer, but its effect on skin friction is comparatively small. The magnetic field enhances both skin friction and heat transfer. The momentum and thermal boundary layer thicknesses become slightly thinner in the presence of wall heating(cooling).

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Effect of Hall Currents and Permeability on Double-Diffusive Convection of Compressible Rivlin-Ericksen Fluid in Rotation

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 37C75

The term ‘double-diffusive convection’ applies to convection in a fluid where there are two diffusing components. The archetypal case is heat and salt where the faster diffusing component is ‘heat’ and the slower diffusing component is ‘salt’. Convection that is dominated by the presence of two components is very common in geophysical systems and oceanography. In the present work we have considered the effect of Hall currents and permeability on a rotating Rivlin-Ericksen elasto-viscous fluid heated and soluted from below. The relevant hydromagnetic equations are linearized using Boussinesq approximation and the perturbations are analyzed in terms of normal modes. A dispersion relation governing the effects of visco-elasticity, salinity gradient, rotation, magnetic field, Hall currents and medium permeability is derived. For stationary convection, Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid due to the vanishing of the visco-elastic parameter. Compressibility is found to postpone the onset of thermosolutal instability. In the absence of Hall currents and rotation, permeability hastens the onset of instability and therefore has the usual destabilizing influence on the thermosolutal instability problem. In the presence of rotation and/or Hall currents though various conditions for stabilizing/destabilizing effect of permeability are derived yet it has been found that for the permissible range of values of various parameters permeability has destabilizing influence. Similarly, the conditions for stabilizing/destabilizing effects of Hall currents are derived and its destabilizing influence for permissible range of various parameters is established. Also, the dispersion relation is analyzed numerically and the results drawn analytically are depicted graphically.

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Inverse Problems for PDE and ODE Systems with Incomplete Information Originated in Molecular Biology

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A variety of both modern theoretical and practical problems of molecular biology are successfully modelled using mathematical apparatus by means of applying of continuous media laws. The obtained mathematical models are represented by the initial-boundary problems for the linear or nonlinear equations in partial derivatives and/or by the systems of the linear or nonlinear ordinary differential equations. A prominent feature of mathematical models in molecular biology is the case of sourcing the initial data (e.g., coefficient-functions of the equations; initial and/or boundary conditions; some dynamic parameters, etc.) that is either incomplete, or obviously inaccurate. Thus, it is required to determine characteristics of the studied molecular process or the phenomenon with admissible accuracy. For example, if it is required to determine the nucleotide sequences of DNA and RNA at incomplete information on the source data in genetic engineering; in protein engineering it is required to identify the relationship of structure and function of proteins. Other examples are the problem of determining the intermolecular interactions in living systems; problem of determining the molecular mechanisms of cell cycle regulation, etc. In terms of

mathematics, in the above-mentioned problems of molecular biology it comes to determine the cause under some known initial data (often inaccurate and/or incomplete), derived from observations of the investigation. In other words, in terms of mathematics, these models, which are described using the language of the theory of differential equations, are considered to be inverse problems.

In this paper some linear and nonlinear inverse problems of molecular biology are investigated, there are resulted both analytical and numerical methods for its solution, which are based on the principles of the Tikhonov regularization method.



Revealing the Density-based Clustering Structure of the SwissProt Database

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There have been several papers dealing with clustering of the SwissProt amino acid sequence database. Most of the methods used involve heuristics at sequence alignment score calculation and/or single-linkage-type or greedy clustering algorithms. In this poster we present a whole different approach: we use (1) exact pairwise alignment score calculation (with an optimized version of the Smith-Waterman algorithm) and (2) the density-based OPTICS [1] clustering algorithm. We also propose a way to assign colours to SwissProt entries based on taxonomy information that may help in visually seeing the composition of clusters consisting of amino acid sequences from different species.

In this work, we classified 389046 sequences occurring in SwissProt release 55.5 using the OPTICS algorithm. We proposed a quality measure that is specifically useful in comparing the reachability plot created by the OPTICS algorithm with an arbitrary reference-clustering. We proposed a colouring scheme that is based on taxonomy information and helps analyze the composition of clusters. We validated our results with the Pfam [2] database and concluded that we obtained clusters of high quality.

Compared to the available, usually greedy sequence clustering algorithms, the proposed clustering method might provide a more precise alternative for sequence clustering. Of course, this comes at an expense of a tolerable increase

of required computing power. Furthermore, we proposed a method to visualize the composition of clusters, making cluster composition and quality evaluation much easier.

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Using ABS Algorithm to Schedule Medical Residents

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This paper presents our experience in using ABS algorithm to develop an on-call schedule for medical residents at Sawai Man Singh Medical College and Hospital, Jaipur. The ABS methods have been used broadly for solving linear and nonlinear systems of equations comprising large number of constraints and variables, thereby saving time and effectively dealing with resulting complexities. Computational results are presented using programming in MATLAB environment. Key challenges are discussed primarily on a few issues. They provide a very large scale combinatorial challenge to the healthcare personnel who are usually from a non-mathematical background. We present a pragmatic approach for finding an optimally feasible solution which may be applicable in other real world problems as well.



Triple Correlation on Groups

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The triple correlation of an ordinary function on the real line is the integral of the product of that function with two independently shifted copies of itself. Triple correlation methods are frequently used in signal processing for treating signals that are corrupted by additive Gaussian noise; in particular, triple correlation techniques perform well when multiple observations of the signal are available and the signal may be translating in between the observations, e.g., a sequence of images of an object translating on a noisy background. What makes the triple correlation particularly suitable for such tasks are three properties: (1) it is invariant under translation of the underlying signal; (2) it is insensitive to additive Gaussian noise; and (3) it retains most of the phase information in the underlying signal. This paper investigates whether properties (1)-(3) of the triple correlation extend to functions on arbitrary locally compact groups, in particular the groups of rotations and rigid motions of euclidean space that arise in computer vision and signal processing.

After defining the triple correlation for any locally compact group by using the group's left-invariant Haar measure, it is easily shown that the resulting object is invariant under left translation of the underlying function and insensitive to additive Gaussian noise. What is more interesting is the question of uniqueness: when two functions have the same triple correlation, how are the functions related? Our results show that for most cases of practical interest, the triple correlation of a function on an abstract group uniquely identifies that function up to a group translation. We show how our results utilize the duality theorems of Pontryagin, Tannaka-Krein [1], Iwahori-Sugiura [2], and Tatsuuma [3]. We

also develop explicit algorithms for recovering bandlimited functions from the triple correlation on the rotation groups in two and three dimensions. Finally we describe the formal relationship between our triple correlation analysis and the Tauberian theorem of N. Wiener concerning the span of translates of a function.

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Use of State-space and Eigenvalue Approaches in Two Temperature Generalized Thermoelasticity in Presence of a Spherical Cavity

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State-space and eigenvalue approaches are used to investigate the problem of thermoelastic interaction in an infinite elastic body with a spherical cavity in the context of two temperature generalized thermoelasticity (2TT) (Youssef: 2006, 2007, 2008). The basic equations have been written in vector matrix differential equation in Laplace transform domain. The numerical inversion of the transform is carried out using Fourier series expansion techniques. The thermoelastic stresses, conductive temperature and thermodynamic temperature, the quantities of physical interest for any thermoelastic interaction problem are shown graphically for two temperature Lord Shulman model and for two models of two temperature Green Nagdhi.

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Modeling the Spread of HIV in a Stage Structured Population: Effect of Awareness

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HIV is a disease caused by a deadly virus HIV (Human Immunodeficiency Virus). Over the past few years HIV is spreading rapidly among the population. Everyday there are thousands of new cases of HIV infection in the world and these occur in almost every country, but its spread is very fast in developing countries, which have limited resources to deal with the spread of this disease [1]. In worldwide, 70% of HIV infections in adults have been transmitted through heterosexual contact and vertical transmission accounts for more than 90% of global infection in infants and children [2].

In this paper, a nonlinear mathematical model is proposed to study the spread of HIV by considering transmission of disease by heterosexual contact and vertical transmission. A stage structured model is proposed and analyzed

by considering total population variable and dividing whole population under consideration into three stages: children, adult and old. Also it is assumed that the rates of recruitment are different in different groups of population. Various equilibria of the model and their stability are discussed. Using stability theory of differential equations and computer simulation, it is shown that due to increase in the awareness of the diseases in the adult class, the total infective population decreases in the region under consideration. Numerical simulation also supports theoretical findings.

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A Moored Ship Motion Analysis with the Resonant Frequency Waves in the POSCO New Harbor

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The POSCO New Harbor (PNH), located at northeast part of Pohang City, has been experienced extreme wave hazards of about 3.0-5.0 meters high in surface elevation due to the wave induced oscillations. Firstly, to find these resonant wave frequencies, theoretical studies are introduced to investigate the wave induced oscillations in the arbitrary shaped harbor, which are based on the reduced wave equation, i.e., Helmholtz equation. Then numerical simulations are conducted to find the resonant frequencies in the PNH. The geometry of the PNH, based on the actual topography and bathymetry data, is constructed and then the numerical scheme is implemented using the boundary matching technique with the Weber's solution of Helmholtz equation. Secondly, to analyze the hydroelastic response of a moored ship due to these resonant frequencies in the PNH, a numerical scheme is developed further. We assumed the constant

depth and incident waves are completely reflected along the coast in addition to the usual assumptions made for free surface problem. The fluid domain is divided into three subdomains which are named as Region I (moored ship), region II (bounded harbor) and region III (outside the harbor). The solutions in the region III are used to provide a numerical radiation condition for the region II and region I. In this method, we introduce a artificial boundary which enclose the ship inside, and define this inner subdomain (region II). The solution in region II is obtained using matching condition at entrance of harbor and boundary condition at region I. The simulation results are compared with the real time measurements of wave heights (WTG) at the specified eight track recorder points inside the PNH and show good agreements.

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Propagation of SH Waves in Multilayered Viscoelastic Medium: A Finite Difference Approach

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The paper deals with the use of finite difference technique on the study of propagation of shear waves in viscoelastic medium. General dispersion relation has been obtained for the case of multilayered case when (n-1) layers lies over

a half space. The result is in agreement for the single/double viscoelastic layers lying over a semi infinite viscoelastic medium. The stability analysis has been done for the used finite difference scheme, also phase and group velocity have been derived using finite difference scheme in terms of dispersion parameter and courant number.

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Effect of Nonhomogeneity on Free Transverse Vibration of Orthotropic Equilateral Triangular Plates with Linearly Varying Thickness

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An analysis and numerical results are presented for transverse vibrations of nonhomogeneous orthotropic equilateral triangular plates of variable thickness using two dimensional boundary characteristic orthogonal polynomials in the Rayleigh-Ritz method on the basis of classical plate theory. Gram-Schmidt process has been used to generate orthogonal polynomials. The nonhomogeneity of the plate is assumed to arise due to linear variations in elastic properties and density of the plate material with the in-plane coordinates. The thickness variation is taken as linear along one direction. The first three natural frequencies for four different combinations of clamped, simply supported and free edges have been computed correct to three decimal places. Effect of nonhomogeneity parameters together with variation in thickness has been studied. Three dimensional mode shapes for specified plate for all the four boundary combinations have been plotted. Results in particular cases have been compared with those available in the literature.

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An Expansion Formula for Wave Structure Interaction Problems in Three Dimensions

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In recent decades, wave structure interaction problems have gained considerable importance to analyze wave interaction with very large floating structures (VLFS) and mobile offshore base (MOB) for utilization of ocean space for various humanitarian activities and military operations in addition to wave interaction with floating ice sheet which finds application in Arctic Engineering. In these class of problems, higher order boundary conditions arise in a natural way on the structural boundary associated with three dimensional Laplace equation which is satisfied in the fluid domain. Various mathematical theory are developed to deal with wave structure interaction problems in two dimensions and the three dimensional problems are analyzed in very special cases for oblique incident waves.

In the present paper, a generalized Fourier type expansion formula in terms of double series along with the corresponding orthogonal mode-coupling relations are derived to deal with wave structure interaction problems in three dimensional fluid domain in which the structure is considered two-dimensional in nature. The present expansion formula is a generalization of the expansion formula developed by Manam et al. (2006) in case of a semi-infinite strip to deal with wave structure interaction problems in water of finite depth. Several identities and results on the convergence of the double series are derived by the direction application of Cauchy residue theorem. The present method of solution can be easily applied to a large class of problems in the area of wave structure interaction in the field of ocean engineering and other fluid structure interaction problems arising in various branches of applied mathematics, engineering and mathematical physics.

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Universality in the Financial Market

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We compute the analytic expression of the probability distributions $F_{I_p,+}$ and $F_{I_p,-}$ of the normalized positive and negative I_p index returns $r(t)$, with periodicity p , see [5, 6]. The main indices I_p that we study are the PSI-20 and the Dow Jones Industrial Average but we also extend our study to north american, european and world wide indices. The periodicity p varies from daily (d), weekly (w) and monthly (m) returns to intraday data (60 min, 30 min, 15 min and 5 min). We define the α re-scaled I_p index positive returns $r(t)^\alpha$ and negative returns $(-r(t))^\alpha$ that we call, after normalization, the α positive fluctuations and α negative fluctuations. We use the Kolmogorov-Smirnov statistical test, as a method, to find the values of α that optimize the data collapse of the histogram of the α fluctuations with the Bramwell-Holdsworth-Pinton (BHP) (see [1]) probability density function. We also study, the probability distributions $F_{ES,+}$ and $F_{ES,-}$ of the normalized positive and negative spot

daily prices or daily returns $r(t)$ of distinct energy sources ES and the probability distributions $F_{ER,+}$ and $F_{ER,-}$ of the normalized positive and negative spot daily prices or daily returns $r(t)$ of distinct exchange rates ER . Since the BHP probability density function appears in several other dissimilar phenomena, our results reveal an universal feature of the stock market exchange, see [2, 3, 4].

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An Investigation into Effect of Electromagnetic Fields on Generalised Couette Flow with Heat Transfer

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One of the basic problem, the plane Couette flow, has been a source of many research workers in dealing with the interplay of various fluid forces and their interaction with the electromagnetic forces. The effect of electromagnetic fields on (a) separation tendency in generalized Couette flow, and (b) heat transfer in generalized Couette flow has been investigated. For the physical insight of problem velocity distribution, temperature field is obtained and with the aid of it the local Nusselt number is derived.



Pinto's Human Decision Bussola

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91A10, 91A05

The main goal in Planned Behavior or Reasoned Action theories, as developed in the works of Ajzen and Baker, is to understand and forecast how individuals turn intentions into behaviors. Almeida-Cruz-Ferreira-Pinto created a game theoretical model for reasoned action, inspired in the works of J. Cownley and M. Wooders. They studied how saturation, boredom and frustration can lead to *split* or *impasse* strategies, and no saturation situations can lead to *no-split* or *heard* strategies. Here, we introduce the Yes-No decision model that is a simplified version of the Almeida-Cruz-Ferreira-Pinto decision model. In this model, there are just two possible decisions $D \in \{Yes, No\}$ that individuals

can take. This model is in the core of psychological, educational and economical models and exhibits the usefulness, and the high complexity, of characterizing the split and no-split strategies that are Nash equilibria (see [1, 2]). Pinto's *thresholds* give a full characterization of no-split Nash equilibria and describe a *hysteretic-like* behavior, a usual concept in dynamics [3], that it is responsible by the occurrence of *catastrophes* consisting of abrupt changes of individuals and collective behavior. The way these thresholds evolve and interact, called *bifurcations* in dynamics, is completely characterized by Pinto's *human decision bussola* that allow us to understand how small changes in psychological or social variables can create or annihilate individuals or collective behavior.

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Efficient and Accurate Numerical Solution for Optimal Control of Reaction-diffusion Systems in Cardiac Electrophysiology

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The focus of this work is on the development and implementation of an efficient numerical technique to solve an optimal control problem related to a

reaction-diffusions system arising in cardiac electrophysiology. The bidomain model equations consist of a linear elliptic partial differential equation and a non-linear parabolic partial differential equation of reaction-diffusion type, where the reaction term is described by a set of ordinary differential equations. The monodomain equations are popular since they approximate, under many circumstances of practical interest, the bidomain equations quite well at a much lower computational expense, owing to the fact that the elliptic equation can be eliminated.

The optimal control problem is considered as a PDE constrained optimization problem. Specifically, we present an optimal control formulation for the monodomain equations with an extra-cellular current as the control variable which must be determined in such a way that excitations of the transmembrane voltage are damped in an optimal manner. In this study, we have chosen the finite element method for the spatial and higher order linearly implicit Runge-Kutta time stepping methods for the temporal discretization to solve the primal and dual problem. A nonlinear conjugate gradient method and a Newton method are compared for solving the optimization problem. A more in depth description will be found in [1, 2]. The adaptive grid refinement (AMR) technique and receding horizon methods are applied to get more efficient and to do longer time horizons respectively. Finally, the numerical results are discussed for higher order methods which show a superlinear convergence.

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Thermal Stratification and Radiation effects on MHD Free Convection Flow Past an Impulsively Started Infinite Vertical Plate

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This work is undertaken to study the free convective flow past an impulsively started infinite vertical plate with thermal stratification and radiation in presence of transverse magnetic field. Shapiro and Fedorovich [3] recently revisited the classical theory of one dimensional flow by introducing the thermal stratification parameter in the energy equation. Magyari et. al. [2] also considered the same problem in porous medium. Deka and Neog [1] studied it with MHD effect. Here we have investigated the combined effects of thermal stratification and radiation effects on magnetohydrodynamics free convection flow past an impulsively started infinite vertical plate. The fluid considered is a gray, absorbing emitting radiation but a non-scattering medium. Pressure work term and the vertical temperature advection are considered in the thermodynamic energy equation. The dimensionless governing equations are solved by Laplace transform technique. Velocity profiles, temperature profiles, skin-friction and the rate of heat transfer are presented graphically and discussed the effects of different physical parameters. The results obtained show that the flow field is influenced appreciably by the thermal stratification and radiation.

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The Subtle Sets Theory (SST) theoretical concepts

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During the 16th century, the concept of subtlety has been theorized and applied by G. Cardano [3]. Relate to this subject he said: “Praevident spiritus quod mihi imminent and emphasis on diagnostic analysis and the forecast” [1]. L. A. Zadeh (1965) defined the concept of “Fuzzy Sets”, which differs from the crowd, in respect with Cantor [5]. Recently, Petre Osmătescu [2], continued the spirit of L. A. Zadeh, and defined the notion of subtle sets and insidious concept of

subtle space, by using the idea of fiber and by the instrument called “operator of act”.

