Flow in a porous visco-elasto-plastic solid.

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A model for porous media flow with hysteretic pressure-saturation relation involving thermodynamic effects and governed by the system

$$(1)\quad \rho_S u_{tt} = \operatorname{div}(B\nabla_s u_t + P[\nabla_s u]) + \nabla p - \beta\nabla\theta + g\,,$$

$$(2)\quad G[p]_t = \operatorname{div} u_t + \frac{1}{\rho_L}\operatorname{div}(\mu(p)\nabla p)\,,$$

$$(3)\quad c_0\theta_t = \operatorname{div}(\kappa(\theta)\nabla\theta) + \|D_P[\nabla_s u]_t\|_* + |D_G[p]_t| + B\nabla_s u_t : \nabla_s u_t + \frac{1}{\rho_L}\mu(p)|\nabla p|^2 - \beta\theta\operatorname{div} u_t\,,$$

has been derived and existence of global strong solutions in 3D for the isothermal case has been proved in [1]. Existence for the full system under suitable hypotheses is proved in [2]. The unknowns are u (displacement of the solid matrix), p (capillary pressure), and θ (absolute temperature). The system contains four hysteresis operators: The degenerate Preisach operator G describing pressure-saturation hysteresis, P describing elastoplastic hysteresis, and the associated dissipation operators D_P and D_G. The main challenge in the existence proof is related to the degeneracy of G which has been handled by means of a hysteretic version of Moser's iterations.

The permeability μ is assumed to depend only on the pressure. A more realistic case of saturation dependence has been considered [3, 4], but existence results have been obtained only if solid-liquid interaction is neglected and if additional time or space regularizing operators are involved.

References

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