Bilinear control of Schrödinger equations

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Let us consider a quantum particle in a potential and an electric field, represented by a Schrodinger equation. It is a control system in which the state is the wave function of the particle and the control is the electric field: one wants to steer the quantum particle from a given initial state, to a desired target state, by applying a suitable control. In the dipolar moment approximation, the control acts bilinearly on the state in the equation. The goal of this talk is to propose an overview of results and technics concerning this problem. We address several questions: exact and approximate controllability, feedback stabilization, influence of the dimension, existence of a positive minimal time required for controllability. First, we recall methods introduced in finite dimension (for ODEs), relying on iterated Lie brackets and discuss their adaptation in infinite dimension (for PDEs). Concerning exact controllability, we propose negative and positive results, depending on the functional frame, the dimension of the space and the assumption on the dipolar moment. Then, we discuss their generalizations. Concerning feedback stabilization, first, we present powerfull methods in finite dimension (for ODEs), relying on Lyapunov functions and LaSalle invariance principle; then, we propose adaptations in infinite dimension (for PDEs).