Assumed to be defined by *the observer* O , there are a lot of elements (systems, subsystems, complex objects, etc.), $\{e_1, e_2, \dots, e_n\}$, with the *characteristic* S . Let be $SYO = \{O\} \cup MO$, where MO is a lot of means of *observation* $SPO = \{e_1, e_2, \dots, e_n\} \cup S$, SO is the *supporting* observed. The *global socio-economic* S , could be estimated by O , who provides a series of criteria $f_1^0, f_2^0, \dots, f_k^0$, $W_{ik}^0 (k = \overline{1, p})$ is the *event*, that consists in estimating the consequences a_{ik}^0 of the criterion f_k ; the observer O attaches the elements $e_i (i = \overline{1, n})$. The characteristic S_0 is as: $S_0 = \{f_1^0, f_2^0, \dots, f_k^0; e_1, e_2, \dots, e_n; a_{ik}^0, i = \overline{1, n}; k = \overline{1, p}\}$, where p is the number of the influencing factors (criteria, tests), e_i are the elements under observation, $a_{ik}^0 =$ consequences of criterion k , estimated by O , for elements i . Both the subtle set and the “global characteristic”, will be denoted by S_0 . The observer O can be outside the element e_i or can be part of this. The first case is when an “invisible” statistics is developed, the second one, when an “apparent” statistics has resulted.

Following [4] and recent contributions, after defining the general concept of subtle set (basic concept, the subtle sets without frequentist (probabilistic) sequences with deterministic/fuzzy appearance), using the membership degree concept and the square root approach, we obtained the following results: Fuzzy appearance paradigm covers a higher specificity than the deterministic appearance approach, and the deterministic appearance thinking is more specific related to the basic concept. According to the concrete necessities of the study, other versions of a subtle set can be also defined, and used in various real-life applications.

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Asymptotic Option Pricing under the CEV Diffusion

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In finance, many option pricing models generalizing the Black-Scholes model do not have closed form, analytic solutions so that it is hard to compute the solutions or at least it requires much of time to compute the solutions. Therefore, asymptotic representation of options of various type has important practical implications in finance. In this paper, asymptotic option pricing is developed to connect the Black-Scholes model and the constant elasticity of variance model. We obtain the relevant results for the European vanilla, barrier, and lookback options and prove the accuracy of each asymptotic formula.

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Analytical Solution for Axi-symmetric Rotating Flow of Newtonian Conducting Fluid Past a Stretching Porous Sheet under a Transverse Magnetic Field

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The steady laminar flow of an electrically conducting fluid over a radial stretching porous sheet rotating with an angular velocity, is considered for investigation in the presence of a transverse magnetic field. The axi-symmetric flow of conducting fluid is induced due to radial stretching of a sheet rotated with an angular velocity which generates the boundary layer type of flow. Introducing the dimensionless quantities the governing partial differential equations are transformed into non-linear ordinary differential equations. An expression for pressure distribution is derived. A series solution is obtained analytically for different existing parameters. The effects of magnetic fields are shown on the radial, peripheral and axial velocity across the boundary layer.



Topologies and Sheaves in Linguistics

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We outline a sheaf-theoretic formalism as a new framework to study the process of interpretation of text written in some unspecified natural language, e.g. in English.

A text is a finite sequence of its constituent sentences, and so it is formally identified with a graph of function. A part of text is a subsequence whose graph is a subset of the whole sequence graph. The *meaning* of a given part is accepted as the content grasped in a particular reading following the reader's attitudes formalized by the term *sense*. Whether a part of an *admissible* (written for human understanding) text X is meaningful or not depends on some

accepted *criterion of meaningfulness*. For such a criterion conveying an idealized reader's linguistic competence, the set of meaningful parts $\mathcal{O}(X)$ is stable under arbitrary union and finite intersection, and hence defines a topology we call *phonocentric*. We argue that the connectedness and the T_0 -separability of phonocentric topology are two *linguistic universals*.

For a given admissible text X and an adopted sense \mathcal{F} , we collect into the set $\mathcal{F}(U)$ all meanings of an open $U \subseteq X$ read in the sense \mathcal{F} . It defines a presheaf $\mathcal{F}: \mathcal{O}(X) \rightarrow \mathbf{Sets}$, acting as $U \mapsto \mathcal{F}(U)$ and $U \subseteq V \mapsto \text{res}_{V,U}: \mathcal{F}(V) \rightarrow \mathcal{F}(U)$, where restriction maps $\text{res}_{V,U}$ are defined following the precept of *hermeneutic circle* "to understand a part in accordance with the understanding of the whole".

Two meanings $s, t \in \mathcal{F}(U)$ should be considered as identical globally on U iff they are identical locally on each U_j of a covering $U = \cup_{j \in J} U_j$ by opens already read. The hermeneutic circle prescribes "to understand the whole by means of understandings of its parts". Whence the generalized *Frege's compositionality principle*:

The presheaf of fragmentary meanings naturally attached to an adopted sense of reading of an admissible text endowed with phonocentric topology is really a sheaf. We define so the *Schleiermacher category* $\mathbf{Schl}(X)$ of sheaves of partial meanings.

The generalized *Frege's context principle* describes the set \mathcal{F}_x of contextual meanings of a phrase $x \in X$ as the inductive limit $\mathcal{F}_x = \varinjlim (\mathcal{F}(U), \text{res}_{V,U})_{U, V \in \mathcal{O}(x)}$. We define this way the category $\mathbf{Context}(X)$ of étale bundles of contextual meanings.

The section-functor and the germ-functor establish a *Frege Duality* of categories $\mathbf{Schl}(X) \rightleftarrows \mathbf{Context}(X)$. So, the belief in one Frege's principle implies the other.

These topologies and sheaves study is formal syntax and semantics in linguistics.

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The Role of Mathematical Models in Epidemic Control and Policy in India

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 92D30,92C60,92C15

In the recent years, mathematical models have been assisting in various public health planning and policies in India, especially for controlling spread of AIDS, Swine flu, Avian Influenza. Most of these models involve representing transmission dynamics of diseases spread through a set of non-linear differential equations and estimating the parameters involved. These models, in principle, can assist in deriving theoretical conditions under which disease spreads in a given population will eventually attain stability, time taken to extinct the infective population etc. In this talk, some of the recent works of the author and also collaborative works with public health experts, medical professionals in planning AIDS control and other infectious diseases will be described ([1], [2], [3], [4]). Some of these models were practically adopted by the government in their respective five year or annual policies. Mathematical population biology has substantially contributed in the developing foundations for the infectious disease epidemiology ([5], [6]). Models have been helping in deriving reproductive rates ([7], [8]). We will also discuss theory of evaluation of our models and their fitting with the observed data.

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Applications of a Number Notation Hyper-format for Science and Engineering

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The use of a homomorphism for defining new numerical methods applicable to the solution of differential equations is proposed, with the aim of possibly increasing the efficiency of computational modeling in presently available microprocessors. This analysis has shown the advantage of using an iterated exponential for building-up the appropriate mapping function applied in the homomorphism. In this case, it is possible to get a significant conversion of the numerical scale, applicable to very large or very small numbers. The solution of this problem is obtained by defining a new format of number notation (the RRH hyper-format) [1]. This new number notation uses the iterative exponential operation, known as “*tetration*”. Unfortunately, *tetration* (*super-power*, *power-tower*, *tower*) and its two inverse operations (*super-root* and *super-logarithm*) cannot be directly employed for the creation of this new numbers format. Nevertheless, introducing a new functional operator “***”, obtained as a homomorphism of addition, and using the *tetration* operation (${}^n x$, in Maurer’s notation) and *super-logarithm* as mapping function (${}^n k = a \Rightarrow n = \text{slog}_k a$), this can be solved, leading to: $a \overset{[k]}{*} b = (\text{slog}_k a + \text{slog}_k b) k$. Based on a *homomorphism of addition* and using *tetration*, the authors have developed a new RRH of a real number $D \in \mathbf{R}$, by putting: $D = m \overset{[k]}{*} ({}^n k)$. The use of a homomorphism, *tetration*, *super-logarithm* and of this number hyper-format notation allows the authors to define an innovative procedure to the solution of some problems. For example, the well known functional equation $f(f(x)) = e^x$, by the authors exactly solved by $f(x) = 1/2 e \overset{[e]}{*} x = (1/2 + \text{slog}_e x) e = (0,5 + \text{sln } x) e$. A new number notation format is therefore proposed, formally similar to the “*floating point*”

format, but using *tetration* instead of *exponentiation*. A practical machine storage format has been implemented. Prototypes of a hyper-calculator and of a number notation hyper-converter have also been developed.

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Effect of Suspended Particles on Thermosolutal Instability in Elastico-Viscous Rivlin-Ericksen Fluid

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The influence of suspended particles on the thermosolutal stability is examined for viscoelastic polymeric solutions. These solutions are known Rivlin-Ericksen fluids and their rheology is approximated by the Rivlin-Ericksen constitutive relations, proposed by Rivlin and Ericksen [1955][2]. For stationary convection, the suspended particles density parameter has a destabilizing effect and the stable solute parameter has a stabilizing effect on the system. These effects have also been shown graphically. Further, Rivlin-Ericksen fluid also behaves like Newtonian fluid [1] and the convection in fluid in the presence of suspended particles sets in earlier than no-particles case. The oscillatory modes are introduced due to the presence of stable solute parameter, suspended particles and viscoelasticity which were non-existent in their absence. The sufficient condition for the non-existence of overstability is also obtained.

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Structure and Spectral Characteristics in Network Redundancy

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Many real-world complex networks, in contrast to random graph models, contain a significant amount of structural redundancy, in which multiple vertices play identical topological roles. Such redundancy arises naturally from the simple growth processes which form and shape many real-world systems. Since structurally redundant elements may be permuted without altering network structure, redundancy may be formally investigated by examining network automorphism (symmetry) groups.

We give a complete description of spectral signatures of redundancy in undirected networks and, in particular, we describe how a networks automorphism group may be used to directly associate specific eigenvalues and eigenvectors with specific network motifs. In addition, we compute the most common of these motifs and demonstrate their presence in a variety of real-world empirical networks.

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Immune Response Dynamics by T Cells

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We analyse the effect of the Regulatory T cells (Tregs) in the local control of the immune responses by T cells in a mathematical model [1]. We obtain an explicit formula for the level of antigenic stimulation of T cells as a function of the concentration of T cells and the parameters of the model [2]. The relation between the concentration of the T cells and the antigenic stimulation of T cells is an hysteresis, that is unfold for some parameter values [3]. We also study an asymmetry in the death rates. With this asymmetry we show that the antigenic stimulation of the Tregs is able to control locally the population size of Tregs [4]. The rate of increase of the antigenic stimulation determines if the outcome is an immune response or if Tregs are able to maintain control. This behaviour is explained by the presence of a transcritical bifurcation [4]. We study the appearance of autoimmunity from cross-reactivity between a pathogen and a self antigen [1] and from bystander proliferation [5]. This model has a good fit to experimental data from infected mice.

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Nonlinear Programming Techniques for the Synthesis of Animal Feed Formulation

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Animal diet is a key variable that must be controlled and managed to achieve the objective of accurate results and meaningful conclusions. Optimization models are the best way to achieve the objective of best nutrient diet. An optimization model consists of an objective function defined on a set of variables, restricted by various kinds of constraints. The diet identified was represented by a set of nutrient weights, subsequently called nutrient variables, each representing a decision variable for the models. In this paper, the nutrient optimization is done by linear programming method and then it is compared to proposed model by non-linear programming. Proposed model with Non-linear programming measures its performance and gives a comparative result with linear programming models. We are focusing here on application of Nonlinear programming and statistical techniques in the field of animal nutrition. It also points out that the application of Nonlinear Programming gives the benefit of simultaneous inclusion of different kind of nutrient ingredient satisfying the feeding standards.

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Modelling Uncertainties for Achieving Service Level Goals in Projects Portfolio

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In the organizational portfolio of projects, there is a need to identify related projects so as to effectively share system resources, save costs, increase benefits and cut down development time by having proper resource utilization. The major interdependencies are of cost, benefit and technical which makes a portfolio risk and takes a considerable amount of judgment for project selection [1]. There are explicit trade-offs of stock out costs but service level is a measure of how effective a company is at supplying demanded services from its stock on hand by preventing chances of losing sales and backorders. There are several methods of expressing quantitative measure of service level [2].

In case of service sensitive items, organizations must model optimal cost control policies, with long and short costs as main criteria of control as they have a direct bearing on service levels. I have related demand, profit, and customers with inventory costs which has a direct bearing on service level and for this I have modified the model given by Ernst and Powell [3]. Actually demand, profit, revenue, and new potential customers are directly related to stocking decisions w.r.t. long/short costs and also bringing in the difference in inventory control costs and bring in the difference in profits. This shows that customers can react independently to changes in stock out and overstock costs. The loss of future profits in no doubt a direct consequence of stock out. I have inferred two results, First One is for optimal service level I have to balance C_s , the cost of probable units required (if short) during lead time AND C_l , the cost of probable units not used (over stock) during lead time. It means $P(C_s) = (1 - P)C_l$ or $P = 1 - C_s/C_l + C_s = C_l/C_s + C_l = \text{Risk}$. P is the probability of needing an item and $1 - P$ is probability of not needing that item. The second one is for portfolio risk that is calculated as square root of D_r , as given by $D_r = (\sum X_i) * (\sum X_j) * (R_{ij}) * (a_i * a_j)$ where R_{ij} is common risk between projects i and j , a_i and a_j are individual risks, and X_i and X_j are portions of portfolio invested in projects. Here i and j varies from 1 to n .

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Pulsatile Two Phase Flow Model of Blood Through Stenosed Artery

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76Txx

A mathematical model for the pulsatile flow of blood through an artery with axially symmetric linear and multiple stenoses in the presence of Lorentz force is considered. Blood in the artery is considered as two-fluid model comprising core and peripheral region. The core region which is suspension of erythrocytes (a non-Newtonian fluid) surrounded by plasma (a Newtonian fluid) in the peripheral region. The flow in the core region is modelled by the Casson fluid model. In view of cardiac flow behavior the pulsatile pressure gradient is considered. The governing equations of motion are solved numerically to compute velocity and the volumetric flow in the presence of the axial symmetric linear/multiple stenoses in the artery. The effects of location of the multiple stenoses, curvature of the stenoses, haematocrit concentration and magnetic field on the pressure drop, plug core radius, Wall Shear Stress (WSS), Oscillating Wall Shear Stress (OWSS), Wall Shear Gradient and the resistance to flow are simulated and analyzed graphically.

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Effect of Suction/Injection and Viscous Dissipation on Unsteady Heat Transfer in Steady Stagnation Point Flow through Porous Media

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Aim of the paper is to investigate unsteady heat transfer in a two-dimensional steady laminar forced stagnation point flow of a viscous fluid through porous medium [1] on a flat permeable plate when the sudden step change (linear/non-linear) in the surface temperature of plate occurs. The fluid is sucked/injected through the plate in the presence of viscous dissipation. The governing boundary layer equations of continuity and momentum for flow are transformed into ordinary differential equations using similarity transformation and solved using Runge-Kutta fourth order method along with shooting technique, while the transformed energy equation is solved using finite difference scheme. The results

obtained are compared with earlier results by Yih[2]. The effect of permeability and other parameters on the skin-friction coefficient, rate of heat transfer, velocity and temperature distributions are discussed numerically and presented through graphs. It is observed that fluid velocity increases with the increase in permeability parameter, Prandtl number or suction/injection parameter at low permeability parameter, but as permeability parameter increases the effect diminishes. Fluid temperature decreases with the increase in permeability parameter, while it increases with increase in Eckert number. Skin-friction coefficient increases with the increase in permeability parameter, suction/injection parameter or Prandtl number.

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Integral Equation Solution for Multi-connected Contact Domain under Nonsymmetrical Loading

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The main integral equation of three-dimensional contact interaction problems is $\varphi(p) + \iint_{\Omega} k(p, \psi(p, \mu))/r d\Omega$, $\mu = \cos \widehat{n, \hat{x}}/r$, [1]. It is the first kind equation in case of smooth surface ($\varphi(p) = 0$). Taking into account roughness by $\varphi(p)$ the second kind equation is obtained, and without friction in symmetrical problems its solution methods are known, for example, a solution for circle was found in [1].

At present work the solution is developed for multi-connected domain Ω with nonsymmetrical density caused by friction or special loading. When the domain Ω is doubly-connected, the equations of its contours depend on small parameter ε . The domain mapping onto annular ring S is obtained. Simple fiber potential expansion by ε is found under conditions that the mapping is one-to-one and continuously differentiable.

Also potential expansion is developed when the density has no circular symmetry:

$$\iint_S \frac{\sigma(\rho) \cos 2m\theta}{r} ds = 2\pi \cos 2m\theta_0 \sum_{n=0}^{\infty} \left[\frac{(2n-1)!!}{(2n)!!} \right]^2 C_{m,n} U_{2n}(\rho),$$

$$U_{2n}(\rho) = \int_a^{\rho_0} \sigma(\rho) \left(\frac{\rho}{\rho_0} \right)^{2n+1} d\rho + \int_{\rho_0}^b \sigma(\rho) \left(\frac{\rho_0}{\rho} \right)^{2n} d\rho.$$

The proof is made by mathematical induction. The expansion convergence is shown and at the bounds also. The similar expansion was made at [2] when the density is presented in view of series on cosines of odd arcs. Such expansion makes easier to use computational methods in [3] for the contact problems with central loading.

So, proposed numerically-analytical method is based on potential expansion, regularization of the first kind Fredholm equation that leads to the second kind, smoothing of the kernels as they have singularity. Then integral equations system could be solved by numerical methods, here the successive approximations is used.

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Shear Wave Propagation in a Heterogeneous Irregular Monoclinic Medium

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The object of the present paper is to investigate horizontally polarized shear waves through the non homogeneous monoclinic layer with an irregularity. The irregularity is taken in the form of parabola at the interface of heterogeneous monoclinic layer and monoclinic semi-infinite medium. The dispersion equation has been obtained in closed form. The effect of size of irregularity and non homogeneity parameter on the dispersion curve is being depicted by means of graphs. It is also observed that the dispersion equation reduces to the standard SH wave equation in the isotropic case, when heterogeneity and irregularity are absent.

