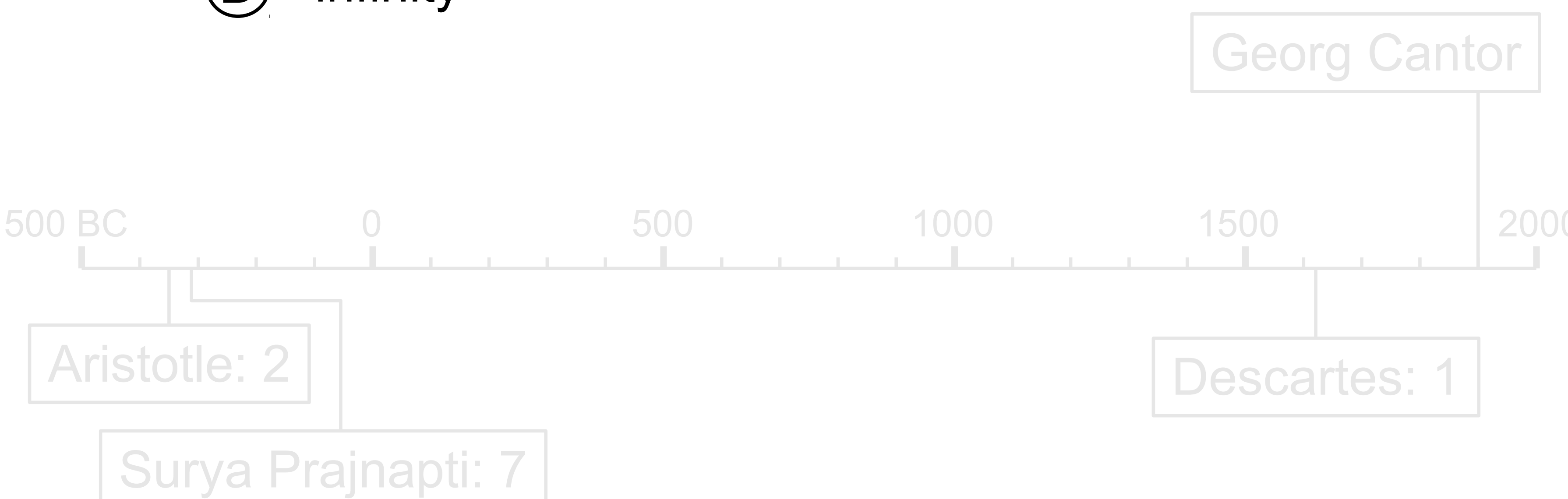
How many infinities are there?

- (A) none
- (B) one
- (C) more than one
- (D) infinity



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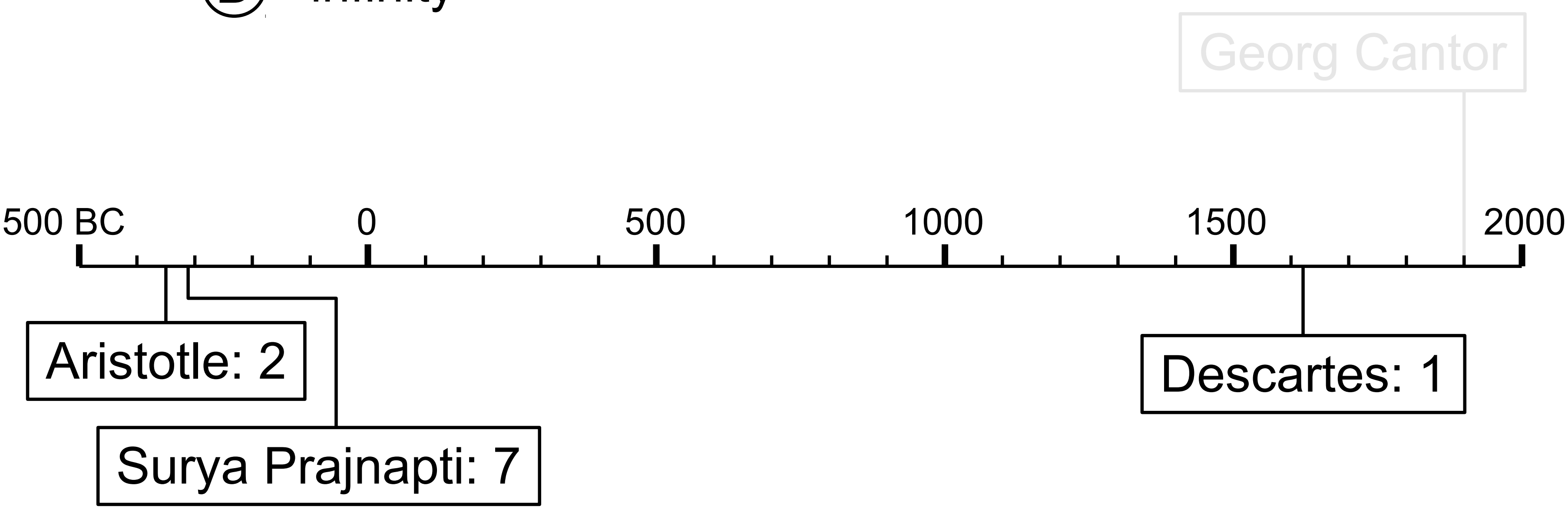

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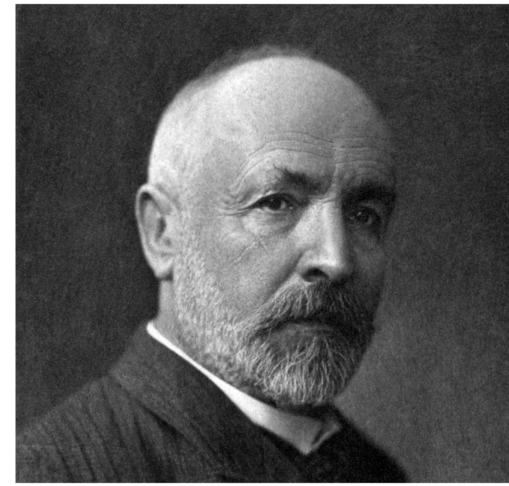

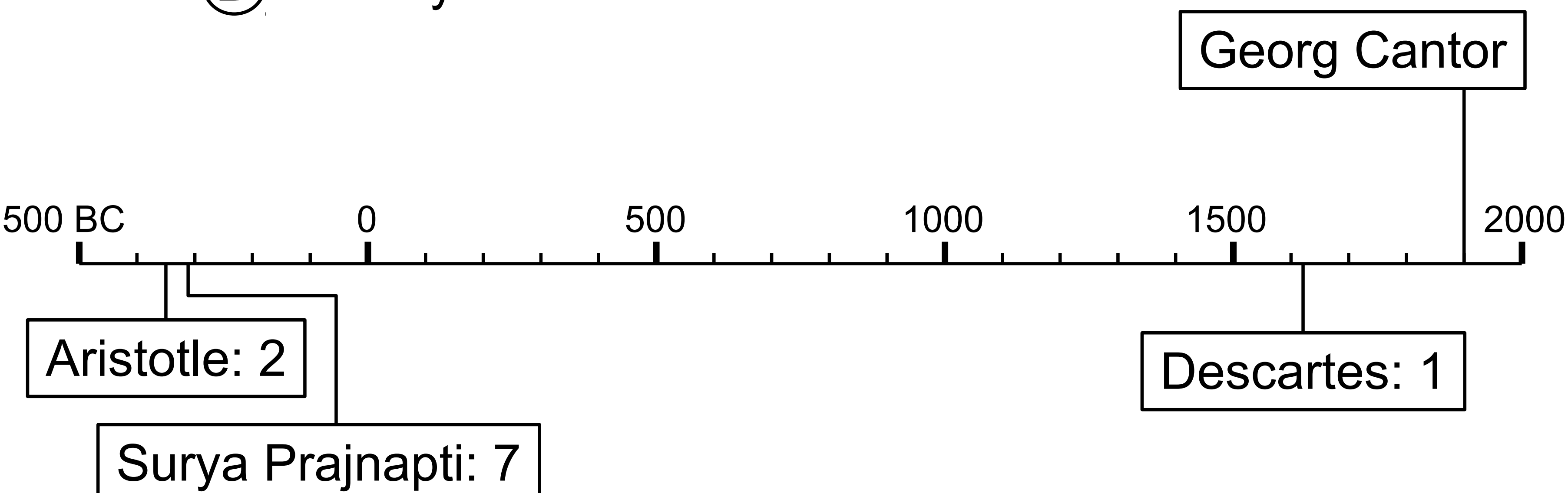

- Infinity ≈ Number
- Number ≈ Property of a set

Assumption 1. *Infinite sets exist*

- How do you count an infinite set?
- When are two sets equal in size?

Assumption 2. *Two sets are equal in size if all elements can be paired together*

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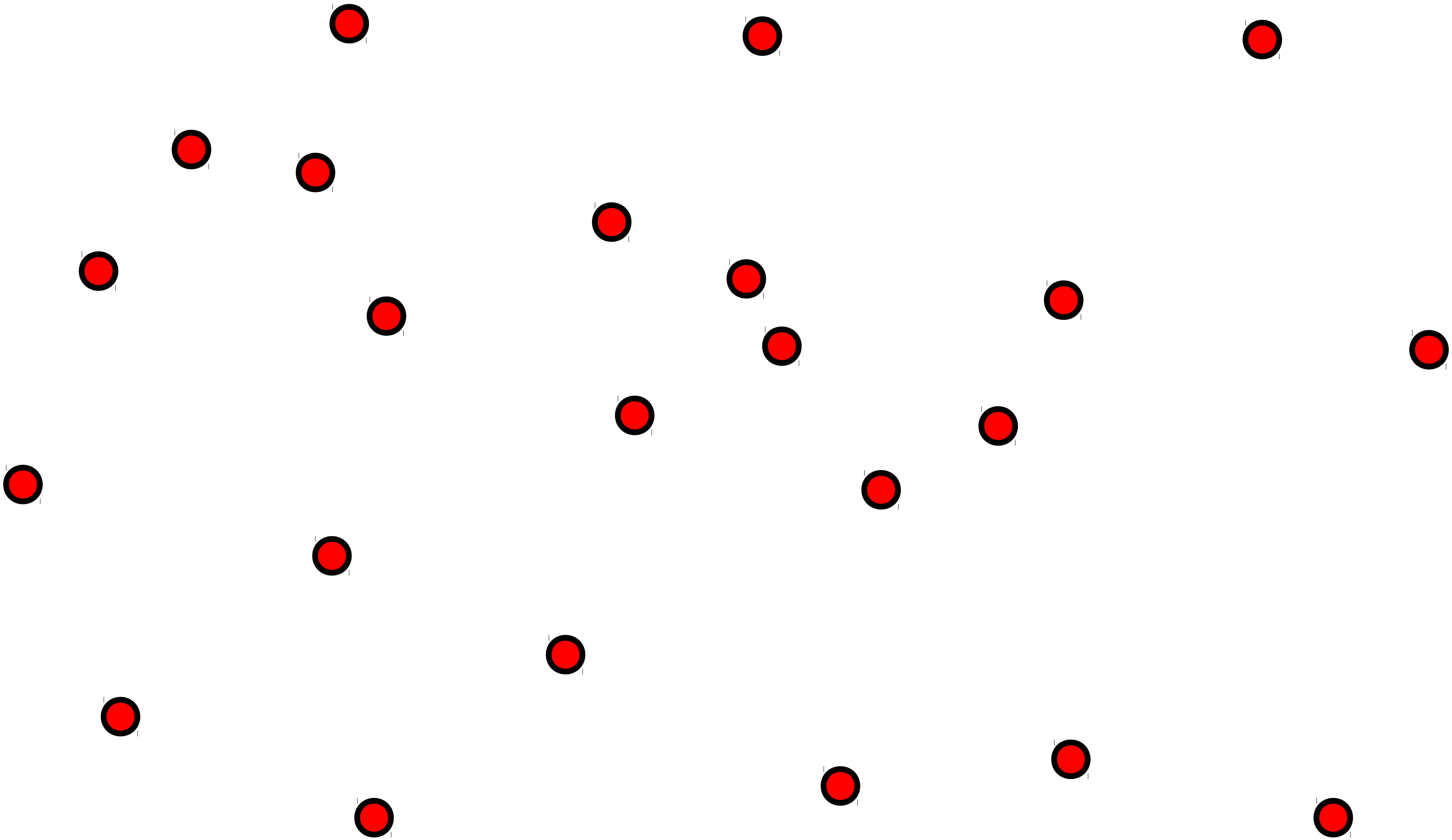

