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On the velocity of the Biot slow wave in a porous medium: Uniform asymptotic expansion

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Abstract

Asymptotic behavior of the Biot slow wave is investigated. Formulae for short- and long-wave approximations of phase velocity of the P2 wave are presented. These asymptotic expansions are compared with exact solution, constructed numerically. It is shown that both expansions fit very well the real velocity of the P2 mode. Procedure for matching of short- and long-wave asymptotic expansions is suggested.

Introduction

This paper develops the ideas of the work presented in [1]. Let us remind that paper [1] is devoted to the asymptotic analysis of the velocity of the Biot slow (P2) wave in a porous medium. Asymptotic expansion, describing the speed of P2 wave in a low frequency range, was constructed with respect to small wave numbers. Bifurcation behavior of P2 wave depending on its wave number was revealed: it was proven analytically that longitudinal wave of the second kind is not propagatory (fully attenuated mode) if its wave number is lower than some critical value. This critical wave number is a bifurcation point, above which longitudinal wave of the second kind begins to propagate. It was shown that slow wave behavior is dominated by permeability of a medium.

The focus of this paper is on the research of domains of validity of short- and long-wave approximations for the phase velocity of the Biot slow wave. On the base of comparison of the asymptotic results with those obtained numerically, the matched asymptotic expansion, which gives a uniformly valid approximation for the speed of P2 wave, is constructed.

1. Mathematical Model

Consider propagation of the bulk waves through an infinite space Ω occupied by a saturated porous medium. In dimensionless variables the set of balance equations describing a fluid-filled porous medium has the following general form ($x \in \Omega,\ t \in [0,T]$) [2,3]:

Mass conservation equations

$$\frac{\partial \rho^F}{\partial t} + \mathrm{div}\big(\rho^F \mathbf{v}^F\big) = 0,$$

$$\frac{\partial \rho^S}{\partial t} + \mathrm{div}(\rho^S \mathbf{v}^S) = 0. \tag{1.1}$$

Here, ρ is the partial mass density, $\mathbf{v}$ is the velocity vector and indices F and S indicate fluid and solid phases, respectively.

Momentum conservation equations

$$\rho^F \left[\frac{\partial}{\partial t} + (v_j^F, \frac{\partial}{\partial x_j})\right] v_i^F - \frac{\partial}{\partial x_j} T_{ij}^F + \Pi(v_i^F - v_i^S) = 0,$$

$$\rho^S \left[\frac{\partial}{\partial t} + (v_j^S, \frac{\partial}{\partial x_j})\right] v_i^S - \frac{\partial}{\partial x_j} T_{ij}^S - \Pi(v_i^F - v_i^S) = 0. \tag{1.2}$$

Here $\mathbf{T}^F$ and $\mathbf{T}^S$ are the partial stress tensors and Π is a positive constant.

Balance equation for the porosity

$$\frac{\partial n}{\partial t} + \big(v_i^S, \frac{\partial}{\partial x_i}\big) n + n_0 \mathrm{div}(\mathbf{v}^F - \mathbf{v}^S) = -(n - n_0), \tag{1.3}$$

where n is the porosity and n_0 is its initial value, assumed to be constant. Stress tensors have the form:

$$\mathbf{T}^F = -p^F \mathbf{1} - \beta(n - n_0)\mathbf{1}, \quad p^F = p_0^F + \kappa(\rho^F - \rho_0^F), \tag{1.4}$$

$$\mathbf{T}^S = \mathbf{T}_0^S + \lambda^S \mathrm{div}\mathbf{u}^S \mathbf{1} + 2\mu^S \mathrm{symgrad}\mathbf{u}^S + \beta(n - n_0)\mathbf{1}. \tag{1.5}$$

Here p^F is the pore pressure; p_0^F and ρ_0^F are the initial values of pore pressure and fluid mass density, respectively; κ is the constant compressibility coefficient of the fluid; β denotes the coupling coefficient of the fluid and solid components; $\mathbf{T}_0^S$ denotes a constant reference value of the partial stress tensor in the skeleton, λ^S and μ^S are the Lamé constants of the skeleton; $\mathbf{u}^S$ is the displacement vector for the solid phase with

$$\mathbf{v}^S = \frac{\partial \mathbf{u}^S}{\partial t}. \tag{1.6}$$

2. Short- and Long-Wave Approximations for the Phase Velocity of the Biot Slow Wave

Let us study the bulk longitudinal waves in an unbounded fluid-filled porous medium (1D case). We focus on the Biot slow wave.