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Stagnation Point Mixed Convection Flow and Mass Transfer in Porous Media Along a Non-isothermal Permeable Vertical Plate in Presence of Heat Source/Sink and Chemically Reacting Specie

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Aim of the paper is to investigate steady mixed convection stagnation point flow of an incompressible viscous fluid through highly porous media and clear fluid

along a permeable non-isothermal vertical plate in the presence of first order homogeneous chemical reaction, which consumes species, and heat source/sink [1]. The governing equations of continuity, momentum, energy and specie diffusion in the boundary layer are transformed into coupled non-linear ordinary differential equations using similarity transformation and are solved using Runge-Kutta fourth order method with shooting technique and results verified in special cases with Yih [2]. It is seen that as the permeability parameter transits from zero to non-zero value, the effect of suction/injection parameter, heat source/sink parameter, chemical reaction parameter, buoyancy, modified buoyancy parameter and surface temperature parameter diminish also the skin-friction coefficient and the fluid velocity increase with the increase of permeability parameter, suction/injection, heat source/sink, buoyancy or modified buoyancy parameter, while they decreases with increase in chemical reaction parameter, Prandtl number or Schmidt number.

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Love Wave at a Layer Medium Bounded by Irregular Boundary Surfaces

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The problem of propagation of Love wave in a corrugated isotropic layer over a homogeneous isotropic half-space has been investigated. The dispersion relation of Love wave propagation in a corrugated layer medium bounded by irregular boundaries is derived. In special cases, the dispersion relation is reduced for the corrugated layers bounded by periodic boundary surfaces, $d \cos(px)$, $d_1 \cos px$ and $d_2 \cos px$. The dispersion relation is found to be a function of the amplitudes of the corrugation, frequency and position parameters of the corrugated boundary surfaces. The results of Ewing et al. [1] and results similar to Noyer [2] are recovered from our analysis. The phase velocity and group velocity of Love waves are computed numerically and the results are depicted graphically.

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Derivation and Geometric Proofs of Corollaries of the Developable Surface Equations and Industrial Applications

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The theory for complex flow ducts bound by developable surfaces, the generic mathematical models, and the algorithms to unfold the same have been discussed at length in a previous paper [1]. The parametric Beta-Theta equation derived for the generators is of a generic nature. Investigations carried out by the authors has shown that numerous simplifications are possible which have many advantages.

- a) They can be explained geometrically,
- b) The equations become very simple for specific cases which are very useful in numerous industrial applications,
- c) These equations are computationally very efficient,
- d) Can serve as a very simple educational aid for teaching Differential Geometry and the physical significance of Gaussian Curvature zero surfaces can be demonstrated.

Based on the generic mathematical model and equations, corollaries have been derived which have immense practical value for the industry. Briefly, the Corollaries can be stated as follows:

Corollary - 1: If the two end curves bounding the developable surface are parallel, the tangent plane reduces to the case of parallel tangents.

Corollary - 2: If the sections are parallel, the Beta-Theta relationship is independent of the lateral shift between end curves.

Corollary - 3: If the two end curves are circular, or elliptic very simple analytical equations are obtained which are computationally much more efficient than the generic equation [1].

Corollary - 4: If the two end curves are planar with any angle Phi between them, the Beta-Theta equation can be shown to be independent of the angle Phi between them. The tangent plane can be geometrically interpreted as tangents intersecting the common axis of intersection of the planes containing the end curves.

The simplified Beta-Theta equations derived from the corollaries increase computational efficiencies many-folds and also form a part of mathematical/graphics software packages developed by the authors.

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Section 19

Mathematics Education and Popularization of Mathematics

Glimpses of 'GANEET DARSHAN'

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 97CXX

This paper is sharing of self innovated method 'GANEET DARSHAN' of teaching-learning process. With the great extent of our sense organs, abstract subject like mathematics can be made enjoyable and meaningful. It helps in learning of mathematics as well as transforming oneself into a wonderful human being.

GANEET DARSHAN involves some activities which make students aware of their qualities and to know their inner potentials. It sharpens their thoughts and provide them a vision towards realities.



The Lilavati's Legend

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The Lilavati is an ancient book on arithmetic written in the twelfth century in which techniques for the solution of problems are simple and easy to use and, moreover, there is a lot of interesting information in the problems presented therein. It was used in India as a textbook for many centuries.

In this poster we are presenting the history of this book with the aim of finding interesting ideas and problems we can use to motivate the transmission of mathematical knowledge, since knowing it will help the teacher, and the pupil, understand mathematics as an activity that is part of peoples' culture.

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Cave Man Math

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The Proposal: "Cave Man Math" is a practicum course focused on aspects of the environment that have occupied human thought for more than 12,000

years: survival and development. At the dawn of the Neolithic Age, a creature possessing great capabilities became dominant. They spoke a language, planted food, kept animals, made tools, cooked meat and survived the winter. As their culture grew, their first concerns were the same as ours: stay out of danger, feed everyone, keep strong and learn how to make life more secure. Studying the questions these early humans faced may now provide students with a valid foundation for studying challenges of the future. The Logistics and Content Life in all ages is characterized by questions. Questions arise from challenges, opportunities, choices, threatening situations, disparate needs or purposes, long-range planning, avoidance and problems of fairness or cooperation. Early society, like traditional societies today, was not free of complications or questions. Mathematics is our most valid way questions are answered and problems are solved. Mathematic ideas generate our best analyses of complex situations, process and relationship. Early humans organized aspects of their society and worked cooperatively on large projects such as building shelter, animal care, hunting and gardening. Students will analyze challenges faced by Homo Sapiens, use mathematical ideas to model them and present their ideas using modern technology. Models constructed by student groups take forms as lists, drawings, charts, graphs, networks, fractals, sequences, designs, diagrams and maps. Students use computers to translate data from traditional societies or animal populations as difference equations, difference equations, logistic growth rates, matrices and graphs. Local elk and sea lion populations provide opportunity for sample data collection of protected herds. Predator-prey studies use data on coyote and mice populations in Yosemite National Park. The course is designed for liberal arts students and carries no prerequisites. It is a practicum and requires active student participation and cooperative learning to develop the mathematic understanding every student needs, in any discipline. Students study the direct connection and influence of mathematics to the environment and to development of a successful society. The goal of the course is to intensify the liberal arts program by developing students' mathematic purview, through non-proscribed active learning experiences. Beginning with these components of life in the Neolithic Age underscores the two fundamental reasons for mathematic study today: (1) Math is power. The natural world can be understood in terms of the natural numbers. Modeling with mathematics is the single reliable way we have to learn about the natural world and create new knowledge. (2) Math is forever. The more we learn, the safer, larger and more secure our lives become.



Mathematics by and for the 21st Century: Goals and Strategies

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 97C

This work pretends to open a window (web page) to the entire mathematical community, to define what are the main goals and strategies, which we as a conscious community should establish, in order to assume (in the best possible way and free from the prejudices of the present) the challenges and tasks that mathematics undertakes today, as key-, transversal-, and guiding-science.

Shouldn't we ask:

- (1) Which actions/changes should finally be done in order to minimize/stop the psychological blockades/social prejudices/scaring myths that blocks/weaken the normal development/expansion/communication of mathematics worldwide? Why don't we replace the myths by the recent scientific truths on psychology, epistemology, brain functioning, ...? Which strategies could be useful to update the mathematical image in the collective unconscious? What should the mathematical community propose to the non-mathematical world?
- (2) Which is the mathematics to be thought/taught in this decade/century/millennium?
- (3) Has mathematics a mystic dimension? Or is such a dimension exclusivity of the natural sciences?

What else should we actually ask? The only desired consensus is that of an urgent need of up-dating!

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The Unique Calendar

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20K

Today, as technology has grabbed the whole arena of life including the human mind; a need for reformation of our mind is felt by various organizations in the world. These organisations are trying to make an awareness of human ability to do arithmetical computations mentally and faster than electronic calculators.

A simple formula for the calendar containing a few steps is demonstrated below. Using this formula any person with the knowledge of basic arithmetic operations and a little logic can calculate the weekday of any given date within a minute. One only requires to remember the twelve codes for the months of the year.

Table 1. Month - Code Table

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Code	7	10	10	6	8	4	6	9	5	7	10	5

Given a date, say 06 Apr 1921, steps are as follows:

- Step 1: Add the date and the month-code that is $06+06 = 12$.
- Step 2: Take the last two digits of the given year, divide it by 4, ignore the remainder and add the quotient with it, that is $21+5 = 26$
- Step 3: Add these two results, that is $12+26 = 38$.
- Step 4: Divide the year by 400 and check the remainder. If the remainder is
 between 0 and 99, subtract 0; if between 100 and 199 subtract 2;
 if between 200 and 299 subtract 4; if between 300 and 399 subtract 6.
 Here, we get remainder as 319. So we subtract 6 and get $38-6 = 32$.
- Step 5: As we have 7 days in a week, divide this result by 7 and take the remainder. Dividing 32 by 7 we get the remainder as 4.

Finally converting the remainder from table 2 we get the day as Wednesday.

Table 2. Conversion Table

Remainder	0	1	2	3	4	5	6
Days	Sat	Sun	Mon	Tue	Wed	Thu	Fri

This formula is valid for all the years except for the first two months, that is January and February, of the leap years. In this case we have to subtract 1 more from the final result to get the exact day.



Polyhedral Differential Geometry

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 97G01

I will describe an undergraduate course in differential geometry that uses piecewise-linear curves and surfaces.

The usual course in differential geometry uses smooth curves and surfaces. The prerequisites for this course are linear algebra and several semesters of calculus. Concepts may be subtle, computations may take work, and proofs may be involved.

The analogous concepts in polyhedral differential geometry are easier to work with. The prerequisites for this course are minimal. Plane curves are polygons (possibly with self-intersections). The curvature at a vertex is simply the exterior angle between the two line segments forming this vertex. It is easy to prove, for instance, that a simple closed polygonal plane curve has total curvature $\pm 2\pi$. For surfaces, the concepts involved in the Theorem Egregium and the Gauss-Bonnet theorem are straightforward, and the theorems are relatively easy to prove.

In fact many of the concepts in smooth differential geometry have polyhedral analogues. I will describe some of these.



Teaching Strategy Adaptation of the Ishikawa's Cause-Effect Diagram for Math Problems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 97D50

In this poster contribution we describe an ADAPTATION of the Ishikawa's Cause-Effect diagram [1] for mathematics problem solutions. Ishikawa's diagram has been used with big success to minimize student difficulties to solve math problems [2]. The algorithm adaptation goes as the following steps:

1. Confront and understand the raised problem.
2. Teachers, acting as mediators prepare general questions: What type of problem is this? Do we know a method to solve it? Which one (Previous ideas)? Student answers are registered at the beginning of the diagram scheme. This part is known as the diagram backbone.
3. At this step, teacher proposes a function, related with a formula to be used. Usually, this relation implies secondary problems, which will require another formula application. Those expressions are registered besides the original formula (backbone formula), generating the adaptation branches of the Ishikawa diagram: solving secondary problems helps to solve the original problem.
4. Then, we substitute numerical values in the original formula, creating a another backbone branch. New secondary problems arise associated with this new branch.
5. Again, we substitute those partial solutions in the backbone; if it is necessary, a new branch is generated, managing it as in the previous cases. We continue with this procedure until we obtain the definitive solution, placing it at the head part of the diagram.
6. Finally, we review and inspect the procedure to catch possible mistakes. The diagram scheme helps greatly because it brings a visual overview of the solution methodology chosen.

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A Dialectical Invariant for a Didactic Approach of Mathematics

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Most of the current problems in the teaching of mathematics emerge from pairs of contradictory dialectical categories. These effectively characterize the problems. When one makes an epistemological study to determine the object of investigation in which this problem is immersed it is possible to find essential pairs of dialectical categories that are deeper with study and should provide us with enough elements for the determination of appropriate didactic actions for solving the researched problem.

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Practical Approach in Mathematics Education

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Mathematics is a practical subject and can be very easily related to the practical life-situations. Therefore this paper recommends that to popularize Mathematics, traditional theoretical approach of teaching Mathematics should be replaced with a practical approach. The paper finds new methods that should be included in Mathematics Education to popularize Mathematics. The suggested

methods are: Classroom Games and Skits, Creating awareness of importance and advantages of acquiring and applying the Knowledge and Skills of Mathematics, Creating awareness of Disadvantages of not acquiring and applying the Knowledge and Skills of Mathematics, Activities to acquire and apply the Knowledge and Skills of Mathematics, E-Group a common platform on Internet to communicate and share with other students what they have learnt, Daily Check-list to keep learning Mathematics and working everyday. By applying the suggested methods in Mathematics Education many benefits to the students are anticipated, like: they will learn to acquire and apply the Knowledge and Skills of Mathematics in a practical way, they will understand the importance of acquiring the Knowledge and Skills of Mathematics and applying them in real lifesituations, they will be prepared to meet future challenges in real lifesituations associated with use of Mathematics, they will understand the importance of team-work and learning cooperatively, they will develop confidence, and above all they will develop interest for learning Mathematics.

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Mathematics Training and Talent Search Programme

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Mathematics Training and Talent Search Programme (M.T. & T.S.) is a national level four week intensive summer training programme in mathematics

which has been running since 1993 in India. This programme is funded by the National Board for Higher Mathematics (NBHM) and is directed by S. Kumaresan since its inception. It has been one of the most significant and successful training programmes in India and has made an impressive impact in mathematical scene in India over the years, especially at undergraduate and post graduate levels. About 170-180 talented students in three levels, selected from all over the country, undergo this training programme every year at various centres in India. The main aim of this programme is to expose bright young minds to the excitement of doing mathematics, to promote independent mathematical thinking, and to prepare them for higher aspects of mathematics.

In this paper, we discuss the main features of this programme, the teaching methodology, its evolution and its impact in mathematical scene in India. We also look at some of main hurdles and challenges faced, in spite of these challenges, this programme continues to live up to its expectations.

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Intuition and Optimization Problems in the Teaching-learning Processes in Basic Education

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The teaching and learning of mathematics should take into account the processes in the mathematical activity and their connection with intuition. Some of the fundamental processes in the mathematical activity are: idealization, generalization and argumentation [1]; and intuition is strongly related to them, to such an extent that it metaphorically can be represented as a vector whose components are these three processes [2]. In daily life, situations in which -naturally or intuitively - an optimal solution is searched are very frequent (when deciding a way to go from one place to another, when choosing a seat at the theater, when making a purchase, etc.) and such situations take place since childhood

(when searching the most preferred toy or the best strategy to win a competition game, etc.) In our research we have found bases to state that just as there is, for example, intuition of probability, it is plausible to state that there is intuition for optimization [2]. However, optimization problems are very scarce in the curriculum for elementary school and high school, in textbooks, and in the math classes at these levels. As a way to strengthen the intuition for optimization of children of the first grades of elementary school, we asked several groups of them to solve some optimization problems. One of them was an adapted linear programming problem with five integer variables, which was solved by second-graders.

We make proposals for the development and training of math teachers, for modifying some methodological approaches of topics like functions, least common multiple and greatest common divisor, and for introducing new math contents in elementary education, such as elements of game theory and of graph theory.

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Promoting Mathematics in Africa: AMMSI Perspective

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The state of mathematics in many African universities is characterized by high student/staff ratios, a small postgraduate classes, limited support by governments for postgraduate training, among other factors [1] and [2]. A number of initiatives have emerged to supplement efforts by established universities to enhance learning and teaching of mathematics, for instance the **African Mathematics Millennium Science Initiative (AMMSI)**[3]. The main objectives of AMMSI include strengthening mathematics teaching, research and applications, and raising general awareness in the importance of mathematics for science and modern nations.

AMMSI has provided partial postgraduate scholarships to supplement the funding obtained by postgraduate students from other sources. It has also awarded fellowships to enable staff visit other African universities in order to engage in research and postgraduate training. In collaboration with other organizations, AMMSI is involved in implementing a project called **Mentoring African Research in Mathematics (MARM)** whose main objective is to promote mentoring relationships between mathematicians in countries with a strong mathematical infrastructure and their African colleagues. The sharpest focus of MARM is on cultivating longer-term mentoring relations between individual mathematicians and students. To date nearly a dozen universities in Africa are participating in the MARM project.

In this paper we present the activities of AMMSI and the impact they have made on the development of mathematics in Africa.

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The Role of Teaching and Learning Settings in Solving Ordinary Differential Equations in Various Contexts

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Reformed Ordinary Differential Equations (ODE) courses' spokesmen underline that traditional teaching and learning of ODE do not provide knowledge sufficient for solving realistic situations problems, as the reformed and computer based courses do. On the other hand, some research indicate that not all computer based courses lead to success in learning and understanding ODE concepts. The aim of this paper is to investigate role of different teaching and learning settings to students' solutions of ODE in mathematical and non-mathematical context. The teaching/learning settings vary from solely tradi-

tional one to those implementing computers as well as some of reformed approaches in process of teaching and learning. Results indicate on proposals to integrate traditional and reformed teaching and learning ODE settings.

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Application of Mathematics in Economics and Management and the appropriate method for teaching mathematics in these fields

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Keywords. Mathematics, Economic, Management, Teaching Mathematics, Test of Hypothesis.

The historical background of using mathematics in economic analyses is divided into three broad and somewhat overlapping periods. The first period is calculus-based marginalist (1838–1947), the second period is set-theoretical, linear models (1948–1961), and third is current period(1961-now).

Teaching calculus to non-mathematics students, like Economics, Management, Accounting, etc., is different from teaching the students with major in pure mathematics. In some universities, the lecturers teach pure theory and don't pay much attention to applications of the topics. Non-mathematics students should learn mathematics as a tool that they need to use in their other courses, and they are not interested in the theory.

Although it is an important tool for economics and management analysis, most of the students are not very interested in mathematics. I show that giving applied examples in Economics and Management will create interests in mathematics among students. Also, by the help of an opinion poll, we observed that the students prefer the second method of teaching mathematics. Also we

observed that students who have learned mathematics using the second method are more successful.