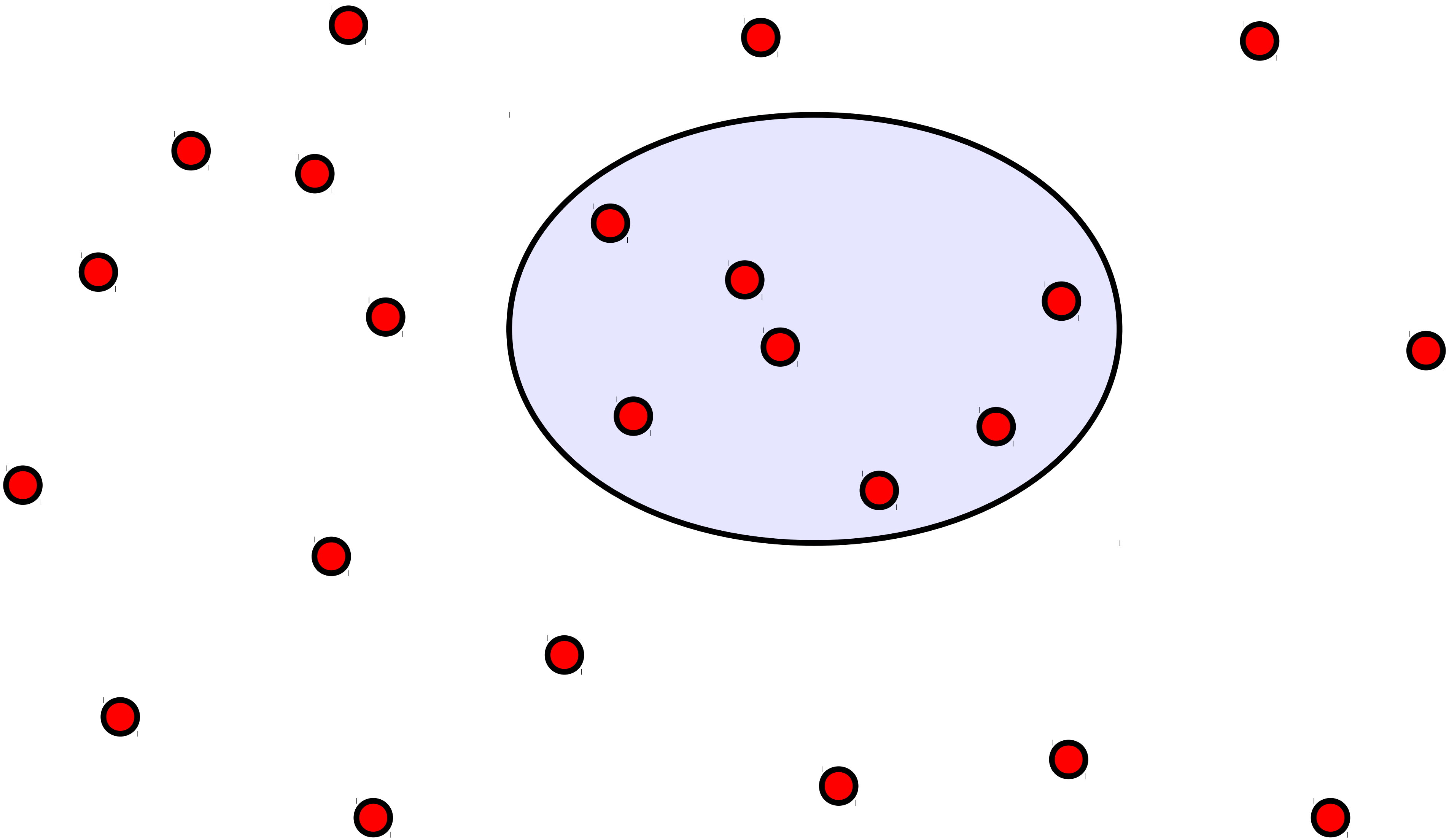

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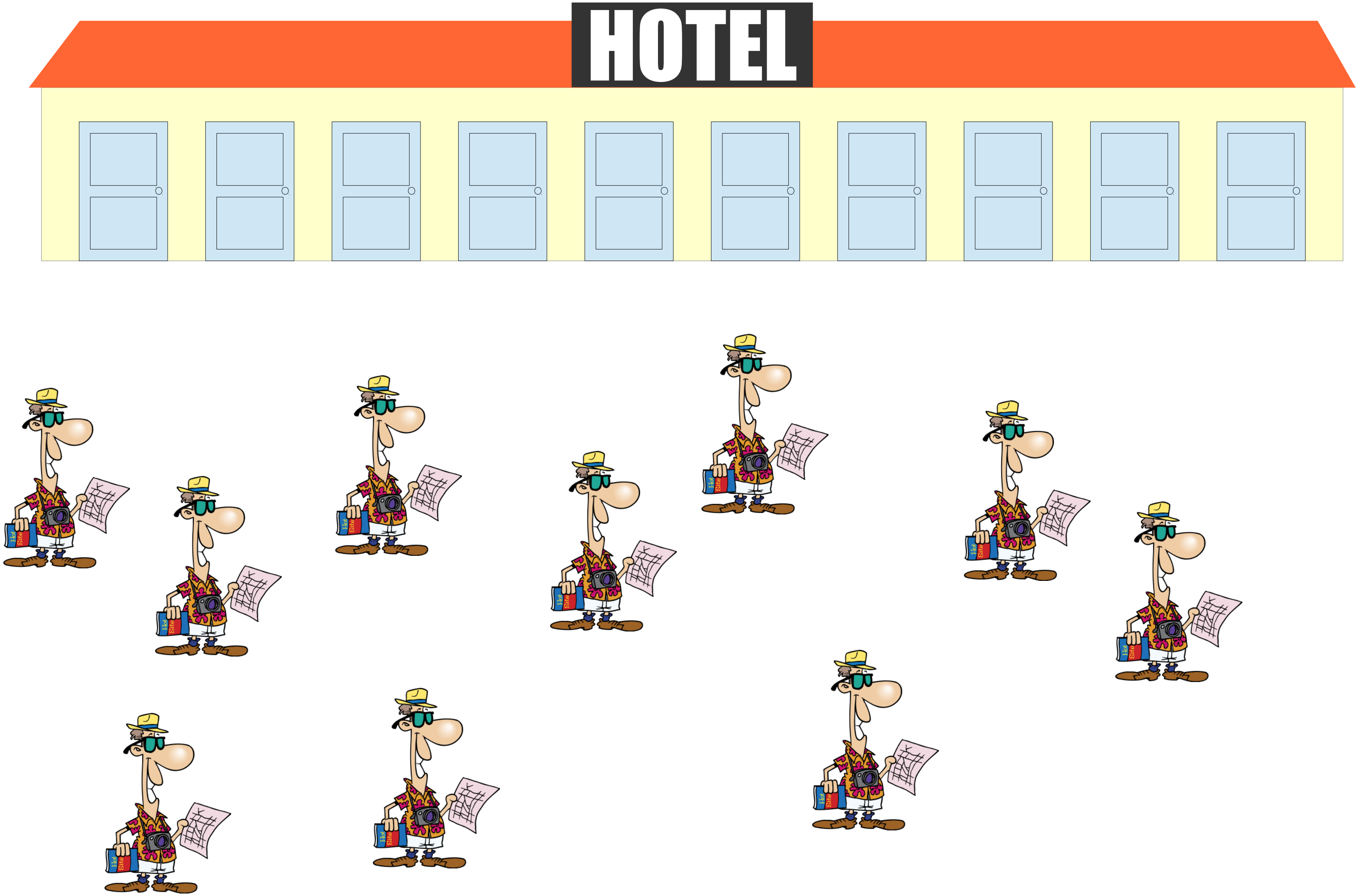
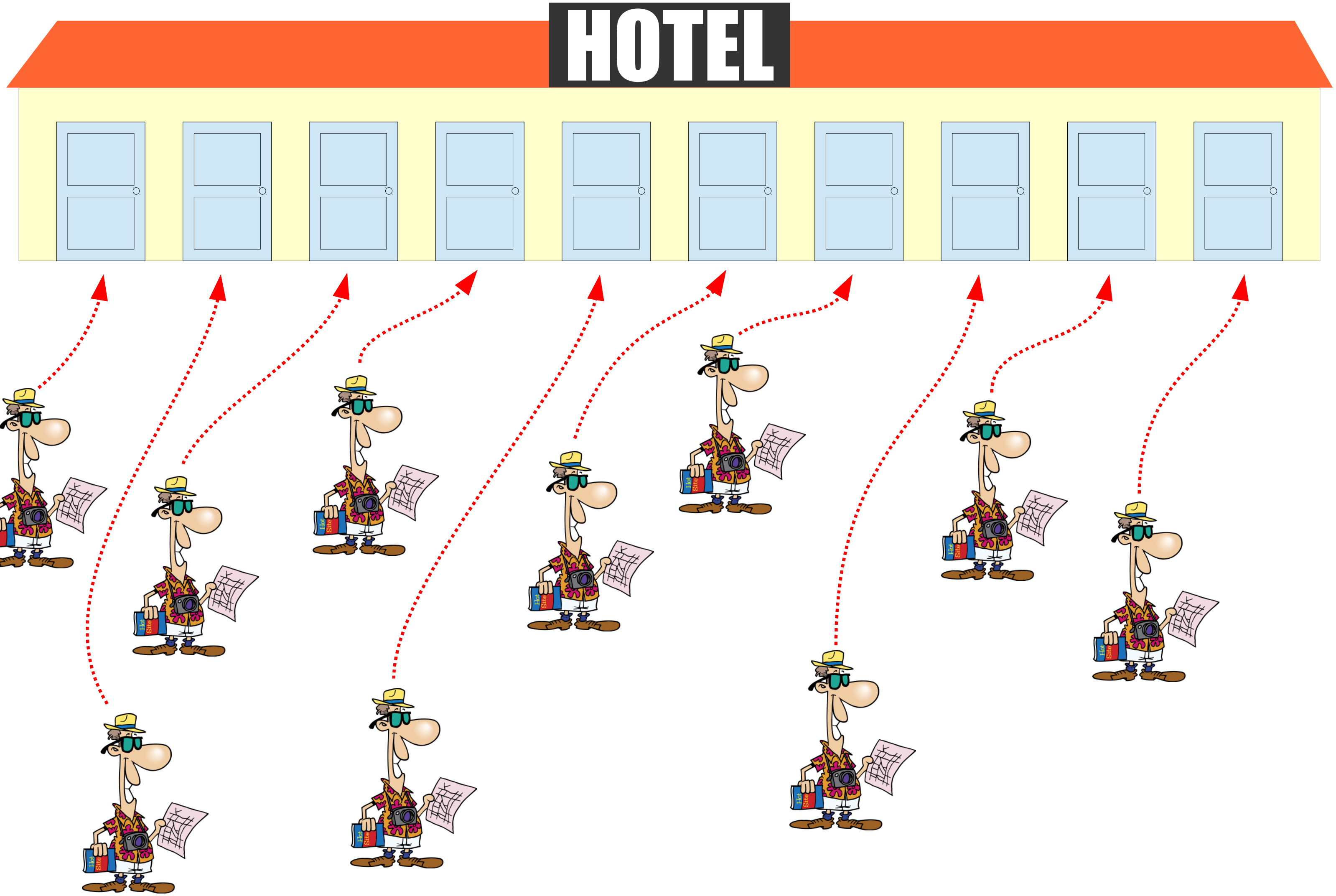