Consider the propagation of the harmonic waves whose frequency is ω and wave number is k. Following standard procedure one obtains the dispersion equation [1,4]:

$$r(\omega^2 - c_f^2 k^2)(\omega^2 - k^2) + i\omega\Pi\left((1+r)\omega^2 - k^2(1 + rc_f^2)\right) = 0. \tag{2.1}$$

Here $r = \rho_0^F/\rho_0^S$ and $c_f = U^F/U_{||}^S$, where $U^F = \sqrt{\kappa}$ is a sound velocity in a fluid and $U_{||}^S = \sqrt{(\lambda^S + 2\mu^S)/\rho_0^S}$ is a velocity of a longitudinal wave in an unbounded elastic medium.

In should be reminded that similar to our previous research [1,3,4], we consider the propagation of elastic bulk waves through an infinite space in the absence of external forces, so that solutions for the system (1.1)-(1.3) are defined uniquely by the Cauchy data (initial value problem). In this case one must set the wave number $k \in R^1$ to be real and define frequency $\omega = \omega(k)$, which can be complex, as a solution of dispersion equation (2.1). Thus, $\mathrm{Re}\omega/k$ defines the phase velocity of a wave and $\mathrm{Im}\omega$ gives its attenuation.

If $k \gg 1$ (high frequency range), the asymptotic expansion for the root of (2.1), corresponding to P2 wave, has the form (note that below $\tilde{\omega} = \omega/k$ and $\tilde{k} = k/\Pi$) [1,4]:

$$\tilde{\omega}_{P2} = \pm c_f - \frac{i}{2r}\frac{1}{\tilde{k}} - \frac{1 - c_f^2(1+4r)}{8r^2(1-c_f^2)(\pm c_f)}\frac{1}{\tilde{k}^2} + O\Big(\frac{1}{\tilde{k}^3}\Big). \tag{2.2}$$

It defines the velocity and attenuation of forward and backward directed P2 wave. Obviously, phase velocity of forward directed P2 wave in high frequency range is given by the following approximate formula:

$$c_{P2}^h \approx c_f - \frac{1 - c_f^2(1+4r)}{8r^2(1-c_f^2)c_f}\frac{1}{\tilde{k}^2} \tag{2.3}$$

If $k \ll 1$ (low frequency range), one obtains for forward and backward directed P2 wave, respectively:

$$\tilde{\omega}_{P2}^f = -i\frac{rc_f^2}{1+rc_f^2}\tilde{k} - i\frac{r^3c_f^4(1+rc_f^4)}{(1+rc_f^2)^4}\tilde{k}^3 + O(\tilde{k}^4), \tag{2.4}$$

$$\tilde{\omega}_{P2}^b = -i\frac{1+r}{r}\frac{1}{\tilde{k}} + i\frac{r(r+c_f)}{(1+r)^2}\tilde{k} + O(\tilde{k}^2). \tag{2.5}$$

Expansions (2.4), (2.5) consist of the imaginary terms only, i.e. the phase velocity of P2 wave is equal to zero and the wave is fully attenuated. However, as it was proven in [1,4], asymptotics (2.3), (2.4) are valid only if wave number k is less then some critical value k_{cr}, i.e. k_{cr} is a bifurcation point in small neighborhood of which solution of equation (2.1) splits into several branches. Critical wave number k_{cr} and corresponding critical frequency are defined asymptotically:

$$k_{cr} \approx c_f\Big(1 + \frac{1}{2rc_f^2}\Big)\Pi, \tag{2.6}$$

$$\omega_{cr} = -i\Pi\Omega_{cr}, \qquad \Omega_{cr} \approx \frac{1}{2r} + 2c_f^2(1 + 3rc_f^2 - 2c_f^2). \tag{2.7}$$

