17th Berlin-Oxford Young Researcher's Meeting on Applied Stochastic Analysis

27^{th} **April** – 29^{th} **April** 2023

Contents

1. Welcome

It is our great pleasure to welcome you to the 17th Berlin-Oxford Young Researchers Meeting on Applied Stochastic Analysis. We hope you enjoy a productive meeting!

Conference organisers
Christian Bayer (WIAS Berlin)
Peter Friz (TU Berlin)
Terry Lyons (University of Oxford)
Elena Gal (University of Oxford)
Nikolas Tapia (WIAS Berlin)
Zhen Shao (University of Oxford)
Thomas Wagenhofer (TU Berlin)

Presentations
All talks will be held in person. They will be 20 minutes and we will have 5 minutes for questions after each talk. On Thursday will be a conference dinner.

Supporting Institutions

DataSig

FOR 2402

WIAS

Stochastic Analysis in Interaction

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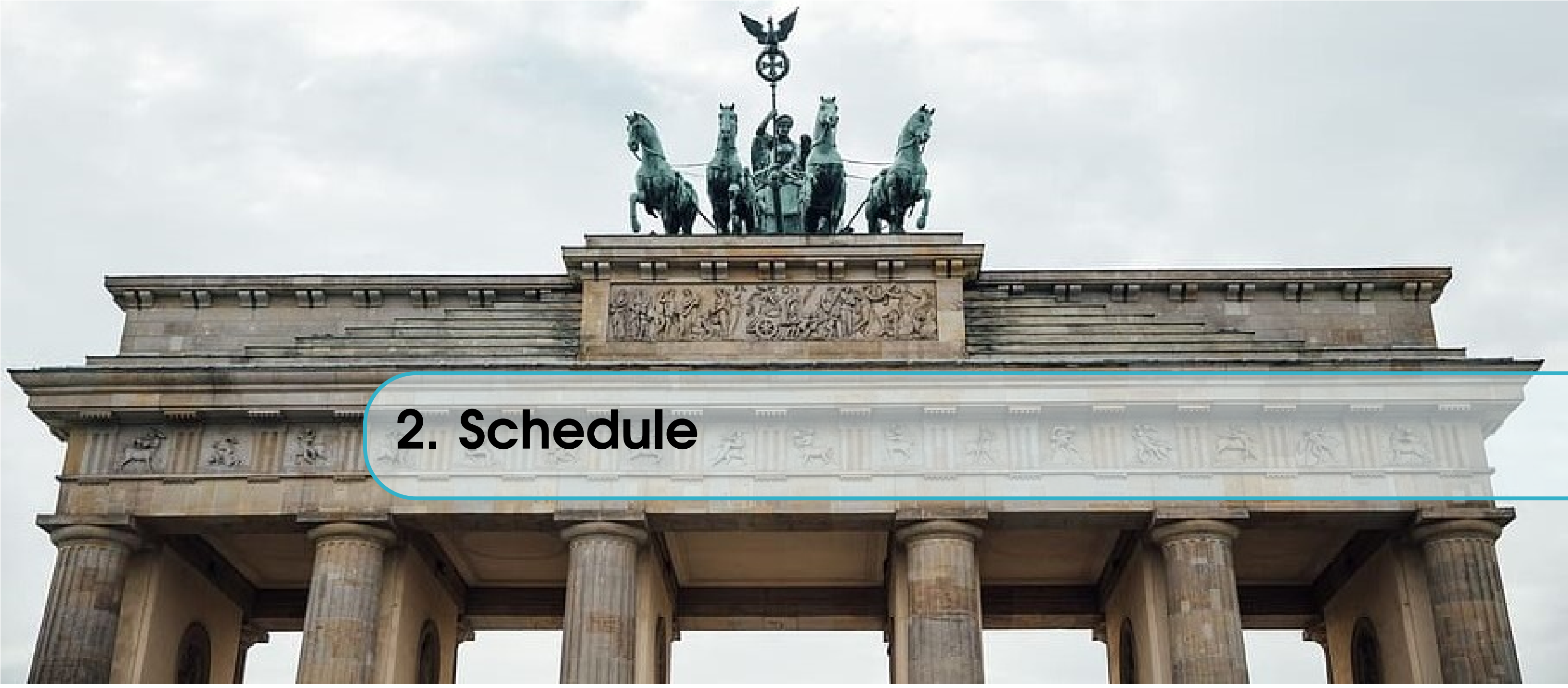