HOTEL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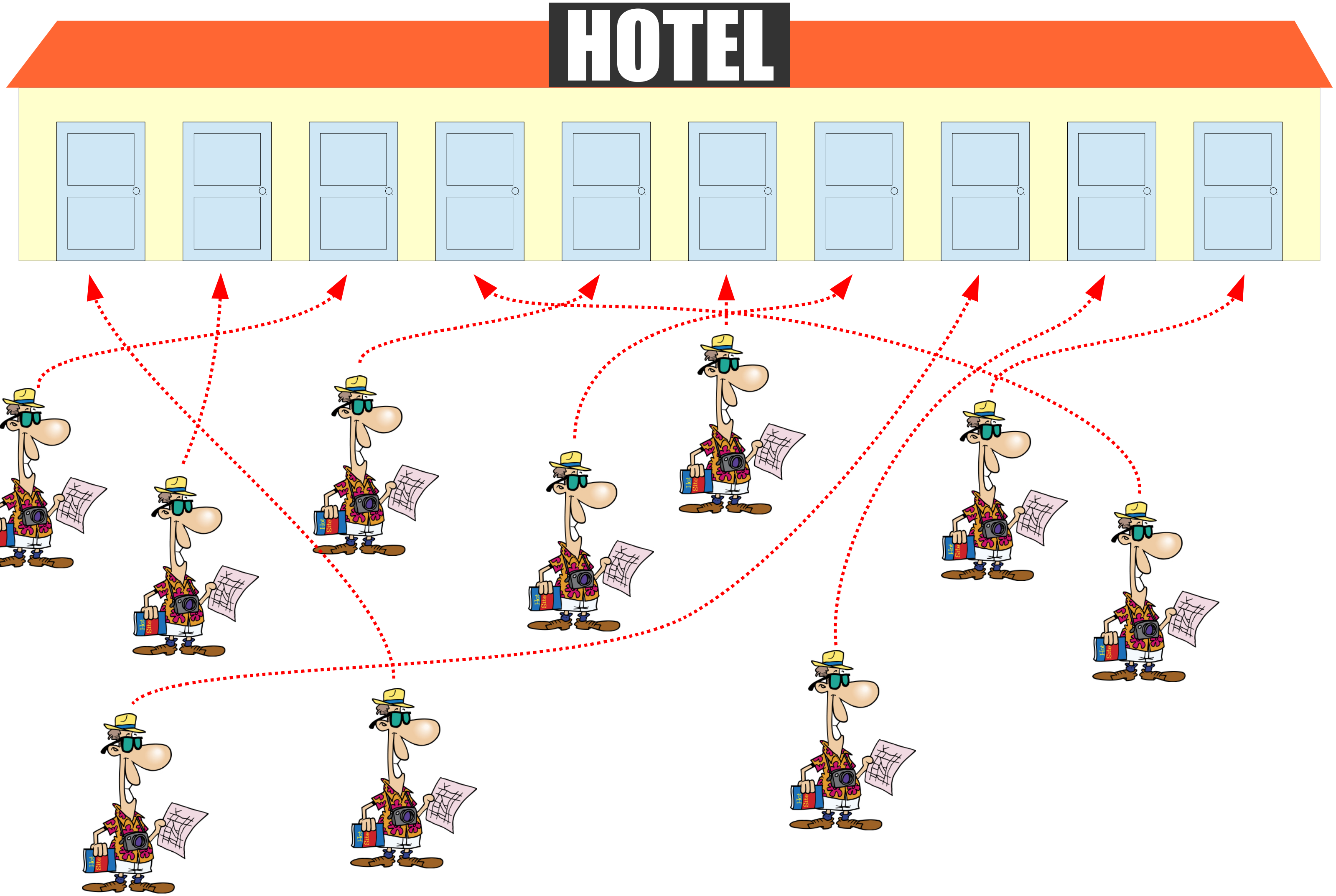
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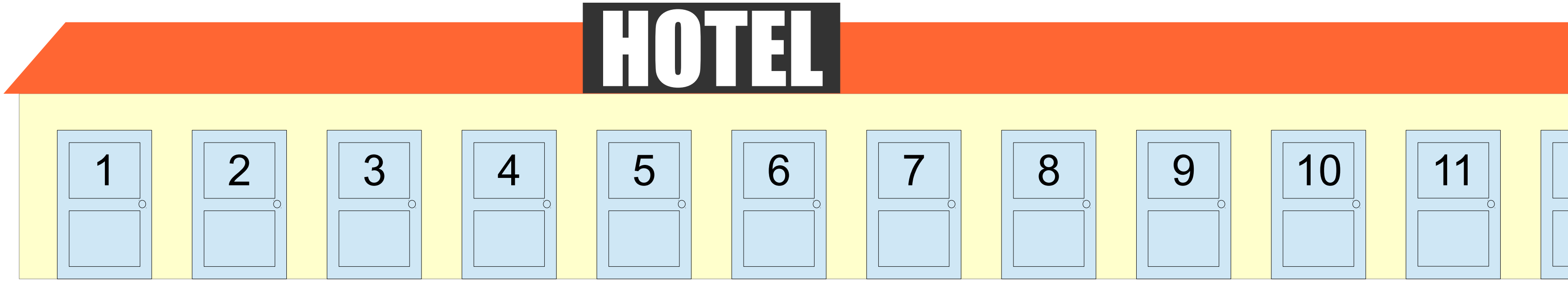
"Aleph-zero"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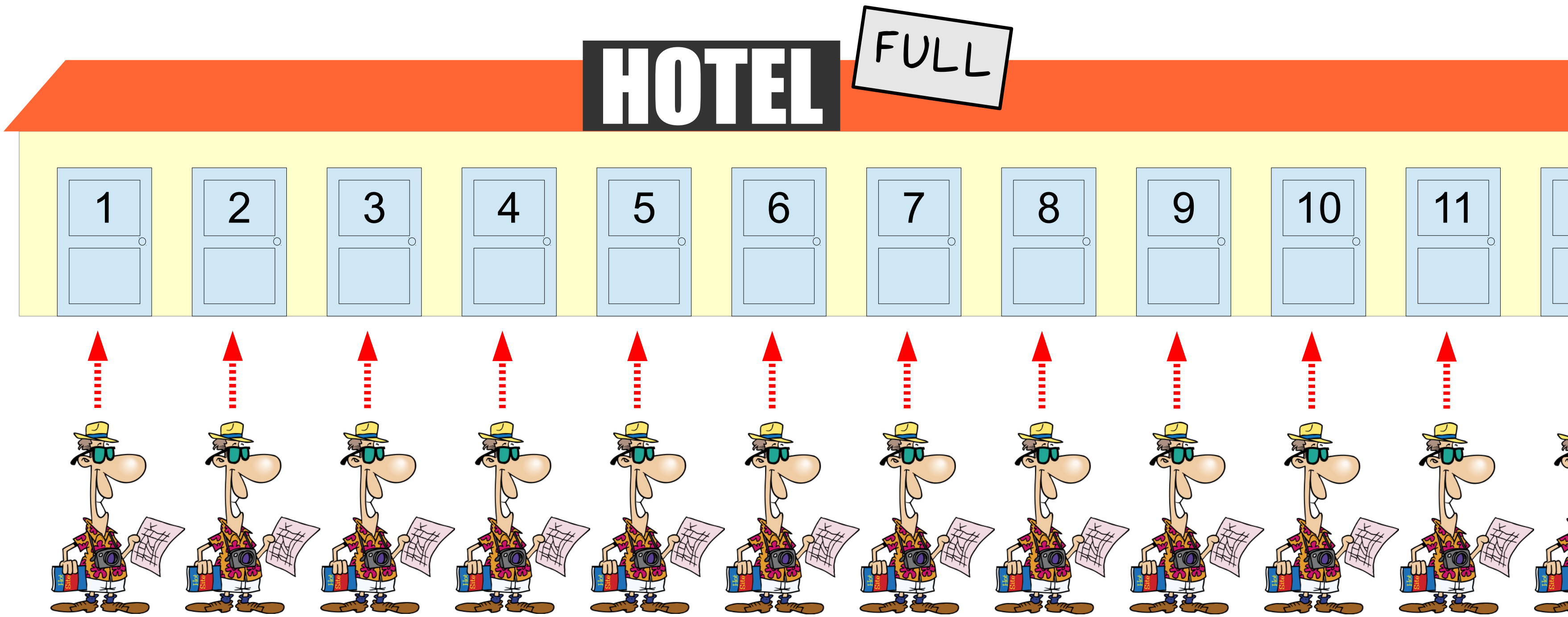
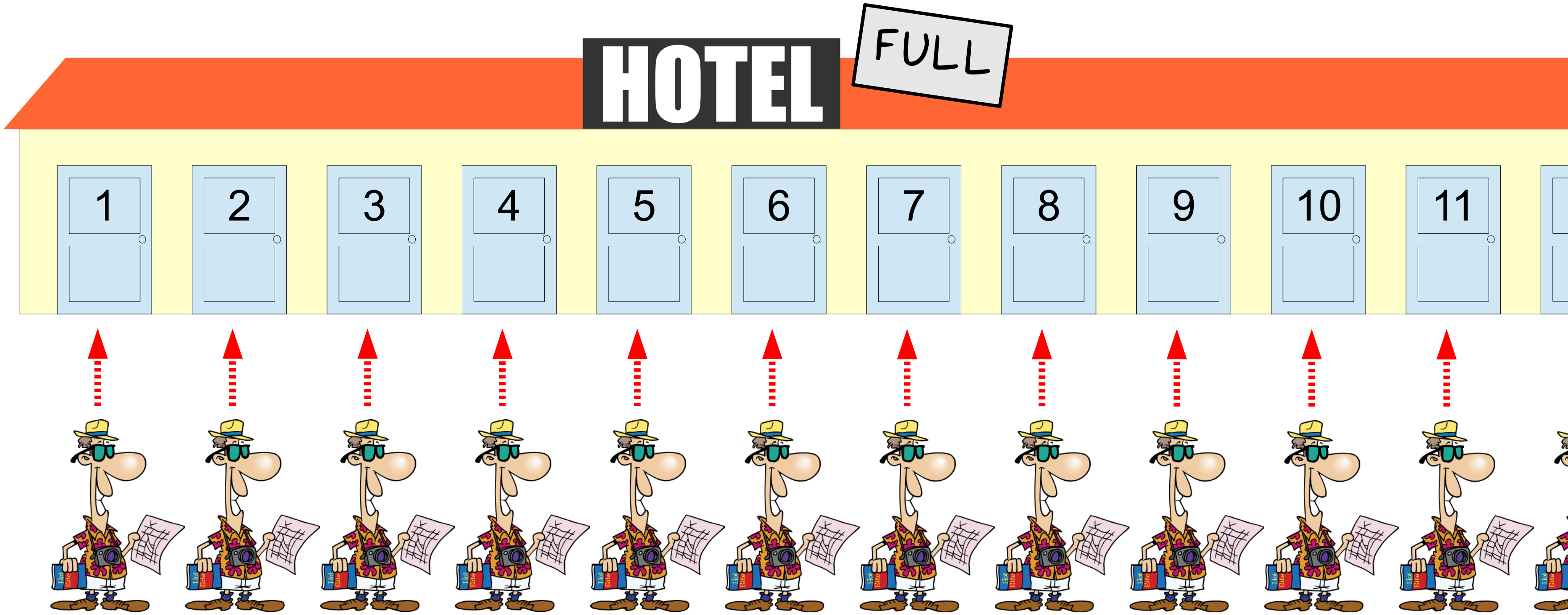


