SPONTANEOUS SYMMETRY-BREAKING, OSCILLATION DEATH AND CHIMERA STATES IN DYNAMICAL NETWORKS

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Symmetry-breaking in dynamical networks

- Motivation and introduction to synchronization in networks
- Oscillation death
- Chimera states as coherence-incoherence transitions
- Chimera death: bridging of chimeras and oscillation death

Chimera of Arezzo
400 BC, Nat. Archeol. Museum Florence
My personal history

- 1980s: hysteresis in semiconductor transport

\[
\frac{dn}{dt} = g(n) - r(n)
\]

Springer Series in Synergetics
Vol. 35 (1987)

Russian translation
(1991)
My personal history

- **1990s:** multistability in semiconductor heterostructures

- current density: \( J_{m\rightarrow m+1}(F_m, n_m, n_{m+1}) \)
- continuity eq.: \( \dot{n}_m = \frac{1}{\epsilon} (J_{m-1\rightarrow m} - J_{m\rightarrow m+1}) \)
- Poisson eq.: \( \epsilon_r \epsilon_0 (F_m - F_{m-1}) = e(n_m - N_D) \)
- voltage (global constraint): \( U = \sum_{m=0}^{N} F_m d \)

- \( N_D \): doping density, \( U \): applied voltage, \( e < 0 \)

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Prengel, Wacker, Schöll PRB 50, 1704 (1994)
My personal history

- **2000s**: slow-fast neural systems: FitzHugh-Nagumo

With activator $u$, inhibitor $v$: $\mathbf{x} = (u, v)^T$

$$F(x) = \left( \frac{1}{\epsilon} \left( u - \frac{u^3}{3} - v \right), u + a \right)$$

- opto-electronic oscillator:

Rosin, Callan, Gauthier, Schöll: EPL 96, 34001 (2011)
My personal history

- **2010s**: networks: multistability of synchronization
  - in-phase synchronization
  - group/cluster synchronization
  - partial synchronization, chimera states, oscillation death
Examples of complex networks

- Brain
- Power grid
- Internet
- Friendships
Complex networks

US power grid
Synchronization in complex networks

- Synchronization and Desynchronization
  - Constructive role for strongly coherent fields:
    - Laser system, ...
      - Synchronization
  - On occasion, undesirable phenomenon:
    - Parkinsonian tremor
    - Swaying motion of London’s Millennium Bridge
      - Desynchronization

Symmetry-breaking in neuronal systems

- Unihemispheric sleep: some birds and dolphins sleep with one half of their brain, while the other half remains awake.

Unihemispheric sleep of bottlenose dolphin (EEG)
B: right,
C: left hemisphere asleep

N.C. Rattenborg et al. / Neuroscience and Biobehavioral Reviews 24 (2000) 817–842
Two different examples of spontaneous symmetry-breaking in dynamical networks:

- Oscillation death
- Chimera states
- Bridging between these two: chimera death
Two different examples of spontaneous symmetry-breaking in dynamical networks:

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Oscillation death: Suppression of oscillations in coupled oscillators by symmetry-breaking steady state

- Example of coupling which breaks the $S^1$ symmetry

\[
\begin{align*}
\dot{z}_1 &= (\lambda + i\omega - |z_1|^2) z_1 + \varepsilon (x_2 - x_1) \\
\dot{z}_2 &= (\lambda + i\omega - |z_2|^2) z_2 + \varepsilon (x_1 - x_2)
\end{align*}
\]

Stuart-Landau model

newly created stable inhomogeneous steady state due to coupling of oscillators:

- Morphogenesis: cellular differentiation

Oscillation death: Symmetry-breaking steady st.

- Analytic result: Bifurcation diagram

\[ z_+ = \frac{1}{2} (z_1 + z_2), \quad z_- = \frac{1}{2} (z_1 - z_2) \]

Secondary oscillation death

Oscillation death: Symmetry-breaking steady st.