2. Schedule

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Location: WIAS Berlin

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12:30–14:00	**Lunch Break**		
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3. Talks and Abstracts

3.1 Path-by-Path Uniqueness for SDEs under Krylov–Röckner Condition

Lukas Anzeletti, CentraleSupélec

Friday
09:55-10:20

We show that any SDE driven by Brownian motion with drift satisfying the Krylov–Röckner condition has exactly one solution in an ordinary sense for almost every trajectory of the Brownian motion, i.e. path-by-path uniqueness holds. Additionally, we show that such SDE is strongly complete, i.e. for almost every trajectory of the Brownian motion, the family of solutions with different initial data forms a continuous semiflow for all nonnegative times. Joint work with Khoa Lê and Chengcheng Ling.

3.2 Varieties of Discrete Signatures

Carlo Bellingeri, TU Berlin

Friday
15:45-16:10

Following the work of Diehl, Ebrahimi-Fard and Tapia on discrete signatures of a time series, in this talk, we will discuss the geometric properties of the discrete signature variety, i.e. the points traced out by discrete signature inside the tensor algebra. In particular, we will give some explicit descriptions of the level 2 and 3 varieties together with some hints on the existence of a universal variety.

3.3 Weak Markovian Approximations of Rough Heston

Simon Breneis, WIAS Berlin

Thursday
16:35-17:00

The rough Heston model is a popular option pricing model in mathematical finance. As the volatility process in this model is neither a semimartingale nor a Markov process, simulation is often costly in practice. To resolve this issue, we approximate the volatility process with an N-dimensional diffusion, yielding a Markovian approximation of the rough Heston model. Previous works have shown that Markovian approximations converge strongly as the number of dimensions N increases, even with superpolynomial rate. However, as the Hurst parameter H becomes small, the rate of convergence, despite being superpolynomial, becomes arbitrarily bad. In this work, we show that we can still achieve weak superpolynomial convergence that does not become arbitrarily slow for H approaching 0.

3.4 Stochastic Equations with Singular Drift Driven by Fractional Brownian Motion

Oleg Butkovsky, TU Berlin

Friday
12:05-12:30

Joint work with Khoa Le and Leonid Mytnik. We consider stochastic differential equation

$$dX_t = b(X_t)dt + dW_t^H,$$

where the drift b is either a measure or an integrable function, and W^H is a d-dimensional fractional Brownian motion with Hurst parameter $H \in (0,1)$, $d \in \mathbb{N}$. For the case where $b \in L_p(\mathbb{R}^d)$, $p \in [1,\infty]$ we show weak existence of solutions to this equation under the condition

$$\frac{d}{p} < \frac{1}{H} - 1,$$

which is an extension of the Krylov-Rockner condition to the fractional case. We construct a counter-example showing optimality of this condition. If b is a Radon measure, particularly the delta measure, we prove weak existence of solutions to this equation under the optimal condition $H < \frac{1}{d+1}$. To establish these results, we utilize the stochastic sewing technique and develop a new version of the stochastic sewing lemma

3.5 Trajectorial Interpretation to the Dissipation Phenomena of Relative Entropy

Jiaming Chen, Courant Institute and Sorbonne Université

Thursday
14:00-14:25

I will present a trajectorial approach, proposed in a recent breakthrough in 2020 by Karatzas/Schachermayer/Tschiderer, to investigate the dissipation phenomena of relative entropy from an Itô-Langevin stochastic dynamical system. Relying on the time-reversal principles of diffusions, this trajectorial approach investigates the pathwise behavior of relevant stochastic processes, and eventually retrieves the known classical de Bruijn inequalities. In essence, this approach provides novel insights and reveals more information from the Itô-Langevin dynamics. Another part is to view the stochastic time-evolution through the lens of the Wasserstein space, under which the geometric feature of steepest descent of the entropy dissipation as well as its exponential decay rate have been revealed.