$\aleph_0$ rooms

$\aleph_0$ guests



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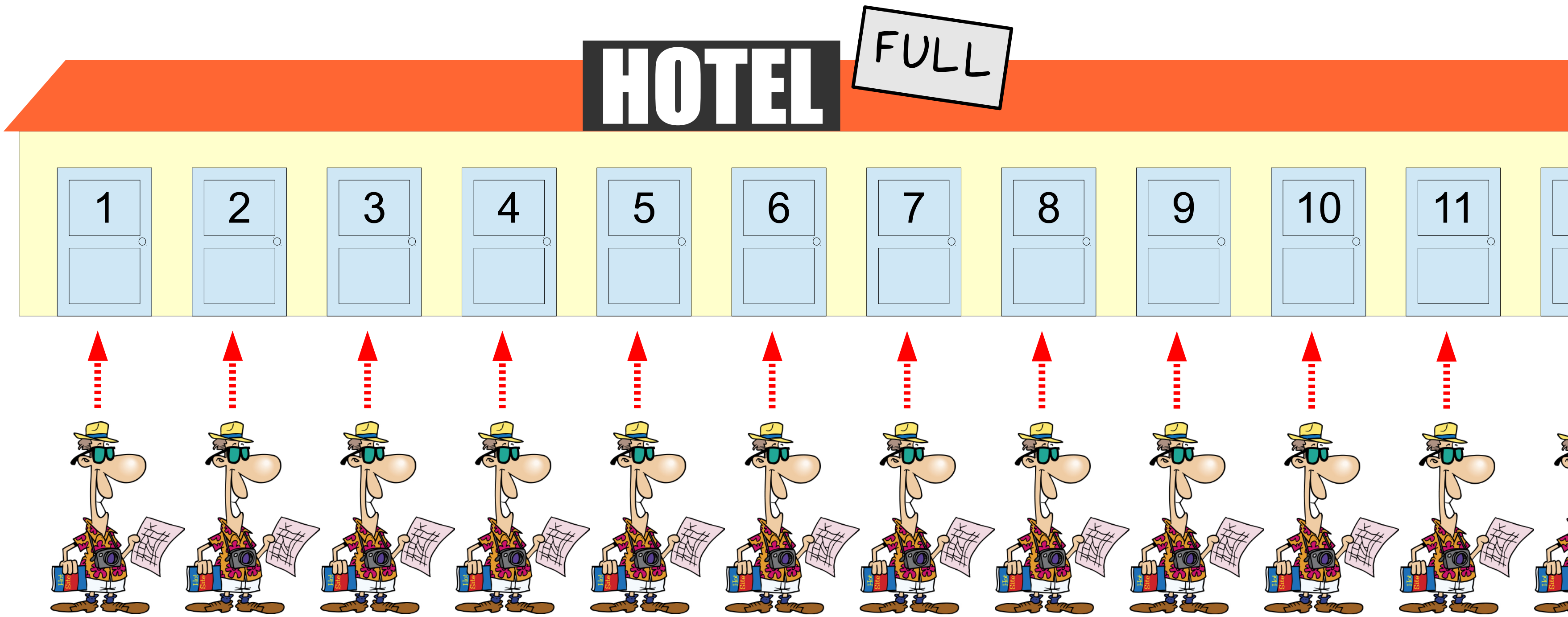
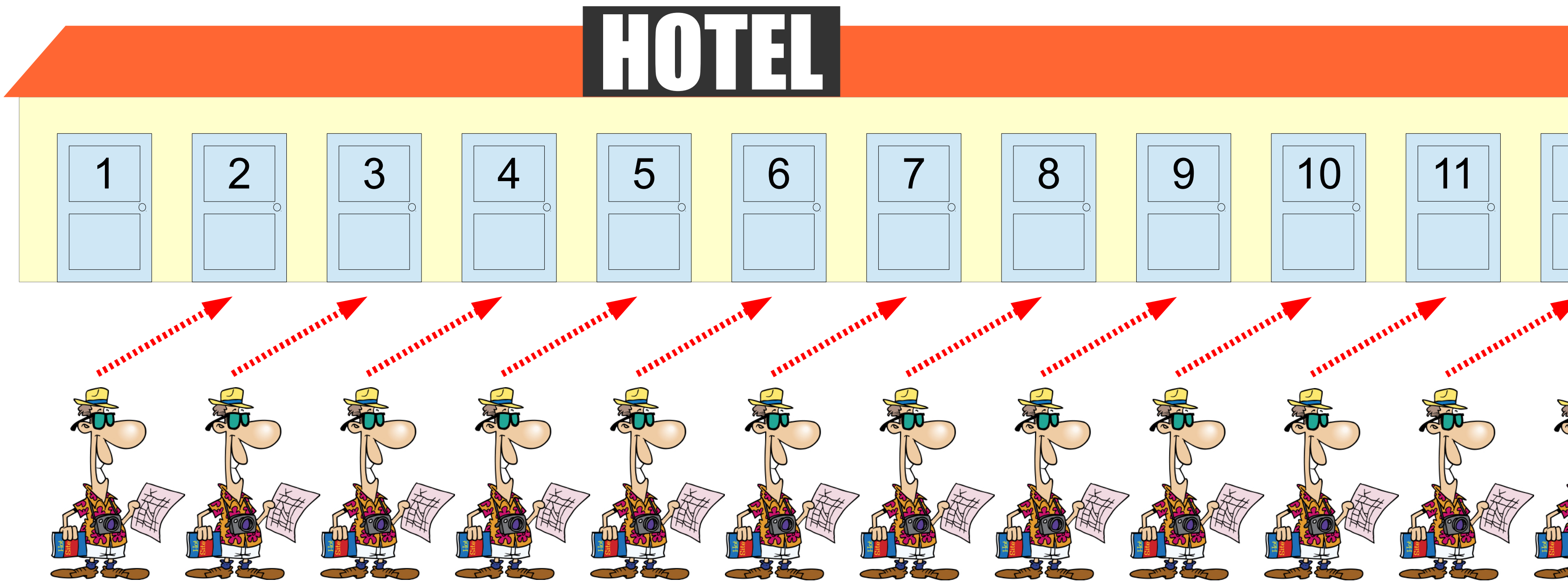


