13th Berlin-Oxford Young Researchers Meeting on Applied Stochastic Analysis

8^{th} June – 10^{th} June 2020

Contents

1. Welcome

It is our great pleasure to welcome you to the **13th Berlin-Oxford Young Researchers Meeting on Applied Stochastic Analysis**. We hope you enjoy a productive meeting!

Conference organisers
Tom Klose, Oleg Butkovsky, Avi Mayorcas, Patric Bonnier.

Scientific committee
Peter Friz, Terry Lyons.

Presentations

All talks will be hosted on Zoom. They will be 25 minutes long including 5 minutes for questions. In addition to the presentations there will be coffee rooms hosted on a Discord server open at all times for discussion.

Supporting institutions

1. Weierstrass Institute for Applied Analysis and Stochastics, Leibniz Institute in Forschungsverbund Berlin e. V.
2. ERC Consolidator Grant
3. FOR2402 - Rough paths, stochastic partial differential equations and related topics (DFG)
4. IRTG 2544 - Stochastic Analysis in Interaction (DFG)
5. Deutsche Forschungsgemeinschaft (German Research Foundation)

2. Data Science and Rough Paths

2.1 Seq2Tens: An efficient representation of sequences by low-rank tensors, *Csaba Toth / Patric Bonnier, Oxford*

Sequential data such as time series, video, or text can be challenging to analyse as the ordered structure gives rise to complex dependencies. At the heart of this is non-commutativity, in the sense that reordering the elements of a sequence can completely change its meaning. We use a classical mathematical object – the tensor algebra – to capture such dependencies. To address the innate computational complexity of high degree tensors, we use compositions of low-rank tensor projections. This yields modular and scalable building blocks for neural networks that give state-of-the-art performance on standard benchmarks such as multivariate time series classification. Joint work with Harald Oberhauser.

2.2 Neural Controlled Differential Equations, *Patrick Kidger, Oxford*

Neural ordinary differential equations are an attractive option for modelling temporal dynamics. However, a fundamental issue is that the solution to an ordinary differential equation is determined by its initial condition, and there is no mechanism for adjusting the trajectory based on subsequent observations. Here, we demonstrate how this may be resolved through the well-understood mathematics of controlled differential equations. The resulting neural controlled differential equation model is directly applicable to the general setting of partially-observed irregularly-sampled multivariate time series, and (unlike previous work on this problem) it may utilise memory-efficient adjoint-based backpropagation even across observations. We demonstrate that our model achieves state-of-the-art performance against similar (ODE or RNN based) models in empirical studies on a range of datasets.

2.3 Acceleration of Descent-based Optimization Algorithms via Caratheodory's Theorem, *Francesco Cosentino, Oxford*

Given a discrete probability measure supported on N atoms and a set of n real-valued functions, there exists a probability measure that is supported on a subset of n+1 of the original N atoms and has the same mean when integrated against each of the n functions. We give a simple geometric characterization of barycenters via negative cones and derive a randomized algorithm that computes this new measure by "greedy geometric sampling". We then propose a new technique to accelerate algorithms based on Gradient Descent using Caratheodory's Theorem. As a core contribution, we then present an application of the acceleration technique to Block Coordinate Descent methods. Experimental comparisons on least squares regression with LASSO regularisation term, show improved performance than the ADAM and SAG algorithms.

2.4 Robustness of Residual Networks via Rough Paths techniques, *Nikolas Tapia, TUB/WIAS*

Residual Neural Networks, introduced by He et al. (2016) have had great success in building deeper neural network architectures while avoiding the problem of vanishing gradients. Inspired by a result of Haber and

Ruthotto (2017), we interpret such a network as a controlled recurrence equation, and derive an estimate for the change in the output layer given two different input layers, for a fixed set of weights. This is achieved by controlling the p-variation of the solution of such a recurrence equation by using a rough paths approach such as the Sewing Lemma and Davie's expansion. This talk is based on joint work in progress with C. Bayer and P. Friz.

3. Theoretical Applications of Signatures

3.1 Optimal stopping: a signature approach, *Sebastian Riedel, WIAS*

We study the optimal stopping problem

$$\sup_{\tau} \mathbb{E}(Y_\tau)$$

where $Y\colon [0,T] \to \mathbb{R}$ is a continuous-time stochastic process and the supremum is taken over all stopping times τ with values in $[0,T]$. Due to its importance in many applications, e.g. in mathematical finance, it is an extensively studied, but still challenging problem. We present a new approach that transforms the problem above into a deterministic optimization problem for which the solution can be calculated with arbitrary precision, assuming only the knowledge of the expected signature of the underlying process.
Joint work with C. Bayer, P. Hager, J. Schoenmakers.