If $k > k_{cr}$, then P2 mode becomes to be propagatory. For any small parameter ϵ and wave number

$$k = k_{cr}\big(1 + \epsilon^2 k_2\big) + \Pi O(\epsilon^3) \tag{2.8}$$

asymptotic expansion for its frequency has the form:

$$\omega_{P2} = \omega_{cr} + \epsilon\omega_1 + \Pi O(\epsilon^2), \qquad \omega_1 = 2k_{cr}\sqrt{k_2/\mathcal{A}} \tag{2.9}$$

where

$$\mathcal{A} = \frac{1 + c_f^2}{c_f^2} + \frac{1 - c_f^2}{c_f^2 g(\Omega_{cr})\sqrt{g(\Omega_{cr})}}\Big(- r^3(1 - c_f^2)^3\Omega_{cr}^3 + 3r^2(1 - c_f^2)^2(1 - rc_f^2)\Omega_{cr}^2$$

$$-3r(1 - c_f^2)(1 + r^2c_f^4)\Omega_{cr} + (1 - rc_f^2)(1 + rc_f^2)^2\Big) > 0,$$

$$g(\Omega) = \Omega^2 r^2(1 - c_f^2)^2 - 2r\Omega(1 - c_f^2)(1 - rc_f^2) + (1 + rc_f^2)^2$$

and $k_2 = O(1)$ with respect to small parameter ϵ.

Therefore, phase velocity of forward directed P2 wave in low frequency range is given by the following approximate formula:

$$c_{P2}^l \approx \epsilon\omega_1/k, \tag{2.10}$$

where ω_1 and k are defined in (2.9) and (2.8), respectively.

Let us note that it is desirable to have an asymptotics, which is valid uniformly everywhere. Thus, construction of matched asymptotic expansion is required.

3. Matching of Long- and Short-Wave Asymptotics

In order to determine an asymptotic expansion for the velocity of the Biot slow wave that is uniformly valid throughout the k-domain of interest, one should investigate domains of validity for the short- and long-wave approximations (2.3) and (2.10) and construct matched asymptotic expansion. If two valid approximations to some function have overlapping domains of validity, they are said to match. While the principles of matching are clear, it is difficult to determine *a priory* the domain of validity of an approximation [5,6]. These ideas are best illustrated by examples, which will be given below.

Let us compare expansions (2.3), (2.10) with the corresponding exact solution of dispersion equation (2.1). It is easy to see that exact solution for the frequency of P2 wave is given by the following implicit formula:

$$k^2 = \frac{1}{2rc_f^2}\Big(r\omega^2(1+c_f^2) + i\Pi\omega(1+rc_f^2)$$
$$+\sqrt{r^2\omega^4(1-c_f^2)^2 - \Pi^2\omega^2(1+rc_f^2)^2 + 2ir\Pi\omega^3(1-c_f^2)(1-rc_f^2)}\,\Big) \tag{3.1}$$

Numerical solution for the velocity of P2 wave $c_{P2} = \mathrm{Re}(\omega)/k$, where frequency $\omega = \omega(k)$ is calculated from (3.1), is presented in Fig.1.

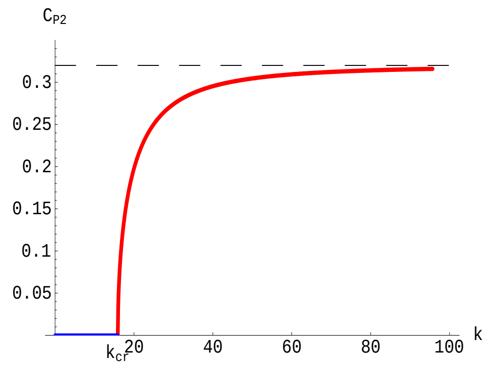

Figure 1: Phase velocity of the P2 wave: $r = 0.1,\ c_f = 0.32,\ \Pi = 1;\ k_{cr} \approx 15.9$

Definition. We say that asymptotics c^l_{P2} and c^h_{P2} are valid in the domains D_1 and D_2 respectively (we refer D_1 and D_2 as domains of relative validity), if in these domains

$$\left|\frac{c^l_{P2} - c_{P2}}{c_{P2}}\right| \le a_0 \quad \text{and} \quad \left|\frac{c^h_{P2} - c_{P2}}{c_{P2}}\right| \le a_0, \tag{3.2}$$

where a_0 is some constant. There exists matched asymptotic expansion with relative accuracy $a_0 \cdot 100\%$, if $D_1 \cap D_2 \neq \emptyset$.

For instance, we may set $a_0 = 0.1$, i.e. relative error does not exceed 10%.

