Am Weierstraß-Institut für Angewandte Analysis und Stochastik spricht im Rahmen des Forschungsseminars Mathematische Statistik

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zu dem Thema

A nonparametric estimation problem for linear SPDEs

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Abstract: It is well-known that parameters in the drift part of a stochastic ordinary differential equation, observed continuously on a time interval $[0, T]$, are generally only identifiable, if either $T \to \infty$, the driving noise becomes small or if a sequence of independent samples is observed. On the other hand, in the case of a linear stochastic partial differential equation

$$dX(t, x) = \vartheta AX(t, x)dt + dW(t, x), \quad x \in \Omega \subset \mathbb{R}^d,$$

(1)

for a nonpositive self-adjoint operator $A$ and an unknown parameter $\vartheta > 0$, [1] showed that consistent estimation of $\vartheta$ is also possible in finite time $T < \infty$, if $\langle X(t, \cdot), e_k \rangle$ is observed continuously on $[0, T]$ for $k = 1, \ldots, N$ as $N \to \infty$, where the test functions $e_k$ are the eigenfunctions of $A$.

Our goal is to study this estimation problem for general test functions $e_k$. Using an MLE-inspired estimator, we extend the results of [1] and give a precise understanding of how the estimation error depends on the interplay between $A$ and the test functions $e_k$. In particular, we show that more localized test functions improve the estimation considerably. It turns out that one local measurement $\langle X(t, \cdot), u_h \rangle$ is already sufficient for identifying $\vartheta$, as long as $h \to 0$, where $u_h(x) = h^{-d/2} u(x/h)$ for a smooth kernel $u$. Central limit theorems are provided, as well. We further show that the same techniques extend to the more difficult nonparametric estimation problem, when $\vartheta$ is space-dependent. Indeed, we can show that $\vartheta(x_0)$ at $x_0 \in \Omega$ is identifiable using only local information. The rate of convergence, however, is affected by the bias, which is non-local and difficult to analyse, even when $T \to \infty$. Possible solutions are discussed, along with questions of efficiency.

References

Interessenten sind herzlich eingeladen!

gez. Prof. Dr. G. Blanchard
Prof. Dr. W. Härdle
Prof. Dr. M. Reiβ
Prof. Dr. V. Spokoiny