3.2 Adapted topologies and higher rank signatures, *Chong Liu, Oxford*

The topology of weak convergence does not account for the growth of information over time contained in the filtration of an adapted stochastic process. For example, two adapted stochastic processes can have very similar laws but give completely different results in applications such as optimal stopping, queuing theory, or stochastic programming. To address this, Aldous introduced extended weak convergence, and subsequently, Hoover and Keisler showed that both weak convergence and extended weak convergence are just the first two topologies in a sequence of topologies that get increasingly finer. By using so-called higher rank expected signatures to describe laws of stochastic processes that evolve in spaces of laws of stochastic processes we derive metrics and embeddings that induce the Hoover–Keisler topology of any given rank. Joint work with Patric Bonnier and Harald Oberhauser.

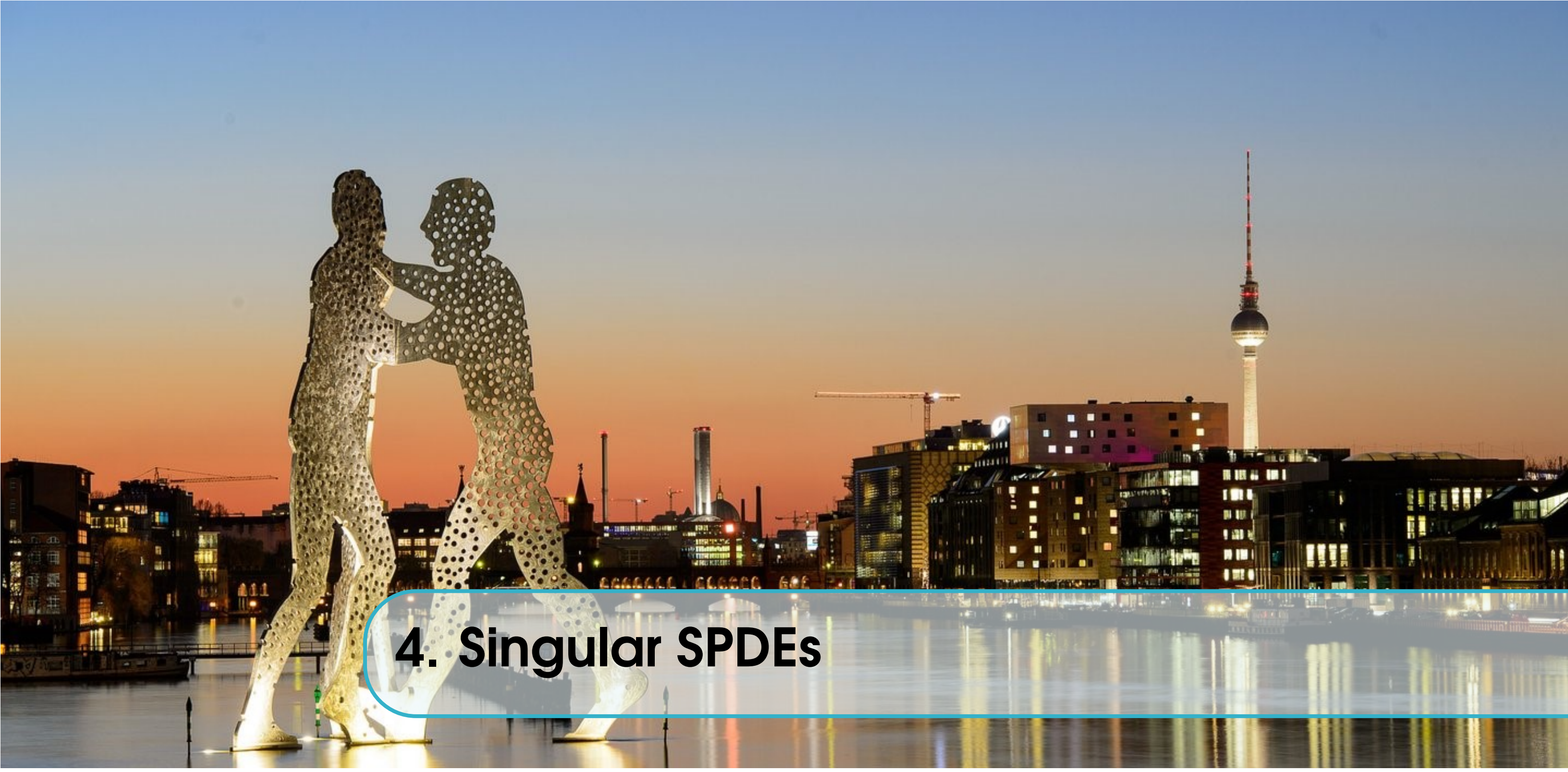
3.3 Unified cumulant signature and Magnus expansion, *Paul Hager, TU Berlin*

Signature cumulants are an efficient way of describing the law of a stochastic process. We present a recursive scheme for calculating the signature cumulants of semimartingale processes, consisting of taking expectations of integrals with respect to the bounded variation part and quadratic variation terms. The derivation is based on an Itô-rule applied to the exponential map in the non-commutative setting. In the one dimensional case we retrieve the recently discovered diamond expansion for the cumulant generating function. In the higher-dimensional and deterministic setting our recursion coincides with the Magnus expansion. This is a joint work with Peter Friz (WIAS, TU Berlin) and Nikolas Tapia (WIAS).

