HomTAp 2017

Homogenization Theory and Applications

Weierstrass Institute for Applied Analysis and Stochastics October 4–6, 2017

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Cover figure: Homogenization of an elastic material with gaseous inclusions (A. Caiazzo)

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HomTAp 2017

Homogenization Theory and Applications

October 4 - 6, 2017

Weierstrass Institute (WIAS) Berlin, Germany

Organizers

Alfonso Caiazzo Martin Heida Sina Reichelt Ben Schweizer

Support

CRC 910: Control of self-organizing nonlinear systems: Theoretical methods and concepts of application CRC 1114: Scaling Cascades in Complex Systems Weierstrass Institute, Berlin (WIAS)



Welcome to the workshop "HomTAp 2017 — Homogenization Theory and Applications" at the Weierstrass Institute (WIAS) Berlin.

The workshop is devoted to homogenization, which is a very powerful tool for the mathematical modeling of heterogeneous materials with applications in biological tissues, composite materials, optics and acoustics (wave equation), porous media, interfaces and transmission problems as well as damage and brittle materials.

The aim of the workshop is to bring together researchers from analysis, numerics and scientific computing and to give them the opportunity to exchange experience in the field of homogenization. In particular, the conference will focus on

- periodic homogenization,
- stochastic homogenization,
- numerical methods for multiscale problems.

Invited lectures, contributed talks, and posters will cover a variety of mathematical approaches towards homogenization such as periodic and stochastic unfolding, Γ -convergence, asymptotic expansion and numerical algorithm for multiscale problems. The invited and contributed talks are scheduled for 45 minutes and 20 minutes, respectively, including discussion. During the quick poster representation each poster is introduced with 1-2 minutes.

Wednesday, October 4th 2017, Afternoon 12:00 - 19:00

12:00 - 13:00	Registration
13:00 - 13:30	Opening
13:30 - 15:00	Plenary (room ESH)
13:30	The homogenization commutator and the pathwise structure of fluctuations
	Antoine Gloria
14:15	Perturbation problems in homogenization of Hamilton-Jacobi equations
	Panagiotis Souganidis
15:00 - 15:30	Coffee Break
15:30 - 17:00	Plenary (room ESH)
15:30	Macroscopic properties of a simple stochastic process with aging: Recursive
	solution
	Thomas Niedermayer
16:15	Effective Maxwell's equations in a geometry with flat split-rings and wires
	Agnes Lamacz
17:00 - 17:30	Quick poster introduction (room ESH)
17:30 - 19:00	Poster Session (room 405/406)

Thursday, October 5th, Morning 9:00 - 12:40

9:00 - 10:30	Plenary (room ESH)
09:00	Homogenization of quasilinear elliptic problems with nonlinear Robin conditions
	and L^1 data
	Patrizia Donato
09:45	Mathematical modeling and multiscale analysis of transport processes through
	membranes
	Maria Neuss-Radu
10:30 - 11:00	Coffee Break
11:00 - 12:40	Parallel Session 1 (room ESH)
11:00	Imperfect transmission problems: Homogenization with weakly converging data
	Sara Monsurrò
11:20	Asymptotic behaviour of a class of controls for imperfect transmission problems
	Carmen Perugia
11:40	Homogenization of the discrete diffusive coagulation-fragmentation equations in
	perforated domains
	Silvia Lorenzani
12:00	Thermoelastic behavior of a composite material with heat jumps and continuous
	deformation across the interface
	Kim Hang Le Nguyen
12:20	Homogenization of reaction-diffusion processes with a nonlinear dynamical
	Wentzell-boundary condition at the interface
	Markus Gahn
11:00 - 12:40	Parallel Session 2 (room 405/406)
11:00	Quantitative homogenization in nonlinear elasticity for small loads
	Stefan Neukamm
11:20	An analysis of variance reduction methods in stochastic homogenization
	Julian Fischer
11:40	Quantitative stochastic homogenization: Large-scale behaviour of a-harmonic
	functions
	Arianna Giunti
12:00	Stochastic discrete approximations of the Mumford-Shah functional
	Matthias Ruf
12:20	A Liouville theorem for stationary and ergodic ensembles of parabolic systems
	Benjamin Fehrmann
12:40 - 14:20	Group Photo & Lunch

Thursday, October 5th, Afternoon 14:20 - 17:45

14:20 - 15:40	Parallel Session 3 (room ESH)
14:20	A numerical primer in 2D stochastic homogenization
	Boris Khoromskij
14:40	Homogenization of periodic hyperbolic systems: Operator error estimates
	Yulia Meshkova
15:00	Stability and energy estimates for periodic frame structures made of thin beams
	Olena Sivak
15:20	Homogenization and dimension reduction for a textile shell
	Stephan Wackerle
14:20 - 15:40	Parallel Session 4 (room 405/406)
14:20	Asymptotic modelling of the wave-propagation over acoustic liners
	Adrien Semin
14:40	A fully homogenized model of Kondaurov's non-equilibrium two-phase flow in
	double porosity media with thin fissures
	Anton Voloshin
15:00	Homogenization in the Hencky plasticity setting
	Martin Jesenko
15:20	Asymptotic rigidity for layered materials and its applications
	Fabian Christowiak
15:40 - 16:15	Coffee Break
16:15 - 17:45	Plenary (room ESH)
16:15	Numerical homogenization by localized orthogonal decomposition and connec-
	tions to the mathematical theory of homogenization
	Daniel Peterseim
17:00	Towards numerical homogenization of multiscale fault networks
	Ralf Kornhuber
18:45	Conference Dinner (Deponie No. 3, Georgenstraße 5)

Friday, October 6th 2017, Morning 09:00 - 12:40

9:00 - 10:30	Plenary (room ESH)
09:00	Homogenization in the presence of defects
	Claude Le Bris
09:45	Homogenization of a thin elastic plate
	Georges Griso
10:30 - 11:00	Coffee Break
11:00 - 12:40	Session 5 (room ESH)
11:00	A software tool for multi-scale model derivation with an application to a micro
	mirror array
	Michel Lenczner
11:20	A two-scale Stefan problem arising in a model for tree-sap exudation
	Malte Peter
11:40	Spectrum of Neumann problems on thin waveguides with strongly corrugated
	boundary
	Guiseppe Cardone
12:00	Homogenization of a semilinear variational inequality in a thick fractal junction
	Taras Meln'yk
12:20	Strong advection and initial layer analysis in passive transport
	Harsha Hutridurga
12:40 - 12:50	Closing

Spectrum of Neumann problems on thin waveguides with strongly corrugated boundary

Guiseppe Cardone

University of Sannio (Italy)

We consider a family $\{\Omega^{\varepsilon}\}_{\varepsilon>0}$ of periodic domains in \mathbb{R}^2 with waveguide geometry and analyse spectral properties of the Neumann Laplacian $-\Delta_{\Omega^{\varepsilon}}$ on Ω^{ε} . The waveguide Ω^{ε} is a union of a thin straight strip of the width ε and a family of small protuberances with the so-called "room-and-passage" geometry. The protuberances are attached periodically, with a period ε , along the strip upper boundary. For $\varepsilon \to 0$ we prove a (kind of) resolvent convergence of $-\Delta_{\Omega^{\varepsilon}}$ to a certain ordinary differential operator. Also we demonstrate Hausdorff convergence of the spectrum. In particular, we conclude that if the sizes of "passages" are appropriately scaled the first spectral gap of $-\Delta_{\Omega^{\varepsilon}}$ is determined exclusively by geometric properties of the protuberances. The proofs (see [1]) are carried out using methods of homogenization theory.

