Cahn-Hilliard-Navier-Stokes systems with moving contact lines

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The motion of an isothermal mixture of two immiscible and incompressible fluids subject to phase separation can be described by the well-known model H (see [4] and references therein). This is a diffuse interface model which consists of the Navier-Stokes equations for the average velocity $\mathbf{u}$ which are subject to a force depending on the difference ϕ of the relative concentrations of the two fluids. The evolution of ϕ is governed by a convective Cahn-Hilliard equation. The system is also known as Cahn-Hilliard-Navier-Stokes (CHNS) system. In a simplified setting (matched densities) the CHNS system reads

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \operatorname{div}\left(2\nu D\left(\mathbf{u}\right)\right) + \nabla p = \epsilon\mu\nabla\phi + \mathbf{h}(t), \tag{1}$$

$$\operatorname{div}\mathbf{u} = 0, \tag{2}$$

$$\partial_t\phi + \mathbf{u}\cdot\nabla\phi - \Delta\mu = 0, \quad \mu = -\epsilon\Delta\phi + \epsilon^{-1}F'\left(\phi\right), \tag{3}$$

in $\Omega\times(0,+\infty)$. Here Ω is a bounded domain in $\mathbb{R}^{\mathrm{N}}$, $\mathrm{N}=2,3$, $\nu>0$ is the kinematic viscosity of the mixture, $D\left(\mathbf{u}\right)$ is the linear deformation tensor, $\mathbf{h}$ is a given external body force, $\epsilon>0$ is related to the thickness of the (diffuse) interface separating the two fluids and F is a suitable double well potential (e.g., $F(s)=\frac{1}{4}(s^2-1)^2$, $s\in\mathbb{R}$).

In the existing literature there are many theoretical results on system (1)-(3) equipped with no-slip (or periodic) boundary conditions for $\mathbf{u}$ and no-flux (or periodic) boundary conditions for ϕ and μ. It was observed (see, e.g., [1]) that the moving contact line, defined as the intersection of the fluid-fluid interface with the solid wall, is incompatible with the no-slip boundary condition (cf. [2] and its references). More precisely, there is a velocity discontinuity at the moving contact line, and the tangential force exerted by the fluids on the solid surface in the vicinity of the contact line becomes infinite. Thus, in immiscible two-phase flows, none of the mentioned boundary conditions can account for the moving contact line slip velocity profiles obtained from simulations. Therefore, new boundary conditions are required to describe the observed phenomena. An example of such conditions are the so-called generalized Navier boundary conditions (GNBC) (see [5] for a variational derivation).

We intend to discuss the existence of a global weak solution to system (1)-(3) endowed with GNBC accounting for some boundary diffusion for ϕ (see [3]). If time permits, we shall also show that any weak energy solution converges to a single equilibrium.

References

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