Convergence of the thresholding scheme for multi-phase mean curvature flow

Felix Otto

Max Planck Institute for Mathematics in the Sciences

We consider the thresholding scheme, a time discretization for mean curvature flow introduced by Bence-Merriman-Osher; and prove a convergence result in the multi-phase case. The result establishes convergence towards a weak formulation in the framework of sets of finite perimeter. The proof is based on the interpretation of the thresholding scheme as a minimizing movement scheme, which means that the thresholding scheme preserves the structure of (multi-phase) mean curvature flow as a gradient flow w. r. t. the total interfacial energy. More precisely, the thresholding scheme is a minimizing movement scheme for an energy functional that Γ-converges to the total interfacial energy. Our proof is similar in spirit to the convergence results of Almgren-Taylor-Wang and Luckhaus-Sturzenhecker of another minimizing movement scheme. In particular, ours is a conditional convergence result, in the sense that we assume that the energy of the approximation converges to the energy of the limit. This is joint work with Tim Laux.