Principles of Professional Writing

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The inspiration for this investigation is the editorial in *Notices of AMS (2002)*: *Although technical writing like teaching is a primary activity of many mathematicians, the principles of good mathematical exposition are rarely made explicit [1].* The reasoned guidance to writers on reporting of research/authoring a book in mathematical sciences presented in [2] runs into 270 principles – each explicitly stated in a crisp sentence. The principles (not exhaustive) are substantiated with examples from the core topics like algebra, analysis, geometry as well as simple applications to physical/social/natural sciences with special reference to history and philosophies of mathematics right up to World Mathematics Year 2000/World Physics Year 2005. The discovery of euphony, elegance, and etiquettes as the three characteristics of professional writing in [2] leads to the **principles (15+40+215) supporting** (i) Euphony – pleasantness to the ears of the audience, facilitating, speakability of a mathematical text in a conference. (ii) Elegance – pleasantness to the eye and the mind of the reader, facilitating printability and readability of the text. (iii) Etiquettes – pleasantness to the professional colleagues through authenticity (observance of international conventions), facilitating the publishability of research and increasing the Science Citation Index of the research paper.

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A Framework for Internet Based Teaching and Learning Mathematics

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Providing uniform standard of education across different parts of a country is a difficult task. One reason is non-availability of suitable teachers/instructors. To bridge this gap to a certain extent we propose an internet based framework that can be used by instructors to design the instructional content. The goals of the framework are supplementing classroom learning and also provide independent home based learning even at remote places where proper teaching infrastructure (e.g. teaching personnel) is not in place.

We leverage the framework for mathematics education by using tools at the content creation level. A storyboard tool is designed for this purpose. This tool consists of various objects used in mathematics such as numbers, symbols, two dimensional figures, three dimensional shapes and special characters. X3D system is considered for this purpose. At the learner end a standard web browser can be used to view the content.

The advantage is that many instructors can collaborate in creating the content enabling translation into different languages and exchange of instruction ideas across different cultures.

The framework will be released under open source and can be bundled with various Linux distributions for inexpensive creation and maintenance.

Technologies: Python, X3D, Java, Openlaszlo, Flash Player, Linux OS, Windows OS, Blender 3D

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Online-Presentation of Linear Algebra

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Understanding mathematics depends to a large extent on the way it is presented. An attempt to facilitate this for linear algebra has been undertaken some time ago (cf. [2]) using a combination of media (web based training software [5], CD-ROM [4], textbook [3]). The content covers a complete basic course for the first year, including additional material which can be used for seminars with graduate students as well.

Technically, the online-script [5] uses the computer-algebra system SINGULAR [1] to interpret a graph of logical dependence between basic parts, to add exercises with random initial data and thus to write for each reader individual textbooks on demand corresponding to his or her recent level of knowledge.

Meanwhile, this has been successfully tested over several years and has led to a recent, refined version which is to be presented in this poster.

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How Artefacts, Motives and Goals Influence the Content of the Mathematical Discourse

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This report is based on a classroom study of 20 4th grade pupils, acting in groups of five, in a Norwegian primary school during a mathematics lesson. The pupils are occupied with a very well defined practical task in that they are going to prepare batter for making waffles, and in this process they are following a given recipe stating the correct amount of milk, flour and other ingredients. The amount of the various ingredients is given in different units according to the nature of the ingredient, and the pupils have different artefacts (mental and physical) available with which they can determine the correct amount. They are free to apply whichever procedure they want. In this presentation I will focus on the process of measuring 15 dl of milk, coming from boxes with 1/4 litre in each.

The analysis of the classroom discourse is based on activity theory (Engeström, 1999; Leont'ev, 1979), and in this analysis an important issue is the tension that can be observed between the motives and goals of the pupils and those of the teacher, and how this tension affects the activity and the actions. This could also be seen in light of what is often referred to as *the problem of transfer* in mathematics education (Evans, 1999). The mathematical content of the classroom episode is connected to measurement; conversion between units and different ways of representing the numbers involved. The analysis of the mathematical content is based on semiotic theory, inspired by Peirce (1998).

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Section 20

History of Mathematics

Słonimski's Theorem and its Implementation in the Calculating Machine

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Chaim Zelig Słonimski was born on March 31, 1810 in Białystok in Eastern Poland and died on May 15, 1904 in Warsaw. Słonimski had wide interests. He wrote several book in Hebrew for astronomy, physics, pure and applied mathematics. He is an author of many Hebrew scientific terms (also mathematical terms).

Słonimski was a very talented inventor. Among other inventions two calculating machines are worth noting. The first machine, invented in 1840, served as a tool for addition and subtraction. The second one carried out multiplication.

The multiplication machine was presented in September 1844 to Berlin Academy of Sciences and in 1845 to Tsarist Academy of Sciences in St. Petersburg. The construction of this machine was based on a theorem in number theory. This theorem, named after its inventor, enabled Słonimski to arrange the table of numbers, which was the construction base for the calculating machine. Thanks to the theorem Słonimski's machine had very simple construction and was cheap. At that time there were only a few calculating machines which were based on such good theoretical background. In 1845 Słonimski won a prize of Tsarist Academy of Sciences. Unfortunately the machine did not survive to our days.

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Residual Analysis versus Analytical Functions

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In his introduction to *Théorie des fonctions analytiques* (1797), Lagrange analyses the metaphysics of infinitesimal calculus and gives John Landen the credit for having worked out a purely analytical method, in which the finite differences of the variables substitute the infinitesimal differences, although he adds: “on doit convenir que cette manière de rendre le Calcul différentiel plus rigoureux dans ses principes lui fait perdre ses principaux avantages, la simplicité de la méthode et la facilité des opérations”.

Preceded in 1755 by *Mathematical Locubrations*, in which the method of fluxions is systematically applied to the solution of algebraic equations and the inverse calculus of fluents, in 1758 John Landen (1719–1790) published *A Discourse concerning the Residual Analysis*, an announcement of the treatise which was to appear in 1764: *The Residual Analysis, a New Branch of the Algebraic Art*. In this Landen intended to release the method of fluxions from the principles derived from the doctrine of motion, and from a basis of pure algebra he would develop methods for immediate application to the main problems of analysis: maxima and minima of functions, curvature, quadrature and rectification of curves. Only finite increments are considered, which are equalized to zero only after having simplified the factor that in their ratio makes them null. An algebraic identity on the differences of rational powers plays a central role [1].

While teaching at the military academy in Turin in the years 1756–59, Lagrange wrote a treatise of Cartesian geometry and differential calculus in which he accomplished a compromise between the Newton method of the “prime and ultimate ratios” and the Leibnitzian notation [2]. Differential calculus and integral calculus were preceded by algebraic calculus of finite differences from

whose formulas those of differential calculus are obtained. The initial concordance then becomes an explicit influence in the basic inspiration of the theory of analytical functions, and even Lacroix was to give credit to Landen for having solved the drawbacks of the theory of fluxions, using a “très-élégant” algebraic identity as its basis [3].

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A Look at Some Research Work of B. G. Pachpatte

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B. G. Pachpatte has contributed to various disciplines in mathematics such as: (1) Analysis and Applications, (2) Analytic Inequalities, (3) Ordinary and Partial Differential Equations, (4) Integral Equations and Inequalities, (5) Finite Difference Equations and Inequalities and (6) Stochastic Analysis and Applications. He has published more than five hundred and fifty research papers in reputed journals. He has written independently five research monographs containing most of his research work. The aim of the present talk is to present his brief biography and to provide an overview of some of his basic discoveries, over the past four decades. In particular, we focus our attention to his fundamental research findings related to some integral and finite difference inequalities and applications; in the hope that it will further broaden developments and the scope of future investigations.

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The Uncountability of the Real Numbers

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No sequence contains all real numbers. The popular proof of this theorem, based on Cantor's second diagonalization procedure, is *not* one of the two proofs published by Cantor himself. Moreover, it depends on the base of the number system used to represent the reals and does not work in binary at all. We propose a most elementary proof which is independent of the number representation employed.

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Proving Classical Impossibility

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The three classical problems (the quadrature of the circle, the trisection of the angle and the duplication of the cube) were formulated in ancient Greece, but it took more than two millennia before they were proved impossible by ruler and

compass (1882 and 1837). In the talk I shall argue that although the impossibility result had been formulated already in late antiquity, it was considered as a kind of meta-result about mathematical problem solving rather than as a mathematical theorem amenable to proof. This only changed in the 17th century when Descartes, Leibniz, Gregory and others formulated impossibility proofs. I shall briefly sketch the main ideas of these “proofs” and point to their main merits and shortcomings. Finally I shall recall the different fate of the 19th century proofs that we now consider the first correct ones.

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Beginnings of Set Theory in Poland

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In the paper the very beginnings of set theory in Poland after the First World War will be considered. The emphasis will be put on the program of developing set theory as well as on the philosophical background of it.

At the end of the First World War after re-establishing of the university in Warsaw Waclaw Sierpiński, Zygmunt Janiszewski and Stanisław Mazurkiewicz became professors of mathematics there. They had similar scientific interests. In 1917 Janiszewski wrote a paper in which he sketched the program of developing mathematics in Poland. He proposed to concentrate on one discipline, more exactly on set theory and related domains and to establish a new journal *Fundamenta Mathematicae* devoted (exclusively) to them as well as to topology and mathematical logic. The first issue appeared in 1920. Janiszewski and others stressed the interconnections between set theory and other domains of mathematics. They understood it as a foundation of mathematics in a methodological (rather than philosophical) sense. This implied the stress put on applications of set theory. One should underline that in the Warsaw school connections of set theory with mathematical logic, the foundations of mathematics and the philosophy of mathematics were stressed. Two philosophers working in the field of

logic, namely Jan Łukasiewicz and Stanisław Leśniewski became members of the editorial board of *Fundamenta*. This contributed to the broadening of the perspective in set-theoretical research. The collaboration of mathematicians with logicians and philosophers in the Warsaw school was a specific phenomenon distinguishing it from other schools. Another typical feature was the fact that the school was an adherent of no definite tendency in the philosophy of mathematics – though one had good knowledge of actual trends and theories. Important was only the correctness and fruitfulness of the applied methods, important were the results and not the particular methods used. It found its expression first of all in the case of investigations on the axiom of choice.

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Numerical Analysis in the 18th Century: the Euler-Mascheroni Constant

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 01A50

Lorenzo Mascheroni (1750-1800) is one of the leading Italian mathematicians of the eighteenth century (*Geometria del Compasso*, 1797). He was also an important politician of the Cisalpine Republic [1]. Mascheroni's deservedly famous works on Mathematical Analysis and Numerical Analysis are the *Adnotationes ad calculum integrale Euleri* published in two parts in 1790 and in 1792 [2]. On 8th January 1790 Mascheroni was very pleased to announce the first part of the work to his friend and correspondent Girolamo Fogaccia [3].

When studying the harmonic sequences, Euler dwelt upon that of the reciprocals of natural numbers, and he found that the limit of its difference from the natural logarithm was a constant, of which he calculated the first six decimal places. Shortly after this, he introduced greater precision to the calculation of the constant:

$$0,5772.56649\ 01532\ 9$$

Euler took up the question once more in his treatise on differential calculus, using, for the calculation of the constant, Bernoulli' numbers. He then came back to the constant in the first volume of the *Institutiones calculi integralis*.

In a final memoir on the subject published posthumously, Euler provided two other ways to calculate the constant without, however, improving his results [4].

Mascheroni succeeded in rectifying Euler's value, reaching the calculation of thirty-two decimal places of which only the first nineteen turned out to be exact:

$$0, 577215 664901 532860 618112 090082 39$$

Considering the fact that in the nineteenth century it was shown that the transcendence of e and of π , the irrationality and transcendence of the Euler-Mascheroni constant, usually denoted by γ , remains the most important open problem [5].

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The Relevance of Bhaskaras's Methods in the Present Context

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Much has been written about Bhaskaracharya's (1150 A.D.) 'Lilavati'. However its relevance to the present day mathematics is not mentioned.

The purpose of this paper is to highlight its relevance in the present context. The general form of Binomial theorem was not known to Bhaskara. But then the way the theorem was used to extract the square and cube roots of a number is ingenious, more so because it is extended to find root of any order.

The similarity and difference between Bhaskara's semi algebraic, and Euclid's purely geometric, demonstrations of Pythagoras theorem is highlighted. The use of algebra in geometry and vice versa in Indian method is amazing.

Geometrisation of square of the sum by Chinese and that of the square of the difference by Bhaskaracharya demonstrates Pythagoras theorem.

The construction of a rectangle equal in area to a cyclic quadrilateral to derive the formula for its area is new, though the derivation is not flawless. The construction of 3 cyclic quadrilaterals (in a circle), given 4 sides, using 2 pairs of similar right angled triangles, is a clever device. The idea of radian measure was used in Astronomical works. The method of doubling the sides of a regular polygon leads to an infinite sequence with limit equal to π . An ingenious guess gives a close approximation for a chord in terms of the arc and vice versa.

The primitive concepts of limits are expressed by an algorithm to deal with expressions containing zero multiples and zero divisors. This algorithm, according to Bhaskara, is used in planetary calculations. The use is illustrated by finding derivatives of $\sin(x)$ and $x^3 + 2 * x - 5$. This is compared with Newton's method. The small difference formula given $\sin(x) = \cos(x)dx$ follows as a corollary from the derivative of $\sin(x)$.

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Mathematical Concepts, Axioms, the Logic and Some of Its Results Viewed from the Philosophical Knowledge of North Indian Saints

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Historically, philosophical knowledge has played an important role in mathematical development. Pythagoras, and later Socrates and his followers have been attributed to have given integers their existence at the mental level, beyond the reach of the senses [1]. Later, in the 16th and 17th centuries, the tradition of philosophy along with mathematical and physical knowledge was continued.

Works of Descartes [2] and Leibniz [3] are in that spirit. Subsequently, the development of mathematics has been pursued independent of philosophy.

Shorn of philosophy, the validity of mathematical knowledge became an important issue in the 20th century. For that purpose, a formal mathematical system was developed by Hilbert and his colleagues. However, Gdel presented his well known result of incompleteness of such a system [4]. It implies that not all the true propositions can be proven to be true by the formal approach only. In other words, mathematical truth cannot be contained in a formal system.

The purpose of this presentation is to examine the implications of the philosophy of North Indian Saints [5-6] to mathematical thought. In addition, an attempt is made to compare the use of basic concepts, axioms, and logic in the formal structure of mathematics, with those implicit in the structure of the system practiced by the North Indian Saints to achieve their absolute knowledge.

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Methods of Interpolation in Indian Astronomy

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Āryabhata (b. 476 C.E.), *Varāhamihira* (b. 485 C.E.), *Brahmagupta* (b. 598 C.E.), *Bhāskara II* (b. 1114 C.E.) and other Indian astronomers have discussed methods of interpolation for evaluating values of trigonometric functions. However, *Brahmagupta* is credited for using second-difference interpolation formula to evaluate various values of Hindu trigonometric functions. It is not known whether he was the pioneer of this kind of formula or he simply used it. The

main purpose of this paper is to present a brief account of the work done on interpolation by *Brahmagupta* in his astronomical work *Khaṇḍakhādya*. Further, we give a remark on its probable impact on other cultural areas.

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Early Printed Arabic Geometry Textbooks

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It has often been argued that interest in study of geometry and more advanced mathematical sciences languished in the Arabic/Islamic community especially because the sciences were said to have been excluded from the madrasa educational system. This interpretation is now known to be too limited, if not actually incorrect. Science and mathematics were regularly taught in the madrasas over the centuries [2]. If anything, the “decline” of the sciences may have come not because they were excluded from but because they were so well integrated into the educational system [1]. This integration enforced specific styles and formats of presentation on the sciences which are often at odds with modern sensibilities.

When print technology became “naturalized” into nineteenth century Ottoman and Islamic societies, mathematics textbooks were produced along with textbooks in other subjects. Initially, these early printed textbooks perpetuated many of the basic features of the manuscript tradition. The same treatises continued to be used for studying mathematics. And even the traditional manuscript format and appearance of the text continued to be followed [3].

This paper will very briefly describe a number of nineteenth century printed editions of traditional mathematical (mainly geometrical) textbooks from Morocco to India, looking both at content and format of presentation. The focus then narrows to an Indian printing of an Arabic geometry textbook (Calcutta, 1826), which broke with the more traditional printed Arabic geometry textbooks in several ways. These changes in presentation style imply a new pedagogical approach in the Arabic Euclidean tradition. The paper concludes with a brief attempt to situate this Calcutta edition of Euclid within the intellectual and mathematical landscapes of the time.

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Euclid’s Number-Theoretical Work

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When people mention the mathematical achievements of Euclid, his geometrical achievements always spring to mind. But, his Number-Theoretical achievements (See Books 7, 8 and 9 in his magnum opus *Elements* [1]) are rarely spoken. The object of this paper is to affirm the number-theoretical role of Euclid and the historical significance of Euclid’s algorithm.

It is known that almost all elementary number-theoretical texts begin with Division algorithm. However, Euclid did not do like this. He began his number-theoretical work by introducing his algorithm. We were quite surprised when we began to read the *Elements* for the first time. Nevertheless, one can prove that Euclid's algorithm is essentially equivalent with the Bezout's equation and Division algorithm. Therefore, Euclid had preliminarily established Theory of Divisibility and the greatest common divisor. This is the foundation of Number Theory. After more than 2000 years, by creatively introducing the notion of congruence, Gauss published his *Disquisitiones Arithmeticae* in 1801 and developed Number Theory as a systematic science. Note also that Euclid's algorithm implies Euclid's first theorem (which is the heart of 'the uniqueness part' of the fundamental theorem of arithmetic) and Euclid's second theorem (which states that there are infinitely many primes and represents undoubtedly a major advance in Ancient Greek). Thus, in the nature of things, Euclid's algorithm is the most important number-theoretical work of Euclid. For this reason, we further summarize briefly the influence of Euclid's algorithm. According to Knuth [3], 'we might call Euclid's method the granddaddy of all algorithms...'. It leads to the conclusion that Euclid's algorithm is the greatest number-theoretical achievement of the Euclidean age.

Remark: For the preprint of paper, it is at <http://arxiv.org/abs/0902.2465>. Recently, by studying Euclid's algorithm and related problems, we obtained some interesting results (<http://eprint.iacr.org/2009/151>; <http://arxiv.org/abs/0912.0147>; <http://arxiv.org/abs/0905.1655>; <http://arxiv.org/abs/0911.3679>). Particularly, we found a special sequence (<http://arxiv.org/abs/0903.1019>) which leads to a new weakened form of Goldbach's Conjecture [2], and a refinement of the function $g(x)$ on Grimm's Conjecture (<http://arxiv.org/abs/0811.0966>), and so on.

