Humboldt-Universität zu Berlin

Weierstraß-Institut für Angew. Analysis und Stochastik

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Forschungsseminars
Mathematische Statistik

Randolf Altmeyer / Markus Reiß
(HU Berlin)

zu dem Thema

A nonparametric estimation problem for linear SPDEs

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Erhard-Schmidt Hörsaal, Mohrenstraße 39, 10117 Berlin

Abstract: It is well-known that parameters in the drift part of a stochastic *ordinary* differential equation, observed continuously on a time interval $[0, T]$, are generally only identifiable, if either $T \to \infty$, the driving noise becomes small or if a sequence of independent samples is observed. On the other hand, in the case of a linear stochastic *partial* differential equation

$$dX(t,x) = \vartheta AX(t,x)dt + dW(t,x), \quad x \in \Omega \subset \mathbb{R}^d, \tag{1}$$

for a nonpositive self-adjoint operator A and an unknown parameter $\vartheta > 0$, [1] showed that consistent estimation of ϑ is also possible in finite time $T < \infty$, if $\langle X(t,\cdot), e_k\rangle$ is observed continuously on $[0, T]$ for $k = 1, \ldots, N$ as $N \to \infty$, where the test functions e_k are the eigenfunctions of A.
Our goal is to study this estimation problem for general test functions e_k. Using an MLE-inspired estimator, we extend the results of [1] and give a precise understanding of how the estimation error depends on the interplay between A and the test functions e_k. In particular, we show that more localized test functions improve the estimation considerably. It turns out that one local measurement $\langle X(t,\cdot), u_h\rangle$ is already sufficient for identifying ϑ, as long as $h \to 0$, where $u_h(x) = h^{-d/2}u(x/h)$ for a smooth kernel u. Central limit theorems are provided, as well. We further show that the same techniques extend to the more difficult nonparametric estimation problem, when ϑ is space-dependent. Indeed, we can show that $\vartheta(x_0)$ at $x_0 \in \Omega$ is identifiable using only local information. The rate of convergence, however, is affected by the bias, which is non-local and difficult to analyse, even when $T \to \infty$. Possible solutions are discussed, along with questions of efficiency.

References

[1] M. Huebner and B.L. Rozovskii. On asymptotic properties of maximum likelihood estimators for parabolic stochastic PDE's. *Probability theory and related fields*, **103**, 1995, 143-163.

Interessenten sind herzlich eingeladen!

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