In Fig.2 exact solution c_{P2}, constructed numerically, is compared with the long-wave asymptotics (2.10) (on the left) and with the short-wave asymptotics (2.3) (on the right). Evidently, before the intersection point the curve, labeled c^l_{P2} (long-wave approximation), fits rather well the curve c_{P2} (numerical solution of dispersion equation (2.1), heavy solid line). Comparing solutions c_{P2} and c^l_{P2} point by point, one can define the domain of validity D_1 of the long-wave approximation: $D_1 = \{k_{cr} \le k \le 23\}$.

>From Fig.2 it is obvious also, that the short-wave asymptotics (2.3) (curve, labeled c^h_{P2}) approximates very well the numerical solution c_{P2} (heavy solid line). Also

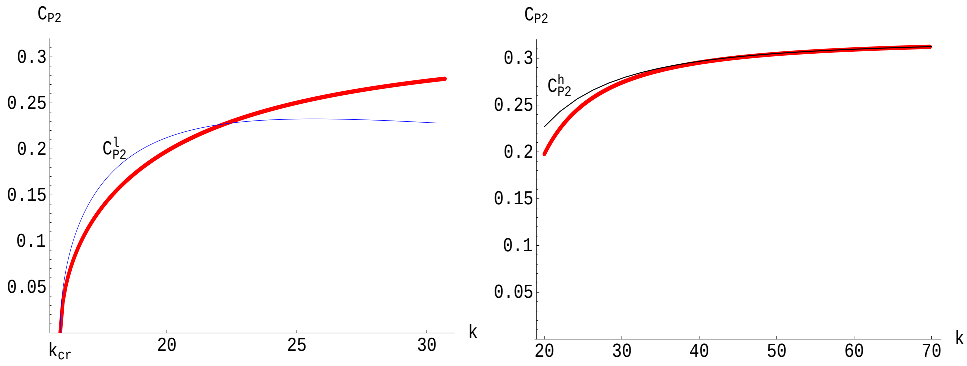

Figure 2: Phase velocity of the P2 wave: comparison of numerical solution c_{P2} (heavy solid line) with the long-wave approximation c^l_{P2} (left) and with the short-wave approximation c^h_{P2} (right); as above $r = 0.1$, $c_f = 0.32$, $\Pi = 1$, and $\epsilon = c_f$

comparing solutions c_{P2} and c^h_{P2} point by point, one determines the domain of validity D_2 of the short-wave approximation: $D_2 = \{22 \leq k\}$.

Closer inspection of the plots in Fig.2 shows that there exists the overlap domain $D = D_1 \cap D_2$, where both long- and short-wave approximations are valid. This gives us a clue to the matching of these asymptotics. Namely, the following function is proposed as matched asymptotic solution:

$$c^m_{P2} = f_0(k)c^l_{P2} + \big(1 - f_0(k)\big)c^h_{P2}, \tag{3.3}$$

where

$$f_0(k) = \frac{1}{2}\Big(1 - \tanh\big(\frac{k - k_0}{\xi}\big)\Big), \quad k_0 \in D, \tag{3.4}$$

and ξ is a matching parameter. Matched asymptotics c^m_{P2} is uniformly valid throughout the k-domain of interest. Comparison of this matched solution with the numerical one (dashed line) is presented in Fig.3. Evidently, c^m_{P2} is in excellent agreement with c_{P2}.

Comment 1. Slight deviation of matched approximation from the numerical solution in the vicinity of $k \approx 24$ (see Fig.3) is caused by the fact that only two terms of asympotics were constructed for the long-wave approximation c^l_{P2}. Thus, one has to take into account the next term of asymptotic expansion for c^l_{P2}. This will certainly improve the accuracy of long-wave approximation c^l_{P2} and, consequently, will result in better agreement between c^l_{P2} and exact solution c_{P2} as well as between matched approximation c^m_{P2} and c_{P2}. It should be noted that one can construct asymptotic expansion c^l_{P2} with any accuracy.

Comment 2. Parameter a_0 is selected so that $D \neq \emptyset$. Obviously, choice of a_0 affects accuracy of an approximation.

