Damage problems in bulk domains and interfaces

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The mathematical investigation of Contact Mechanics has been extensively developed over the last decades and it includes *adhesive contact* and *delamination* models. Their analysis is relevant for several mechanical and engineering problems, ranging from fractures in brittle materials, to the investigation of earthquakes, to the study of layered composite structures in machine designing and manufacturing. Indeed, the interface regions between the various laminates are fundamental for the strength and stability of the structural elements, and the degradation of the adhesive substance on such regions may lead to material failure. It turns out that the damage theory can be successfully used for describing adhesive contact between solids, in terms of a suitable internal variable accounting for the state of the adhesion. This approach is in fact mainly due to M. Frémond (see [1]). It is closely related to the theory of phase change problems in nonsmooth thermomechanics [6]. These problems have recently attracted remarkable attention, and been widely investigated, both in the case of *rate-independent* evolution for the adhesion parameter and, in the case of *rate-dependent*, or *viscous*, evolution, [2]–[4].

We discuss and present some recent results on a model for adhesive contact with friction between a thermo-viscoelastic body and a rigid support. A PDE system, consisting of the evolution equations for the temperatures in the bulk domain and on the contact surface, of the momentum balance, and of the equation for the internal variable describing the state of the adhesion, is recovered. Unilateral boundary conditions are rendered by a generalization of the Signorini law to the case of adhesive contact. In addition dissipative frictional effects are accounted for due to the presence of non-smooth boundary operators. The existence of global-in-time weak solutions to the associated initial-boundary value problem is proved , mainly by passing to the limit in a carefully tailored time-discretization scheme. Finally, we discuss an asymptotic dimensional reduction analysis for a damage model written in thin domains which formally justify the limit equations we have used for surface damage in contact problems [5].

References

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