

Efficient and reliable numerical approximation of transport equations with small diffusion

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Nonstationary and nonlinear systems of coupled convection-diffusion-reaction equations

$$\partial_t u_i + \sum_{j=1}^m \mathbf{b}^{ij} \cdot \nabla u_j - \nabla \cdot (\mathbf{A}_i \nabla u_i) + r_i(\mathbf{u}) = f_i, \quad \text{for } i = 1, \dots, m, \quad (1)$$

are studied in various environmental and technical applications. Accurate, reliable and efficient simulations of solutions of the equations are often desirable in technical and environmental studies and engineering and are important from the point of view of physical realism.

In applications of practical interest, the size of the diffusion in (1) is often smaller by several orders of magnitude compared to the size of the flow field. In such cases the system (1) admits solutions with sharp moving fronts, interior or boundary layers and complicated structures where important physical and chemical phenomena take place. Then, the numerical approximation of the set of equations becomes a challenging task.

In this talk higher order finite element approximations of the model system with streamline upwind Petrov-Galerkin (SUPG) stabilization and additional shock capturing stabilization are studied carefully. Error analyses as well as numerical performance tests are presented and discussed. It is illustrated that spurious oscillations as well as over- and undershoots caused by convection-dominance can be reduced significantly by the proposed approach. Generalized systems of stronger coupled equations are further addressed.