On gradient structures for reversible Markov chains and the passage to Wasserstein gradient flows

Karoline Disser(1), **Matthias Liero**(1),

(1) Weierstrass Institute, Berlin, Germany

In this talk we discuss the limit passage from reversible, time-continuous Markov chains to the one-dimensional Fokker–Planck equation with linear drift. In [Mie11] and [Maa11] it was shown that Markov chains satisfying the reversibility condition (also called detailed balance condition) have entropic gradient structures. More precisely, the evolution of a reversible Markov chain on the finite state space $\{1, \ldots, n\}$ and with intensity matrix $\mathbb{A}_n$ can be written as

$$\dot{u} = \mathbb{A}_n u = -\mathbb{K}_n(u)\mathrm{D}E_n(u). \tag{1}$$

Here, the driving functional E_n is the relative entropy and $\mathbb{K}_n(u) = \mathbb{G}_n(u)^{-1}$ denotes the state-dependent, symmetric, and positive semi-definite Onsager matrix, which is the inverse of the metric tensor $\mathbb{G}_n(u)$.

In particular, reversible Markov chains arise as finite volume discretizations of the Fokker–Planck equation. Using the entropy/entropy-dissipation formulation of (1) we show that solutions of (1) converge to a solution of the Wasserstein formulation of the Fokker–Planck equation when the fineness of the partitions goes to zero. Here, we only use the gradient structures of the systems and prove a Γ-convergence result for the relative entropy and dissipation potentials. Finally, we address the question of a generalization to higher dimenions.

References

[Maa11] J. Maas. Gradient flows of the entropy for finite Markov chains. *J. Funct.Anal.*, 261, no. 8, 2250–2292, 2011.

[Mie11] A. Mielke. A gradient structure for reaction-diffusion systems and for energy-drift-diffusion systems. *Nonlinearity*, 24, 1329–1346, 2011.