3.4 Rotation Invariants of Paths Through Iterated Integral Signatures, *Rosa Preiß, TU Berlin*

We are concerned with rotation (i.e. SO(d)) invariants of d-dimensional paths in terms of their iterated integral signature, a topic originally discussed in a paper by Joscha Diehl and Jeremy Reizenstein. The mathematical foundations date back much longer though, to the works of Dieudonné, Gardner and Weyl, among others.

I will start my talk with a discussion of the simplest and arguably most fundamental examples of path invariants, namely the signed area in dimension two and the d-dimensional signed volume in higher dimension, as well as the distance from starting point to end point. Then, I will continue with the introduction of the well-established theory of moving frames from differential geometry into our setting, and point out how it can be used to derive an algorithm for a fundamental (i.e. functionally independent) set of rotation invariants of a path.
This talk aims to advertise an ongoing project together with Joscha Diehl, Michael Ruddy and Nikolas Tapia.



4. Singular SPDEs

4.1 Large n limit of the $O(N)$ linear sigma model via stochastic quantization, *Rongchan Zhu, Beijing*

In this talk we consider large N limits of a coupled system of N interacting Φ^4 equations posed over $\mathbb{T}^d$ for $d = 1,2$, known as the $O(N)$ linear sigma model. Uniform in N bounds on the dynamics are established, allowing us to show convergence to a mean-field singular SPDE, also proved to be globally well-posed. Moreover, we show tightness of the invariant measures in the large N limit. For large enough mass, they converge to the (massive) Gaussian free field, the unique invariant measure of the mean-field dynamics, at a rate of order $1/\sqrt{N}$ with respect to the Wasserstein distance. We also consider fluctuations and obtain tightness results for certain $O(N)$ invariant observables, along with an exact description of the limiting correlations in $d = 1$. This talk is based on joint work with Hao Shen, Scott Smith and Xiangchan Zhu.

4.2 The Anderson Hamiltonian on a two-dimensional manifold, *Antoine Mouzard, Rennes*

We construct the continuous Anderson hamiltonian driven by a white noise on a compact two-dimensional manifold. We use the paracontrolled calculus to define a dense domain that depends on an enhanced noise built through a renormalisation step. It yields a self-adjoint operator with pure point spectrum and we have estimates for the eigenvalues. Different applications can be considered as the study of the nonlinear stochastic Schrödinger equation.

4.3 Synchronisation by noise for the stochastic quantisation equation in dimensions 2 and 3, *Pavlos Tsatsoulis, MPI Leipzig*

The stochastic quantisation equation (also known as stochastic Allen-Cahn equation) in dimensions two and three is given by

$$\begin{cases} (\partial_t - \Delta)u = -\left(u^3 - 3\infty u\right) + u + \xi \\ u|_{t=0} = f, \end{cases} \tag{SQE}$$

where ξ is space-time white noise and f is some initial condition of suitable regularity. Here, the term $-3\infty u$ is reminiscent of renormalisation, otherwise (SQE) is not well-posed in dimensions two and three due to the low regularity of space-time white noise. It is known that the deterministic analogon

$$\begin{cases} (\partial_t - \Delta)u = -u^3 + u \\ u|_{t=0} = f \end{cases}$$

of (SQE) has finitely many unstable solutions. In this talk I will discuss how the presence of noise implies uniform synchronisation, that is, any two trajectories approach each other with speed which is uniform in the initial condition. More precisely, I will explain how a combination of "coming down from infinity" estimates and order-preservation can be used to obtain uniform synchronisation with rates. This will be a special case

of a more general framework which implies quantified synchronisation by noise for white noise stochastic semi-flows taking values in Hölder spaces of negative exponent.
The talk is based on a joint work with Benjamin Gess.

4.4 Towards a low temperature expansion for Φ^4_3, *Trishen Gunaratnam, Bath / Imperial*

The ϕ^4_3 model undergoes phase transition as the inverse temperature/coupling constant is varied. This was famously established by Fröhlich, Simon, and Spencer in 1978, using soft arguments based heavily on reflection positivity. In upcoming joint work with Ajay Chandra and Hendrik Weber, we establish Peierls'-type contour bounds for low temperature ϕ^4_3 (analogous to those obtained by Glimm, Jaffe, and Spencer for ϕ^4_2 in 1975). This gives an alternative proof of the phase transition. However, our methods are more quantitative than those of Fröhlich, Simon, and Spencer: they allow us to initiate the study of phase segregation in ϕ^4_3.
At the heart of these contour bounds are estimates on the partition function for low temperature ϕ^4_3 that are uniform in the temperature. The difficulty in these bounds comes from handling ultraviolet divergences whilst simultaneously preserving the structure of the low temperature ϕ^4_3 potential. In this talk, we explain how this achieved by combining an expansion and coarse-graining procedure with the recent stochastic control approach to ultraviolet stability for ϕ^4_3 due to Barashkov and Gubinelli (2018). To finish, we comment on possible extensions of these bounds to a full low temperature expansion for ϕ^4_3.