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Section 21

Mathematical Software

KNOPPIX/Math: Open Source Desktop Environment for Mathematics

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KNOPPIX/Math is a project to archive free mathematical software and documents and offer them on KNOPPIX, a bootable CD/DVD that contains a collection of GNU/Linux software. The KNOPPIX project was started in Germany by Klaus Knopper. KNOPPIX can be used for Linux demos, educational presentations and system recovery. KNOPPIX/Math provides a desktop for mathematics that can be set up easily and quickly. This project began in February 2003. The newest product is “KNOPPIX/Math/2010 The next generation”, which contains many open source mathematical software: CoCoA, GeoGebra, Macaulay2, Maxima, Reduce, Risa/Asir, Sage, and Singular, . . . Once you run the live system, you can enjoy a wonderful world of mathematical software without needing to make any installations yourself. KNOPPIX/Math also includes many documents, sample files and flash movies. As an experimental attempt, KNOPPIX/Math has full-text search capability for mathematical documents.

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A C++ Class Library for Statistical Set Processing

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MRS is an open C++ class library for statistical set processing. The goal of MRS is to improve the accuracy and reliability of numerical results in computational statistical problems and provide a cohesive object-oriented framework for set-valued and set-oriented computational statistics. This is generally achieved by extending arithmetic beyond the built-in data types and applying fixed-point theorems. MRS has a suite of methods at the interface of randomized algorithms, interval analysis, validated numerics and extended arithmetics over tree-based multi-dimensional metric data structures with nice algebraic properties.

Computational statistical applications with the MRS library include exact samples from highly multi-modal target densities in a trans-dimensional setting, importance sampler from pseudo and quasi random numbers by adaptive bisections in the domain, and L_1 consistent high-dimensional density estimators for massive data problems on the basis of data-dependent randomized priority queue and posterior mean of Markov chain over dense histogram spaces.

This C++ class library is the mathematical software behind recent applications [1, 2, 3] and the source code is publicly available at <http://www.math.canterbury.ac.nz/~r.sainudiin/codes/mrs/>.

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Sage: Creating a Viable Open Source Alternative to Magma, Maple, Mathematica, and Matlab

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I started the Sage mathematical software project (<http://sagemath.org>) in 2005 to provide a powerful free open source alternative to Magma, Maple, Mathematica, and Matlab. In 2007, Sage was awarded first prize for scientific software in the Trophees du Libre competition. Sage has since grown dramatically, with well over 200 developers around the world.

Sage can be used to study a wide range of subject areas: algebra, calculus, elementary and advanced number theory, cryptography, numerical computation, commutative algebra, group theory, combinatorics, graph theory, exact linear algebra and much more. Sage combines nearly 100 open source software packages such as Python, Singular, Pari, GAP, etc., and seamlessly integrates their functionality into a common experience. Sage is well suited for both education and research. The interface is a notebook in a web browser or the command line. Using the notebook, Sage connects either locally to your own Sage installation or to a Sage server on the network. Inside the Sage notebook you can create embedded graphics, typeset mathematical expressions, add and delete input, and share your work across the network.

This talk will explain the history and motivation behind Sage and demonstration how Sage is useful to working mathematicians.



Graphsoft

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The software “Graphsoft” is developed to help the researchers in graph theory. With the help of this software they can input graphs for their programs using the mouse. There is no need to input the graph by its adjacency matrix or by its incidence matrix. With the help of this software they can check the results, before trying to prove them, by verifying the results by drawing as many graphs as needed with the desired properties.

With the help of this software one can draw a graph, using the mouse, and save the graph. One can retrieve any already saved graph and edit the graph by inserting new vertices or edges, or deleting any of the existing vertices or the edges. One can assign labels and colors to the vertices and the edges. The software also gives the \LaTeX file for the graphs drawn which can be used as input files in any \LaTeX project.

The software is developed in C++. The program (the source code) is also given with the software. So one can easily modify the program so as to design a software for one's own use. Some examples of such modifications, with proper instructions, are also available with the software. The size of the software is very small. No supporting files are needed and there is no restriction on the configuration of the computer to use the software.

As the size of the software is very small and as no supporting files are needed, the software is easily portable. Among all the available such softwares, this is the simplest, the smallest, easily modifiable, and user friendly one which is available with the source code.



Supplement

Section 2

Algebra

Matrices over Dedekind Domains as Sums of k -th Powers

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For $n \geq k \geq 2$, and for $k = 3, 4$, $n \geq 2$, we find a necessary and sufficient condition for every $n \times n$ matrix over a dedekind domain D to be a sum of k -th powers of matrices over D . We also deduce the discriminant criterion (see [2] and [1]) for every matrix over the ring of integers of a number field to be a sum of k -th powers in these cases. This work is motivated by [4] and [3].

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On Semirings and Ordered Semirings

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Let $(S, +, \cdot)$ be a semiring and $(S, +, \cdot, \leq)$ be a totally ordered semiring (t.o.s.r.). In this paper we study the properties of zero-square semirings. Also properties of semirings in which $ab = a + b + ab$ for all a, b in S are studied. It is established that in a semiring with IMP (Integral Multiple Property), if $(S, +)$ is a band then the following are true.

1. (S, \cdot) is a band.
2. If $(S, +, \cdot)$ is a Positive Rational Domain (PRD) then S reduced to singleton set.
3. If (S, \cdot) is quasi commutative then (S, \cdot) is commutative.

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Stably Free Modules over $R[X]$ of Rank $> \dim R$ are Free

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We prove constructively that for any finite-dimensional ring R and $n \geq \dim R + 2$, the group $E_n(R[X])$ of elementary matrices of size $n \times n$ with entries in $R[X]$

acts transitively on the set $Um_n(R[X])$ of unimodular vectors of length n with entries in $R[X]$. In particular, we obtain, without any Noetherian hypothesis, that for any finite-dimensional ring R , all finitely generated stably free modules over $R[X]$ of rank $> \dim R$ are free.



Section 4

Algebraic and Complex Geometry

On the Minimal Number of Generators of an Ideal of General Points in a Projective Space, \mathbf{P}^4

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Let S be a general set of s points in \mathbf{P}^4 , and R the homogeneous coordinate ring of \mathbf{P}^4 . Then the ideal of S , I_S has a minimal free resolution of the form:

$$0 \longrightarrow F_3 \longrightarrow F_2 \longrightarrow F_1 \longrightarrow F_0 \longrightarrow I_S \longrightarrow 0$$

where $F_p = R(-d-p)^{a_{p-1}} \oplus R(-d-p-1)^{b_p}$,

d is the smallest integer such that $s \leq h^0(\mathbf{P}^4, \mathcal{O}_{\mathbf{P}^4}(d))$,

$a_p = h^0(\mathcal{I}_S \otimes \Omega_{\mathbf{P}^4}^{p+1}(d+p+1))$, $b_p = h^1(\mathcal{I}_S \otimes \Omega_{\mathbf{P}^4}^{p+1}(d+p+1))$ and

$\binom{d+3}{4} < s \leq \binom{d+4}{4}$, with $0 \leq p \leq 3$, $a_{p-1} = \binom{d+4}{4} - s$, when $p = 0$.

C Walter in [2] proved for what set of s points in \mathbf{P}^4 does $a_p b_p \neq 0$ for some p . In this paper I prove that either $a_0 = 0$ or $b_0 = 0$ by proving maximal rank for the map: $H^0(\mathbf{P}^4, \Omega_{\mathbf{P}^4}(d+1)) \longrightarrow \bigoplus_{i=1}^s \Omega_{\mathbf{P}^4}(d+1)|_{S_i}$ by use of the methods of Horace, used for \mathbf{P}^3 in [1] to prove bijectivity for a specific number of fibres and then maximal rank for a general set.

We wish to prove that μ is of maximal rank and as a consequence we have the following theorem.

Theorem 1. *Suppose we have a general set S , of s points in \mathbf{P}^4 , $s \geq 5$ such that the map $\mu : H^0(\mathbf{P}^4, \Omega_{\mathbf{P}^4}(d+1)) \rightarrow \bigoplus_{i=1}^s \Omega_{\mathbf{P}^3}(d+1)|_{S_i}$ is of maximal rank then the homogeneous ideal $I_S \subset \mathbf{k}[X_0, X_1, X_2, X_3, X_4]$ has $(4s - \frac{1}{6}d(d+2)(d+3)(d+4))$ number of minimal generators of degree $d+1$ and $(\frac{1}{6}d(d+2)(d+3)(d+4) - 4s)$ number of minimal relations of degree d .*

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Section 5

Geometry

Secondary Chern-Euler forms and the Law of Vector Fields

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The Law of Vector Fields is a relative Poincaré-Hopf theorem first proved by Morse [2]. It expresses the Euler characteristic of a manifold X with boundary M in terms of the total indices of a generic vector field V and the inner part $\partial_- V$ of its tangential projection on the boundary as

$$\text{Ind } V + \text{Ind } \partial_- V = \chi(X). \quad (1)$$

Endowing X with a Riemannian metric, we give in [3] two differential-geometric proofs of this topological theorem. In his famous proof [1] of the Gauss-Bonnet theorem, Chern constructed a differential form Φ on the tangent sphere bundle STX to transgress the Euler curvature form Ω of X , i.e., $d\Phi = -\Omega$. In terms of the secondary Chern-Euler form Φ , the Law of Vector Fields (1) is equivalent to

$$\int_{\vec{n}(M)} \Phi - \int_{\alpha_V(M)} \Phi = \text{Ind } \partial_- V, \quad (2)$$

where \vec{n} is the outward unit normal of M and α_V is the normalized V .

A first proof of (2) is achieved by constructing a chain on $STX|_M$ away from the singularities of $\partial_- V$ connecting $\alpha_V(M)$ to $\vec{n}(M)$ and applying Stokes' theorem. A second proof employs a detailed study of Φ on $STX|_M$, which may be of independent interest. More precisely, we explicitly construct a differential form Γ that, away from the outward and inward unit normals, transgresses Φ up to a pullback form. That is, on $STX|_M \setminus (\vec{n}(M) \cup (-\vec{n})(M))$, we have

$$\Phi - \pi^* \vec{n}^* \Phi = d\Gamma,$$

where $\pi : STX|_M \rightarrow M$ is the natural projection. Then an application of Stokes' theorem again proves (2).

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Section 6

Topology

The Gromov-Hausdorff Hyperspace of the Unit Interval

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The Gromov-Hausdorff distance d_{GH} was introduced by M. Gromov [2]. It turns the set of all isometry classes of compact metric spaces into a metric space. If X and Y are two compact metric spaces, then $d_{GH}(X, Y)$ is defined to be the infimum of all Hausdorff distances $d_H(i(X), j(Y))$ for all metric spaces M and all isometric embeddings $i : X \rightarrow M$ and $j : Y \rightarrow M$. Clearly, the Gromov-Hausdorff distance between isometric spaces is zero; it is a metric on the set GH of isometry classes of compact metric spaces. The metric space (GH, d_{GH}) is called the Gromov-Hausdorff hyperspace. It is a challenging open problem to understand the topological structure of this metric space. The talk contributes towards this problem.

For a metric space X , we denote by $\text{GH}(X)$ the subspace of GH consisting of the classes $[E]$ whose representative E is a subset of X .

In this paper we prove that the Gromov-Hausdorff hyperspace $\text{GH}([0, 1])$ of the unit interval is homeomorphic to the Hilbert cube, thus giving a positive answer to Question 1307 from the book of open problems [4].

As usual, by the Hilbert cube we mean the product $Q = \prod_{k=1}^{\infty} \{I_k \mid I_k = [0, 1]\}$ endowed with the product topology.

Our proof is based on some facts and methods of the theory of Hilbert cube manifolds (see [3] and [5]) and the equivariant theory of retracts [1].

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Pairwise Metacompact Spaces

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After the introduction of Bitopological Spaces by Kelly [3] in 1963, many researchers investigated compactness and other weaker covering properties in Bitopological spaces using different types of open covers. Various versions of paracompactness were introduced by many authors (See [2, 4]). In [1], Bose et.al. introduced paracompactness in terms of locally finite families and pairwise parallel open covers and obtained a characterization for that. In this paper we define point finite families and define pairwise metacompactness in terms of parallel open refinement. A characterization of metacompactness in bitopological spaces together with many related results are obtained. Further, some other relationships of the introduced concept with other covering properties and its behaviour under various types of mappings are also investigated.

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Section 7

Lie Theory and Generalizations

Naturally Graded non p -filiform Leibniz Algebras

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Leibniz algebras appear as a generalization of Lie algebra and are one of the new algebras introduced by Loday in connection with the study of the periodicity phenomena in algebraic K -Theory [4]. The naturally graded algebras constitute, in certain way, the basic structure of the algebra which we are considering and so they are interesting in this line of investigation. This family of Leibniz algebras plays a fundamental role in the cohomological study of the nilpotent Leibniz algebras [5]. The natural gradation of nilpotent Leibniz algebras, the subspaces of gradation, and the existence of an appropriate homogeneous basis (needed to obtain the classification) are derived from the central descending sequences ([3]). Recently, several papers are focused to the study of some interesting families of Leibniz algebras, such as p -filiform and quasi-filiform Leibniz algebras (see [1, 2]). These algebras have their characteristic sequences equal to $(n-p, 1, 1, \dots, 1)$ and $(n-2, 2)$ with $\dim(L) = n$.

In this work we study a subclass of naturally graded Leibniz algebras with nilindex $n-3$. In this case, for the characteristic sequence we have the following

three possibilities: $(n - 3, 1, 1, 1)$, $(n - 3, 2, 1)$ and $(n - 3, 3)$. We will show the classification of those with characteristic sequence is equal to $(n - 3, 3)$. To highlight that along the work we have used the software *Mathematica*.

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On Maximum Length of Nilpotent Leibniz Algebras

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The notion of length of a Lie algebra was introduced by Gomez, Jimenez-Merchan and Reyes in [3]. In this work they distinguished a very interesting family: algebras admitting a graduation with the greatest possible number of non-zero subspaces, the so-called algebras of maximum length. Leibniz algebras appear as a natural generalization of Lie algebras (Loday,[4]) and the concept of length can be defined in a similar way in this setting. It is therefore expected that Leibniz algebras of maximum length will play a similar role to the Lie case. The cohomological properties of Leibniz algebras have been widely studied (see for example [2] and [5]). A remarkable fact of the algebras of maximum length is the relative simplicity of the study of these properties [1]. The study of the

classification of non associative nilpotent Lie algebras is too complex. In fact, it appeared two centuries ago and it still remains unsolved. As to Leibniz algebras the problem is analogous thus we will restrict our attention to two important families of Leibniz algebras: p -filiform and quasi-filiform.

In this work we will study the length of the quasi-filiform Leibniz algebras. The classification of null-filiform, filiform and 2-filiform case are already done. We will show the classification of the algebras with characteristic sequence $(n - 2, 2)$ in order to completely classify the Leibniz algebras of maximum length with nilindex $n - p$, with $0 \leq p \leq 2$. For this study we will extend the naturally graded quasi-filiform Leibniz algebras, which are well-known in this line of investigation.

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Section 8

Analysis

Differential Properties of Orthogonal Matrix Polynomials

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In the last years new families of orthogonal matrix polynomials on the real line have been found along with their orthogonality measure. Typically they are joint eigenfunctions of some fixed differential operator with matrix coefficients. We give an overview of the techniques that have led to these examples in the last years, focusing on new phenomena which are absent in the scalar theory. We also show applications that these families have in quantum mechanics, quasi-birth-and-death processes or time-and-band limiting problems.



The Proof of Koebe's General Uniformisation Theorem for Planar Riemann Surfaces and its Application

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The object of the talk is to present a transparent and complete proof of Koebe's General Uniformisation Theorem which asserts that "Every planar Riemann surface is biholomorphic to a domain in Riemann sphere". We also indicate

how Koebe's Theorem can be used to construct compact Riemann surfaces of every genus (explicitly in case of $g = 1$) in a very concrete way.

We show here how, for a relatively compact domain with good boundary, the planarity condition just means that the boundary curves generate the homology of the domain. In the sequel we use this result along with the beautiful construction of Weyl [3] to prove that the boundary curves form an *integral* basis for the homology of the domain.

Finally we show that the method of proof used in [1] (to prove that plane domains of finite connectivity with analytic boundary are biholomorphic to circular-slit annuli) can be carried over for domains with analytic boundary on planar Riemann surface. However, our proof for the injectivity of the constructed mapping function seems to be new, and we feel it is more satisfactory than the one given in [1].

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Fixed Point Theorems for Various Classes of 1-set Contraction Mappings

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Let X be a real Banach space, D an open subset of X with $0 \in D$ and T a mapping defined on the closure \bar{D} of D and taking values in X . It is well known that if D is bounded and T is a condensing mapping, then the Leray-Schauder boundary condition is sufficient to guarantee the existence of a fixed point of T . Indeed this result is due to Petryshyn [5] who observed that it follows easily from Nussbaum's degree theory [4]. In a recent paper [3] A.J. Melado and C.H. Morales have introduced and studied a new condition called the Interior Condition, which resembles the Leray-Schauder condition. However unlike the Leray-Schauder condition which is a boundary condition, the interior condition

holds for the interior points, which lie near the boundary of the domain D of the map T . This interior condition is formulated as follows.

A mapping $T : \overline{D} \rightarrow X$ satisfies the **Interior Condition** if there exists $\delta > 0$ such that $T(x) \neq \lambda x$ for $x \in D^*$, $\lambda > 1$ and $T(x) \notin \overline{D}$, where $D^* = \{x \in D : \text{dist}(x, \partial D) < \delta\}$.

The fixed point theorem similar to the one obtained by Petryshyn does not necessarily hold true under this new interior condition. Therefore a more restricted class of domains called strictly star-shaped sets is introduced, in order to obtain fixed points.

In section two of this paper we extend the main result obtained by Melado and Morales to 1-set contraction maps and use our result to deduce new fixed point theorems for various other classes of mappings. In section three we have fixed point results for nonexpansive, LANE, and uniformly strictly contractive maps with compact or completely continuous perturbations under the interior condition.