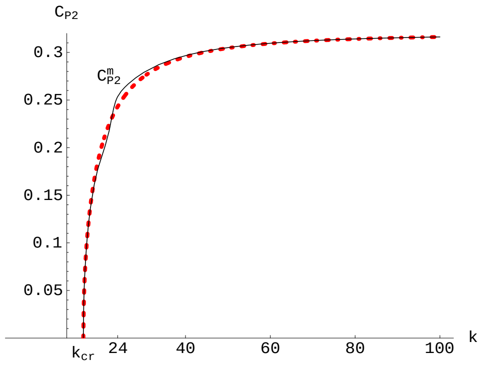

Figure 3: Phase velocity of the P2 wave: comparison of matched approximation c^m_{P2} (thin curve) with the numerical solution c_{P2} (dashed curve); as above $r = 0.1,\ c_f = 0.32,\ \Pi = 1,\ \epsilon = c_f;\ \ k_0 = 22.5,\ \ \xi = 1$

Comment 3. Usually, more complicated technique is applied for matching of outer and inner expansions. Namely, the outer expansion is expressed in terms of the inner variable and re-expanded. It should agree with the inner expansion evaluated for large variable (in our case this variable is k). We use simpler procedure for matching of outer and inner expansions because of the fact that only two first terms in expansions c^l_{P2} and c^h_{P2} were derived, that is not enough for usual technique. Moreover, in order to obtain uniform asymptotic solution for the system (1.1)-(1.3) in the weak sense, it suffices to construct matched asymptotics in norm C^0. Examples of such construction are given in this paper.

Above example was analyzed for the following dimensionless parameters, entering dispersion equation (2.1): $r = 0.1,\ c_f = 0.32,\ \Pi = 1$. They correspond to the following real parameters of the porous saturated medium (water-saturated sandstone) [7]:

porosity: $n = 0.2$;

initial value of the partial mass density of the fluid phase: $\rho^F_0 = 0.2 \cdot 10^3\ \frac{kg}{m^3}$ (note, that $\rho^F_0 \approx n\rho^{FR}_0$, where ρ^{FR}_0 is the initial value of the real mass density of the fluid phase, $\rho^{FR}_0 = 10^3\ \frac{kg}{m^3}$);

initial value of the partial mass density of the solid phase: $\rho^S_0 = 2.0 \cdot 10^3\ \frac{kg}{m^3}$ (note, that $\rho^S_0 \approx (1-n)\rho^{SR}_0$, where ρ^{SR}_0 is the initial value of the real mass density of the solid phase, $\rho^{SR}_0 = 2.65 \cdot 10^3\ \frac{kg}{m^3}$);

sound velocity in the fluid: $U^F = 1450\frac{m}{s}$;

velocity of the longitudinal wave in the skeleton: $U^S_{||} = 4450\frac{m}{s}$;

$\Pi = \mu^f/\mathcal{K}$ (as it follows from the dimensional analysis), where μ^f is a viscosity of a liquid, and $\mathcal{K}$ is a permeability of a porous medium: $\mathcal{K} = 1000\ mDarcy$, $\mu^F = 10^{-3}\ \frac{kg}{m \cdot s}$.

In Fig.4 phase velocity of the P2 wave is evaluated in physical variables. The following observation can be made: wave length λ of propagatory modes is very small, namely $\lambda < \lambda_{cr} = 0.4\ cm$. Thus, we can conclude that in fact P2 waves do not exist since their wave length is smaller than the characteristic size of the medium. This result may have the following explanation. From (2.6) it is clear that k_{cr} is directly proportional to parameter Π, entering momentum balances (1.2). In the model under research (1.1)-(1.3) Π is assumed to be constant. Traditionally, as in the classical Biot model [8], $\Pi = \mu^f/\mathcal{K}$, where permeability of a porous medium $\mathcal{K}$ is calculated from the measurements of the flow rate and pressure drop on the base of the Darcy law (at low frequencies) or on the base of the Forchheimer law (at high frequencies). In considered above example (water-saturated sandstone) one has $\Pi = 10^9\ \frac{kg}{m^3 \cdot s}$, that results in big values for k_{cr}. Consequently, we may presuppose that Π in the form $\Pi = \mu^f/\mathcal{K}$ with permeability $\mathcal{K}$, obtained by standard methods, cannot be used for the model (1.1)-(1.3).