$\aleph_0$ rooms

$\aleph_0 + 1$ guests



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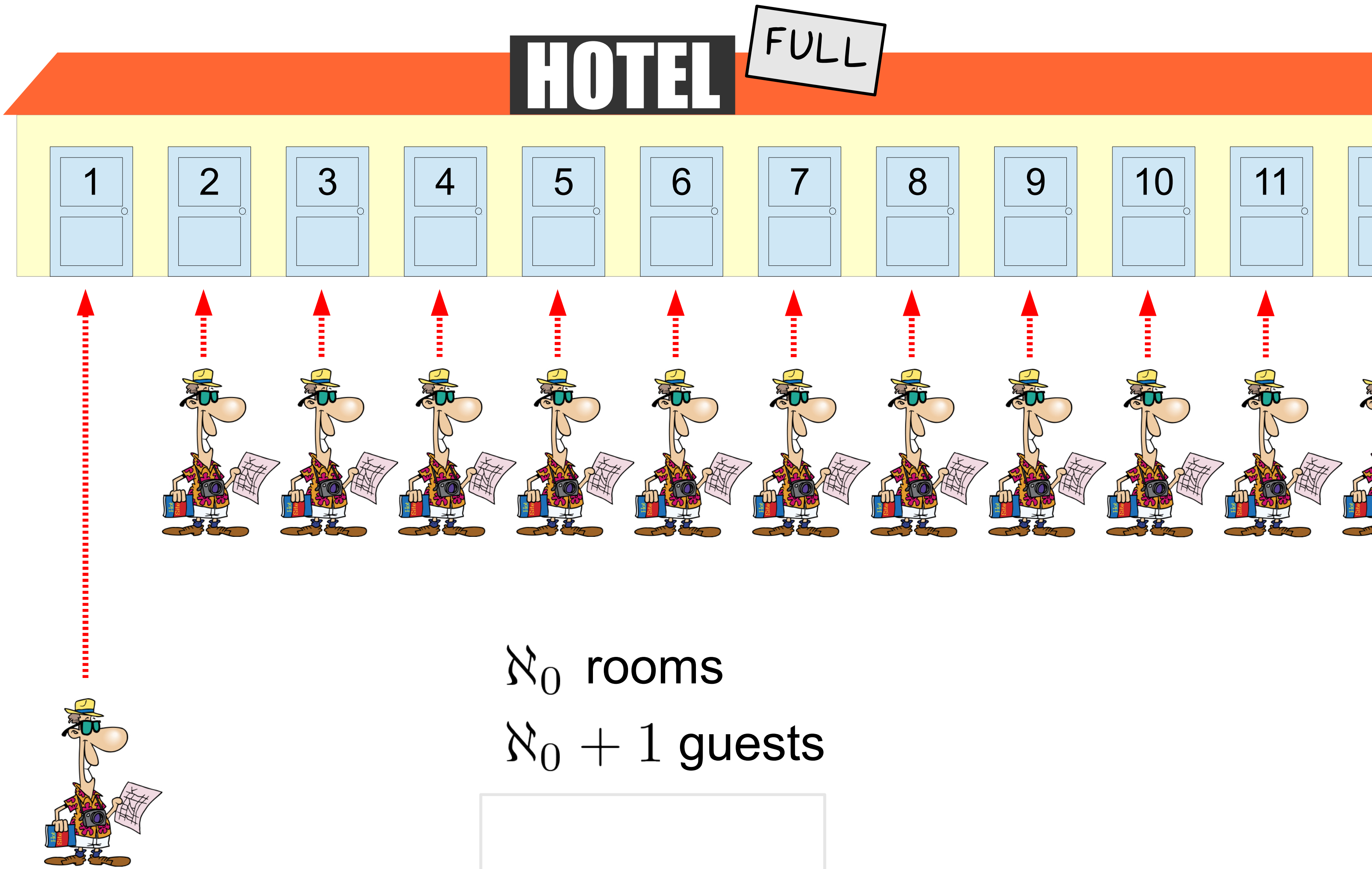

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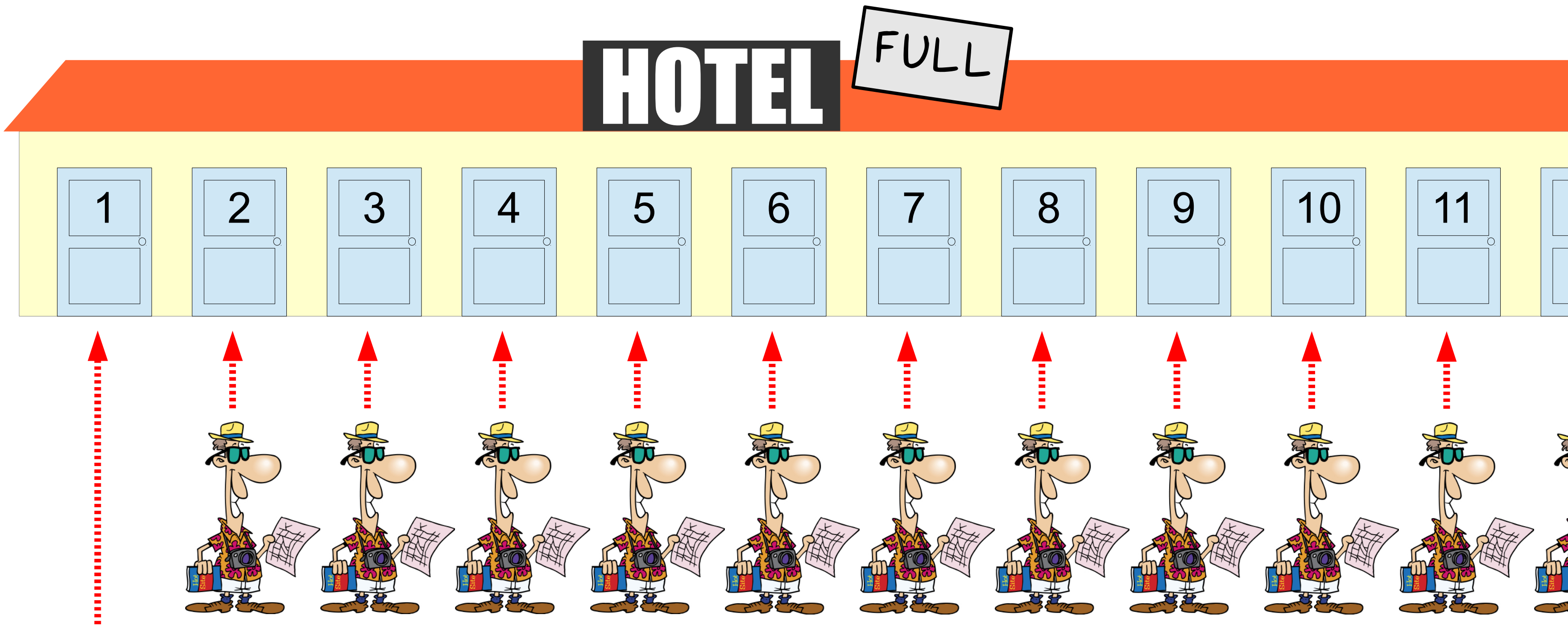
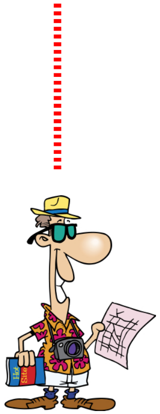
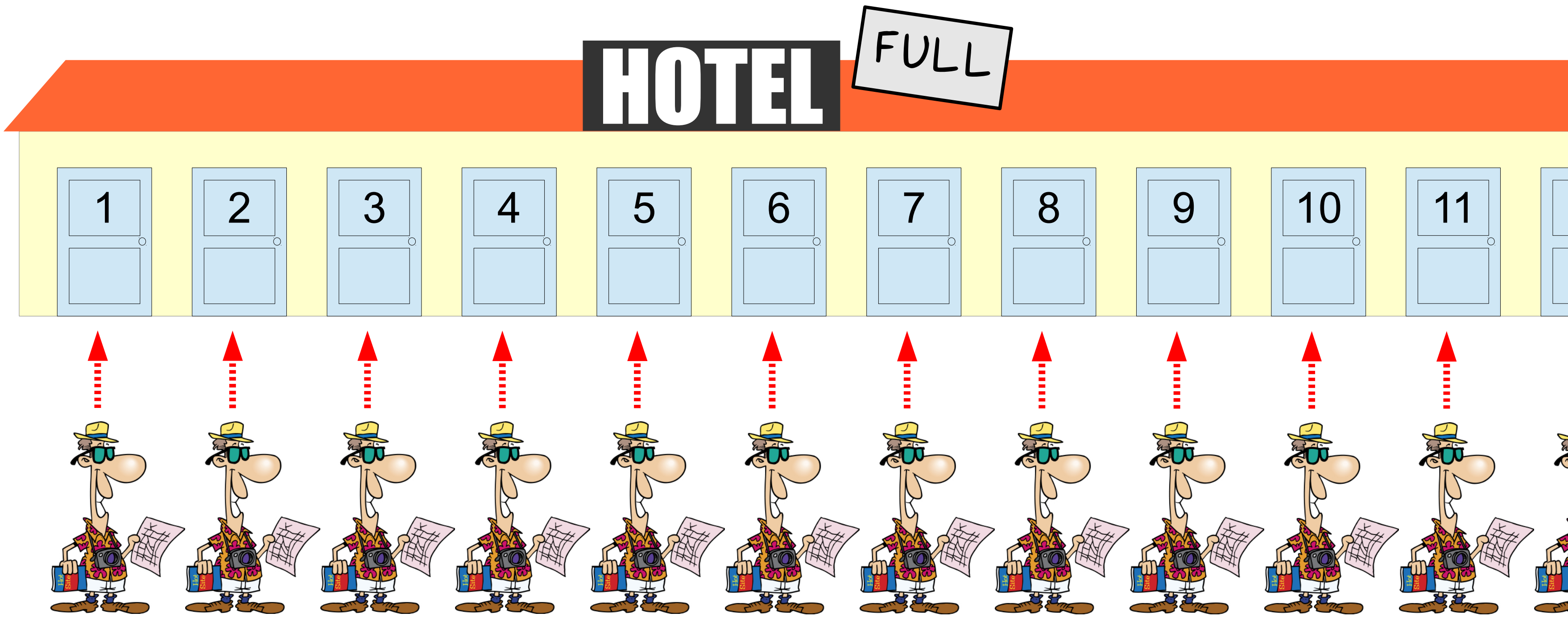