- Delayed coupling can control the threshold for oscillation death

\[
\dot{z}_1 = (\lambda + i \omega - |z_1(t)|^2) \, z_1(t) + \varepsilon \left( x_2(t - \tau) - x_1(t) \right) \\
\dot{z}_2 = (\lambda + i \omega - |z_2(t)|^2) \, z_2(t) + \varepsilon \left( x_1(t - \tau) - x_2(t) \right)
\]

Stuart-Landau model

\[\lambda = 1\]

\[\lambda = 3.5\]

Two different examples of spontaneous symmetry-breaking in dynamical networks:

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Chimera states in networks of identical oscillators with nonlocal coupling

- Spatially coexisting domains of coherent/phase-locked and incoherent/desynchronized oscillators
- Chimera in **Greek mythology**: fire-breathing monster with three heads: lion’s head, goat’s head, serpent’s head
- Prototype behavior of system on the transition from complete coherence to complete incoherence
- Essential: nonlocal coupling of range \( r \) between local and global coupling
Chimera states in networks of identical phase oscillators

- **Theory:** Kuramoto and Battogtokh 2002
  Abrams and Strogatz 2004
Chimera states in networks of identical oscillators

- **Theory:** Kuramoto and Battogtokh 2002
  Abrams and Strogatz 2004

Experimentally verified only recently (2012 - 2014):

- **Optical experiment:** Spatial light modulator
  Hagerstrom, Murphy, Roy, Hövel, Omelchenko, Schöll, Nature Phys. 8, 658 (2012)

- **Chemical experiment:** Light-sensitive BZ reaction

- **Mechanical experiment:** coupled pendula
  Martens, Thutupalli, Fourriere, Hallatschek, PNAS 110, 10563 (2013)

- **Electronic experiment:** frequency-modulated delay oscillator
  Larger, Penkovsky, Maistrenko, PRL 111, 054103 (2013)

- **Electrochemical experiment:** electro-oxidation of Si
  Schmidt, Schönleber, Krischer, Garcia-Morales, Chaos 24, 013102 (2014)
Networks with nonlocal coupling

Local coupling

Global coupling

Nonlocal (intermediate) coupling

Coupling radius
\[ r = \frac{P}{N} \]

\( P \) – number of coupled nearest neighbors
\( N \) – total number of elements in network
Dynamics of networks with nonlocal coupling of range $r$

\[ \dot{X} = F(X) + \frac{\sigma}{2P} (G \otimes H)X \]

$X = (X_1, \ldots, X_N)$ – state vector
$F$ – dynamics of individual element
$H$ – local interaction matrix
$G$ – coupling matrix (network topology)

Here $G$ – circulant matrix with rows
\[ (-2P, 1, \ldots, 1, 0, \ldots, 0, 1, \ldots, 1) \]
\[ g_{ii} = -2P \]

$\sigma$ – coupling strength
$P$ – number of coupled neighbors (in each direction)
$N$ – total number of elements

Coupling radius
\[ r = P/N \]
Time-discrete maps (logistic map) with step-like coupling function

\[ z_i^{t+1} = f(z_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} \left[ f(z_j^t) - f(z_i^t) \right] \]

- \( z_i \) - state variables, \( i = 1, \ldots, N \)
- \( P \) - number of coupled nearest neighbors (in each direction)
- \( \sigma \) - coupling strength
- \( N \) - number of elements
- \( t \) - discrete time

**Periodic boundary conditions:**
\[ z_{N+1} = z_1 \]

**Local dynamics:**
\[ f(z) = az(1 - z), \quad a = 3.8 - \text{chaotic} \]

Spatially coherent states

Snapshots:

- $k=0$
- $k=1$
- $k=2$
- $k=3$

Wavevector $k$

$z_i^t (i = 1, ..., N)$ - coherent on the ring $S^1$ as $N \to \infty$ if for any point $x \in S^1$

$$\lim_{N \to \infty} \lim_{t \to \infty} \sup_{i,j \in U^N_\delta(x)} |z_i^t - z_j^t| \to 0, \text{ for } \delta \to 0,$$

$\longrightarrow$ scan $(\sigma, r)$-plane
Bifurcation diagram

coherence-incoherence tongues:

CIB = Coherence-Incoherence Bifurcation

Bifurcation parameters:

\[ r = \frac{P}{N} \] - coupling radius,
\[ \sigma \] - coupling strength
Coherence-incoherence transition ($r=0.32$)

Analytical results: critical coupling strength

Continuum limit (large N), period-2 dynamics:

\[ z_{1-j}(x) = (1 - \sigma)f(z_j(x)) + \frac{\sigma}{2r} \int_{x-r}^{x+r} f(z_j(y))dy. \]

Transition from coherence to incoherence: Profile becomes discontinuous (infinite slope) at some point \( x \rightarrow \) neglect coupling term