Joint work with Andrii Khrabustovskyi (Karlsruhe Institute of Technology).

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Asymptotic rigidity for layered materials and its applications

Fabian Christowiak¹, Carolin Kreisbeck²

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We investigate the asymptotic behavior of variational models for bilayered elastic composite materials with one stiff component as the layer thickness tends to zero. While the rigidity of individual layers can be characterized by common generalizations to the classical Liouville theorem, which states that every local isometry of a domain corresponds to a rigid body motion, global rigidity fails due to the softer material components.

In this contribution, we present a new type of asymptotic rigidity lemma, which shows that if the layers are sufficiently stiff, the class of admissible macroscopic deformations is rather restricted. In the presence of a local volume preservation condition, it only comprises globally rotated shear deformations in layer direction. Furthermore, we identify the optimal scaling between layer thickness and stiffness, using suitable bending constructions.

Building on this result, we determine explicitly the homogenized Γ -limits for two models in nonlinear elasticity and finite crystal plasticity. To that end, the non-convex (approximate) differential inclusion constraints are addressed by tailor-made nested laminates along with tools from convex integration while the delicate issue of localization is overcome by exploiting the essentially one-dimensional character of the problem.

Homogenization of quasilinear elliptic problems with nonlinear Robin conditions and L^1 data

Patrizia Donato

Université de Rouen (France)

We present here some recent homogenization results obtained in collaboration with O. Guibé (Université de Rouen) and A. Oropeza (University of the Philippines Diliman). They concern the homogenization of a class of quasilinear elliptic problems in a periodically perforated domain Ω_{ε} , with L^1 data and nonlinear Robin conditions on the boundary of the holes. Since we deal with L^1 data, we cannot have solutions in $H^1(\Omega_{\varepsilon})$. Therefore, we use here the convenient notion of renormalized solutions. For the homogenization, we use the Periodic Unfolding Method but we can only apply it to the truncated solutions, which are in $H^1(\Omega_{\varepsilon})$. Hence, as a main difficulty, we have to carefully describe the limits of the truncated unfolded solutions and of their gradients. This allow us to prove that we obtain at the limit, an unfolded renormalized problem, as well as a homogenized problem in Ω .

A Liouville theorem for stationary and ergodic ensembles of parabolic systems

Peter Bella¹, Alberto Chiarini², Benjamin Fehrman³

¹Universität Leipzig, Mathematisches Institut (Germany) ²Université d'Aix-Marseille (France) ³Max-Planck-Institut für Mathematik in den Naturwissenschaften (Germany)

We will discuss a first-order Liouville theorem for random ensembles of uniformly parabolic systems under the mere qualitative assumptions of stationarity and ergodicity. The method of proof effectively separates the probabilistic and deterministic aspects of the argument through the introduction of an extended corrector. The statistical properties of the environment are used to prove the sublinearity of the large-scale L^2 -averages of this corrector, which subsequently provides the starting point for a Campanato iteration. The latter is used to establish, almost surely, an intrinsic large-scale $C^{1,\alpha}$ -regularity estimate for caloric functions.

An analysis of variance reduction methods in stochastic homogenization

Julian Fischer

IST (Austria)

The theory of stochastic homogenization of elliptic PDEs predicts that a medium whose heat conductivity varies randomly on a small scale behaves on large scales like a homogeneous medium with a constant effective heat conductivity. An important approach for the computation of such effective coefficients in stochastic homogenization is the so-called representative volume element (RVE) method: A sample volume of the random medium is chosen, say, a cube with side length L; by solving the equation of the homogenization corrector on this sample volume, one may obtain an approximation for the effective coefficient. However, the resulting approximation for the effective coefficient is a random quantity, as it depends on the sample of the random medium. It turns out that the leading-order contribution to the error of the RVE approximation is actually caused by the random fluctuations of the RVE approximation around its expected value. To increase the efficiency of numerical methods, it is therefore desirable to reduce the variance of the approximation. We provide an improved rigorous analysis of the variance reduction approaches in stochastic homogenization introduced by Blanc, Le Bris, Legoll, and Minvielle.

Homogenization of reaction–diffusion processes with a nonlinear dynamical Wentzell-boundary condition at the interface

Markus Gahn¹, Maria Neuss-Radu², Peter Knabner²

¹Interdisciplinary Center for Scientific Computing (IWR), Center for Modelling and Simulation in the Biosciences (BIOMS), University of Heidelberg (Germany)
² Freidrich-Alexander-Universität Erlangen-Nürnberg (Germany)

We are dealing with the periodic homogenization of a system of microscopic nonlinear reactiondiffusion equations in a multi-component porous medium, where the components are separated by an interface. One component is connected, the other ones are disconnected and consist of periodically distributed inclusions. The differential equations in the different domains are coupled by a nonlinear dynamical Wentzell-boundary condition, i. e. the normal fluxes at the interface are given by a reaction-diffusion equation with nonlinearities depending on the solutions on both sides of the interface.

For this system we derive a macroscopic model using the two-scale convergence and the unfolding method for periodic domains and on periodic surfaces. To cope with the diffusion terms in the Wentzell-boundary condition new two-scale compactness results for functions with regular traces on disconnected surfaces are derived. For the convergence of the nonlinear reaction-rates, especially the transmission conditions on the interface between the components, strong two-scale compactness results are developed using an unfolding argument and a Banach-valued compactness theorem of Kolmogorov type.

Large-scale behaviour of *a*-harmonic functions

Peter Bella¹, Arianna Giunti², Felix Otto¹

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We are interested in quantifying the random homogenization of the elliptic (and possibly vectorial) operator $-\nabla \cdot a\nabla$ on \mathbb{R}^d , d > 2. We do this by comparing the large-scale behaviour of a-harmonic functions and functions which are harmonic with respect to the homogenized operator $-\nabla \cdot a_{hom} \nabla$. More precisely, we want to investigate the spaces of *a*-harmonic functions which grow at infinity with a certain algebraic rate $k \in \mathbb{N}$, or of a-harmonic functions in a neighbourhood of infinity which decay with rate d - 2 + k. In the Euclidean case, namely for functions which are a_{hom} -harmonic, these sequences of spaces are known to have a nice algebraic structure: By Liouville principles, the spaces of growing harmonic functions are the spaces of harmonic polynomials. In the case of decaying harmonic functions in exterior domains, the quotient spaces taken between the two neighbouring orders d - 2 + k and d - 1 + k are spanned by the k-th homogeneous derivatives of the Green function [3]. We develop criteria on a under which these sequences of spaces on the Riemannian side, namely *a*-harmonic functions, behave like in the Euclidean counterpart. Our main aim is to construct isomorphisms between spaces of growing/decaying a-harmonic functions and the corresponding spaces of $a_{\rm hom}$ -harmonic functions, which are canonical with a certain degree of precision $m \in \mathbb{N}$ (i.e. if we take quotients with respect to elements which grow/decay m orders less/more).

