Energy-reaction-diffusion systems

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We derive thermodynamically consistent models of reaction-diffusion equations coupled to a heat equation. While the total energy is conserved, the total entropy serves as a driving functional such that the full coupled system is a gradient flow. The novelty of the approach is the Onsager structure, which is the dual form of a gradient system, and the formulation in terms of the densities and the internal energy. In these variables it is possible to assume that the entropy density is strictly concave such that there is a unique maximizer (thermodynamical equilibrium) given linear constraints on the total energy and suitable density constraints.

We consider two particular systems of this type, namely, a diffusion-reaction bipolar energy transport system, and a drift-diffusion-reaction energy transport system with confining potential. We prove corresponding entropy-entropy production inequalities with explicitely calculable constants and establish the convergence to thermodynamical equilibrium, at first in entropy and further in L^1 using Cziszár-Kullback-Pinsker type inequalities.