A free boundary problem for the flow of viscous liquid bilayers

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This talk addresses modeling and simulation of thin-film flows for viscous bilayers [1]. A formal gradient structure for the corresponding free boundary problem will be presented and used to develop a numerical algorithm to solve the transient problem of the form

$$\begin{aligned} \partial_t u - \nabla \cdot (m(u)\nabla\pi) &= 0 \qquad \text{in } \omega(t) \\ \partial_t \mathbf{u} - \nabla \cdot (M(\mathbf{u})\nabla\boldsymbol{\pi}) &= \mathbf{0} \qquad \text{in } \Omega \setminus \omega(t) \end{aligned}$$

with $\mathbf{u} = (u, u_+)$ and $\boldsymbol{\pi} = (\pi, \pi_+)$ with $\pi = \delta E/\delta u$, $\pi_+ = \delta E/\delta u_+$ and degenerate mobilities m, M, where $\omega(t)$ is part of the unknowns. The proper treatment of the underlying contact line problem and coupling the PDE on ω with the one on its complement are the main mathematical challenges [2]. Numerical solutions will be used to compare with experiments [3], limitations and perspectives will be discussed.

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References

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