Constructing such isomorphisms amounts to estimating the relative growth/decay of the difference of a growing/decaying *a*-harmonic function *u* and a (canonically "corrected") Euclidean counterpart *v*. In particular, estimating this relative rate is closely related to estimating the homogenization error, i.e. the error in the two-scale expansion $v + \phi_i \partial_i \phi + \psi_{ij} \partial_{ij} v$, which involves the first and second-order correctors associated to $-\nabla \cdot a\nabla$. In [2], which may be considered a further development and an upgrade of [1], we show that there is a direct correspondence between the rates of the sublinear growth at infinity of both the first an second order correctors and the smallness of the relative homogenization error. In addition, we construct a canonical isomorphism between the Riemannian and Euclidean spaces with a degree m = 2 of precision. We expect that this canonical isomorphism may be upgraded to a fixed order of precision m, provided we assume that there exist sublinear correctors up to the m-th order. In the case of stochastic homogenization, this implies that we have a constraint on m given by the dimension d.

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- [2] P. Bella, A. Giunti and F. Otto, Effective multipoles in random media, arXiv:1708.07672.
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The homogenization commutator and the pathwise structure of fluctuations

Antoine Gloria

Université Pierre et Marie Curie, Paris (France)

In this talk I will introduce the notion of homogenization commutator. The fluctuations at large scales of this stationary field characterize in a pathwise sense the fluctuations at large scales of standard quantities like the flux and the gradient of the corrector, and of the solution operator. In the first part of the talk, I will address this pathwise structure in two typical settings: Gaussian coefficients with integrable correlations and Gaussian coefficients with non-integrable correlations. In the second part of the talk, I will describe the large scale fluctuations of the homogenization commutator: White noise for integrable correlations, and fractional white noise for non-integrable correlations. This is based on joint works with M. Duerinckx, J. Fischer, S. Neukamm, and F. Otto.

Homogenization of a thin elastic plate

Georges Griso

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Let $\Omega_{\delta} = \omega \times (-\delta, \delta)$ be a plate whose thickness is 2δ and mid-surface ω a bounded domain in \mathbb{R}^2 with Lipschitz boundary. Our aim is to give the asymptotic behavior of the following problem depending of two small parameters ε and δ :

Find
$$u_{\varepsilon,\delta} \in \mathbf{H}_{\Gamma_{0,\delta}}(\Omega_{\delta}) \doteq \left\{ v \in H^{1}(\Omega_{\delta}; \mathbb{R}^{3}) \mid v = 0 \text{ sur } \Gamma_{0,\delta} \doteq \partial \omega \times (-\delta, \delta) \right\},\$$

$$\int_{\Omega_{\delta}} a_{ijkl} \left(\frac{x_{1}}{\varepsilon}, \frac{x_{2}}{\varepsilon}, \frac{x_{3}}{\delta} \right) \gamma_{ij}(u_{\varepsilon,\delta}) \gamma_{kl}(v) \, dx = \int_{\Omega_{\delta}} f_{\delta}(x) \, v(x) \, dx$$
$$\forall v \in \mathbf{H}_{\Gamma_{0,\delta}}(\Omega_{\delta})$$

where

- $a_{ijkl} \in L^{\infty}(Y \times (-1,1))$, $Y \doteq (0,1)^2$ and satisfy the usual conditions
- $a_{ijkl} Y \text{-periodic,}$ $\gamma_{ij}(v) = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).$

We study the asymptotic behavior of the sequence $\{u_{\varepsilon,\delta}\}_{(\varepsilon,\delta)}$ in these cases:

- 1 $\varepsilon \to 0$ (δ fixed) then $\delta \to 0$,
- 2 $\delta \rightarrow 0$ (ε fixed) then $\varepsilon \rightarrow 0$,
- **3** $(\varepsilon, \delta) \to (0, 0)$ and $\lim_{(\varepsilon, \delta) \to (0, 0)} \frac{\varepsilon}{\delta} = \theta \in [0, +\infty].$

It is well known that cases 1 and 2 give different results. We show that case 1 is identical to $\theta = 0$, while case 2 is identical to $\theta = +\infty$. In case 3 we obtain a continuum of models (with respect to θ). The techniques developed here could be used in any problem of reduction of dimension and homogenization.

Strong advection and initial layer analysis in passive transport

Thomas Holding¹, Harsha Hutridurga², Jeffrey Rauch³

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Suppose $\mathbf{b}(x)$ is a given smooth divergence free field. The main objective of this work is to understand the behaviour of the solution $u^{\varepsilon}(t, x)$ to

(1)
$$\partial_t u^{\varepsilon} + \frac{1}{\varepsilon} \mathbf{b}(x) \cdot \nabla u^{\varepsilon} - \Delta u^{\varepsilon} = 0 \qquad \text{in } (0, \infty) \times \Omega$$

in the $\varepsilon \ll 1$ regime, i.e., to study the effect of strong incompressible flow on the diffusion in Ω . The evolution in (1) is supplemented with initial and boundary conditions. In this work, we develop a technique of multiple scale asymptotic expansions along mean flows and a corresponding notion of weak multiple scale convergence (in the spirit of two-scale convergence) to achieve this objective. Essential idea in our work is to recast the evolution in (1) along the flow coordinates. More precisely, we study the family $u^{\varepsilon}(t, \Phi_{t/\varepsilon}(x))$ rather than $u^{\varepsilon}(t, x)$ where $\Phi_{\tau}(x)$ is the flow associated with the field $\mathbf{b}(x)$. The associated jacobian matrix is denoted $J(\tau, x)$. Note that we have introduced an new time scale $\tau := t/\varepsilon$ which we call the *fast time variable*. Studying the $\varepsilon \to 0$ limit then corresponds to studying certain averages in the fast time variable. Our theory uses the notion of *ergodic algebra with mean value*. Loosely speaking, some of the results that we have obtained are

- (a) Under the assumption that $|J(\cdot, x)| < \infty$, the family $u^{\varepsilon}(t, \Phi_{t/\varepsilon}(x))$ converges in certain weak sense to a limit which solves a diffusion equation.
- (b) In scenarios where $J(\tau, x) \sim \tau$, the family $u^{\varepsilon}(t, \Phi_{t/\varepsilon}(x))$ converges in certain weak sense to a limit which solves a *degenerate* diffusion equation.
- (c) The weak $L_t^2 H_x^1$ -limit of the family $\{u^{\varepsilon}\}$ solves a parabolic problem with a constraint that it belongs to the null space of the operator $\mathbf{b} \cdot \nabla$.

The analysis corresponding to (a) are contained in [2]. An initial layer analysis linking the results (b) and (c) is part of a work in preparation [3]. Our work has some intricate links to the relaxationenhancing phenomena studied in [1].

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^[2] T. Holding, H. Hutridurga, J. Rauch, Convergence along mean flows, SIAM J. Math. Anal. 49(1), pp. 222–271, (2017).

Homogenization in Hencky plasticity

Martin Jesenko¹, Bernd Schmidt²

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In perfect small strain elastoplasticity (i.e. with zero hardening) the material behaviour beyond the elastic regime is modelled by a flat relation between stress and deviatoric strain. After passing the yield surface, the stored energy thus grows linearly in the deviatoric strain, yet still quadratic in the hydrostatic strain.