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Section 9

Functional Analysis and Applications

Rational Approximation on Closed Curves

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A problem an approximation of classes of function determined only on the boundary of domain takes important place simultaneously with studying approximation by means of polynomials analytic in the domain G and with some conditions on the boundary Γ of functions.

Obviously, generally speaking, it is impossible to approximate such classes of function by means of polynomials. Therefore, in this case, usually different forms of rational functions or so called generalized polynomials are used as approximation aggregate. My followers D. Israfilov, I. Botchaev and me studied problems on approximation of function determined only on the boundary of domain by means of rational functions of the form $R_n(z) = P_n(z, \frac{1}{z})$.

In the given report, we consider a rational function of the form $R_n(z) = P_n(z, \bar{z})$ as an approximate aggregate. For this case, analogies of Jackson's direct theorems on closed curves of complex plane are proved.

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Some Classes of Operators Related to p-hyponormal Operator

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We present an alternative proof of a result of M. Fujii, D. Jung, S.H. Lee, M.Y. Lee, and R. Nakamoto. The authors obtained a result that “if $T(p, p)$ for $p > 0$, then T is p -paranormal”, treating x as unit vector. T. Ando has proved that an operator T is paranormal if and only if $T^{*2}T^2 + 2k T^*T + k^2 \geq 0$ for all real k . Taking a quadratic form analogous to this we have established the result for all x . We have also introduced a new class $*A(p, q)$ parallel to $A(p, q)$ (for $p, q > 0$) and established its monotonicity.

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Iterative Method for Finite Family of Hemi Contractions

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Iterative methods for approximating fixed points of nonexpansive mappings have been extensively studied (see e.g. [1, 2, 3, 5]). But, iterative methods for approximating pseudocontractive mappings are far less developed than those of nonexpansive mappings. However, on the otherhand pseudo contractions have more powerful applications than nonexpansive mappings in solving nonlinear inverse problems. In recent years many authors have studied iterative approximation of fixed point of strongly pseudocontractive mappings. Most of them used Mann iteration process [4]. But in the case of pseudocontractive mapping it is well known that Mann iteration fails to converge to fixed point of lipschitz pseudocontractive mappings in compact convex subset of a Hilbert space. Hemicontractive mappings is an important generalization of pseudocontractive mappings.

In the present paper, we consider problem of finding a common fixed points of finite family hemicontractive mappings. We suggest a new algorithm for solving this problem. Strong convergence of this algorithm for a finite family of hemicontractions will be proved which generalizes the recent result of [6] and others.

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Section 10

Dynamical Systems and Ordinary Differential Equations

Exponential Decay of Semigroups for Second Order Non-selfadjoint Linear Differential Equations

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In Hilbert space H consider a second-order linear differential equation

$$u''(t) + Du'(t) + Au(t) = 0, \quad u(0) = u_0 \quad (1)$$

under the following assumptions

- (A) Operator $A : \mathcal{D}(A) \subset H \rightarrow H$ is m -sectorial and $\Re(Ax, x) \geq a_0(x, x)$ for some positive a_0 ($x \in \mathcal{D}(A)$).

Since A is m -sectorial there exist a self-adjoint positive definite operator T and a self-adjoint $S \in \mathcal{L}(H)$, such that

$$\Re(Ax, x) = (T^{1/2}x, T^{1/2}x) \geq a_0(x, x), \quad A = T^{1/2}(I + iS)T^{1/2}.$$

By H_s ($s \in \mathbb{R}$) denote a collection of Hilbert spaces generated by a self-adjoint operator $T^{1/2}$. By $(\cdot, \cdot)_{-1,1}$ denote a duality pairing on $H_{-1} \times H_1$ and by $\|\cdot\|_s$ denote a norm on H_s .

(B) D is a bounded operator $D \in \mathcal{L}(H_1, H_{-1})$, and

$$\beta = \inf_{x \in H_1, x \neq 0} \frac{\Re(Dx, x)_{-1,1}}{\|x\|^2} > 0.$$

Denote $D_1 = \Re D, D_2 = \Im D \in \mathcal{L}(H_1, H_{-1})$ and $D = D_1 + iD_2$.

Theorem *Let the assumptions (A) and (B) hold and for some $k \in (0, \beta)$ and $m \in (0, 1]$*

$$\omega_1 = \inf_{x \in H_1, x \neq 0} \frac{\frac{1}{k}(D_1x, x)_{-1,1} - \|x\|^2 - \frac{1}{4m} \|(\frac{1}{k}\tilde{S} - D_2)x\|_{-1}^2}{\|x\|^2} \geq 0.$$

Then for all $u_0, u_1 \in H_1, Du_1 + Au_0 \in H$ there exists a unique solution of (1) and

$$\|u(t)\|_1^2 + \|u'(t)\|^2 \leq \text{const} \cdot \exp\{-2k\theta t\} (\|u_0\|_1^2 + \|u_1\|^2), \quad t \geq 0$$

where

$$\theta = \min \left\{ \frac{\omega_1}{2}, \frac{1-m}{\omega_2} \right\} \geq 0$$

and

$$\omega_2 = \sup_{x \in H_1, x \neq 0} \frac{\|x\|_1^2 + k(D_1x, x)_{-1,1} + k^2\|x\|^2}{\|x\|_1^2}$$



Coexistence of Competing Predators in Coral Reef Ecosystem

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An analysis is made on a four dimensional mathematical model where there is a constant rate of flow of input nutrient. An organism is introduced in the model which is growing on that nutrient. Two other predators are also introduced on that organism. The predators at the second and third trophic levels belong to the same species, though of different age groups. The predator at the third trophic level exhibits a distinct cannibalistic attitude to the predator of the second trophic level. A local stability of the system is obtained when one or more predators goes extinct. Under appropriate circumstances a positive rest point of the system is obtained. Computer simulations have been carried out to illustrate different analytical results.

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Asymptotic Properties of Some Nonlinear Differential Equations

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Asymptotic properties of solutions have been considered for some nonlinear differential equations. The paper deals with investigation of bounded solutions, oscillatory solutions and another asymptotic properties. We consider some equation and we obtain necessary conditions and sufficient conditions for existence of certain monotonic, oscillatory solutions, and estimates eigenvalue for operators of higher order.

For the nonlinear equation

$$(r(x)y^{(n)})^{(n)} = yf(x, y) \tag{1}$$

we obtain necessary and sufficient conditions for existence of certain monotonic, oscillatory solutions.

As special cases we will consider generalizations of the Emden-Fowler equation

$$(r(x)y^{(n)})^{(n)} = f(x)|y|^\gamma y \quad (2)$$

where $\gamma > 0$. We also consider the Emden-Fowler equation

$$y'' = Ax^\sigma y^n, A = \text{const}, \sigma = \text{const} \quad (3)$$

which arises in a number of physical problems, connected with problems of gas dynamics. We obtain asymptotic formulas for all positive solutions of the equation. (see [3],[5]).

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Oscillation Criteria for Second Order Nonlinear Neutral Type Dynamic Equations on Time Scales

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We are concerned with the second-order nonlinear neutral type dynamic equations on a time scale \mathbb{T} , which may be an arbitrary closed subset of the reals. The study of dynamic equations on time scales has been created in order to unify the study of differential and difference equations. The oscillatory behavior of solutions of the second-order nonlinear dynamic equations of neutral type is discussed by employing the Riccati transformation technique and some oscillation criteria are established for the dynamic equations. Our results in the special case when $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{N}$ extend and improve some well-known oscillation results for second-order nonlinear neutral delay differential and difference equations and are essentially new on the time scales $\mathbb{T} = h\mathbb{N}$, $h > 0$, $T = q^{\mathbb{N}}$ for $q > 1$, $\mathbb{T} = \mathbb{N}^2$, etc. Some examples are considered to illustrate our main results.



Applications of Noethers Theorem to Non Linear Oscillators Through Canonical Transformations

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Second order nonlinear ordinary differential equations are of importance in dynamics. We propose a method of deriving first integrals of second order nonlinear ordinary differential equations through canonical transformations. Every second order ordinary differential equation can be represented by a Hamiltonian function and the corresponding non-conservative force though the representation is not unique. Hamilton's canonical equations of motion for non-conservative dynamical system are derived in the new canonical variables [1]. We define new Hamiltonian function to incorporate non-conservative force term appearing in the generalized momentum equation. Non-conservative force in the new canonical variables is defined to retain the form of Hamilton's canonical equations. Thus a family of Hamiltonian functions and the corresponding non-conservative force is generated. These families represents the given second order ordinary differential equation in new coordinate system. Application of Noether's theorem [2] to this family gives the first integral of given ordinary differential equation. With this approach the first integrals of equations representing nonlinear Harmonic oscillators, Helmholtz oscillator with friction, Duffing Vander Pol oscillator, Emden Fowler equation and generalized Emden Fowler equation are constructed.

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Section 11

Partial Differential Equations

Invariant Solutions of the Mixed Korteweg-de Vries Equation Arising in Stratified Fluids

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We consider an equation combining the Korteweg-de Vries (KdV) and modified Korteweg-de Vries (mKdV) equations, or simply the combined KdV-mKdV equation. Exploiting the Lie-point symmetries and conservation laws of the underlying equation, we construct the invariant solutions thereof. Invariant solutions allow reduction of partial differential equations (PDEs) into ordinary differential equations (ODEs) which can be solved either analytically or numerically.



Section 12

Mathematical Physics

Unsteady Convectively Driven Flow Past an Infinite Vertical Cylinder in Presence of Chemical Reaction

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76R10, 80A20

An exact solution to the problem of unsteady one dimensional free convective flow over an infinite vertical cylinder under the combined buoyancy effects of heat and mass transfer along with chemical reaction is presented. The dimensionless unsteady governing equations are solved by using the usual Laplace-transform technique. Numerical computations for velocity, temperature, concentration profile, skin friction, Nusselt number, Sherwood number are obtained for various set of physical parameters viz. chemical reaction parameter, Prandtl number, Schmidt number, buoyancy ratio parameter and time and computed results are presented in graphs.



String Cosmological Universes with the Bulk Viscosity in Relativistic Cosmology

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In this paper we study cosmological models with particles attached to them in LRS BI type space time. The dynamical and physical properties of such universes are studied, and the possibility that during the evolution of the universe the strings disappear leaving only the particles is also discussed here. It is found that the bulk viscosity plays a great role in the evolution of the universe. In these models we can find critical instant of time when there is a "Bounce". The models we study here are found to be of inflationary type and since a desirable feature of a meaningful string cosmological model is the presence of an inflationary epoch in the very early stages of evolution, our models can be thought of as realistic universes.

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Section 13

Probability and Statistics

Convolution Type Stochastic Volterra Equations in Hilbert Space

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The talk deals with stochastic Volterra equations of the form

$$X(t) = X_0 + \int_0^t a(t-\tau) A X(\tau) d\tau + \int_0^t \Psi(\tau) dW(\tau), \quad t \geq 0, \quad (1)$$

in a separable Hilbert space H . In (1), $X_0 \in H$, $a \in L^1_{\text{loc}}(\mathbb{R}_+; \mathbb{R})$, A is a closed linear unbounded operator, $\Psi(t)$ is an appropriate operator-valued process and $W(t)$ is a cylindrical Wiener process.

Let us note that the equation (1) contains a big class of equations and is an abstract version of several problems.

To the study of the equation (1) we use the resolvent approach, which enables us to obtain results in an analogous way as in the semigroup approach usually applied to stochastic differential equations. It is worth to emphasize that in our, resolvent case, new difficulties arise because the solution operator corresponding to the Volterra equation (1) in general does not create any semigroup. So, in consequence, powerful semigroup tools are not available in our case.

The aim of the talk is to present results concerning fundamental and the most important questions related to the equation (1), like the existence of strong solutions to (1) and some kind of regularity of these solutions.

We shall point out some consequences and complications coming from deterministic and stochastic convolutions connected with the stochastic Volterra

equations (1). Next, we will show, how to overcome these difficulties for some classes of kernel functions $a(t)$, $t \geq 0$, and the operators A , relevant for applications.

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On Dynamic Renyi Cumulative Residual Entropy Measure

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 62N05, 62B10

Shannon entropy [5], given by $H(X) = -\int_0^\infty f(x) \log f(x) dx$, plays an important role in the context of information theory. An alternative notation of entropy called cumulative residual entropy (CRE) defined by $\xi(X) = -\int_0^\infty \bar{F}(x) \log \bar{F}(x) dx$ is proposed in Rao et al. [4]. This measure is based on cumulative distribution function (CDF) rather than probability density, and is thus, in general more stable, since the distribution function is more regular because it is defined in an integral form unlike the density function, which is defined as the derivation of the distribution.

These entropy measures are not applicable to a system which has survived for some unit of time. Ebrahimi [2] considered the Shannon entropy of the residual lifetime $X_t = [X - t | X > t]$ as a dynamic measure of uncertainty. Asadi and Zohrevand [1] have considered the *dynamic cumulative residual entropy (DCRE)*, defined as the cumulative residual entropy of the random variable $X_t = [X - t | X > t]$. Extending the concept of cumulative residual entropy, we consider generalized cumulative residual information measure based on Renyi entropy [3] for a continuous random variable which is useful in life testing. The exponential, the pareto and finite range distributions, which are commonly used

in the reliability modeling have been characterized in terms of the proposed dynamic entropy measure.

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Non-additive Entropy Measure Based Residual Lifetime Distributions

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In the statistical physics literature Shannon entropy [2] is called Gibbs entropy, and is meant to be the amount of “disorder” of a system. Shannon entropy has been extended in several ways. One particular generalization is q -entropy, known as Tsallis nonextensive entropy [3]. The Tsallis nonextensive entropy of the statistical physics literature exactly matches with the Havrda-Charvat α entropy [1] of information theory.

The present communication considers non-additive Havrda and Charvat entropy measure for residual lifetime distributions. It is shown that the proposed measure determines the lifetime distribution uniquely. We characterize residual lifetime distributions and draw the probability curves for the distributions characterized for some specific values of the parameters.

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Section 14

Combinatorics

Polynomial Classes of Permutations Avoiding Exactly Two Patterns

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Albert, Atkinson and Brignall. [1] completely characterize sets Π such that the class of permutations avoiding the patterns of Π has polynomial growth. They also provide a complete classification when the set Π has size 2 or 3. When the set contains two permutations α and β , only two cases are possible. One of these cases (α increasing and β decreasing) has been thoroughly studied. The other case (α increasing and β quasi-decreasing) still presents some interesting open questions. In particular, Albert, Atkinson and Brignall provide bounds for the degree δ of the polynomial expressing the growth of the corresponding permutation class in terms of the sizes of permutations α and β . We provide more accurate bounds for this degree, showing in particular that δ grows at most linearly with respect to the size of α , whereas the upper bound provided by Albert et al. still left open the possibility that such dependency could be quadratic.

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Section 16

Numerical Analysis and Scientific Computing

Difference Schemes for Singularly Perturbed Boussinesq System

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65N06 · 65N12 · 65N15 · 65N30

Keywords. Boundary layer, Boussinesq system, Difference scheme, Singular perturbation, Uniform convergence.

In this study, we investigate the numerical solutions of the initial-boundary value problem for singularly perturbed Boussinesq system. First, asymptotic estimates for the original problem are established. Next, two level fitted difference method on a special non-uniform mesh, for the numerical solution of this problem is presented. The difference scheme is shown to converge to the continuous solution uniformly with respect to the perturbation parameter.



Streamline-Diffusion Finite Element Method for Singularly Perturbed Coupled Elliptic-Elliptic Transmission Boundary Value Problem

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We consider the singularly perturbed coupled elliptic-elliptic transmission boundary value problem, where in one subregion the coefficient of the second derivative is a small parameter. The solution of such problems typically contain strong interior layer rather than boundary layer. Parameter-uniform error bounds for the solution and its first scaled derivative are established using the Streamline-Diffusion Finite Element Method (SDFEM) on piecewise uniform meshes. We prove that the method is almost second order convergence for solution and first order convergence for its derivative in the maximum norm, independently of the perturbation parameter. Numerical results are provided to substantiate the theoretical results.



Numerical Solution of the Swift-Hohenberg Equation using Quintic B-spline Collocation Method

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In the present paper, a numerical method is proposed for the numerical solution of nonlinear Swift-Hohenberg equation

$$u_t + u_{xxxx} + 2u_{xx} + (1 - \alpha)u - u^3 = 0$$

This equation is widely used as a model for the study of various issues in pattern formation. It has been used to model patterns in simple fluids (e.g. Rayleigh-Bénard convection) and in a variety of complex fluids and biological materials, such as neural tissues. The equation is solved using the quintic B-spline collocation scheme on the uniform mesh points with appropriate initial and boundary conditions. The scheme is based on the Crank-Nicolson formulation for time integration and quintic B-spline functions for space integration. Computed results are obtained on domain $[0, L]$ and for different values of α and time T . By means of numerical simulations it is shown how different values of α and parameter L give different profiles. Solutions are depicted graphically and are compared with those already available in the literature. It is shown that quintic B-spline collocation method can effectively be used to solve these kind of non-linear equations.

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An Adaptive Cubic Spline Approach to Solve a Second Order Singularly Perturbed Boundary Value Problem

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A fourth order difference scheme using adaptive cubic spline for solving a self adjoint singularly perturbed two point boundary value problem of the form

$$\begin{aligned}\epsilon y'' &= q(x)y + r(x) \\ y(a) &= \alpha_0 \quad y(b) = \alpha_1\end{aligned}$$

is presented. Our scheme leads to a tri diagonal linear system. The convergence analysis is given. This method gives second and fourth order convergence depending upon the choice of parameters A_1 , A_2 , A_3 and A_4 . Numerical illustrations are given to verify the theoretical analysis of our method.

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XFT: A Fast Discrete Fractional Fourier Transform

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In recent years there has been a growing interest in the fractional Fourier transform driven by its large number of applications. The literature in this field follows two main routes [1]–[8]. On the one hand, the areas where the ordinary Fourier transform has been applied are being revisited to use this intermediate time-frequency representation of signals, and on the other hand, fast algorithms for numerical computation of the fractional Fourier transform are devised. In this paper we derive a Gaussian-like quadrature of the continuous fractional Fourier transform. This quadrature is given in terms of the Hermite polynomials and their zeros. By using some asymptotic formulas, we rewrite the quadrature as a chirp-fft-chirp transformation, yielding a fast discretization of the fractional Fourier transform and its inverse in closed form. We extend the range of the fractional Fourier transform by considering arbitrary complex values inside the unitary circle and not only at the boundary. We find that, the chirp-fft-chirp transformation evaluated at $z = i$, becomes a more accurate version of the **fft** which can be used for non-periodic functions.