3.6 Towards Abstract Wiener Model Spaces

Gideon Chiusole, TU Munich/TU Berlin

Saturday
09:55-10:20

The theory of Abstract Wiener Spaces is the basis for many fundamental results of Gaussian measure theory: Large Deviations, Cameron-Martin theorems, Malliavin Calculus, Support theorems, etc. Analogues of these classical theorems exist also in the context of Gaussian Rough Paths and Regularity Structures. It is our goal to investigate the role of an “enhanced” Cameron-Martin subspace in this setting. In particular, we present two approaches to a generalization based on Large Deviation theory and apply them to examples of Rough Path theory and Regularity Structures.

3.7 Branched Itô formula and Itô-Stratonovich Correction

Emilio Ferrucci, University of Oxford

Friday
16:10-16:35

The Itô lemma and the Itô-Stratonovich correction formula are fundamental results of stochastic analysis. Branched rough paths constitute a theory of controlled differential equations driven by irregular signals, which allows for arbitrarily ill-behaved integration-by-parts formulae. In this talk we explain how cofreeness of the Connes-Kreimer Hopf algebra yields a canonical Itô formula that extends the one for continuous semimartingales. Moreover, we define a natural isomorphism between the Connes-Kreimer Hopf algebra and the shuffle Hopf algebra, which we conjecture to be unique in a certain sense. We show Hoffman’s isomorphism between quasi-shuffle and shuffle to be a quotient of ours, which therefore restricts to the well-known Itô-Stratonovich correction map for continuous semimartingales. Finally, we explain how our work differs from that of other authors on similar questions. This talk is based on joint work with Carlo Bellingeri and Nikolas Tapia.

3.8 Controlled Rough Paths as Integrals of Cocyclic One-Forms

Martin Geller, University of Oxford

Saturday
12:05-12:30

We establish a connection between the work of Danyu and Lyons (2018) and that of Gubinelli (2002), by showing how integrals of cocyclic one-forms are in one-to-one correspondence with controlled rough paths. This enables to apply results about cocylic one-forms to controlled rough paths, including local approximations. To derive our results, we also needed to develop a method of perturbing rough paths in the direction of other rough paths, which is valuable for other questions than the main ones that we considered.

3.9 Weak Well-Posedness for an SDE with Singular, Divergence-Free, Random Drift

Lukas Gräfner, FU Berlin

Friday
11:15-11:40

On the 2d torus, we consider a Gaussian free field ξ and the SDE

$$dX_t = dB_t + b(X_t)dt, b = \rho(-\Delta)\nabla^{\perp}\xi, \tag{3.1}$$

where the Brownian motion B is independent of ξ and ρ is a Fourier multiplier.
In general, equation (3.1) is singular since b is almost surely distribution-valued and the meaning of the evaluation $b(X_t)$ is unclear. For initial conditions with $L^2(Leb_{\mathbb{T}^2})$-density, we show weak-wellposedness for this equation in the sense that classically well-defined approximations of the equation converge in law to a unique limit. The main tool is a singular perturbation result for semigroups on certain Hilbert spaces which solves a linear backward PDE corresponding to equation (3.1).
Although we cannot treat the case $\rho \equiv 1$, the multiplier can be chosen in such a way that b has critical regularity -1, almost-surely.
This is joint work with Nicolas Perkowski

3.10 McKean–Vlasov Equations with Rough Common Noise and Quenched Propagation of Chaos

Antoine Hocquet, TU Berlin

Thursday
14:50-15:15

I will show well-posedness and propagation of chaos for McKean–Vlasov equations with rough common noise and progressively measurable coefficients. The results are valid under minimal regularity assumptions on the coefficients, in agreement with the respective requirements of Itô and rough path theory. To achieve these goals, I will introduce the framework of rough stochastic differential equations recently developed by K. Lê, P. Friz and myself.