$\aleph_0$ rooms

$\aleph_0 + 1$ guests

$$\boxed{\aleph_0 + 1 = \aleph_0}$$


HOTEL
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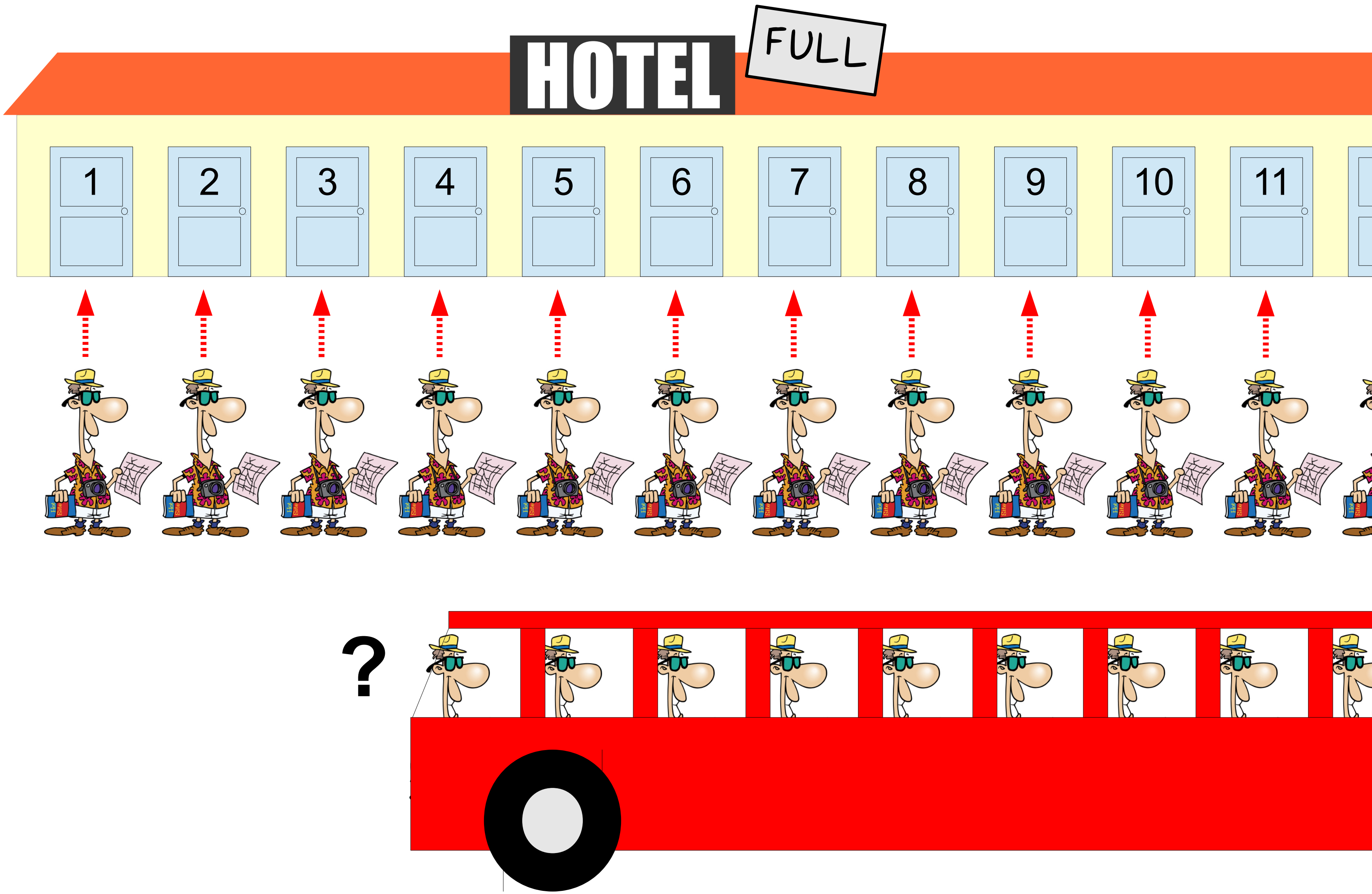
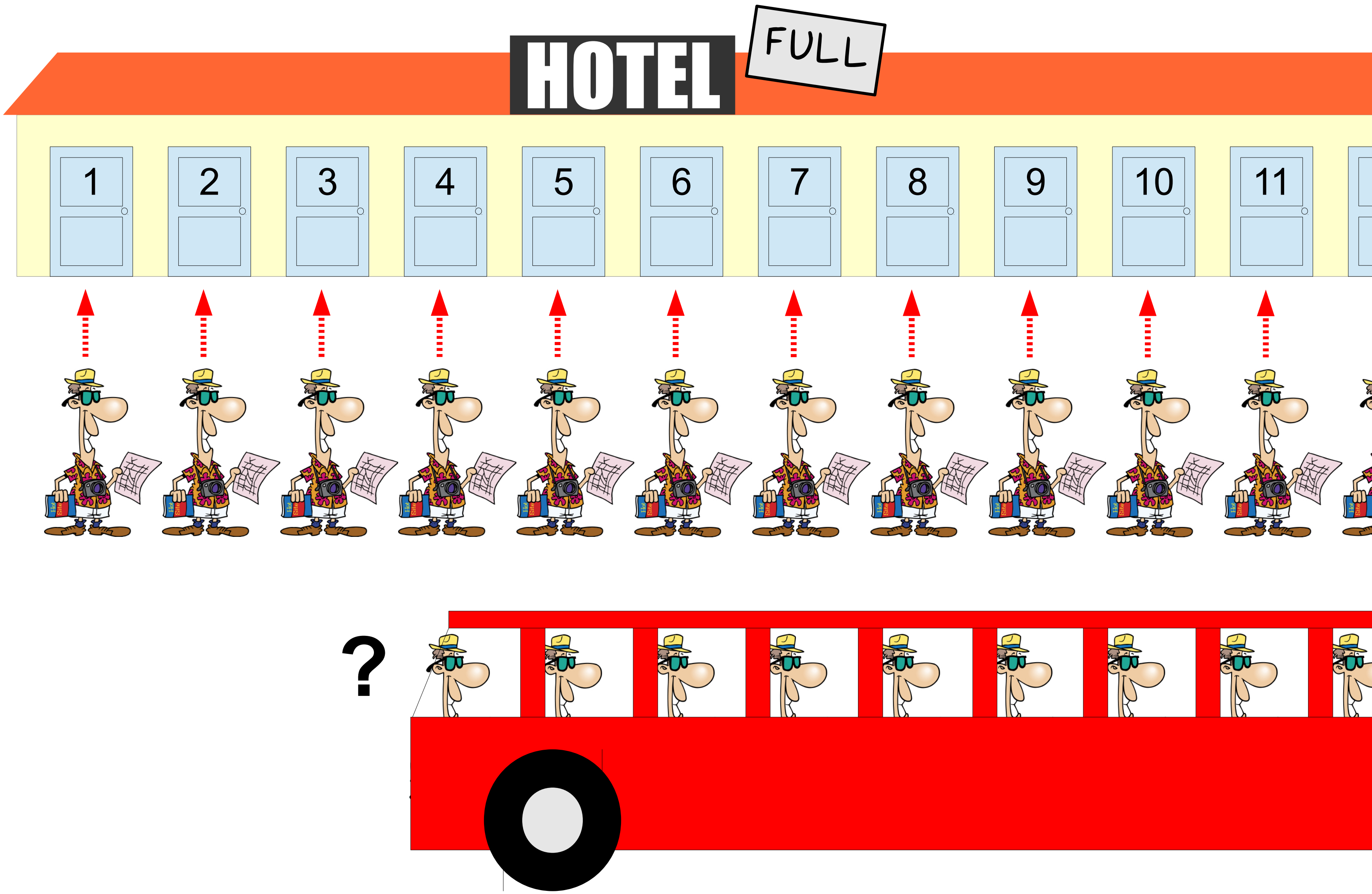
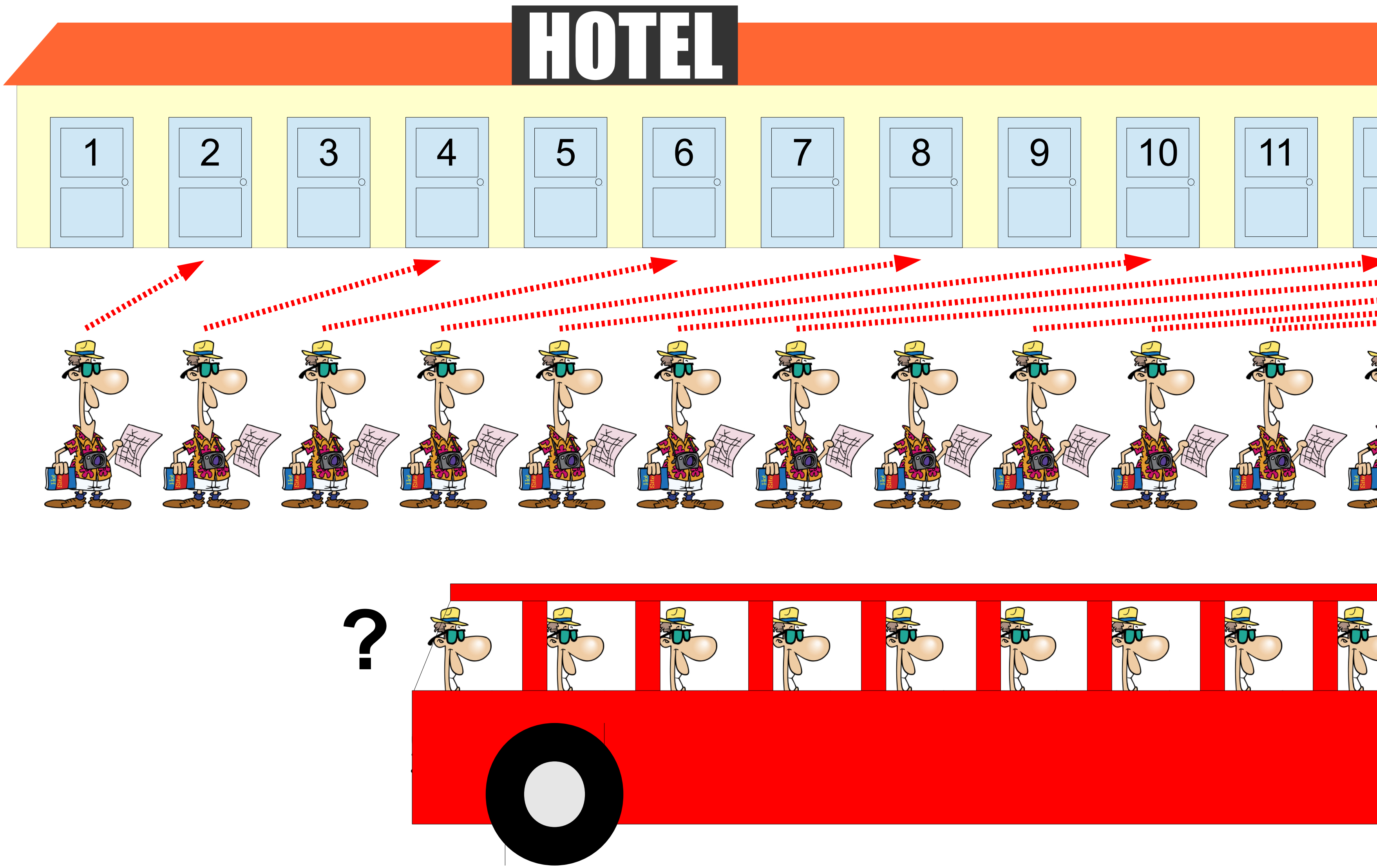