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Reasons Why the ode45 Performs Better than the ode15s in Some Stiff Problems

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65L05, 65L06, 65L60

We have numerically simulated the solution of the linear diffusion equation and wave equation, which are both time-evolution partial differential equations (PDE). We have used a class of Galerkin method [5] and the Matlab ODE solvers ode45 and ode15s. The solver ode45 is based on an embedded Runge-Kutta method of orders 5(4) proposed in [1] and it is useful when solving non-stiff problems, due to its minor stability regions. It uses an error control based on local extrapolation as mentioned in [2] and an error estimation per step unit [3]. The solver ode15s is based on the Backward Differentiation Formulae (BDF) which are methods of orders 1:5 proposed in [4]. They have good stability properties, hence, they are efficient in solving stiff differential equations, whenever the eigenvalues do not lie near the imaginary axis. The ode15s uses the local truncation error as the error estimation in each step, as detailed in [2].

As the elements of the discretization increase, the eigenvalues differ significantly from each other, so the problem becomes stiff. Predictably, the ode15s behaves better than the ode45 when solving the heat equation, although the ode45 performs better with the wave equation. The reason is that the error estimation of the ode15s does not let the algorithm take major steps when solving rapidly oscillating problems such as the wave equation.

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Numerical Studies of Thermal Stratification in Cylindrical Vessel

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Keywords. Thermal Stratification, Higher order schemes, Turbulence model, Flow pattern, Interfacial velocity

Thermal stratification has been studied numerically for axi-symmetric turbulent flow in a cylindrical vessel. The governing equations for different turbulent model (k-epsilon and k-omega) have been discretized by using finite volume method. Semi-implicit method for pressure linked equations (SIMPLE) algorithm has been used. Various numerical schemes such as; 2nd order upwind scheme, Power law and Quick have been used. The effect of turbulent model parameters on flow characteristics has been studied. It is interesting to note that by varying the values of turbulent model parameters a good agreement between predicted and experimental results have been observed. Furthermore, it is observed that, k-omega model predicts a better result in comparison with k-epsilon model. The obtained results have also been compared with the result obtained by Open source code such as Open FOAM and commercial code such as FLUENT.

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Numerical Study of a Geometric Flow of Curves that Develop Singularities in Finite Time

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65D10, 65D30, 65N35, 65T50, 76B47

In order to understand the evolution of a vortex patch, Goldstein and Petrich obtained in [2] the following geometric flow:

$$\mathbf{X}_t = -k_s \mathbf{n} - \frac{1}{2} k^2 \mathbf{T}. \quad (1)$$

The motion happens only in the tangential and normal directions, so an initial planar curve remains planar for all t . Hence, we can identify $z \equiv \mathbf{X}$, and z satisfies:

$$\begin{cases} z_t = -z_{sss} + \frac{3}{2} \bar{z}_s z_{ss}^2, & z(s, t) \in \mathbb{C}, \quad (s, t) \in \mathbb{R}^2, \\ |z_s|^2 = 1, \end{cases} \quad (2)$$

where s is the arc-length parameter. Denoting with $\kappa(s, t)$ the curvature of $z(s, t)$, κ satisfies the modified Korteweg-de Vries (mKdV) equation:

$$\kappa_t + \kappa_{sss} + \frac{3}{2} \kappa^2 \kappa_s = 0. \quad (3)$$

Looking for self-similar solutions of (3), Perelman and Vega proved in [3] that (2) has a one-parameter family of regular solutions that develop a corner-shaped singularity at finite time. We give a method for reproducing numerically the evolution of those solutions, as well as the formation of the corner, showing several properties associated to them [1].

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Sums of Reciprocal of Generalized Factorial

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 39A

In this paper, authors obtain some results on generalized reciprocal factorial using generalized difference operator $\Delta_{\pm\ell}$, for the positive real ℓ . Also we derive the formulae for the sums of reciprocal of generalized factorial in number theory using inverse operator. Suitable examples are provided to illustrate the main results.

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Solution of Axial Dispersion Model Using Orthogonal Hermite Collocation Method

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Axial dispersion models have been used for decades to describe mass transport processes of solid and semi solid particles in chemical and process industries, namely, the displacement of an initially homogeneous solute from a porous medium of finite thickness by the introduction of a less concentrated solvent.

The numerical solution of the axial dispersion model involving Langmuir adsorption isotherm (non linear) is attempted via orthogonal Hermite collocation method. The discretized non-linear differential equations are solved using MATLAB software. The results are given in dimensionless form for the exit concentration of solute leaving the packed bed of pulp fibers. The purpose of this paper is to provide accurate numerical solution to the mathematical models involving longitudinal dispersion in porous medium.

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Analysis of the Finite Element Method for the Neumann and Transmission Problems

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We study theoretical and practical issues arising in the implementation of the Finite Element Method for a strongly elliptic second order equation with jump discontinuities in its coefficients on a polygonal domain Ω that may have cracks or vertices that touch the boundary. We consider in particular the equation $-\nabla \cdot (A\nabla u) = f \in H^{m-1}(\Omega)$ with mixed boundary conditions, where the matrix A has variable, piecewise smooth coefficients. We establish regularity and Fredholm results and under some additional conditions, we also establish well-posedness in weighted Sobolev spaces. When Neumann boundary conditions are imposed on adjacent sides of the polygonal domain, we obtain the decomposition $u = u_{reg} + \sigma$, into a function u_{reg} with better decay at the vertices and a function σ that is locally constant near the vertices, thus proving well-posedness in an *augmented* weighted space. The theoretical analysis yields interpolation estimates that are then used to construct improved graded meshes recovering the *quasi-optimal* rate of convergence for piecewise polynomials of degree $m \geq 1$. Several numerical examples will be presented.

See [1, 2, 3, 4] and references therein for related discussions on a-priori analysis of elliptic PDEs and numerical treatments for different aspects regarding the finite element method.

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High Frequency Localized Solutions and Filtering Mechanisms for Classical and Non-conforming Semi-discretizations of the Wave Equation

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We study the propagation properties of the solutions of classical (finite difference - FD) and non-conforming (discontinuous Galerkin - DG) space semi-discretizations of the $1 - d$ wave equation on an uniform grid of size h . The continuous wave equation has the so-called observability property: for a sufficiently large time, the total energy of its solutions can be estimated in terms of the energy concentrated in the exterior of a compact set (cf. [2]). For the FD scheme, we show that this fails to be true, uniformly on h , whatever the observability time is, so that the observability constant blows-up at an arbitrarily large polynomial order. We construct high frequency wave packets that propagate along the discrete bi-characteristic rays of Geometric Optics with a group velocity arbitrarily close to zero. We also explain how these constructions can be adapted to the P_1 -DG approximation (cf. [1]), for which the Fourier symbol of the Laplacian is a matrix, having two eigenvalues: a physical one and a spurious one. Each of them contains wave numbers where the corresponding group velocity vanishes.

The second purpose is to describe several filtering mechanisms for the DG semi-discretization, aimed to recover the uniformity of the observability constant with respect to h in suitable subspaces of numerical solutions, which is a quite well understood topic for classical approximations (cf. [3]). For the DG methods, the most feasible filtering strategy consists in first choosing the jump of the numerical initial data to vanish and then to apply the bi-grid algorithm. It has the advantage of being applicable in the physical space and guarantees the energy of solutions to be concentrated in the low frequencies of the physical branch.

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Darcy Mixed Convection in a Fluid Saturated 3D Porous Enclosure with a Centrally Buried isothermal Cubical Structure under Suction Effect

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Study of convective process in porous media has always been a topic of great interest. Several problems with simple to complex geometries have been solved in two dimensions. However, some of the real life problems like thermal insulation of buildings, environmental chambers, coal gasification, geothermal heating due to nuclear waste disposal, etc., would require three-dimensional analysis. In applications such as nuclear waste disposal, one will be interested in tracing three-dimensional convection process due to three-dimensional hot objects buried deep in earth. Here such three-dimensional hot objects are assumed to be cubical in shape and the convection due to such objects in a cubical enclosure is analyzed. In many of the engineering applications such as solar central receivers exposed to wind currents, electronic devices cooled by fans, heat exchangers, vented enclosures filled with micro spheres, which require a detailed study on mixed convection process the spatial location of inlet/outlet windows become very vital [3]. In this study our numerical investigation on Darcy mixed convection in a porous enclosure with a injection on bottom surface and fluid suction at the top surface of the cube.

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An Integral-equation Approach to Solve Time-dependent Heat Conduction Problems

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In this work we develop an algorithm for solving the time-dependent heat conduction equation [4]

$$c_p \rho \partial_t T - k T_{,ii} = 0$$

in an analytical, exact fashion for a two-component domain. The formal solution of the problem is obtained by the Green's function approach [1, 2]. As an intermediate result an integral-equation for the temperature history at the domain interface is formulated which can be solved analytically. The method is applied to a classical engineering problem, i.e. to a special case of a *Stefan-Problem* [6]. The Green's function approach in conjunction with the integral-equation method is very useful in cases where strong discontinuities or jumps occur. The system parameters and the initial conditions of the investigated problem give

rise to two jumps in the temperature field. Purely numerical solutions are obtained by using the *FEM* (finite element method) [3] and the *FDM* (finite difference method) [5] and compared with the analytical approach. At the domain boundary the analytical solution and the FEM-solution are in good agreement, but the FDM results show a significant smearing effect.

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Septic B-spline Collocation Method for Sixth Order Boundary Value Problems

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In this paper, sixth order boundary value problems is solved numerically by collocation method. The solution is approximated as a linear combination of septic B-spline functions. The septic B-splines constitute a basis for the space of septic polynomial splines. In the method, the basis functions are redefined into a new set of basis functions which in number match with the number of selected collocation points. To test the efficiency of the method, several numerical examples of sixth order linear and nonlinear boundary value problems are solved by the proposed method. Numerical results obtained by the proposed method are in good agreement with the exact solutions available in the literature.

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Efficiency of Direct Parallel Algorithm applied to Thermohydrodynamic Lubrication

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This study examines the parallel computing as a tool to minimize the execution time in the optimization applied to thermohydrodynamic (THD) lubrication. The objective of the optimization is to maximize the load capacity of a slider bearing with two design varibales with minimum friction coefficient. A global optimization method, DIviding RECTangles (DIRECT) algorithm is used. The first approach is to apply the parallel computing within the THD model in a shared memory processing (SMP) environment to examine the parallel efficiency of fine - grain computation. After that, a distributed parallel computing in the search level was conducted by use of the standard DIRECT algorithm. Then the algorithm is modified to a version suitable for effective parallel computing. It is found that the standard DIRECT algorithm is an efficient sequential but less parallel computing friendly method. When the modified algorithm is used the slider bearing optimization, a parallel efficiency of 98

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An Efficient Root-finding Method with Eighth-order Convergence

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65H05, 65D05

In this paper, we derive a higher order multipoint method for solving nonlinear equations. The methodology is based on Ostrowski's method and further developed by using inverse interpolation process. The adaptation of this strategy increases the order of Ostrowski's method from four to eight and its efficiency index from 1.587 to 1.682. The method is compared with its closest competitors in a series of numerical examples. Moreover, theoretical order of convergence is verified on the examples.

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Unsteady MHD Mixed Convection Heat Transfer over a Stretching Surface Embedded in a Porous Medium with Heat Source/sink Using Mesh-Free Method

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In this paper, the MHD mixed convection heat transfer of viscous fluid over an unsteady stretching sheet placed in a porous medium in the presence of heat source/sink have been examined. A uniform magnetic field is applied transversely to the direction of the flow. The problem has scientific and engineering applications, for instance, in a melt-spinning process, where the quality of the final product greatly depends on the rate of cooling and the process of stretching. The mathematical model of problem is highly non-linear whose analytical solution is very hard to find out, so the only choice left is approximate numerical solution.

A variety of grid or mesh based numerical techniques such as the FDM, FVM, FEM etc. are developed by researcher to solve these types of problems. Although grid based method are most general numerical method but the discretization, meshing and re-meshing of complex geometries of the problems are very difficult and expensive. To overcome these problems, a number of Meshfree methods [1] have been developed in last two decades, which has been successfully used to solve various types of problems in different areas of engineering and science.

Most of the studies of stretching sheet were restricted to the steady state conditions. In this paper we consider the unsteady case of stretching surface problem. The time dependent nonlinear differential equations governing the problem have been transformed by a similarity transformation into a system of non-linear ordinary differential equations, which are solved numerically by Element Free Galerkin method (Meshfree method). Some of the result has

been compared with finite element method. Finally, excellent validation of the present numerical results has been achieved with the earlier steady state results of [2] for local Nusselt number.

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Wavelets Associated with Non-uniform Vector-valued Multiresolution Analysis

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A generalization of multiresolution analysis has been introduced and studied by Gabardo and Nashed, see references [1], [4], [5], [6] and [8]. On the other hand vector-valued multiresolution analysis and associated vector-valued wavelets have been investigated in [2], [3] and [7]. Motivated by these papers authors have introduced the concept of non-uniform vector-valued multiresolution analysis. Besides various results related to this new concept a necessary and sufficient condition for existence of vector-valued wavelets has been obtained.

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A Parameter-uniform Numerical Method for a Singularly Perturbed Initial Value Problem of a Linear System of First Order Ordinary Differential Equations with Discontinuous Source Terms

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65L05, 65L11, 65L12, 65L20, 65L50

A linear system of singularly perturbed first order ordinary differential equations with discontinuous source terms is considered in the form

$$\vec{L}\vec{u}(t) = E\vec{u}'(t) + A\vec{u}(t) = \vec{f}(t), \quad t \in \Omega = (0, T]$$

with $\vec{u}(0)$ prescribed. Here \vec{u} and \vec{f} are column n -vectors and $\vec{f}(d-) \neq \vec{f}(d+)$ is assumed. E and $A(t)$ are $n \times n$ matrices, $E = \text{diag}(\vec{\varepsilon})$, $\vec{\varepsilon} = (\varepsilon_1, \dots, \varepsilon_n)$. The parameters $\varepsilon_i, i = 1, \dots, n$ are assumed to be distinct and for convenience, the ordering $0 < \varepsilon_1 < \varepsilon_2 < \dots < \varepsilon_n < 1$ is assumed. Also \vec{f} has a jump at $t = d$ and the solution \vec{u} is continuous at d .

The solution of the system exhibits n overlapping initial layers as well as interior layers near the point of discontinuity d , where $0 < d < T$. A numerical method that resolves the layers and convergent in $\bar{\Omega}$, uniformly in all the

perturbation parameters is constructed. The method uses a classical finite difference scheme on a piecewise uniform Shishkin mesh and is convergent in the maximum norm. Related works are found in [1].

Numerical illustrations are presented in support of the theory.

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Properties of Infinite Series Solutions of Second Order Generalized Difference Equation

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In this paper, the authors discuss properties of infinite series solutions of second order generalized difference equation $\Delta_\ell^2 u(k) + f(k, u(k)) = 0$, $k \in [0, \infty) = \cup_{j \in [0, \ell)} N_\ell(j)$, where Δ_ℓ is the generalized difference operator, f and u are real valued functions.

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Parameter-Uniform Hybrid Numerical Scheme for Singularly Perturbed Problems of Mixed Parabolic-Elliptic type

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Here we propose a hybrid numerical scheme for the following class of singularly perturbed mixed parabolic-elliptic problems posed on the domain $G^- \cup G^+$, $G^- = (0, \xi) \times (0, T]$, $G^+ = (\xi, 1) \times (0, T]$:

$$\begin{cases} \left(\frac{\partial u}{\partial t} - \varepsilon \frac{\partial^2 u}{\partial x^2} + b(x, t)u \right)(x, t) = f(x, t), & (x, t) \in G^-, \\ \left(-\varepsilon \frac{\partial^2 u}{\partial x^2} - a(x, t) \frac{\partial u}{\partial x} + b(x, t)u \right)(x, t) = f(x, t), & (x, t) \in G^+, \end{cases}$$

where $0 < \varepsilon \ll 1$ is a small parameter and the coefficients a , b are sufficiently smooth functions and the source term f is sufficiently smooth on $G^- \cup G^+$ such that $a(x, t) > 0$, $x > \xi$, $b(x, t) \geq 0$ on $\bar{G} = [0, 1] \times [0, T]$, with suitable initial, boundary and the interface conditions at $x = \xi$. Such kind of problems describe, for example, an electromagnetic field arising in the motion of a train on an air-pillow. In general, the solutions of this class of problems possess both boundary and interior layers. Due to the presence of layers, classical numerical methods on equidistant mesh usually fail to decrease the maximum point-wise error as the mesh is refined, until the mesh parameter and the perturbation parameter have the same order of magnitude.

To solve these problems, we discretize the time derivative by the classical backward-Euler method. While for the spatial discretization of the problem, we use the classical central difference scheme on the first subdomain and we propose a hybrid finite difference scheme (a proper combination of the midpoint upwind scheme in the outer regions and the classical central difference scheme in the interior layer regions) on the second subdomain. At the point of discontinuity, a second-order one-sided difference approximations are used to keep the continuity of the spatial derivative. The proposed method is analyzed on a layer resolving piecewise-uniform Shishkin mesh and is shown to be ε -uniformly convergent with almost second-order spatial accuracy in the discrete supremum norm, provided that the perturbation parameter ε satisfies $\varepsilon \leq N^{-1}$. Here, N is the number of mesh-intervals in the spatial direction. Finally numerical results are presented to validate the theoretical results.



Numerical Study of Visco-elastic Fluid Flow over an Exponentially Stretching Sheet

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Most of the fluids which are used in industry are non-Newtonian and particularly visco-elastic in nature. So, in recent years, the study of visco-elastic fluids gains the attention of researchers. In polymer processing applications, it is essential to consider flow over a stretching sheet as mentioned by Rajagopal et.al. [1]. Several Mathematicians and the Scientists obtain the analytical solutions similar to [2]. However, the constitutive equations of this type of fluid flows are highly non linear. So, getting analytical solution may not be always possible.

