Title: The Dirichlet problem for elliptic equations without the maximum principle

Speaker: Tom ter Elst (Auckland)

Abstract:

The maximum principle plays an important role for the solution of the Dirichlet problem. Now consider the Dirichlet problem with respect to the elliptic operator

$$-\sum_{k,l=1}^{d} \partial_k \, a_{kl} \, \partial_l - \sum_{k=1}^{d} \partial_k \, b_k + \sum_{k=1}^{d} c_k \, \partial_k + c_0$$

on a bounded open set $\Omega \subset \mathbb{R}^d$, where $a_{kl}, c_k \in L_\infty(\Omega, \mathbb{R})$ and $b_k, c_0 \in L_\infty(\Omega, \mathbb{C})$. Suppose that the associated operator on $L_2(\Omega)$ with Dirichlet boundary conditions is invertible. Note that in general this operator does not satisfy the maximum principle. We define and investigate a solution of the Dirichlet problem with data in $C(\partial\Omega)$. We show that it coincides with the Perron solution, in case the maximum principle would be available. In the general setting we characterise this solution in different ways: by approximating the domain by smooth domains from the interior, by variational properties, by the pointwise boundary behaviour at regular boundary points. We also investigate for which boundary data the solution has finite energy. We show that the solution is obtained as an H_0^1-perturbation of a continuous function on $\overline{\Omega}$. This is new even for the Laplacian.

This is joint work with Wolfgang Arendt and Manfred Sauter.

References:

W. Arendt and A.F.M. ter Elst, The Dirichlet problem without the maximum principle. *Annales de l'Institut Fourier* **69** (2019), 763–782.

W. Arendt, A. F. M. ter Elst and M. Sauter, The Perron solution for elliptic equations without the maximum principle. *Math. Ann.* (2023). https://doi.org/10.1007/s00208-023-02761-0