$\aleph_0$ rooms $\quad 2 \times \aleph_0$ guests



$\aleph_0$ rooms

$2 \times \aleph_0$ guests



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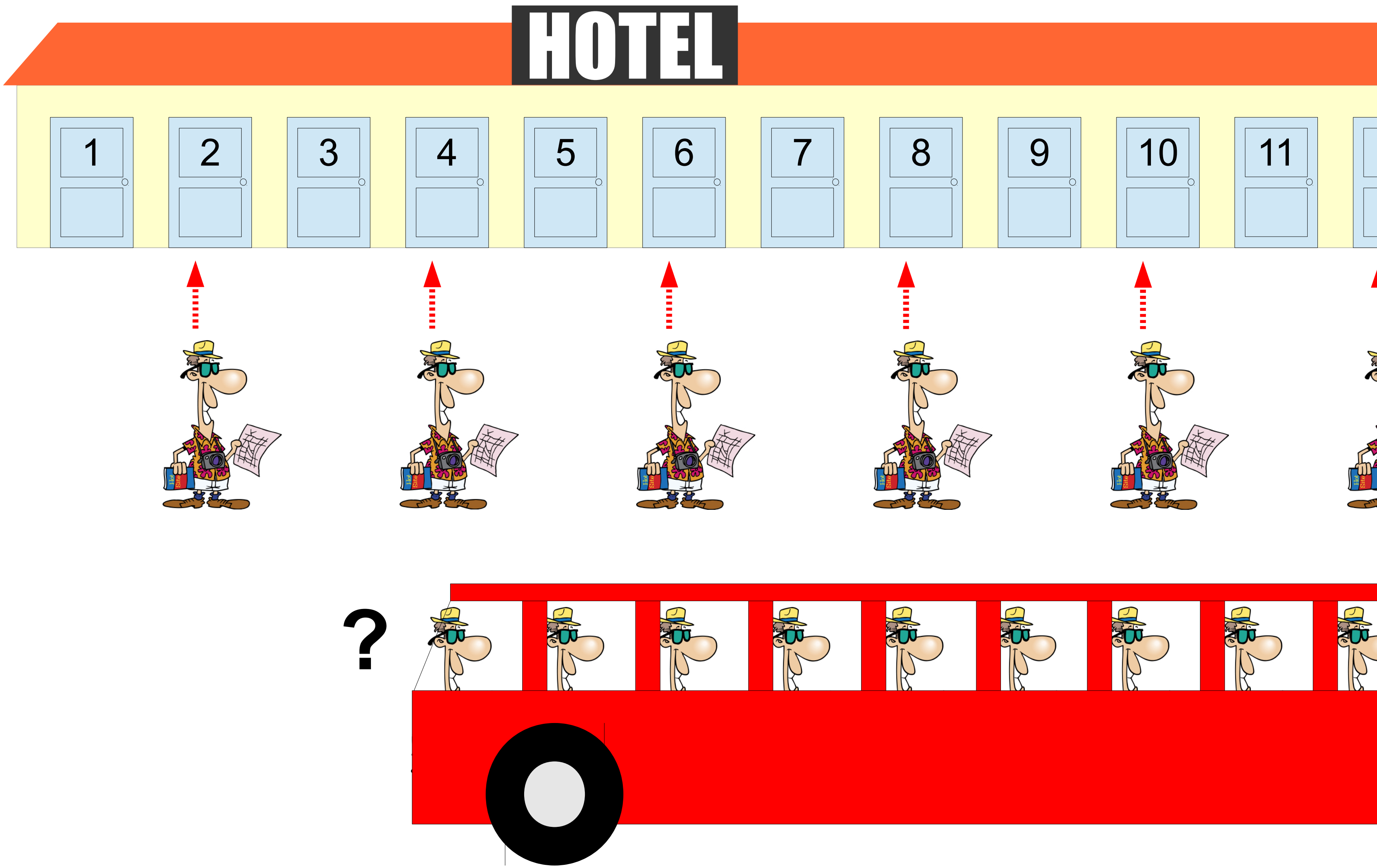

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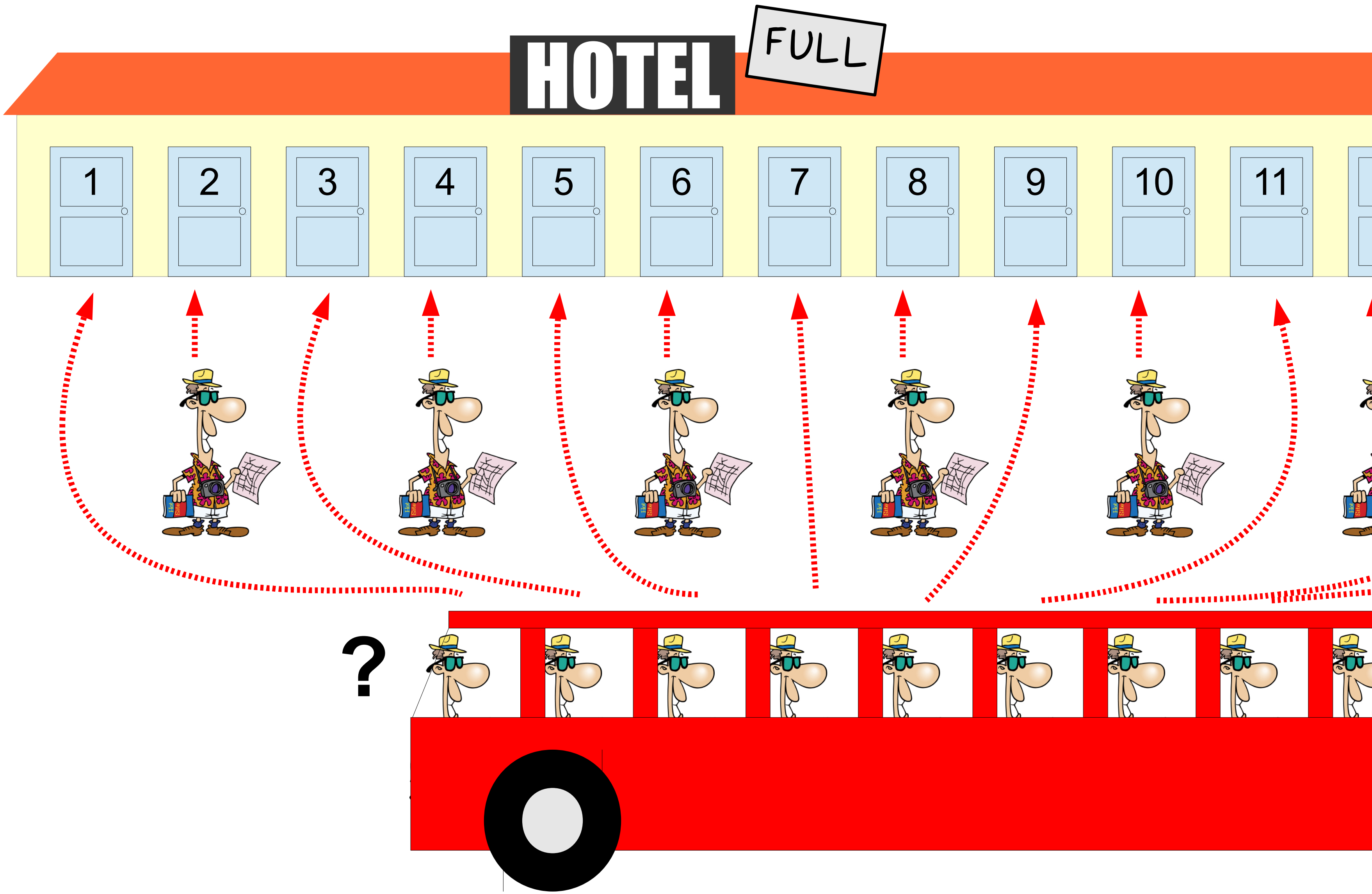
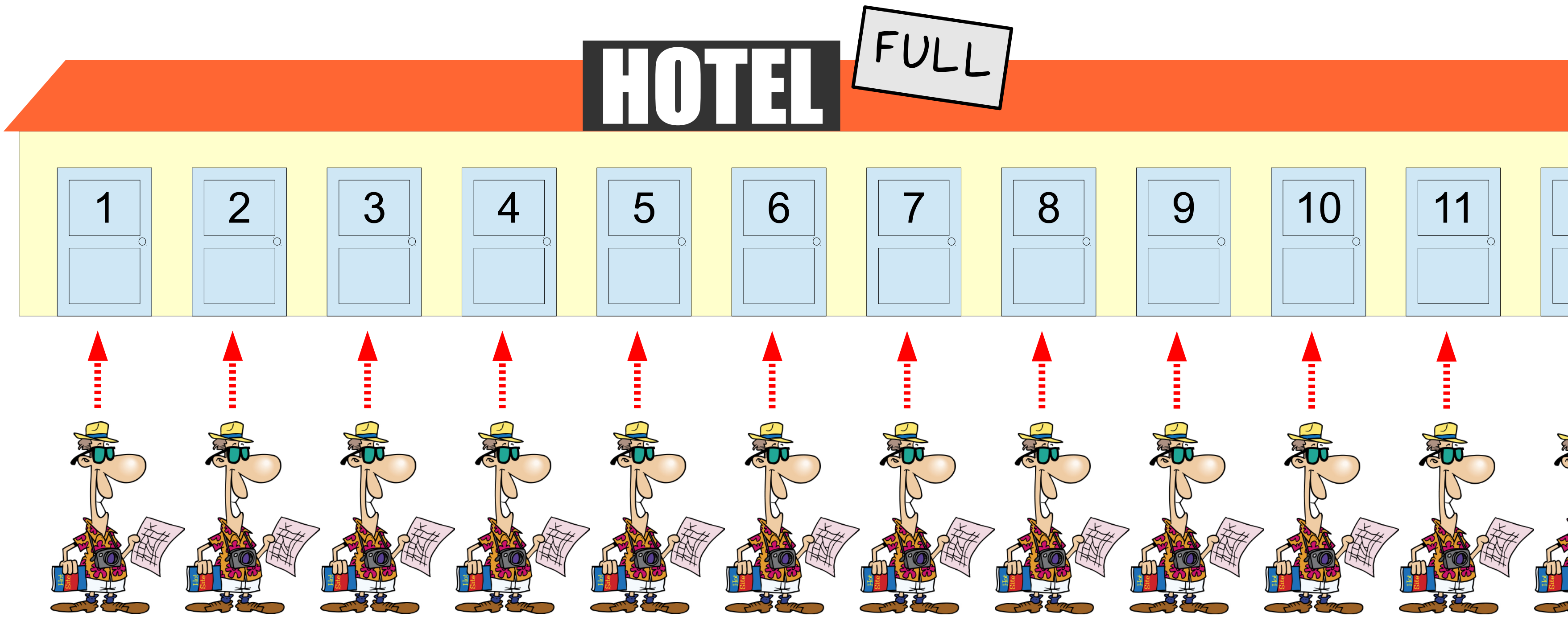
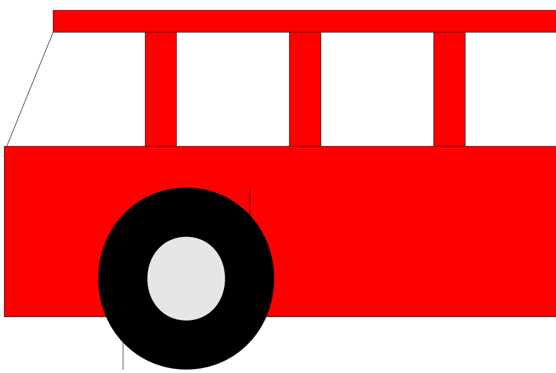