A static description of this 'pseudoelastic' regime through stored energy functionals with mixed linear-quadratic growth is referred to as the Hencky plasticity model. Due to the linear growth conditions, the Hencky plasticity functional is not coercive on Sobolev spaces, and more general displacements in the space BD of functions of bounded deformation have to be taken into account. We will address the problem of deriving an effective functional for a heterogeneous material in the Hencky plasticity model. As usual, we will look at the behaviour of the family

$$\mathcal{F}_{\varepsilon}(u) = \int_{\Omega} f\left(\frac{x}{\varepsilon}, \nabla u(x)\right) dx$$

for $\varepsilon \to 0$ with f being periodic in the first variable. The main difficulty are non-standard growth properties of the energy density, and thus the standard results are not applicable. We will present the known results, introduce a suitable space for the domain of the homogenized functional and derive an integral formula for a rather general class of densities. Thus we will generalize the result obtained by Demengel and Qi in [1] for convex densities. Our results are gathered in [2].

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A numerical primer in 2D stochastic homogenization

Venera Khoromskaia, Boris N. Khoromskij, Felix Otto

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Following [2], we discuss the numerical scheme for the discretization and solution of 2D elliptic equations with strongly varying piecewise constant coefficients arising in stochastic homogenization [1]. An efficient stiffness matrix generation scheme based on assembling of the local tensor (Kronecker) product matrices is described. Spectral properties of the discrete stochastic operators are studied by estimation of the density of spectrum for the family of stochastic realizations. The resulting large linear systems of equations are solved by the preconditioned CG iteration with the convergence rate that is independent of the grid size and the variation in jumping coefficients (contrast) [3, 4]. The numerical analysis on the convergence rates in stochastic homogenization theory is provided. This includes the calculation of the homogenized coefficient matrix and the subsequent estimation on the limit of empirical average. The approach allows to analyze the asymptotic of empirical average for the homogenized matrix coefficient and the spectral properties of the family of stochastic elliptic operators for a wide range of intrinsic model parameters. The convergence rates on the Representative Volume Element Matrix to derive the homogenized coefficient rigorously established in [1] are reproduced by our numerical experiments. The proposed tensor-based numerical method allows to compute extensive series of stochastic realizations for a large size of the representative volume element, L, using only MATLAB on a moderate computer cluster. Numerical illustrations will be presented.

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Towards numerical homogenization of multiscale fault networks

Ralf Kornhuber

Freie Universität Berlin (Germany)

Numerical homogenization tries to approximate the solution of partial differential equations with dominating fine-scale structures by functions from modified finite element spaces. Utilizing the framework of subspace decomposition, we present and analyze a class of new methods for elliptic problems with strongly oscillating coefficients that is closely related to the approach of Malqvist and Peterseim [Math. Comp. 83, 2014] together with corresponding iterative counterparts. As a first step towards numerical homogenization of multiscale fault networks with rate and state dependent friction, we then discuss extensions of our approach to elliptic problems with linear transmission conditions on multiscale networks of interfaces. Our findings are illustrated by numerical computations. This is joint work with Joscha Podlesny und Harry Yserentant.

Effective Maxwell's equations in a geometry with flat split-rings and wires

Agnes Lamacz

Technische Universität Dortmund (Germany)

Propagation of light in heterogeneous media is a complex subject of research. It has received renewed interest in recent years, since technical progress allows for smaller devices and offers new possibilities. At the same time, theoretical ideas inspired further research. Key research areas are photonic crystals, negative index metamaterials, perfect imaging, and cloaking.

The mathematical analysis of negative index materials, which we want to focus on in this talk, is connected to a study of singular limits in Maxwell's equations. We present a result on homogenization of the time harmonic Maxwell's equations in a complex geometry. The homogenization process is performed in the case that many (order η^{-3}) small (order η^1), flat (order η^2) and highly conductive (order η^{-3}) metallic split-rings are distributed in a domain $\Omega \subset \mathbb{R}^3$. We determine the effective behavior of this metamaterial in the limit $\eta \searrow 0$. For $\eta > 0$, each single conductor occupies a simply connected domain, but the conductor closes to a ring in the limit $\eta \searrow 0$. This change of topology allows for an extra dimension in the solution space of the corresponding cell-problem. Even though both original materials (metal and void) have the same positive magnetic permeability $\mu_0 > 0$, we show that the effective Maxwell system exhibits, depending on the frequency, a negative magnetic response. Furthermore, we demonstrate that combining the split-ring array with thin, highly conducting wires can effectively provide a negative index metamaterial.

Homogenization in the presence of defects

Claude Le Bris

École Nationale des Ponts et Chaussées, CERMICS, Marne La Vallée (France)

We overview a series of joint works with Xavier Blanc (Université Paris Diderot), Pierre-Louis Lions (Collège de France), and also Marc Josien (Ecole des Ponts), about homogenization theory for elliptic equations in the presence of defects that break, typically, the periodicity of the coefficients. We show that under appropriate assumptions, the quality of approximation and the rates of convergence of periodic homogenization are preserved. We address both equations in divergence form and equations not in divergence form, such as in particular advection-diffusion equations. A comparison is made with equations different in nature, such as Hamilton-Jacbi equations (joint works with Pierre Cardaliaguet (Université Paris Dauphine) and Panagiotis Souganidis (University of Chicago).

Thermoelastic behavior of a composite material with heat jumps and continuous deformation across the interface

Kim Hang Le Nguyen

Universität Ulm (Germany)

In this work, we consider a hyperbolic-parabolic coupled system in a two-component domain. This models the thermoelastic processes occurring in heterogeneous materials consisting of two constituents separated by an ε -periodic interface with imperfect thermal contact. This property results in temperature jumps on the interface. We assume that the interfacial heat conductance depends on a real parameter γ and the displacement is continuous across the interface. Here, the interaction of the deformation and thermal process inside material is expressed via the Duhamel-Neumann law. We first investigate the unique existence of the solution to the problem using semigroup theory and give some a prior estimates. Then, we shall study the asymptotic behavior of the solution as $\varepsilon \rightarrow 0$ and obtain the homogenized problems for different values γ . For this aim, we use the time-dependent periodic unfolding method for two-component domains which is adapted from the unfolding method in some recent works (see [1, 2, 3]). The complexity of the coupled mixed system of equations of different types on the two-component domain makes the problem quite interesting.

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A software tool for multi-scale model derivation with an application to a micro mirror array

Walid Belkhir, Nicolas Ratier, Nguyen Nhat Binh Trinh, Michel Lenczner

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The presentation will report progresses in the development of MEMSALab (for MEMS Array Lab) and its first application to an array of micro mirrors, see [3].

This software package aims at automatically generating families of multi-scale models issued from partial differential equations and is grounded on a concept of reusability. It is expected that it will be applicable to general devices but we currently focus on simulation of arrays of micro and nanosystems.

The technique of model construction combines principles issued from asymptotic methods as the two-scale convergence [5], [4] and from rewriting techniques developed in computer science. A multiscale model of a complex system is built incrementally by the method of *Extension and Combination* [6], [1], and [2] that has been taylor made.