3.11 Scaling Limits of Stochastic Transport Equations on Manifolds

Wei Huang, FU Berlin

Friday
16:35–17:00

We consider the stochastic transport equations driven by multiplicative noises(smooth in space, white in time) on a manifold. Under certain conditions, we obtain different scaling limits depending on the initial data. If the initial data is regular, the limit satsifies a deterministic heat equation with some quantitative estimates on the convergence rate. However, if we consider the stationary solution with space white noise as initial data, the limit satisfies a stochastic heat equation with additive noise.

3.12 Interacting Particles and Market Capitalization Curves

Florian Huber, University of Vienna

Thursday
16:10-16:35

Motivated by the robustness of the so-called market capitalization curve, our goal is to study the behaviour of equity market models on a macroscopic scale. This is done by extending the volatility stabilized market models studied by Fernholz and co-authors and allowing for simple correlation structure induced by a common noise term. Letting the number of companies approach infinity, we show that the limit of the empirical measure of the N-company system converges to the unique solution of a degenerate, non-linear SPDE. The obtained limit also possess a representation as a conditional probability of the solution to a certain McKean-Vlasov SDE for which we obtain uniqueness in law by a superposition result as well as Besov regularity of its laws with respect to the Lebesgue measure. Lastly, we investigate the fluctuations of the particle system around the limiting SPDE. This is joint work in progress with Christa Cuchiero.

3.13 Post-Lie Algebras of Derivations and Regularity Structures

Jean-David Jacques, Sorbonne Université

Saturday
09:30–09:55

Post-Lie algebra structures are a generalization of Pre-Lie algebras. They have their roots in geometry and correspond to the algebraic properties satisfied by the covariant derivative in the case of a flat and constant torsion connection.
In my talk, I will provide a brief overview of the new theory of regularity structures developed by F. Otto and colleagues, and discuss how post-Lie algebra structures arise in this framework

3.14 Numerical Analysis for Singular SDEs-Milstein Scheme

Chengcheng Ling, TU Vienna

Friday
11:40-12:05

We study the L^p rate of convergence of the Milstein scheme for SDEs when the drift coefficients possess only Hölder regularity with order smaller than one. If the diffusion is elliptic and sufficiently regular, we obtain rates consistent with the additive case. The proof relies on regularisation by noise techniques, particularly stochastic sewing, which in turn requires (at least asymptotically) sharp estimates on the law of the Milstein scheme, which we attempt to achieve through Malliavin calculus.
This talk is a base on a joint work with Máté Gerencsér (TU Wien) and Gerald Lampl (TU Wien).

3.15 Selection Problem, Peano Example and Non-Markovian Noise

Łukasz Mądry, Université Paris-Dauphine

Friday
10:20-10:45

We study the selection problem (alternatively - zero noise limit) for the Peano example with non-Markovian Gaussian noise. We extend the approach of Delarue-Flandoli 14' to provide a description of dynamics in this context. To this end, we marry recent progress in regularisation by noise with techniques coming from the study of ergodicity of fractional SDEs (Hairer 05', Panloup-Richard 20') to establish the convergence rate to the zero noise solution. Joint work with Paul Gassiat (Univ. Paris-Dauphine)

3.16 Almost Sure Averaging for Evolution Equations Driven by Fractional Brownian Motions

Bin PEI, Friedrich Schiller University Jena (FSU Jena)

Friday 09:30-09:55

We apply the averaging method to a coupled system consisting of two evolution equations which has a slow component driven by fractional Brownian motion (FBM) with the Hurst parameter $H_1 > 1/2$ and a fast component driven by additive FBM with the Hurst parameter $H_2 \in (1 - H_1, 1)$. The main purpose is to show that the slow component of such a couple system can be described by a stochastic evolution equation with averaged coefficients. Our first result provides a pathwise mild solution for the system of mixed stochastic evolution equations. Our main result deals with an averaging procedure which proves that the slow component converges almost surely to the solution of the corresponding averaged equation using the approach of time discretization. To do this we generate a stationary solution by a exponentially attracting random fixed point of the random dynamical system generated by the fast component.