$\aleph_0$ rooms $2 \times \aleph_0$ guests

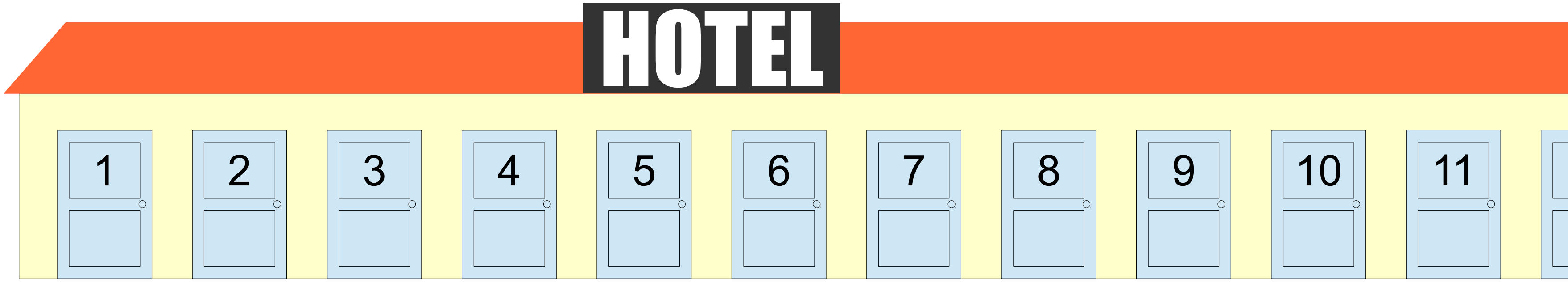




$$\boxed{2 \times \aleph_0 = \aleph_0}$$

$\aleph_0$ rooms

$2 \times \aleph_0$ guests


HOTEL
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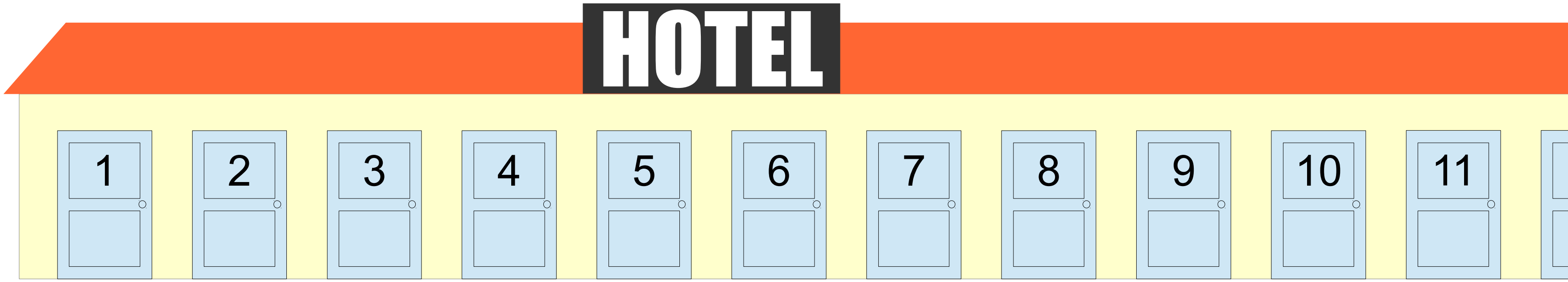
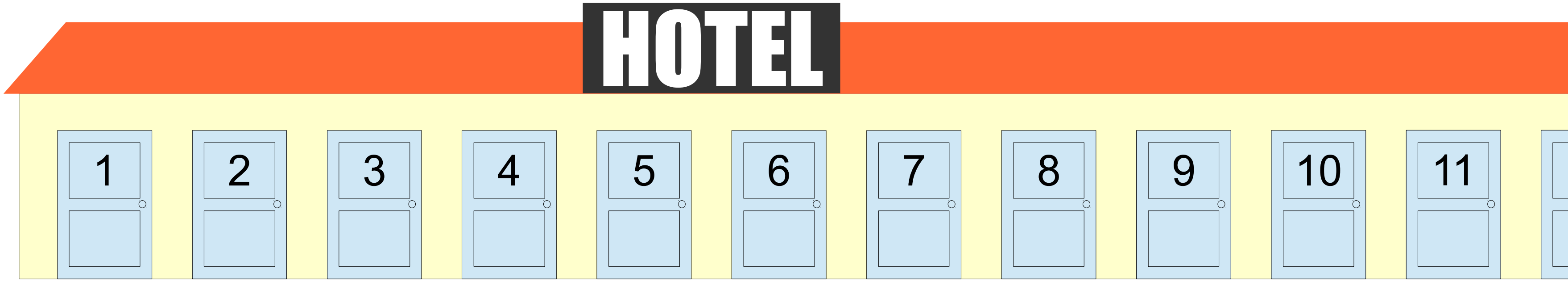


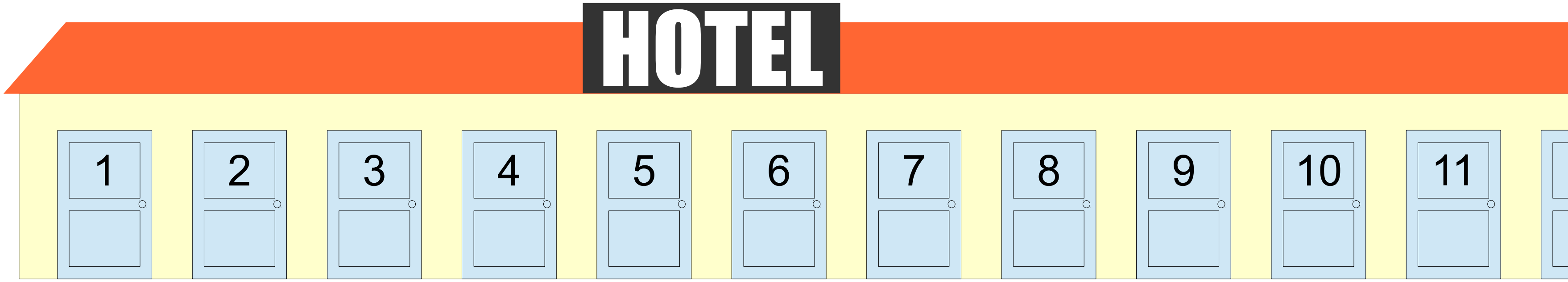
$\aleph_0$ rooms

2 guests



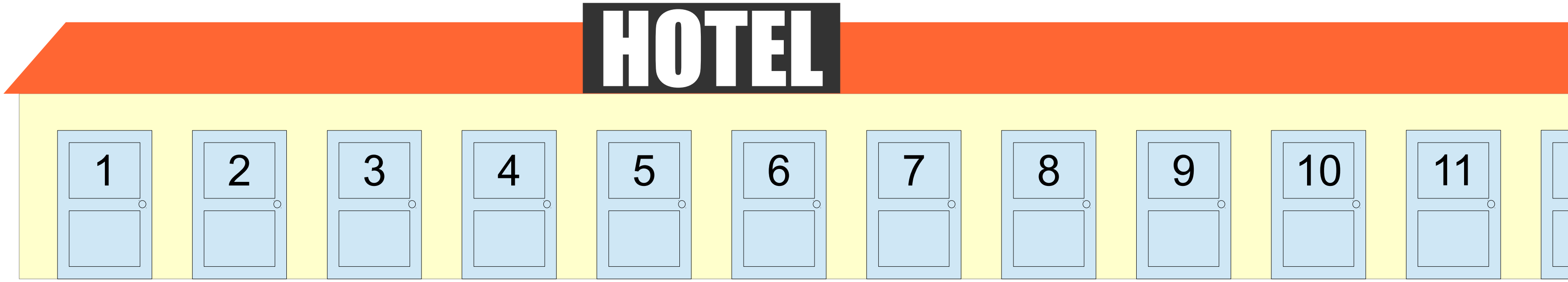
$\aleph_0$ rooms

2×2 guests



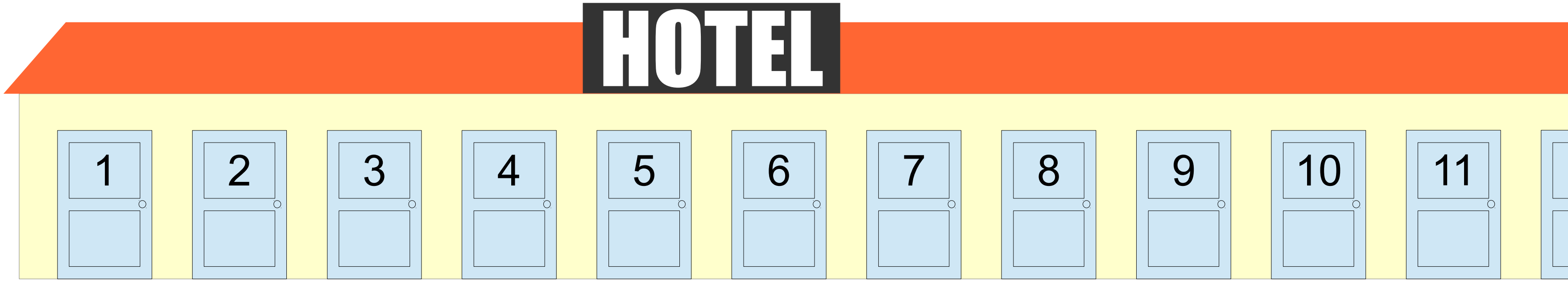
$\aleph_0$ rooms

$2 \times 2 \times 2$ guests



$\aleph_0$ rooms

$2 \times 2 \times 2 \times \ldots = 2^{\aleph_0}$ guests



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$$\boxed{2^{\aleph_0} > \aleph_0}$$

More weird facts

$2^{\aleph_0} > \aleph_0$ but how big exactly?

➡ **undecidable!**

$2^x > x$ true for any x (Cantor)

➡ **there are always bigger infinities!**

The set of all infinities does not exist (Russell)

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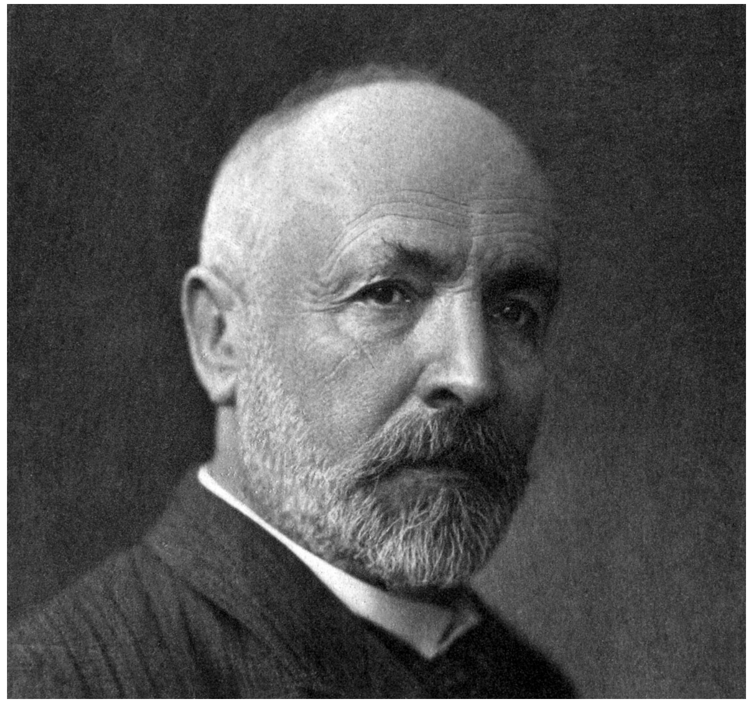

Georg Cantor
(1845-1918)