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Homogenization of the discrete diffusive coagulation-fragmentation equations in perforated domains

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In this work, we study the homogenization of a set of Smoluchowski's discrete coagulationfragmentation-diffusion equations in a periodically perforated domain. The system of evolution equations considered describes the dynamics of cluster growth, that is the mechanisms allowing clusters to coalesce to form larger clusters or break apart into smaller ones. The approach of two clusters leading to aggregation is assumed to result only from diffusion. Since the size of clusters is not limited a priori, the system of reaction-diffusion equations that we consider is made of an infinite number of equations.

The structure of the chosen equations, coupled with a non-homogeneous Neumann condition on the boundaries of the holes, is useful in investigating several typical phenomena arising in porous media (as deposition effects or chemical reactions taking place on the surface of the pores) [1], [2] or in the field of biomedical research (as the agglomeration and diffusion of β -amyloid peptide, a process associated with the development of Alzheimer's disease) [3].

Our homogenization result, based on Allaire-Nguetseng two-scale convergence, is meant to pass from a microscopic model (where the physical processes are properly described) to a macroscopic one (which takes into account only the effective or averaged properties of the system).

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Homogenization of a semilinear variational inequality in in a thick fractal junction

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A semilinear variation inequality is considered in a thick fractal junction Ω_{ε} , which is the union of a domain Ω_0 and a lot of joined thin trees situated ε -periodically along some manifold on the boundary of Ω_0 . The trees have finite number of branching levels. The following non-linear boundary conditions of the Signorini type:

 $u_{\varepsilon} \leq g_i, \quad a_i \partial_{\nu} u_{\varepsilon} + \varepsilon^{\alpha_i} h_i(u_{\varepsilon}) \leq 0, \quad (u_{\varepsilon} - g_i) \left(a_i \partial_{\nu} u_{\varepsilon} + \varepsilon^{\alpha_i} h_i(u_{\varepsilon}) \right) = 0$

is given on the boundaries of the branches from the *i*-th branching layer; $\{\alpha_i\}$ are real parameters. The asymptotic analysis of this problem is made as $\varepsilon \to 0$, i.e., when the number of the thin trees infinitely increases and their thickness vanishes. The passage to the limit is accompanied by special intensity factors $\{\varepsilon^{\alpha_i}\}$ in the boundary conditions. We establish qualitatively different cases in the asymptotic behaviour of the solution depending on the value of parameters $\{\alpha_i\}$. For each case a convergence theorem is proved, the corresponding homogenized problem is found and the existence and uniqueness of its solution in an anisotropic Sobolev space of multi-sheeted functions is justified. To study this problem, the methods developed in [1, 2] are used.

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Homogenization of periodic hyperbolic systems: Operator error estimates

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In $L_2(\mathbb{R}^d; \mathbb{C}^n)$, we consider a selfadjoint second order differential operator $A_{\varepsilon} = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$, $\varepsilon > 0$. The matrix-valued function $g(\mathbf{x})$ is assumed to be periodic, bounded and uniformly positive definite. Next, $b(\mathbf{D}) = \sum_{j=1}^d b_j D_j$ is a first order differential operator with constant coefficients. The symbol $b(\boldsymbol{\xi})$ is subject to some condition which ensures strong ellipticity of the operator A_{ε} . We study the behavior of the operator $A_{\varepsilon}^{-1/2} \sin(\tau A_{\varepsilon}^{1/2}), \tau \in \mathbb{R}$, as $\varepsilon \to 0$. In [1], M. Sh. Birman and T. A. Suslina proved that

(1)
$$\|\cos(\tau A_{\varepsilon}^{1/2}) - \cos(\tau (A^0)^{1/2})\|_{H^2(\mathbb{R}^d) \to L_2(\mathbb{R}^d)} \leq C\varepsilon(1+|\tau|).$$

Here $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ is the effective operator with the constant positive effective matrix g^0 . The results of this type are called operator error estimates in homogenization theory.

Later M. A. Dorodnyi and T. A. Suslina [2] showed that estimate (1) is sharp with respect to the type of the operator norm. On the other hand, in [2], under some additional assumptions on the operator, the result (1) was improved with respect to the type of the norm. In [1, 2], by virtue of the identity $A_{\varepsilon}^{-1/2} \sin(\tau A_{\varepsilon}^{1/2}) = \int_{0}^{\tau} \cos(\tilde{\tau} A_{\varepsilon}^{1/2}) d\tilde{\tau}$ and the similar identity for the effective operator, from (1) as a (rough) consequence it was deduced that

$$\|A_{\varepsilon}^{-1/2}\sin(\tau A_{\varepsilon}^{1/2}) - (A^0)^{-1/2}\sin(\tau (A^0)^{1/2})\|_{H^2(\mathbb{R}^d) \to L_2(\mathbb{R}^d)} \leq C\varepsilon(1+|\tau|)^2.$$

Our first result is improvement of this estimate. Our second result is approximation for $A_{\varepsilon}^{-1/2} \sin(\tau A_{\varepsilon}^{1/2})$ with the corrector $K(\varepsilon, \tau)$ taken into account:

$$\begin{aligned} \|A_{\varepsilon}^{-1/2}\sin(\tau A_{\varepsilon}^{1/2}) - (A^{0})^{-1/2}\sin(\tau (A^{0})^{1/2})\|_{H^{1}(\mathbb{R}^{d}) \to L_{2}(\mathbb{R}^{d})} &\leq C_{1}\varepsilon(1+|\tau|) \\ \|A_{\varepsilon}^{-1/2}\sin(\tau A_{\varepsilon}^{1/2}) - (A^{0})^{-1/2}\sin(\tau (A^{0})^{1/2}) - \varepsilon K(\varepsilon,\tau)\|_{H^{2}(\mathbb{R}^{d}) \to H^{1}(\mathbb{R}^{d})} \\ &\leq C_{2}\varepsilon(1+|\tau|). \end{aligned}$$

The constants C_1 and C_2 are controlled explicitly in terms of the problem data. The results are applied to homogenization for the solutions of the hyperbolic equation $\partial_{\tau}^2 \mathbf{u}_{\varepsilon}(\mathbf{x}, \tau) = -A_{\varepsilon} \mathbf{u}_{\varepsilon}(\mathbf{x}, \tau) + \mathbf{F}(\mathbf{x}, \tau)$.

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Imperfect transmission problems: Homogenization with weakly converging data

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The aim of the talk is to describe the asymptotic behavior, as $\varepsilon \to 0$, of an elliptic problem with rapidly oscillating coefficients in an ε -periodic two component composite with imperfect inclusions of size ε . On the interface we prescribe a jump of the solution proportional to the conormal derivative by means of a function of order ε^{γ} . This work extends to the case of weakly converging data some previous results by P. Donato and S. Monsurrò, obtained when a fixed datum or strongly converging data are considered.

This homogenization result, interesting in itself, could have useful applications in the study of the exact controllability of a hyperbolic problem set in the same kind of domain and with the same jump condition on the interface. Indeed, when studying the exact controllability, via HUM method, one needs to exploit some homogenization results applied to a transposed problem. To do that, it is necessary to study the asymptotic behavior of a stationary problem, with weakly converging data.

