A topological obstruction related to nematic shells: Morse's index formula for VMO vector fields

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In this talk, we consider the following problem. Given a compact manifold $N \subseteq \mathbb{R}^d$ with boundary, which are the maps $\mathbf{g} \colon \partial N \to \mathbb{S}^{d-1}$ that can be extended to a unit vector field $\mathbf{u}$, defined on N, belonging to — say — the Sobolev space $W^{1,p}$? This question is motivated by the analysis of variational models for a thin film of nematic liquid crystals spread on a surface. In the continuous setting, the answer depends on a topological obstruction, namely, Morse's index formula. Inspired by Brezis and Nirenberg's work on the topological degree [1, 2], we extend Morse's formula to the class of VMO (Vanishing Mean Oscillation) functions. This yields a characterisation of the boundary data $\mathbf{g}$ that admit an extension with the required properties.

References

[1] H. Brezis and L. Nirenberg, Degree theory and BMO. I. Compact manifolds without boundaries, *Selecta Math. (N.S.) 1* **2** (1995), 197–263.

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