Multiplying the eqs for even and odd time steps:

Assume: \( z_0(x)=z_1(x)=z^* \) at turning points \( x=x_1 \) with fixed point of map \( z^*=f(z^*) \)

\[ \sigma_c = 1 - \frac{1}{|f'(z^*)|} \]

Universal result for critical coupling strength \( \sigma_c \)
**Application of analytical results:**

**Logistic map**

Logistic map \( f(z) = az(1-z) \), \( f'(z) = a(1-2z) \)

Analytical approximation: \( z_0(x) = z_1(x) = z^* \)
with fixed point of map \( z^* = f(z^*) = 1 - \frac{1}{a} \)

Critical coupling strength

\[
\sigma \approx 1 - \frac{1}{a - 2}
\]


**Optical experiment: spatial light modulator**

Cosine map \( f(z) = \pi a (1 - \cos z) \), \( f'(z) = \pi a \sin z \)

Analytical approximation: \( z_0(x) = z_1(x) = \phi^* \)
with fixed point of map \( \phi^* = f(\phi^*) = \phi^* \)

Critical coupling strength:

\[
\epsilon_c = 1 - \frac{1}{(\pi a)|\sin \phi^*|}
\]

Good agreement with experiment

Experimental realization

Liquid crystal spatial light modulator

Simulation

\[ z_i^{t+1} = az_i^t (1 - z_i^t) \]

Experiment

\[ z_i^{t+1} = \pi a (1 - \cos z_i^t) \]

A. M. Hagerstrom, T. E. Murphy, R. Roy, P. Hövel, I. Omelchenko, and E. Schöll:

Comparison with time-continuous systems

Logistic map

\[ z_{i+1} = az_i (1 - z_i) \]

\( a = 3.2 \) (periodic), \( r = 0.1 \)

Rössler model

\[ \dot{x}_i = -y_i - z_i \]
\[ \dot{y}_i = x_i + ay_i \]
\[ \dot{z}_i = b + z_i(x_i - c) \]

\( a = 0.42, b = 2, c = 4, r = 0.3 \)

Structure of coherence–incoherence tongues

Two different examples of spontaneous symmetry-breaking in dynamical networks:

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Novel phenomenon: Chimera death

- Bridging between chimera states and oscillation death

Coexisting domains of (i) spatially coherent oscillation death
(ii) spatially incoherent oscillation death

\[ \dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} (\text{Re}z_k - \text{Re}z_j) \]

Stuart-Landau model coupling breaks $S^1$ symmetry

- Amplitude chimeras
  P=4

- In-phase synchronization
  P=5

- Chimera death
  P=33

A. Zakharova, M. Kapeller, E. Schöll: PRL 112, 154101 (2014)
Amplitude chimeras

- Spatially incoherent amplitudes, phases are correlated

\[ P = 4, \sigma = 14 \]

Snapshots at \( t = 1000 \)

\[ \text{center of mass } y_{CoM} = \int_0^T y_i(t) dt / T \]

Distance between the center of mass of each oscillator and the origin

- Incoherent domains:
  displacement of center of mass of oscillations

A. Zakharova, M. Kapeller, E. Schöll: PRL 112, 154101 (2014)
Direct transition from amplitude chimera to chimera death

$\sigma = 26$

P=4: amplitude chimera

P=5: chimera death

- Multicluster-chimera

A. Zakharova, M. Kapeller, E. Schöll: PRL 112, 154101 (2014)
Dynamic regimes

- AC: amplitude chimera (oscillations)
- N-CD: N-cluster chimera death (inhomog. steady state)

Amplitude dynamics - inhomogeneous steady states
Partially coherent spatio-temporal patterns

A. Zakharova, M. Kapeller, E. Schöll: PRL, in print (2014), arXiv1402.0348
Conclusions

Symmetry breaking in nonlocally coupled networks

- Oscillation death: quenching of oscillations by coupling
  -> inhomogeneous steady state

- Chimera states: splitting in spatially coherent + incoherent domain
  Universal mechanism for coherence-incoherence transition via
  chimera states: critical coupling strength
  logistic map, cosine map, Rössler oscillator, optical experiment

- Chimera death: coherence-incoherence
  patterns of inhomogeneous steady states:
  Bridging between chimera states and oscillation death

A. Zakharova, M. Kapeller, E. Schöll: PRL 112, 154101 (2014)
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