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Quantitative homogenization in nonlinear elasticity for small loads

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We consider a nonlinear elastic composite with a periodic micro-structure described by the nonconvex integral functional

$$I_{\varepsilon}(u) = \int_{\Omega} W\left(\frac{x}{\varepsilon}, \nabla u(x)\right) - f(x) \cdot u(x) \, dx,$$

where $u: \Omega \to \mathbb{R}^d$ is the deformation, $f: \Omega \to \mathbb{R}^d$ is an external force, $\varepsilon > 0$ denotes the size of the micro-structure, and W(y, F) is a stored energy function which is periodic in y. As it is well-known, under suitable growth conditions, $I_{\varepsilon} \Gamma$ -converges to a functional with a homogenized energy density $W_{\text{hom}}(F)$, which is given by an *infinite-cell formula*. Under appropriate assumptions on W (namely, $p \ge d$ -growth from below, frame indifference, minimality at identity, non-degeneracy and smoothness in a neighborhood close to the set of rotations) and on the microstructure, we show that in a neighbourhood of rotations the homogenized stored energy function W_{hom} is of class C^2 and characterized by a *single-cell homogenization formula*. Moreover, for small data, we establish an estimate on the homogenization error, which measures the distance between (almost) minimizers u_{ε} of I_{ε} and the minimizer of the homogenized problem. More precisely, we prove that the L^2 -error as well as the H^1 -error of the associated two-scale expansion decays with the rate $\sqrt{\varepsilon}$.

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Mathematical modeling and multiscale analysis of transport processes through membranes

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In this presentation, we develop multiscale methods for the derivation and analysis of effective models in environments containing membranes. At the microscopic level, where membranes are modeled as thin heterogeneous layers, the model consists of nonlinear reaction-diffusion equations within each subdomain. At the macroscopic level, membranes are reduced to interfaces, and effective transmission conditions and/or effective equations at these interfaces are derived. It turns out that the form of the effective laws at the interface depends on the scaling of the microscopic system as well as of the type of microscopic transmission conditions imposed at the bulk-layer interface.

For the derivation of macroscopic (effective) models, we first generalize the concept of weak and strong two-scale convergence to flat membranes with periodic structure. This allows to derive cell-problems, which approximate at zeroth order the processes in the membrane. For the derivation of effective transmission conditions at the interfaces, we define test-functions of boundary-layer type, adapted to dimension reduction. In case of curved membranes, we introduce the notion of locally periodic functions on manifolds and thin layers. Furthermore, we define the notion of two-scale convergence with respect to charts.

Parts of the presented results are obtained jointly with Markus Gahn (Heidelberg), Willi Jäger (Heidelberg), and Peter Knabner (Erlangen).

Macroscopic properties of a simple stochastic process with aging: Recursive solution

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The coupling of stochastic growth and shrinkage of one-dimensional structures to random aging of the constituting subunits defines a stochastic process which captures the essential features of the nonequilibrium assembly of cytoskeletal filaments. Because of correlations, previously employed mean-field methods fail to correctly describe effective macroscopic properties such as filament growth. We study an alternative formulation of the full master equation of the process. An ansatz for the steady-state solution leads to a recursion relation which allows for the calculation of all emergent quantities with increasing accuracy and in excellent agreement with stochastic simulations. In particular, we compute the force-velocity relation and the stall force, i.e. the pushing force at which filament growth vanishes. It turns out this force is much smaller than previously calculated from mean-field equations.

Asymptotic behaviour of a class of controls for imperfect transmission problems

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We deal with the asymptotic behaviour of the sequence of a particular class of controls related to a second order linear evolution equation defined in two-component composites with ε -periodic disconnected inclusions of size ε in presence of a jump of the solution on the interface that varies according to a parameter $\gamma \leq 1$. The case $\gamma = 1$ is the most interesting and delicate one, since the homogenized problem is represented by a coupled system of a P.D.E. and an O.D.E. no more symmetric, giving rise to a memory effect (see [1]). Due to the lack of symmetry, usual results either about existence and uniqueness of the solutions or characterization of the controls (see [4, 5]) do not directly apply. Our main result proves that the control and the corresponding solution of the ε -problem converge to the control of the homogenized problem and to the corresponding solution respectively.

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Numerical homogenization by localized orthogonal decomposition and connections to the mathematical theory of homogenization

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This talk aims to bridge existing theories in numerical and analytical homogenization. For this purpose, the localized orthogonal decomposition method, which can be linked to subspace correction techniques, is reinterpreted by means of a discrete integral operator acting on standard finite element spaces. The exponential decay of the involved integral kernel motivates the use of a diagonal approximation which leads to a piecewise constant effective coefficient. In a periodic setting, this computable effective coefficient is proved to coincide with the classical homogenization limit. An a priori error analysis shows that the corresponding effective numerical model is appropriate beyond the periodic setting whenever the computed effective coefficient satisfies a certain homogenization criterion, which can be verified a posteriori. The new representation of the numerical homogenization method turns out to be particularly attractive for computational stochastic homogenization.

A two-scale Stefan problem arising in a model for tree-sap exudation

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The study of tree-sap exudation, in which a (leafless) tree generates elevated stem pressure in response to repeated daily freeze-thaw cycles, gives rise to a multiscale problem involving heat and multiphase liquid/gas transport. By assuming a periodic cellular structure based on an appropriate reference cell, we derive a homogenized heat equation governing the global temperature field on the scale of the tree stem, with all the remaining physics relegated to equations defined on the reference cell. The particular form of our homogenized temperature equation is obtained using periodic-homogenization techniques with two-scale convergence, which we apply rigorously in the context of a simpler two-phase Stefan-type problem corresponding to a periodic array of melting cylindrical ice bars, which is prototypical in the sense that it also relates to similar problems such as water uptake beyond capillary suction in concrete structures. Numerical simulations are performed to validate the results and draw conclusions regarding the phenomenon of sap exudation.

Stochastic discrete approximations of the Mumford-Shah functional

Matthias Ruf

Université libre de Bruxelles (Belgium)

In this talk we present a Γ -convergent discrete approximation of the Mumford-Shah functional. The discrete functionals act on functions defined on stationary stochastic lattices and take into account nearest neighbor interactions through a non-convex potential. In this setting the geometry of the lattice determines the anisotropy of the limit functional. Thus we can use statistically isotropic lattices to approximate the Mumford-Shah functional via stochastic homogenization. This approach works in any dimension.

Asymptotic modelling of the wave-propagation over acoustic liners

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We will consider the acoustic wave propagation in a channel separated from a chamber by a thin periodic layer. This model stand for micro-perforated absorbers which are used to supress reflections from walls. Due to the smallness of the periodicity a direct numerical simulation, e.g. with the finite element method (FEM), is only possible for very large costs. Based on homogenization techniques we find impedance transmission conditions [1], which integrated into numerical methods like the FEM or the boundary element method leads to much lower computational costs. For liners of finite length their endings have a significant impact to the macroscopic absorption and this effect is a-priori not considered with the transmission conditions. We aim to describe the interaction of the thin periodic layer with the singularities from its endings asymptotically when the periodicity and layer thickness δ tend to zero [4]. For this, the Kondratiev theory for corner singularities (which is based on the Mellin transform) has to be extended to infinite cones with periodic layers [3] in the spirit of Nazarov [2].