3.17 Rough PDEs for Local Stochastic Volatility Models

Luca Pelizzari, WIAS Berlin

Thursday 15:45-16:10

In this work we aim to use rough partial differential equations (RPDEs), to solve European pricing problems in general local stochastic volatility (LSV) models. For non-Markovian volatility processes, classical (deterministic) PDE approaches are not applicable. Using tools from rough path theory, we show that conditional on the Brownian motion that drives the volatility, the price dynamics possess a Markovian nature. In particular, we relate the conditional dynamics to linear, pathwise RPDEs, and provide a Feynman-Kac representation for the pathwise solutions. Using the tower property, we conclude that European prices can be characterized as expected values of the pathwise RPDE solutions. Finally, we present numerical examples in several rough LSV models, using finite-difference schemes for RPDEs

3.18 Non-Redundancy of the Log-Signature Revisited

Rosa Preiß, Uni Potsdam

Saturday 11:15–11:40

The iterated-integral signature itself is highly redundant as a tensor sequence, due to Ree's shuffle relation. In order to get rid of that redundancy, it is standard procedure to look at the log-signature. And indeed, thanks to the Chen-Chow theorem, any finite truncation of the log-signature is non-redundant, as in all algebraically possible values are reached through bounded variation paths.
This talk poses the completely open and maybe surprising problem of redundancy of the log-signature as an infinite tensor sequence. While we cannot provide an answer yet, we think it's essential to understand and more researchers should be aware of how this is different from what the Chen-Chow theorem deals with.

3.19 Signature SDEs with Jumps

Francesca Primavera, University of Vienna

Friday 14:25-14:50

Signature-based models have recently entered the field of stochastic modeling, in particular in Mathematical Finance. The choice of the signature as main building block is mainly explained by a universal approximation theorem according to which continuous functionals of paths can be approximated by linear functionals of the time-extended signature. Relying on these approximation results, we introduce a generic class of jump diffusion models via the so-called signature SDEs with jumps and elaborate on their tractability properties. As a special case, we focus on jump-diffusions with entire characteristics, an extension of the class of polynomial processes for which expected values of entire functions of the process' marginals are expressed as power series expansions in terms of the process' initial value. This talk is based on joint work with Christa Cuchiero and Sara Svaluto-Ferro.

3.20 Planar Regularity Structures

Ludwig Rahm, NTNU

Saturday
10:20-10:45

Branched rough paths, used to solve ODEs on $\mathbb{R}$, have been generalised in two different directions. In one direction, there are regularity structures aimed at solving SPDEs on $\mathbb{R}$. In the other direction, there are planarly branched rough paths to solve ODEs on homogeneous spaces. This talk will combine these two directions to construct planar regularity structures, for (S)PDEs on homogeneous spaces.

3.21 Decision-Theoretic Aspects of Stochastic Differential Games

Emanuel Rapsch, TU Berlin

Thursday
12:05-12:30

Stochastic differential games provide a classical model at the interface of stochastic analysis and interactive decision (alias game) theory. The basic idea is that players "continuously" control the coefficients of a stochastic differential equation with respect to a criterion given in terms of its solution. However, several problems arise in this context. First, how should exogenous noise (e.g. deriving from a solution to a stochastic differential equation) on the one hand and strategic dependence on the other be modelled? What does it mean to take decisions in continuous time against the backdrop of continuous noise? And what does this imply in particular for stochastic differential games, and which role may pathwise solution concepts play? In this talk I would like to explains these problems, which in parts have already been addressed in the literature and are part of my current doctoral research, supervised by Christoph Belak.

3.22 Neural Signature Kernels as Infinite-Width Limits of Neural Controlled Differential Equations.

Cristopher Salvi, Imperial College London

Friday
14:50–15:15

Motivated by the paradigm of reservoir computing, I will consider randomly initialized neural controlled differential equations and show that in the infinite-width limit and under proper rescaling of the neural vector fields, these architectures converge weakly to Gaussian processes indexed on pathspace and with covariances satisfying certain PDEs varying according to the choice of activation function. In the special case where the activation is the identity, the equation reduces to a linear PDE and the limiting kernel agrees with the original signature kernel.