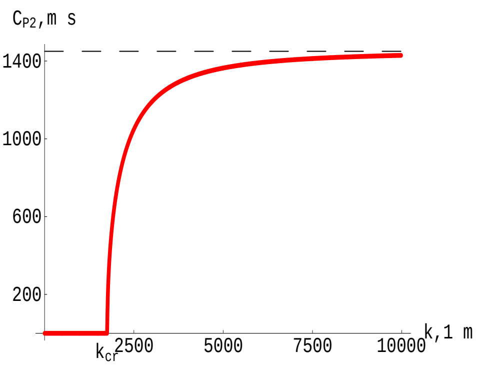

Figure 4: Phase velocity of the P2 wave, evaluated in physical variables: $k_{cr} \approx 1755.54\ m^{-1}$ (water-saturated sandstone)

Next let us consider another example of practical importance, namely sandstone, saturated by normal oil. In this case $\rho_0^F = 1.2 \cdot 10^2\ \frac{kg}{m^3}$ ($\rho_0^{FR} = 0.6 \cdot 10^3\ \frac{kg}{m^3}$), $U^F = 1200\frac{m}{s}$; $\mu^F = 10^{-2}\ \frac{kg}{m \cdot s}$ [7]. The other parameters remain as in the previous example. Thus, corresponding dimensionless parameters take the following values: $r = 0.06$, $c_f = 0.27$, $\Pi = 10$. In Figs.5-7 numerical solution for the velocity of the P2 wave and its comparison with long- and short-wave asymptotics as well as with matched asymptotic expansion are presented. Fig.6 demonstrates, that both long- and short-wave asymptotics approximate very well the numerical solution c_{P2}. Existence of the overlap domain, where both long- and short-wave approximations are valid, allows us to construct matched asymptotic expansion (see Fig.7). As in the first example, deviation of matched approximation from the numerical solution is caused by the fact that only two terms of asympotics were constructed for the long-wave approximation c_{P2}^l. In order to improve an accuracy, one has to take into account the next term of asymptotic expansion for c_{P2}^l.

In Fig.8 one can see phase velocity of the P2 wave, evaluated in dimensional variables. As in the first example, we conclude that in fact P2 waves do not exist since

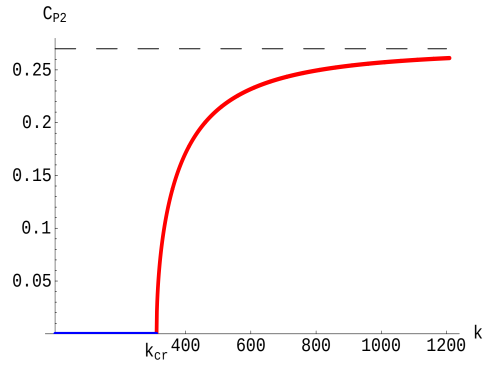

Figure 5: Phase velocity of the P2 wave: $r = 0.06,\ c_f = 0.27,\ \Pi = 10;\ k_{cr} \approx 311.1$

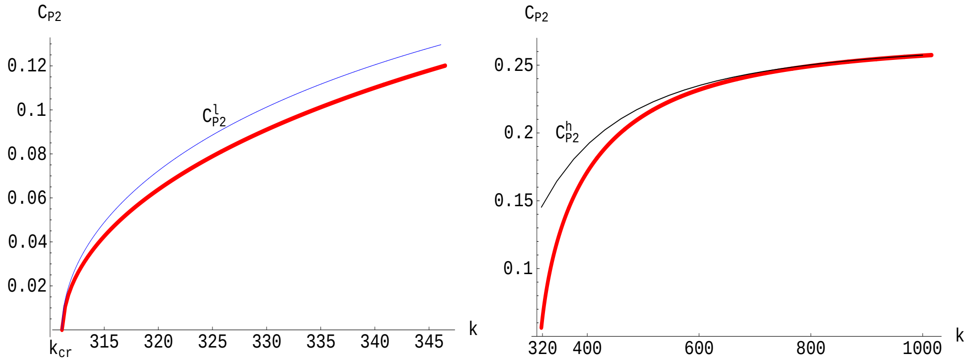

Figure 6: Phase velocity of the P2 wave: comparison of numerical solution c_{P2} (heavy solid line) with the long-wave approximation c^l_{P2} (left) and with the short-wave approximation c^h_{P2} (right); $r = 0.06,\ c_f = 0.27,\ \Pi = 10$, and $\epsilon = c_f^2$

wave length is much smaller than the characteristic size of the medium.