4.5 1D geometric wave equation perturbed by fractional Brownian sheet, *Nimit Rana, Bielefeld*

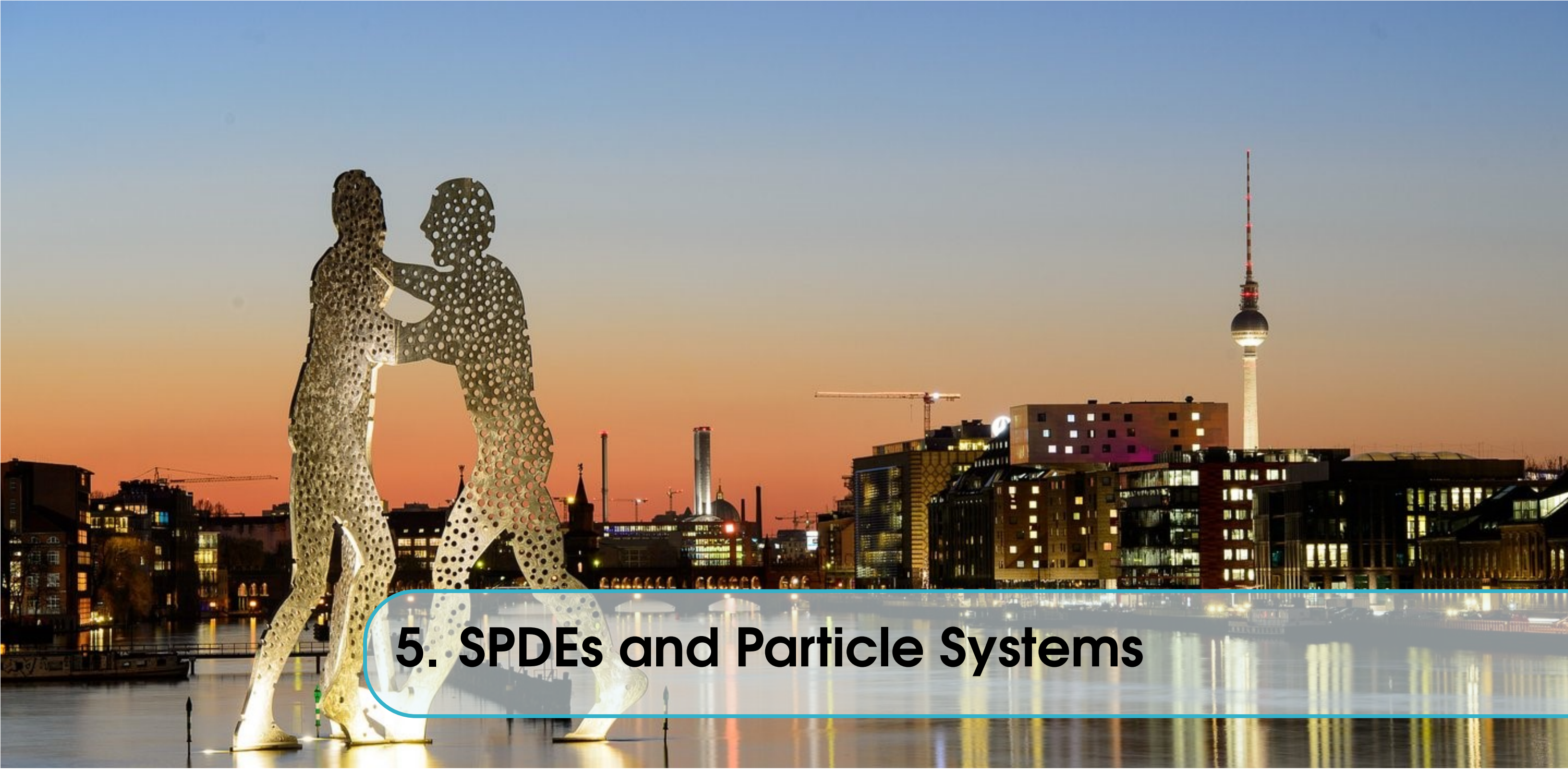
We aim to show the existence of a unique local solution to a stochastic geometric wave equation on the one dimensional Minkowski space $\mathbb{R}^{1+1}$ with values in an arbitrary compact Riemannian manifold. We consider a rough initial data in the sense that its regularity is lower than the energy critical.

4.6 A stochastic Taylor-like expansion for the generalised Parabolic Anderson Model, *Tom Klose, TU Berlin*

We discuss a *stochastic Taylor-like expansion* for the generalised Parabolic Anderson Model (gPAM). Despite the historical term "stochastic", our argument is purely deterministic and relies on the theory of regularity structures to expand gPAM in the noise intensity parameter. After a heuristic discussion, we rigorously set up the fixed-point argument in a suitable space of modelled distributions, differentiate its solution map, and characterise the terms in the expansion. Time permitting, we explain how to obtain estimates for the remainder term and apply our result to the study of precise Laplace asymptotics, for which it is one of the key steps. Joint work with Peter Friz (TU Berlin & WIAS).