3.23 Attention on Signature-Type Features: Beating Quadratic Costs

Leonard Schmitz,University of Greifswald

Thursday
11:15-11:40

Transformer models have demonstrated impressive results in natural language processing, one of the most prominent being GPT-3 by OpenAI. The main transformer building blocks are simple (self)-attention units. We show that the time complexity of the latter can be reduced by a magnitude for low-rank decompositions of the input data. The signature provides such low-rank data when used as a feature of sequences. We present recent results of an efficient matching mechanism for subsequences which combines attention and the signature method. This is joint work with Joscha Diehl.

3.24 A Study of Transformed Brownian Rough Paths

Jiajie Tao, University College London

Friday
14:00-14:25

We provide a general methodology to study the law of linear functionals applied on the elements of the signature of Brownian motion by considering the coupled process Brownian motion and the transformed process. We established the relationship between the characteristic function and the expected signature of the joint process, which can be expressed as the solution to a parabolic PDE. Finally, we derive the PDE that uniquely determines the characteristics function. As an example, we derive the characteristic function of joint distribution of multi-dimensional Brownian motion and the corresponding Levy area at any fixed time horizon.

3.25 Generalized Permutation Patterns in Time Series Analysis

Emanuele Verri, University of Greifswald

Saturday
11:40-12:05

Inspired by discrete time series analysis we define a bialgebra on finite interval partitions. This is work in progress with Joscha Diehl.

3.26 Neural Controlled Differential Equations: Memory

Benjamin Walker, University of Oxford

Thursday
11:40-12:05

Neural controlled differential equations (NCDEs) are a powerful approach to multivariate, irregularly sampled, time-series modelling. When combined with the Log-ODE method, they demonstrate good performance on long time series. Inspired by the current state-of-the-art methods, LMU and S4, this talk explores whether the performance of NCDEs can be further improved by using the log-signature of the input path as an NCDE's memory.

3.27 A Characterization of Intermittency in Continuum Parabolic Anderson Model

Huanyu Yang, FU Berlin and WIAS Berlin

Thursday
14:25-14:50

We consider the parabolic Anderson problem:

$$\begin{aligned} \partial u(t,x) &= \Delta u(t,x) + \xi(x)u(t,x), && (t,x) \in (0,\infty)\times\mathbb{R}^d, \\ u(0,x) &= \delta_0(x) && x \in \mathbb{R}^d, \end{aligned} \tag{3.2}$$

where ξ is a smooth homogeneous Gaussion field on $\mathbb{R}^d$. We prove the geometric intermittency, i.e. with probability one, as $t\to\infty$, the overwhelming contribution of the total mass $\int_{\mathbb{R}^d} u(t,x)dx$ comes from a slowly increasing number of islands which are located far from each other. These islands are local regions of those high peaks of the field ξ in a box of side length $t\log t$ for which the (local) principal Dirichlet eigenvalue of the Anderson Hamiltonian $\Delta+\xi$ close to the top of the spectrum in the box. This is the joint work with Nicolas Perkowski.

4. Participants

Henri Elad Altman
Lukas Anzeletti
Peter Bank
Carlo Bellingeri
Simon Breneis
Leonie Brinker
Oleg Butkovsky
Jiaming Chen
Henry Chiu
Gideon Chiusole
Emilio Ferrucci
Thomas Gaskin
Ioannis Gasteratos
Martin Geller
Lukas Gräfner
Emanuela Gussetti
Paul Hager
Antoine Hocquet
Teodor Holland
Wei Huang
Florian Huber
Jean-David Jacques
Hannes Kern
Vaios Laschos
Chengcheng Ling
Łukasz Mądry
Adrian Martini
Toyomu Matsuda
Nicolas Moench
Bin Pei
Luca Pelizzari
Nicolas Perkowski
Rosa Preiß
Francesca Primavera
Ludwig Rahm
Emanuel Rapsch
Cristopher Salvi
Leonard Schmitz
Florin Suciu
Yuchen Sun
Jiajie Tao
Mazyar Ghani Varzaneh
Emanuele Verri
Benjamin Walker
Weile Weng
Huanyu Yang
Huilin Zhang
Willem van Zuijlen