Comment 4. General solution of the Cauchy problem for the system (1.1)-(1.3) has the form:

$$U(x,t) = \sum_{j=1}^{4} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{g}_j(k) \exp\big(i(kx - \omega(k)t)\big)\, dk, \tag{3.5}$$

where $U(x,t)$ is a vector-function of the unknown variables ρ^S, ρ^F, v^S, v^F and vectors $\widehat{g}_j(k)$ are the linear combinations of the Fourier transforms of the initial data. Let us consider the part of the solution (3.5), which corresponds to the Biot slow wave:

$$U_{P2}(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{g}_{P2}(k) \exp\big(i(kx - \omega_{P2}(k)t)\big)\, dk. \tag{3.6}$$

One can prove that for sufficiently large time t (more precisely for $\Pi t \gg 1$) the main input to this integral is due to its value in the vicinity I_0 of $k = 0$, where $\mathrm{Re}\omega_{P2} = 0$ and $\mathrm{Im}\omega_{P2} = -\frac{rc_f^2}{1+rc_f^2}\frac{k^2}{\Pi} + O(k^3)$ (see (2.4)). Thus, one finds the leading term of

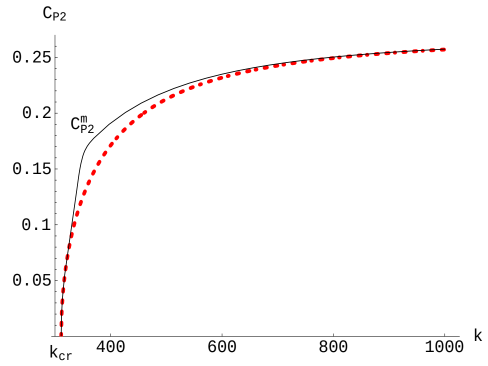

Figure 7: Phase velocity of the P2 wave: comparison of matched approximation c^m_{P2} (thin curve) with the numerical solution c_{P2} (dashed curve); $r = 0.06$, $c_f = 0.27$, $\Pi = 10$, $\epsilon = c_f^2$; $k_0 = 340$, $\xi = 10$

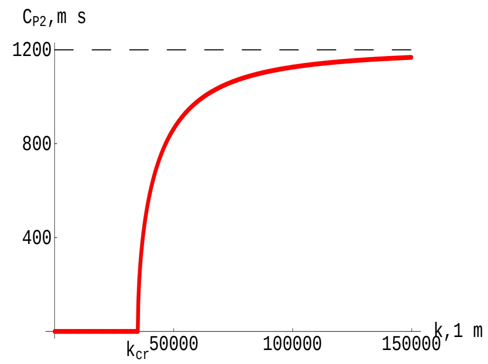

Figure 8: Phase velocity of the P2 wave, evaluated in physical variables: $k_{cr} \approx 34989.45\ m^{-1}$ (oil-saturated sandstone)

asymptotic expansion for (3.6):

$$U^{as}_{P2}(x,t) = \frac{1}{\sqrt{2\pi}} \int_{I_0} \widehat{g}_{P2}(k) \exp\big(ikx - \frac{rc_f^2}{1+rc_f^2} \frac{k^2}{\Pi} t\big) dk. \tag{3.7}$$

It is not difficult to show that $U^{as}_{P2}(x,t)$ satisfies the following parabolic equation

$$\frac{\partial}{\partial t} U^{as}_{P2}(x,t) = \frac{rc_f^2}{1+rc_f^2} \frac{1}{\Pi} \frac{\partial^2}{\partial x^2} U^{as}_{P2}(x,t), \tag{3.8}$$

with the initial data

$$U^{as}_{P2}(x,t)_{|t=0} = g_{P2}(x).$$

Thus, for t large limit behavior of the P2 wave is not hyperbolic. It is described by the parabolic equation (3.8) with the diffusion coefficient $D_{P2} = \frac{rc_f^2}{1+rc_f^2} \frac{1}{\Pi}$. Obviously,

D_{P2} differs from the corresponding coefficient in the Darcy law, which equals $\frac{1}{\Pi}$. Parabolic behavior of the Biot slow wave near $k = 0$ arises from the fact that there exists k-domain, where $\mathrm{Re}\omega_{P2} = 0$.

4. Conclusions

The results presented in the paper concern construction of uniform asymptotic expansion for the velocity of the Biot slow wave. It was shown that derived earlier long- and short-wave asymptotics for the velocity of the P2 wave [1] approximate well exact solution. Moreover, existence of the overlap domain, where both long- and short-wave approximations are valid, allowed one to construct matched asymptotic expansion. Procedure for the construction of uniform asymptotic expansion with prescribed accuracy was suggested. It was demonstrated graphically that matched asymptotics results in rather good approximation of exact solution.

Acknowledgments

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