4.7 Aging for stationary KPZ equation, *Tal Orenshtein, TU Berlin*

Aging is a property of non-equilibrium dynamics that can be expressed in terms of the asymptotic behavior of the two-times correlation function. In the talk we shall discuss aging for models in the stationary KPZ universality class, and present an explicit, universal, aging function. The Cole-Hopf solution to the stationary KPZ equation and the partition functions of the semi-discrete directed polymers in a Brownian environment in the intermediate disorder regime are two examples. As a comparison, models in the Edwards-Wilkinson class such as the linear stochastic heat equation - either in its discrete version or in the continuum - and the Ginzburg-Landau models, are all satisfying aging with an explicit, universal, aging function. The talk is based on a joint work with Gregorio Moreno-Flores (PUC Chile) and Jean-Dominique Deuschel (TU Berlin).



5. SPDEs and Particle Systems

5.1 The uniqueness problem for global stationary solutions to the semidiscrete stochastic heat equation, *Tobias Hurth, Neuchâtel*

Via the Feynman-Kac formula, the study of solutions to the stochastic heat equation naturally leads to a model for directed polymers in a random potential. In this talk, we investigate global and stationary in time solutions to the semidiscrete stochastic heat equation in spatial dimension 3 or higher and with small noise. A new factorization formula for the point-to-point partition function of the associated polymer model implies uniqueness for the global stationary solution up to a random time-dependent renormalization. The talk is based on a project with Kostya Khanin and Beatriz Navarro Lameda

5.2 Ergodicity via controllability, *Vahagn Nersesyan, Versailles*

We will discuss the problem of long-time behaviour for PDEs perturbed by a very degenerate bounded noise. We will start with an abstract sufficient condition for ergodicity formulated in terms of the controllability of the underlying deterministic equation. Then we will explain how this condition is checked in the cases of the Navier-Stokes and complex Ginzburg-Landau equations. This talk is based on joint works with S. Kuksin and A. Shirikyan

5.3 Mixing for Hamiltonian Monte Carlo in infinite dimensions, *Cecilia F. Mondaini, Drexel*

The analysis of convergence/mixing rates for testing sampling efficiency in Markov Chain Monte Carlo (MCMC) algorithms, a fundamental question for establishing their range of applicability, has gained increased attention recently. Of particular interest are MCMC methods that are designed to be well-defined in infinite dimensions, a property that allows them to overcome the curse of dimensionality when applied to corresponding finite-dimensional approximations. We analyze such question for an infinite-dimensional version of the Hamiltonian/Hybrid Monte Carlo algorithm, for which mixing rates had been an open problem until being very recently addressed via an exact coupling approach. Our proof uses the weak Harris theorem together with a generalized coupling argument, providing a flexible methodology to establish mixing rates for other MCMC algorithms. Furthermore, as an application of our general result, we show that all required assumptions can be verified in the context of a Bayesian inversion approach to advection-diffusion type PDEs. This is a joint work with Nathan Glatt-Holtz (Tulane U).

5.4 Large-scale regularity in stochastic homogenization, *Benjamin Fehrman, Oxford*

The first-order Liouville theorem states that the only sub-quadratic harmonic functions on the whole space are the linear functions. In this way the Laplacian characterizes the natural geometry of Euclidean space and identifies the natural coordinate functions. In stochastic homogenization we consider random coefficient fields that describe, for instance, the conductivity of a material with small-scale impurities. The behavior of these environments is almost surely understood using homogenization correctors, which define the natural coordinate functions in the random geometry of the space. In this talk, I will explain new results based on the role of correctors in homogenization, large-scale regularity, and first-order Liouville theorems.

5.5 Interpolation results for pathwise Hamilton-Jacobi equations, *Benjamin Seeger, Dauphine*

I will show how interpolation methods can be used to make sense of pathwise Hamilton-Jacobi equations for a wide range of Hamiltonians and driving paths. The various function spaces describe regularity (including Sobolev, Besov, Holder, and variation) as well as structure. I will also discuss some criteria for a function to be representable as a difference of convex functions, a class which plays an important role in the theory of pathwise Hamilton-Jacobi equations.

5.6 Pathwise regularisation of McKean-Vlasov Equations, *Avi Mayorcas, Oxford*

I will present a work in progress with F. Harang of Oslo University in which we demonstrate that by introducing a suitably rough perturbation in both the dynamics and law of a McKean-Vlasov process we recover the classical results of A. Sznitman when the interaction kernel is any Holder-Besov distribution, i.e it is allowed to be arbitrarily far from the classical Lipschitz requirement. In the same vein as L. Galeati & M. Gubinelli '20 and F. Harang & N. Perkowski '20 we obtain this result for a wide class of perturbations that are suitably regularising.

