Global-in-time well-posedness for a phase field system describing rate-dependent damage phenomena

M. Hassan Farshbaf-Shaker and Christian Heinemann

Weierstrass Institute, Berlin, Germany

In this talk we are going to investigate global-in-time well-posedness for a pde/inclusion system in 2D which models damage processes in viscoelastic media according to Kelvin-Voigt rheology. The system consists of the following relations

$$\mathbf{u}_{tt} - \operatorname{div}\big(\mathbb{C}(\chi)\varepsilon(\mathbf{u}) + \mathbb{D}(\chi)\varepsilon(\mathbf{u}_t)\big) = \boldsymbol{\ell} \qquad \text{in } \Omega\times(0,T), \tag{1a}$$

$$\chi_t - \Delta\chi_t - \Delta\chi + \partial I_{(-\infty,0]}(\chi_t) + \frac{1}{2}\mathbb{C}'(\chi)\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) + f'(\chi) \ni 0 \qquad \text{in } \Omega\times(0,T) \tag{1b}$$

supplemented with initial conditions for $\mathbf{u}$, $\mathbf{u}_t$ and χ, homogeneous Neumann boundary conditions for χ and χ_t and the nonhomogeneous boundary condition

$$\big(\mathbb{C}(\chi)\varepsilon(\mathbf{u}) + \mathbb{D}(\chi)\varepsilon(\mathbf{u}_t)\big)\cdot\nu = \mathbf{b} \qquad \text{on } \partial\Omega\times(0,T), \tag{2}$$

where $\boldsymbol{\ell}$ and $\mathbf{b}$ model external volume and boundary forces. Complete degeneration is prevented by assuming that the viscosity tensor $\mathbb{D}(\cdot)$ is uniformly bounded from below by a positive constant.

In order to establish existence and uniqueness of strong solutions we present a special time-discretization scheme and enhanced a priori estimates from [**?**] designed for boundary conditions of type (**??**) and for differential inclusions as in (**??**) forcing the uni-directionality $\chi_t \le 0$.

References

[1] M.H. Farshbaf-Shaler and C. Heinemann, *A phase field approach for optimal boundary control of damage processes in two-dimensional viscoelastic media,* Math. Models Methods Appl. Sci., **25** (2015), 2749–2793.