Entropic solutions for systems of PDEs arising in complex fluids dynamics

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The main aim of the talk will be to show how to reinterpret the concept of weak solution satisfying a suitable energy conservation and entropy inequality - recently introduced by E. Feireisl and coauthors for a problem of heat conduction in fluids - in order to deal with certain classes of complex fluids dynamics.

In many cases indeed the resulting PDE systems dysplay high order nonlinearities due mainly to quadratic forcing terms. The main idea consists in replacing, in the weak formulation, these PDEs by an equality representing energy conservation complemented with a differential inequality describing production of entropy. In this way, the thermodynamical consistency is preserved, but the entropic formulation is more tractable mathematically.

This solution notion has been already successfully applied to the analysis of non-isothermal liquid crystals models:

1 E. Feireisl, M Frémond, E. Rocca, G. Schimperna, A new approach to non-isothermal models for nematic liquid crystals Arch. Ration. Mech. Anal. 205 (2012), no. 2, 651-672

2 E. Feireisl, E. Rocca, G. Schimperna, A. Zarnescu, Evolution of non-isothermal Landau-de Gennes nematic liquid crystals flows with singular potential, Comm. Math. Sci., 12 (2014), 317–343

3 E. Feireisl, E. Rocca, G. Schimperna, A. Zarnescu, Nonisothermal nematic liquid crystal flows with the Ball-Majumdar free energy, WIAS Preprint No. 1865, (2013)

and to the case of the evolution of a non-isothermal mixture of two different viscous incompressible fluids of the same density:

4 M. Eleuteri, E. Rocca, G. Schimperna, On a non-isothermal diffuse interface model for two-phase flows of incompressible fluids, manuscript (2013).