5.7 Fragility of the supercooled Stefan problem with noise, *Andreas Sojmark, Imperial*

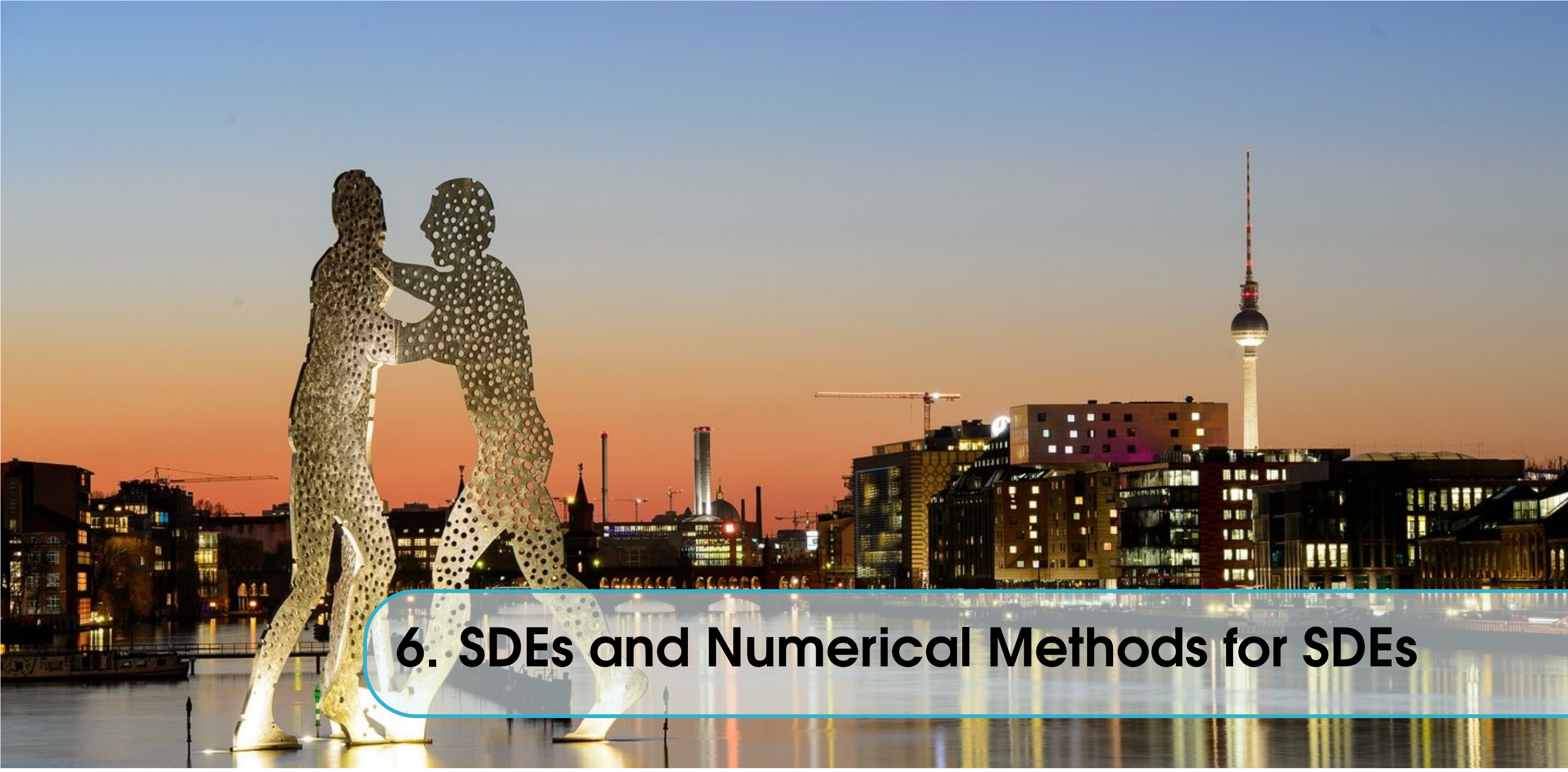
In this talk, we will consider a stochastic version of the supercooled Stefan problem with a Brownian transport term. In short, a supercooled liquid occupies the positive half-line with the freezing front eating its way into the liquid (starting from the origin). The latter yields a positive feedback effect which may create 'blow-ups': the speed of the front can diverge to infinity and degenerate into a point mass. In the deterministic problem, such blow-ups are entirely avoided if the initial profile of the liquid is not too concentrated near the origin. However, once a Brownian transport term is added, concentrations far away from the origin can also cause a blow-up, and thus blow-ups become an inherent part of the problem. We will show how the occurrence of blow-ups coincide with a natural notion of fragility for the system, and we will show how the system can become fragile precisely when the initial density reaches above a critical value. Finally, we will show that uniqueness of solutions is guaranteed whenever the initial density is everywhere below this critical value, whereas uniqueness remains an open problem in the face of fragility.