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Stability and energy estimates for periodic frame structures made of thin beams

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We consider a periodic frame structure consisting of thin beams (of radius r) and study the energy estimates for this problem as the period ε , and the radius r of the beams tend to zero. The decomposition of the displacement field into the extensional and bending components is used, to obtain a priori estimates for beams between the nodes and then the complete microscopic displacement field is extended by the multilinear interpolation. Two types of unfolding operators are introduced to deal with different parts of the decomposition. Different estimates w.r.t. the relation between the period of the frame and the beam's thickness, as well as w.r.t. design of the frame's graph and stability conditions are discussed.

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Perturbation problems in homogenization of Hamilton-Jacobi equations

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The talk is about the behavior of the ergodic constant associated with convex and superlinear Hamilton-Jacobi equation in a periodic environment which is perturbed either by medium with increasing period or by a random Bernoulli perturbation with small parameter. The outcome is a first order rder Taylor's expansion for the ergodic constant which depends on the dimension d. When d = 1 the rst order term is non trivial, while in dimensions larger than 2 is always 0. Although such questions have been looked at in the context of linear uniformly elliptic homogenization, the results are the first of this kind in nonlinear settings. This is joint work with Pierre Cardaliaguet and Claude LeBris.

A fully homogenized model of Kondaurov's non-equilibrium two-phase flow in double porosity media with thin fissures

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We consider a two-phase incompressible non-equilibrium flow in fractured porous media in the framework of Kondaurov's model, wherein the mobilities and capillary pressure depend both on the real saturation and a non-equilibrium parameter satisfying a kinetic equation. The medium is made of two superimposed continua, a connected fracture system, which is assumed to be thin of order $\varepsilon \delta$, where δ is the relative fracture thickness and an ε -periodic system of disjoint cubic matrix blocks. We derive the global behavior of the model by passing to the limit as $\varepsilon \to 0$, assuming that the block permeability is proportional to $(\varepsilon \delta)^2$, while the fracture permeability is of order one, and obtain the global δ -model. In the δ -model we linearize the cell problem in the matrix block and letting $\delta \to 0$, obtain a macroscopic non-equilibrium fully homogenized model, i.e., the model which does not depend on the additional coupling. The numerical tests show that for δ sufficiently small, the exact global δ -model can be replaced by the fully homogenized one without significant loss of accuracy.

In this work we continue the study of the global Kondaurov model started in the papers [1, 2] and generalize the results obtained earlier in [3, 4].

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Homogenization and dimension reduction for a textile shell

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The wrinkling of textiles is an interesting behavior to investigate and optimize. In a first step a model of a woven and hence periodic textile as a three dimensional elasticity problem with contact between yarns is considered. On the microscopic scale we use the decomposition of displacement, possible for both the linear [1] and non-linear case [2], to obtain Korn-like inequalities for the single fibers depending on the small parameters. Upon these decomposed displacements we can define a plate-like displacement by using only the information in the contact nodes and interpolation in between them to arrive at estimates for the whole structure. These are then the basis for the technique of periodic unfolding to obtain the homogenized limit equation.

To give rise to the mentioned wrinkling of the textile, the plate-model requires some non-linear behavior. In particular the von-Karman-plate gives by its semi-linearity the desired coupling between the in-plane and out-of-plane or bending and membrane displacements. For this we discuss the necessary energy and its properties to generate the von-Karman in the limit as the starting point for the optimization.

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Upscaling of coupled geomechanics, flow, and heat, in a poro-elastic medium in the quasi-static situation

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Motivated by geothermal energy storage in the subsurface, we undertake the formal derivation of Biot's equations for a poro-elastic medium completely saturated by a fluid, coupled with energy conservation equations for both the fluid and solid phase. We assume small displacements of the solid structure (i.e. linear strain), a fully connected void and grain space, incompressible fluid, heat transfer dominated by transport, and that the fluid/solid interface coupling conditions may be linearized. We start with the pore scale model, and apply the technique of homogenization to derive the upscaled model in the case of periodically distributed pores. Assuming the homogenization ansatz holds true, we obtain a fully coupled system of equations on the macro-scale accounting for the effects of geomechanics, heat transfer, and fluid flow within a fully saturated porous material.

Analysis and homogenization of a thermoelasticity problem with a-priori known phase transformations

Michael Eden

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We consider a linear, fully coupled thermoelasticity model for a highly heterogeneous, two-phase medium. The medium in question consists of a connected matrix with disconnected, initially periodically distributed inclusions separated by a sharp interface undergoing an a-priori known interface movement due to phase transformation. For this model, we investigate well-posedness and a priori estimates w.r.t. to a scale parameter ε . Via a two scale convergence argument, we rigorously show that the ε -dependent solutions converge to solutions of a corresponding upscaled model with distributed time-dependent microstructures. Finally, we present corrector estimates specifying the convergence rate of the ε -dependent solutions to the homogenized solution that can be established in some simplified scenarios. This research is joint work with A. Muntean (Karlstad University, Sweden)

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A concept of convergence applicable to some homogenization problems

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We will present a concept of convergence, called very weak multiscale convergence, a concept that is applicable to, for instance, homogenization of a variety of evolution problems and to detect hidden scales in for example elliptic problems. The concept of very weak multiscale convergence was first introduced by Holmbom in 1997 but the name "very weak multiscale convergence"was first used in the more general setting by Flodén, Holmbom, Olsson and Persson in 2010. When homogenizing certain parabolic problems, sequences that are not bounded in the L^2 -norm appear and therefore the usual weak and multiscale convergences can not be used alone. The concept of very weak multiscale convergence can actually handle such sequences and it is therefore useful in the homogenization procedure. The reason why we can get a very weak multiscale limit even for sequences where we do not have a multiscale limit is that we use a more restricted class of test functions. The difference between weak convergence, multiscale convergence and very weak multiscale convergence is that the first one captures the global trend of the sequence, the second one captures both the global trend and the microoscillations and the last one omits the global trend but captures the microoscillations. An example of a problem where weak and multiscale convergences are not sufficient is

$$\partial_{t} u_{\varepsilon} (x,t) - \nabla \cdot \left(a \left(\frac{x}{\varepsilon^{2}}, \frac{x}{\varepsilon^{3}}, \frac{t}{\varepsilon}, \frac{t}{\varepsilon^{4}}, \frac{t}{\varepsilon^{7}} \right) \nabla u_{\varepsilon} (x,t) \right) = f(x,t) \text{ in } \Omega_{T},$$

$$u_{\varepsilon} (x,0) = u_{0} (x) \text{ in } \Omega,$$

$$u_{\varepsilon} (x,t) = 0 \text{ on } \partial \Omega \times (0,T)$$

In the homogenization procedure for this problem, sequences of the type $\{\varepsilon^{-k}u_{\varepsilon}\}$ will appear, which are not bounded in $L^2(\Omega_T)$. Here, very weak multiscale convergence will be the key to handle this difficulty.

On periodic homogenisation of a stationary, quasilinear problem

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We present the current state of affairs in carrying out the periodic homogenisation of a stationary and quasilinear boundary value problem following a method proposed by S. Reichelt. Therein, periodic unfolding and the theory of pseudo-monotone operators constitute our main tools.

