Recent results on some diffuse-interface models for incompressible binary fluids with nonlocal interaction

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Abstract:

In the talk we shall present the last results on some diffuse-interface models for flow and phase separation of binary fluids which are based on the coupling of the Navier-Stokes equations with the nonlocal Cahn-Hilliard equation. The nonlocal Cahn-Hilliard/Navier-Stokes (CHNS) system has been studied analytically in a series of papers (cf. [4, 7, 8, 9, 10, 6, 11]) We shall first review a recent result on existence of dissipative global weak solutions for a nonlocal CHNS type system which describes the situation where the two fluids have different densities (cf. [5]). This system represents the nonlocal version of a model derived by H. Abels, H. Garcke and G. Grün in [3] and studied analytically by H. Abels, D. Depner and H. Garcke in [1, 2]. We shall mention the main difficulties connected with this model as far as, in particular, regularity and uniqueness are concerned. This will lead us to consider this system with singular double-well potential and degenerate mobility in 2D. For the nonlocal CHNS system with degenerate mobility, double-well singular potential and matched densities a result concerning existence of strong solutions in 2D will then be presented. This will concern, in particular, the physically relevant and mathematically challenging situation where the viscosity depends on the order parameter. Finally, with the regularity result at hand, we shall study an associated optimal distributed control problem and derive first order necessary optimality conditions. These last results are contained in a work in progress with M. Grasselli and J. Sprekels.

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