5.8 The stochastic FKPP equation with dormancy and duality to on/off branching coalescing Brownian motion, *Florian Nie, TU Berlin*

We introduce a new class of stochastic partial differential equations (SPDEs) with seed bank modeling the spread of a beneficial allele in a spatial population where individuals may switch between an active and a dormant state. Incorporating dormancy and the resulting seed bank leads to a two-type coupled system of equations with migration between both states. We first discuss existence and uniqueness of seed bank SPDEs and provide an equivalent delay representation that allows a clear interpretation of the age structure in the seed bank component. The delay representation will also be crucial in the proofs. Further, we show that the seed bank SPDEs give rise to an interesting class of "on/off" moment duals. In particular, in the special case of the F-KPP Equation with seed bank, the moment dual is given by an "on/off-branching Brownian motion". This system differs from a classical branching Brownian motion in the sense that independently for all individuals, motion and branching may be "switched off" for an exponential amount of time after which they get "switched on" again. On/off-branching Brownian motion shows qualitatively different behaviour to classical branching Brownian motion and is an interesting object for study in itself. Here, as an application of our duality, we show that the spread of a beneficial allele, which in the classical F-KPP Equation, started from a Heaviside intial condition, evolves as a pulled traveling wave with speed $\sqrt{2}$, is slowed down significantly in the corresponding seed bank F-KPP model. In fact, by computing bounds on the position of the rightmost particle in the dual on/off-branching Brownian motion, we obtain an upper bound for the speed of propagation of the beneficial allele given by $\sqrt{\sqrt{5}-1} \approx 1.111$ under unit switching rates. This shows that seed banks will indeed slow down fitness waves and preserve genetic variability, in line with intuitive reasoning from population genetics and ecology.

5.9 Well-posedness for Nonlinear Stochastic Transport PDEs Coming from Fluid Dynamics, *Oana Lang, Imperial*

In this talk we will present well-posedness properties for three classes of nonlinear SPDEs driven by transport noise: the stochastic 2D Euler equation, the stochastic great lake equation, and the stochastic rotating shallow water system. These models have been derived from fluid dynamics principles using stochastic variational approaches and they play an important role in the theoretical analysis of stochastic data assimilation systems.



6. SDEs and Numerical Methods for SDEs

6.1 High order numerical simulation of the underdamped Langevin diffusion, *James Foster, Oxford*

The underdamped Langevin diffusion (ULD) is an important model in statistical mechanics and has recently seen applications within computation statistics. In this talk, I will discuss approximations of ULD that employ ordinary differential equations. By using ODEs, we can capture the key terms in the Taylor expansion of ULD without explicitly requiring further derivatives of the potential. Moreover, this framework is practical as it enables one to harness state-of-the-art ODE solvers. To conclude the talk, I will give numerical evidence that our approach can achieve high orders of convergence

6.2 Numerical aspects for RODEs with unbounded drift, *Yue Wu, Oxford*

In this talk we discuss the implicit numerical schemes of the rough differential equation with an autonomous vector field which satisfies a one-sided Lipschitz condition. Following the perspective of dynamical system for numerical analysis, we propose backward Euler, implicit Milstein and implicit third-order Milstein methods in terms of semi-flows with suitable orders of convergence. We also consider the stochastic case, where the driving path is Gaussian. In this setting, motivated by Deya-Neuenkirch-Tindel's work, we propose the simplified version of implicit schemes and showed the corresponding convergence rates.

6.3 Large and moderate deviations for stochastic Volterra systems, *Alexandre Pannier, Imperial*

We provide a unified treatment of pathwise large and moderate deviations principles for a general class of multidimensional stochastic Volterra equations with singular kernels, not necessarily of convolution form. Our methodology is based on the weak convergence approach by Budhijara, Dupuis and Ellis. We show in particular how this framework encompasses most rough volatility models used in mathematical finance and generalises many recent results in the literature.

6.4 A monotone operator approach to SDEs with additive noise in the Young regime, *Florian Bechtold, Sorbonne*

A popular approach in studying nonlinear evolution problems (a prime example being given by the evolution problem associated with the p-Laplacian) is by the use of the theory of (maximally) monotone operators. Typically in this setup, a right-hand side is required to enjoy some L^p in time regularity, therefore excluding for example the consideration of white in time, colored in space noise.

Focusing on the finite dimensional setting (that is in studying ODEs instead of PDEs), we show how one can modify this approach in order to relax the regularity constraint on the right hand side to H^{-s} for $s \in (0,1/2)$. In particular, this relaxation can be interpreted as a pathwise approach to stochastic differential equations with an additive noise whose sample paths enjoy regularity H^s for $s \in (1/2,1)$.

6.5 Noiseless regularisation by noise, *Lucio Galeati, Bonn*

One of the main questions in regularisation by noise phenomena is to understand under which conditions the introduction of an additive perturbation w (usually sampled as a stochastic process) allows to restore well-posedness of a given ODE $\dot{x} = b(x)$. Davie first addressed the following questions: what are the analytical properties of the path w providing a regularizing effect? Which classes of stochastic processes satisfy them? Catellier and Gubinelli answered them by introducing the key concepts of averaging operators and nonlinear Young integrals; remarkably, this allows to provide a consistent solution theory even when b is merely distributional. In this talk I will present recent extensions of their work; similarly to rough path theory, the identification of such properties allows to split the problem into two distinct steps, a probabilistic and an analytic one. It also allows to provide regularisation results for generic functions (in the sense of prevalence), without the need of statistical assumptions on the perturbation w. Based on a joint work with Massimiliano Gubinelli.