Some existence and homogenization results for a class of singular problems in perforated domains

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We present some existence and homogenization results for the solution of a quasilinear elliptic problem with a singular nonlinearity in a bounded perforated domain with a nonlinear boundary Robin condition on the holes. The quasilinear equation presents a term which is singular in the u-variable (u being the solution), which is the product of a continuous singular function ζ and a fuction f whose summability depends on the growth of ζ near its singularity.

To show the existence result, we firstly prove some a priori estimates and, as usual in the literature, we find a solution by approximating our problem with a sequence of nonsingular problems with a bounded nonlinearity. For the uniqueness and boundedness of the solution, some additional hypotheses on the data are required (see [1] for details).

Then, we study the asymptotic behaviour, as ε goes to zero, of the same kind of problem with oscillating coefficients in a domain perforated by ε -holes of ε -size. In order to study the quasilinear term, which affects also the study of the singular term near its singularity, we prove a suitable convergence result following the techniques in [2] and [3]. The main tool for the homogenization proof consists in proving that the gradient of the solution behaves like that of a suitable linear problem associated with a weak cluster point, as $\varepsilon \to 0$. This idea was originally introduced in the literature for the homogenization of nonlinear problems with quadratic growth with respect to the gradient. Here the additional difficulties due to the singular term and the boundary nonlinear term are trated by means of the periodic unfolding method, introduced by Cioranescu-Damlamian-Griso in [4] and successively adapted in literature to perforated domains.

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On the large-scale boundary regularity of random elliptic operators

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We are interested in large-scale regularity properties of linear elliptic operators with random heterogenous coefficients fields. In the whole-space there are classical counterexamples to Schauder theory in the setting of general heterogeneous coefficient fields; for example, in the systems case there is the famous counter-example of De Giorgi that shows that *a*-harmonic functions (where *a* is a particular uniformly elliptic coefficient field) must not even be locally bounded. By introducing randomness into the coefficients the hope is to benefit from homogenization effects. In particular, assuming that the ensemble of coefficient fields is stationary and ergodic, one can show that there exists a homogenization corrector (a random field) that has a stationary gradient and is sublinear for almost-every coefficient field. For a fixed coefficient field the sublinear corrector may then be used to transfer regularity from the homogenized operator onto the heterogeneous one at large scales. In the whole-space case this analysis has been performed by Gloria, Neukamm, and Otto.

This poster is concerned with the large-scale *boundary* regularity. We will address three situations: Operators on the half-space with coefficients that are the restriction of a random coefficient field on the whole-space in the case of both homogeneous Dirichlet and homogeneous Neumann boundary data; and the situation of a randomly perforated domain (with the additional assumption that the holes are disjoint). All three cases are treated by first constructing an appropriate sublinear corrector (that satisfies certain boundary conditions) and then performing a Campanato iteration *à la* Gloria, Neukamm, and Otto.

A multiscale approximation of a Cahn–Larché system with phase separation on the microscale

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Motivated by phase-separation processes observed in lipid monolayers in film-balance experiments, the starting point of the considered model is the Cahn–Hilliard equation coupled with the equations of linear elasticity, the so-called Cahn–Larché system. Owing to the fact that the mechanical deformation takes place on a macrosopic scale whereas the phase separation happens on a microscopic level, a multiscale approach is imperative. We assume the pattern of the evolving microstructure to have an intrinsic length scale associated with it, which, after nondimensionalisation, leads to a scaled model, suitable for periodic-homogenisation techniques. For the full nonlinear model the homogenised problem is then obtained by using the method of asymptotic expansion. Furthermore, we present a linearised Cahn–Larché system and use the method of two-scale convergence to obtain the associated limit problem, which turns out to be of the same distributed-microstructure type as in the nonlinear case. Properties of the limit model will be discussed.

Weak solvability results for a pore-scale elliptic model with nonlinear Fourier condition and variable scaled production terms

Thi Kim Thoa Thieu, Anh Khoa Vo

Gran Sasso Science Institute, L'Aquila (Italy)

We consider a pore-scale elliptic model with nonlinear Fourier condition and variable scaled production terms. This model is one of the standard models including the stationary diffusion, aggregation and surface deposition of a colloidal concentration in a porous/strongly heterogeneous medium. Our target is constructing a reliable linearization scheme under a weak formulation that allows us, by arguing the choices of scaling and stabilization parameters, to prove the weak solvability of the microscopic model at each scaling factor.

Multi-scale modeling and simulation of a micro-mirror array

Nguyen Nhat Binh Trinh

FEMTO-ST Besancon (France)

Micro-mirror arrays abbreviated as MMAs play an important role in our life. They are widely used in many fields such as optical devices, tunable lasers hard disk storage, projectors, imaging technologies and so on. In our work, we utilize the two-scale transform method (also called unfolding method) to treat the homogenization of one- and two-dimensional periodic arrays of three-dimensional cells. The system is governed by a system of nonlinear electromechanical equations. In the poster, we will present a model involving the electrical field only and that takes into account the periodicity and the boundary layer effects near the faces and near their edges. We will present simulation results for a one-dimensional array, as well as simulation and optimization results for a single cell taking into account the fully nonlinear electromechanical behavior and thus the pull-in phenomenon. In particular, we will report simulations of the bounces of the mirror on the substrate when the actuation voltage exceeds the pull-in voltage.

This is joint work with Duy Duc Nguyen, Michel Lenczner, Frédéric Zamkotsian, and Scott Cogan.

Stochastic unfolding and homogenization

Mario Varga

Technische Universität Dresden (Germany)

In this presentation we provide a simple homogenization procedure for energy driven problems involving stochastic rapidly-oscillating coefficients. Our intention is to extend the periodic unfolding method to the stochastic setting. Specifically, we recast the notion of stochastic two-scale convergence in the mean by introducing an appropriate stochastic unfolding operator. This operator admits similar properties as the periodic unfolding operator, leading to an uncomplicated method for stochastic homogenization. The presentation is based on a joint work with Martin Heida and Stefan Neukamm.

Strong advection and initial layer analysis in passive transport

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We study a nonlinear coupled parabolic-pseudoparabolic system posed in a mixture theory framework that is able to describe the corrosion of the concrete microstructure due to sulfuric acid. The concrete corrosion is modelled as a changing-in-time viscoelastic mixture within a reaction-diffusion setting. Currently, periodic homogenization is being applied to this model with respect to several of the possible geometric scalings.

Homogenization of the generalized Poisson–Nernst–Planck problem in a two-phase medium

Victor A. Kovtunenko, Anna V. Zubkova

University of Graz, Institute for Mathematics and Scientific Computing (Austria)

We investigate a generalized Poisson–Nernst–Planck system of nonlinear partial differential equations. It describes concentrations of multiple charged particles with an overall electrostatic potential. The generalized model is supplied by volume and positivity constraints and quasi-Fermi electrochemical potentials depending on the pressure. The two-phase periodic medium consists of a porous space and disjoint solid particles. At the interface between the solid and pore parts, inhomogeneous conditions depending on interfacial jumps of the concentrations are given. The nonlinearity of the system, coupling phenomena, and nonlinear interface conditions bring the most difficulties to the homogenization procedure. Based on the asymptotic methods, periodic unfolding, and compensated compactness, we arrive at the homogenized problem and provide residual error estimates.

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