A gradient flow approach to large deviations for diffusion processes

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In the 80s, De Giorgi introduced the notion of abstract gradient flows, which allowed to define a notion of solutions to ordinary differential equations of the form $\dot{x} = -\nabla F(x)$ on metric spaces (rather than Riemannian manifolds for the usual definition). In 2005, Ambrosio, Gigli and Savaré showed that when we consider the space of probability measures on $\mathbb{R}^d$ endowed with the Wasserstein metric, this notion allows to give an alternate formulation for Fokker-Planck equations. These equations are the PDEs whose solutions are the flow of marginals of solutions of stochastic differential equations of the form

$$dX_t = -\nabla H(X_t)dt + \sqrt{2}dB_t.$$

In this talk, I will explain how we can use this notion to study large deviations for sequences of SDEs. The main result is that proving a large deviation principle is equivalent to studying the limit of a sequence of functionals that appear in the abstract gradient flow formulation for Fokker-Planck equations. As an application, I will show how to obtain large deviations from the hydrodynamic scaling limit for a system of interacting continuous spins in a random environment.