6.6 Rare exit events near a repelling equilibrium, *Hong-Bin Chen, NYU*

Consider an ODE and a bounded domain in a Euclidean space. Assume, in that domain, the ODE has only one equilibrium and it is repelling. Under white noise perturbation, there is a positive probability that the dynamics emitting from the equilibrium point eventually exits the domain. We will discuss the exact asymptotics of rare exit events, e.g. exit locations and exit times, in the vanishing noise limit. It turns out that the decay rates of such rare events are polynomial powers of the noise magnitude, different from the exponential decay in the Freidlin-Wentzell large deviation theory. This talk is based on the joint work with Yuri Bakhtin

7. Rough Paths

7.1 Singular rough paths, *Carlo Bellingeri, TU Berlin*

We introduce the class of singular Hölder paths and singular controlled rough paths. These spaces arise naturally from the context of singular modelled distributions, one of the main technical notions in regularity structures. However, the simplified setting allow to describe most of their properties, showing some interesting connections with other branches of rough analysis. In particular, we will apply them to study the SLE trace and the rough-volatility regularity structure.

7.2 A rough path approach to the stochastic Landau-Lifshitz-Gilbert equation, *Emanuela Gussetti, Bielefeld*

The Landau-Lifshitz-Gilbert equation describes the behaviour of a ferromagnetic material on a bounded domain. I will present a result on existence and uniqueness of strong solutions to this problem posed on a one dimensional domain and perturbed by a linear multiplicative noise driven by a general rough path. A Wong-Zakai type convergence result will be also discussed. This is a joint work with M. Hofmanova and A. Hocquet.

7.3 Rough paths on Hilbert spaces, *Erland Grong, Bergen*

We consider weakly geometric rough paths in Hilbert spaces of Hölder index greater then 1/3. We consider the Cartnot-Carathéodory distance on the space where the rough paths take value and show that it is a geodesic distance. We use this result do investigate the relationship between weakly geometric rough paths and rough paths in this setting and give some applications.

7.4 Variational principles for fluid dynamics on rough paths, *James-Michael Leahy, Imperial*

We introduce constrained variational principles for fluid dynamics on rough paths. The advection of the fluid is constrained to be the sum of a vector field which represents coarse-scale motion and a rough (in time) vector field which parametrizes fine-scale motion. The rough vector field is regarded as fixed and the rough partial differential equation for the coarse-scale velocity is derived as a consequence of being a critical point of the action functional.

The action functional is perturbative in the sense that if the rough vector field is set to zero, then the corresponding variational principle agrees with the reduced (to the vector fields) Euler-Poincare variational principle introduced in Holm, Marsden, and Ratiu (1998). More precisely, the Lagrangian encodes the physics of the fluid and is a function of only the coarse-scale velocity.

By parametrizing the fine-scales of fluid motion with a rough vector field, we preserve the pathwise nature of deterministic fluid dynamics and establish a flexible framework for stochastic parametrization schemes. The main benefit afforded by our approach is that the system of rough partial differential equations we derive satisfy essential conservation laws, including Kelvin's circulation theorem. This talk is based on recent joint work with Dan Crisan, Darryl Holm, and Torstein Nilssen.

7.5 A Stroock-Varadhan rough martingale problem, *Antoine Hocquet, TU Berlin*

Using a compactness criterion inspired from that of Stroock-Varadhan '79, we show existence of a family of measures $P_{s,x}$ solving the martingale problem for a rough generator of the form $a^{ij}(t,x)D_{ij} + dX/dt b^i(t,x)D_i$ where X is a 2 step rough path. This is part of a joint work with Peter Friz and Khoa Lê.

7.6 Variational estimates for martingale transforms, *Pavel Zorin-Kranich, Bonn*

We show that a pair consisting of a càdlàg rough path ($2 < p < 3$) and a càdlàg martingale can be a.s. lifted to a joint rough path, with area terms given by Itô integrals. This result is based on a variation norm bound for discrete martingale transforms that also provides a simplified approach to the construction of Itô integrals in the càdlàg setting. Joint work with P. Friz.

8. Schramm–Loewner Evolution

8.1 Quasi-Sure Stochastic Analysis through Aggregation and SLE, *Vlad Margarint, NYU Shanghai*

In the first part of the talk, I will give an overview of the Quasi-Sure Stochastic Analysis through Aggregation as developed in the Mathematical Finance community. In the second part of the talk, I will present a link with SLE Theory. Specifically, I will show how using the Quasi-Sure Stochastic Analysis through Aggregation we can study some aspects of the SLE Theory quasi-surely.