This paper deals with the numerical study of visco-elastic fluid flow over an exponentially stretching sheet [3] using the method of quasilinearization [4] [5]. The higher order non-linear momentum equation is converted as system of simultaneous first order equations and the solution of the boundary value problem is obtained using quasilinearization technique. The velocity profiles are drawn for various values of visco-elastic parameter. It is observed that with very approximate initial guesses, only in six to seven iterations good accuracy up to five decimal places is obtained.

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The Exact Orders of the Computational Widths

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 41A25, 41A46

The paper is devoted to the study of computational widths, the idea of definition of which is in optimization on the set of computational units [1]–[5].

We study the problems of numerical integration and differentiation (including the quasi-Monte Carlo method), the problems of recovery of functions, and discretization of solutions of partial differential equations.

Computational units are defined by functionals, supplying numerical information on the studied function or on the initial and boundary conditions, which are then processed through the algorithm that depends on the same variable, as the approximated operator.

Note that with appropriate specifications of the general definition of computational width [1] we obtain the entire arsenal of problems in approximation theory and numerical analysis: Fourier series, bases, wavelets, interpolations, etc.

We obtain two-sided estimates of the same order for computational widths in the case when all possible linear functionals and algorithms serve as the carriers of information [1]–[2].

Numerical integration is represented by two methods based on algebraic number theory and the tensor product of functionals in combination with harmonic analysis [3]–[5].

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Finite Element Analysis of Three-step Taylor Galerkin Approximation for Singularly Perturbed Convection-Diffusion Equation

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In the present study, we propose a Three-step Taylor Galerkin Finite Element Scheme for a Singularly Perturbed Convection-Diffusion problem. Traditional finite element techniques with linear shape functions do not give rise to uniformly convergent methods for Singularly Perturbed differential equations on a uniform mesh. Here we have used exponentially fitted shape functions to generate a uniformly convergent scheme. In Three-Step Taylor Galerkin Method time

discretization is carried out prior to spatial discretization. This leads to higher order accuracy in the numerical solution. Further, the method is also known for its inherent upwinding capability. In the present work, the error estimates for the proposed scheme has been derived and it has been shown that the proposed method is of third order accurate in time and linear in space. Numerical results have been presented for convection-dominated singularly perturbed problems.

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Clenshaw-Curtis-Filon-type Method for Highly Oscillatory Bessel Transforms and Applications

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 65D32, 65R20

A Clenshaw-Curtis-Filon-type method $Q_s[f]$ for highly oscillatory Bessel transform

$$I[f] = \int_a^b f(x)C_m(rx)dx \quad \text{or} \quad I[f] = \int_a^b f(x)D_m(-irx)dx$$

is considered, which is based on a special Hermite interpolation polynomial in the Clenshaw-Curtis points that can be efficiently evaluated in $O(N \log N)$ operations, where N is the number of nodes of the Clenshaw-Curtis points in the integral interval. Moreover, the error bounds related to frequency and approximation of the polynomial for this quadrature

$$O\left(\frac{\max_{0 \leq j \leq s+2} \left\{ \|f^{(j)}(x) - p_{N+2s}^{(j)}(x)\|_\infty \right\}}{r^{s+5/2}}\right), \quad 0 \notin [a, b]$$

or

$$O\left(\frac{\max \left\{ \|f^{(s+1)}(x) - p_{N+2s}^{(s+1)}(x)\|_\infty, \|f^{(s+2)}(x) - p_{N+2s}^{(s+2)}(x)\|_\infty \right\}}{r^{s+2}}\right), \quad 0 \in [a, b].$$

is given. In particular, this method can be easily applied to computation of a class of Volterra integral equations containing highly oscillatory Bessel kernels

$$\int_a^x (x-t)^m J_m(r(x-t))y(t)dt = g(x), \quad x \in [a, b], \quad (1.a)$$

or

$$\int_a^x (x-t)^{m/2} J_{m/2}(r\sqrt{x-t})y(t)dt = g(x), \quad x \in [a, b]. \quad (1.b)$$



Section 17

Control Theory and Optimization

A New Solution Concept of Matrix Games in Fuzzy Environment

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In this paper, we deal with a two-person zero - sum game with fuzzy payoffs. The aim of the paper is to extend the decision theory framework of Bellman and Zadeh [1] to a game theoretic platform so as to solve those problems in game theory, that are imprecise in nature using current techniques of Fuzzy Mathematics. Since fuzziness can exist if the components of the game are specified with some impreciseness or when the players have their own subjective perception of the game, the constraints faced by the players as well as the outcomes of the game warrant fuzzy mathematical treatments [2]. Hence by assuming that the components of the game involve subjective perception on the part of the players, we develop a descriptive theory to analyze games with imprecise characteristics using fuzzy tools. We provide illustrations and numerical examples to study the adequacy of the theory.

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Vibration Control of a Vehicle with Passengers Using Hybrid Genetic Algorithm

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 70F15

In this paper, a new hybrid method based on Region Reduction Division Criteria (RRDC) and Advanced Real Coded Genetic Algorithm (ARCGA) has been proposed for solving the suspension design problem. The dynamical model of a half-car with two passengers' seat suspensions, one at the front and the other at the rear position of the vehicle has been considered. The developed hybrid method has been applied to minimize the vibration, experienced by the passengers due to road bump and irregular terrain when the vehicle runs over the road with uniform velocity. In order to find the optimal solutions/design parameters of the suspension system we have formulated a non-linear constrained optimization problem in which the bouncing transmissibility of the sprung mass at the center of mass has been minimized in time domain with respect to technological constraints and the constraints which satisfy the performance as per ISO 2631 standards [1]. The solutions/parametric values of the suspension so obtained have been compared with the existing suspension parameters by simulating the vehicle model over the roads. Also, the vibration behavior of the passengers over different roads has been studied graphically using the convex combination of the two sets of solutions obtained after optimization. These results have been then compared with the results obtained using existing suspension parameters via simulation. It has been shown that the use of the convex combination suspension parameters improves performance over all the road conditions.

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Hotelling Model with Uncertainty on the Production Cost and Networks

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In order to understand the role of information in the price competition we study the Hotelling model [1] with uncertainty in the production cost of both firms. The incomplete information consists in each firm to know its production cost but to be uncertain about the competitor cost. We find that the Bayesian Nash Equilibrium (see [2, 3]) prices does not depend on the distributions of the production costs of the firms except on their first moments, and that the prices of each firm, at equilibrium, are proportional to the expected cost of both firms and to their own costs. The corresponding profits increase monotonously with the expected cost of both firms and decreases with its own cost. We also do the ex-ante versus ex-post analysis of the profits.

As an application of our result, we choose to introduce a new network model where the nodes are firms competing along the edges (markets) where the consumers are allocated. The firms compete according to the Hotelling's model in each link. We investigate the effects of the network structure on firms' prices and profits. We assume that each firm's production cost depends only upon the degree of the firm's node. Thus, when firms have information about their competitors' nodes they also know their production costs. We first analyze the benchmark case where every firm knows its node degree and its direct rivals' degree nodes. This is the case where all the firms have complete information about the network. In incomplete information each firm only knows its node degree and the probability distribution of the degrees of the nodes in the network. As a corollary of the Bayesian Nash equilibrium, we determine explicitly the Bayesian Nash equilibrium prices and associated equilibrium expected profits of each firm in the network as a function of a firm's degree node.

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Section 18

Mathematics in Science and Technology

Modelling of Washing Zone of Brown Stock Washer

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Process of washing of porous structure of compressible and cylindrical particles fibers has been modelled through an axial dispersion model involving Peclet number (Pe) and Biot number (Bi). Non linear Langmuir adsorption isotherm has been followed to relate bulk fluid and intra-pore solute concentrations. Model equations comprising a set of differential algebraic equations have been solved using orthogonal collocation in conjunction with finite elements. Lagrangian interpolating polynomials has been taken as base functions. Displacement washing was simulated using a lab scale washer and experiments were performed on pulp beds composed of wheat straw. Model predicted values have been used to calculate the efficiency of the washer and displacement ratio.

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The Variation of the Gravitational Constant and the Anomalous Acceleration of the Pioneer Spacecrafts

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 83B99

For nearly two decades J.D. Anderson and coworkers at the Jet Propulsion Laboratory have observed, confirmed and reconfirmed an inexplicable acceleration $\sim 10^{-8} - 10^{-9} \text{cms/s}^2$ in the Pioneer 10 and Pioneer 11 spacecrafts which are leaving the solar system. An independent analysis of the data was performed by the Compact High Accuracy Satellite Motion Program using different algorithms. This too confirms the anomalous acceleration. Anderson and coworkers have considered various possible causes for this anomaly such as additional gravitation due to the Kuiper belt or radiation pressure from solar winds and so on and have eliminated these possibilities. Thus the Pioneer anomaly poses a major challenge to celestial mechanics, and clearly new ideas need to be invoked. We suggest in this paper that this Pioneer anomaly maybe a footprint of the time variation of the gravitational constant.



Correspondence between Monsoon Rainfall over India and Sunspot Numbers: A View Through Spectral Analysis and Neural Network

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The present work reports studies on the association between the mean annual sunspot numbers [1] and the summer monsoon rainfall over India. Fascinating property of sunspots is the approximate 11-year cycle [2] and its association with the meteorological events is well discussed in the literature [3]. The statistical properties of both of the time series have been studied in the present work and it has been found that although the sunspot numbers exhibit persistence, the mean annual summer monsoon rainfall does not have any persistence. The cross correlations have also been studied. After Box-Cox transformation [4], spectral analysis [5] has been executed and it has been found that both of the time series have an important spectrum at the fifth harmonic. A neural network model [6] has been developed on the data series averaged continuously by five years and the neural network could establish a predictor-predictand relationship between the sunspot numbers and the mean yearly summer monsoon rainfall over India.

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A Novel Approach to Cancer Detection

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There are several types of tumors that invade the surrounding normal tissue by diffusion. Radical treatment is not possible to such diffusive tumors. There are several mathematical models of cancer in the literature. Most of them are using the statistical modeling of cell behavior. But it requires high magnification images of surgically removed human tissue. This article is on a new method for cancer detection using low magnification images. Based on the location of the cells in a low magnification image of a tissue sample, surgically removed from a human patient, it is possible to construct a graph G with nodes as cells, called cell graph [1]. By analyzing the physical features of the cells, for example color and size, we can assign a membership value to the nodes of G . This value will range over $(0, 1]$ depending on the nature of the cell; that is healthy, inflammatory or cancerous. Also, arcs of G can assign a membership value based on the distance between the cells [2, 3]. Thus the cell graph can be converted to a fuzzy graph in this manner. A new fuzzy graph clustering method is introduced to cluster fuzzy cell graphs. Applying the new fuzzy clustering procedure to such a fuzzy graph, the cancerous cell clusters can be detected at the cellular level in principle. This process, classifies cell clusters in a tissue into different phases of cancer, depending on the distribution, density and the fuzzy connectivity of the cell clusters within the tissue. Moreover this process helps in examining the dynamics and progress of the cancer qualitatively.

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R&D Dynamics on Costs

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 91A10, 91A20, 91A50

We consider a Cournot competition model where two firms invest in R&D projects to reduce their production costs. This competition is modeled by a two-stage game (see d'Aspremont and Jacquemin [1]). In the first subgame, two firms choose, simultaneously, the R&D investment strategy to reduce their initial production costs. In the second subgame, the two firms are involved in a Cournot competition with production costs equal to the reduced cost determined by the R&D investment program. We use an R&D cost reduction function inspired by the logistic equation which was first introduced in Ferreira et al [2]. The main differences between this cost function and the standard R&D cost reduction function (see [1]) are explained in that same paper. For the first subgame, consisting of an R&D investment program, we observe the existence of four different Nash investment equilibria regions that we define as follows (see [2]): a competitive Nash investment region C where both firms invest, a single Nash investment region S_1 for firm F_1 , where only firm F_1 invests, a single Nash investment region S_2 for firm F_2 , where only firm F_2 invests, and a nil Nash investment region N , where neither of the firms invest.

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Mathematical Modeling of the Sample Solvent Effects on the Chromatography Peak Shape of Analytes

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 76E, 76R, 76S

When the viscosity of the injected sample in a liquid chromatographic column is different from that of the carrier liquid, a hydrodynamic instability occurs at the interface of both the fluids where the more viscous fluid is displaced by the less viscous one. The latter penetrates into the more viscous zone, forming some kind of fingers which grow as they migrate. This may cause a distortion of the peak shapes and, generally, a decrease in separation performances. Evidence of this phenomenon in present-day liquid chromatographic columns has been clearly provided by the in situ optical visualization experiments [1]. We present a mathematical modeling on the influence of viscous fingering instability [2] due to a difference between the viscosity of the displacing fluid and that of the sample solvent on the spatiotemporal dynamics of the concentration of a passive solute initially dissolved in the injected sample and undergoing adsorption on the porous matrix. Such a three component system is modeled using Darcy's law for the fluid velocity coupled to mass-balance equations for the sample solvent and solute concentrations [3]. The influence of the various parameters that

control the viscous fingering phenomenon, especially the adsorption parameter k' , are shown. Numerical simulations appear to be a particularly well suited tool for unraveling the separated influences of the various effects that affect the peak shapes of the analyte zones, like viscous fingering effects and solvent strength effects which interplay in experimental studies.

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Thermal Stability of a Strong Exothermic Chemical Reaction in a Cylindrical Pipe with Variable Thermal Conductivity and Heat Loss

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 80A25; 80A30; 80M35

In many industrial and engineering systems, spontaneous explosions may occur due to internal heating in combustible materials such as industrial waste fuel, coal, hay, wool wastes and so on. This may lead to a huge loss of life and properties. In order to prevent the thermal runaway scenario, a theoretical evaluation of the critical regimes thought of as regimes separating the regions of explosive and non explosive ways of chemical reactions is extremely necessary and important. In this paper, the thermal stability analysis for a strong exothermic chemical reaction in a cylindrical pipe with variable thermal conductivity and heat loss is presented. Approximate solutions are constructed for the governing nonlinear boundary-value problem using perturbation technique together with a special type of Hermite-Pad approximants. The important properties of the temperature field including bifurcations and thermal stability criteria are discussed.

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Section 19

Mathematics Education and Popularization of Mathematics

Students' Algebraic Thinking in Middle School Level

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 97A

The aim of this study was to analyze students' algebraic thinking. The study was conducted in two Indian middle schools located in Delhi and Gurgaon, India from March to October 2009, and was carried out on grade 6, 7, and 8 levels where elementary algebra was being introduced. The participants of the study were school students and their mathematics teachers. Problems related to algebraic topics were given to the students, and their algebraic thinking were studied through three different lenses: problem solving, representation, and reasoning skills. Based on the students' worksheets, classroom observations, and questionnaires collected from students and teachers, it can be figured out that the students' algebraic thinking in middle school level are quite good. This study also showed that the students with good problem-solving skills' ability were those who could manipulate their language translation and well-organized their representations in verbal, picture, sentences, etc.

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Addressing Pedagogical Challenges in an Inquiry Based Classroom

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The use of problem-based mathematical tasks in classrooms presents a multi-dimensional pedagogical challenge for teachers. Designing well-thought out tasks is the first step. Subsequently, some other important questions that teachers must address are: What other heuristic strategies must be used alongside? What role is played by the construction of mathematical representations in working through such mathematical tasks? How should students' unique ways of interpreting, representing and resolving such questions be handled so as to promote the overall understanding of the discipline? My presentation will address some of these questions in the context of middle and high school content. I will discuss the use of problems that may be used with specific content material and questioning skills that can facilitate the development of problem solving strategies in students, and focus on how such problems can challenge students and assist them in developing a variety of connections, particularly those among different representations of the same problem.

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Section 20

History of Mathematics

Jaina Mathematician of Canonical Group

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 20K

India is a religious country. In ancient India people give much importance to the religious ceremonies. They evaluate the utility of any knowledge by its use in understanding the religion and its philosophy and performing religious ceremonies. Due to these special circumstances the religious centres and religious texts are very important source to bring out the developments of knowledge in any field. In the old age the hermitages were the centres of research and higher learning. In the group of Jaina saints (Sramana Sangh) many saints (Acharyas or Munis) also composed several religious and technical texts for the benefit of sangh and society. Religious literature and literature which was useful in understanding these philosophical texts and useful in performing religious ceremonies is also composed here. Jaina literature is very vast and varied. It includes many branches of knowledge but so far we have not given much attention towards these books. The structure of cosmos and System of Karma is available in Jaina literature elaborately. In this process a lot of mathematics is involved. Some work to explore it has been done by B.B. Datta (1), A.N. Singh (2), Takao Hayashi (3), R.C. Gupta (4), Anupam Jain (5), Padmavathamma, Pragati Jain (6), Dipak Jadhav (7), and N. Shivakumar (8). In the present paper we present a glimpse of the mathematics found in the Jaina canonical literature. Few of them are following: Sthananga Sutra, Bhagavati Sutra, Anuyogadvara Sutra, Uttaradhyayan Sutra, Jambudvipa Prajnapti, Visheshavashyaka Bhasya, Tiloyapannatti, Dhavala, Trilokasara, Gommatasara, Jambudvipaprajnaptisamgaho, Lokavibhaga.

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Vedic Binary Systems and Fibonacci Numbers

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2000 MATHEMATICS SUBJECT CLASSIFICATION. 01A32; 01A55

Vedic metrics is one of the six limbs of Vedas. Its age is as old as Vedas. However, the oldest surviving composition on metrics is *Chandaḥ Sūtram* (The Science of Meters) composed by *Ācārya Piṅgala Nāga*, younger brother of the great grammarian *Pāṇini*. This is one of the most glorious texts in the history of ideas which is of vital significance both to scholars of prosody and mathematics. The eighth chapter of this important text deals with various applications of mathematics to study the sequencing and position of meters of a finite syllabic-order. The purpose of this talk is to discuss *Piṅgala's varṇic* and moric binary systems and their mapping to decimal number system. Depth and originality of traditional way of learning may well be understood from the fact that the moric binary system is not available in modern text-books of mathematics and computer science.

The talk is organized in four sections. The first section offers some fundamentals to Sanskrit Prosody. The intent of the second section is to present *Piṅgala* binary numbers and a brief discussion on **Vaṛṇic Meru** (mountain)

generally called Pascal's triangle. The third section offers a new class of binary numbers and its mapping with decimals. Finally, we give a remark on probable impact of Sanskrit Prosody on Chinese literature.

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