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ANISOTROPIC ERROR ESTIMATES FOR PDE DEFINED ON SURFACES

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Key words: anisotropic meshes, error estimators, finite elements

Abstract.

There are many practical differential problems related to the resolution of Partial Differential Equations (PDEs) defined on surfaces embedded in $\mathbb{R}^3$, such as mean curvature flow surface [5], diffusion flow [6], Willmore flow [7], etc.. Since this kind of problem the domain deals with a two dimensional surface embedded in the three dimensional spaces, *all* the classical differential operators have to be suitably modified to take into account the curvature of the computational domain.

In this framework we consider the so called Laplace-Beltrami equation, defined [1]:

$$\left\{\begin{array}{rcll} -\Delta_\Gamma u & = & f & \text{on } \Gamma \\ u & = & 0 & \text{on } \partial\Gamma \end{array}\right. , \qquad (1)$$

where Γ is an arbitrary two-dimensional surface embedded in $\mathbb{R}^3$ and $f : \Gamma \Rightarrow \mathbb{R}$. We generalize the theory provided in [2], [3], [4] and we provided an *anisotropic error estimator* for problem as (1). This new error estimator consists in three different contributions:

- an *almost best-approximation term*, typical of a finite element discretization;
- a *geometric error term*, related to the discretization of the surface;
- a *data approximation term*, due to the approximation of f on the discretization of Γ.

Moving from to this estimator, we settle a metric-based anisotropic mesh adaptation procedure which essentially employs local operations (node smoothing, edge collapsing, edge splitting and edge flipping) to adapt the initial mesh. Since an anisotropic estimator takes into account the directional features of the solution at hand, we will obtain adapted meshes whose elements are suitably oriented to match the intrinsic directionality of the function defined on the surface, and of the surface itself.

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