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On modification of the Newton's law of gravity at very large distances

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ABSTRACT. We discuss a Modified Field Theory (MOFT) in which the number of fields can vary. It is shown that when the number of fields is conserved MOFT reduces to the standard field theory but interaction constants undergo an additional renormalization and acquire a dependence on spatial scales. In particular, the renormalization of the gravitational constant leads to the deviation of the law of gravity from the Newton's law in some range of scales $r_{\min} < r < r_{\max}$, in which the gravitational potential shows essentially logarithmic $\sim \ln r$ (instead of $1/r$) behavior. In this range, the renormalized value of the gravitational constant G increases and at scales $r > r_{\max}$ acquires a new constant value $G' \sim G r_{\max}/r_{\min}$. From the dynamical standpoint this looks as if every point source is surrounded with a halo of dark matter. It is also shown that if the maximal scale $r_{\max}$ is absent, the homogeneity of the dark matter in the Universe is consistent with a fractal distribution of baryons in space, in which the luminous matter is located on thin two-dimensional surfaces separated by empty regions of ever growing size.

1. Introduction

It is well established that dark matter gives the leading contribution to the matter density of the Universe (e.g., see [1]). Apart from some phenomenological properties of the dark matter (e.g., it is non-baryonic, cold, etc.) the problem of its nature remains still open. Particle physics suggests various hypothetical candidates for dark matter. However, while we do not observe such particles in direct laboratory experiments there remains the possibility to avoid or replace the dark matter paradigm. The best known attempt of such kind is represented by the phenomenological algorithm suggested by Milgrom [2], the so-called MOND (Modified Newtonian Dynamics). This algorithm suggests replacing the Newton's law of gravity in the low acceleration limit $g \ll a_0$ with $g_{\mathrm{MOND}} \sim \sqrt{g a_0}$ (where g is the gravitation acceleration and a_0 is a fundamental acceleration $a_0 \sim 2 \times 10^{-8} cm/\, s^2$). This, by construction, accounts for the two observational facts: the flat rotation curves of galaxies and the Tully-Fisher relation $L_{gal} \propto v_c^4$ which gives $M_{gal} \propto L_{gal} \propto v_c^4$ (where L_{gal}, M_{gal}, and v_c are the galaxy's luminosity, mass, and rotation velocity respectively). The MOND was shown to be successful in explaining properties of galaxies and clusters of galaxies [3] and different aspects of MOND attract the more increasing attention, e.g., see Refs.[4, 5, 7]. However, MOND presumes a nonlinear regime (e.g., at low accelerations the force $F \propto \sqrt{M}$) and was criticized in Ref. [7].

A more conservative model was suggested in Ref. [8] which presumes the existence of an additional attraction between baryons with logarithmic potential

$$U \propto \Lambda b_1 b_2 \ln(r) \qquad (1)$$

where b is the baryon number and Λ defines a characteristic scale $r_0 \sim 1/\Lambda \sim 5kpc$ on which this potential starts to dominate over gravity. This model contains basic features of MOND (at least roughly) but fails when confronting with gravitational lensing by clusters. To explain lensing this extra force must act like gravity and, in fact, be gravity. We note that if this additional potential cuts off at very large distances, the effects of the extra potential will not, in fact, be distinguishable from that of dark matter

It appears that the standard particle physics does not possess fields able to produce interactions which has a range of scales with logarithmic behavior. In the present paper we, however, show that the logarithmic potentials appear naturally in the so-called Modified Field Theory (MOFT) suggested in Ref. [9] to account for the spacetime foam effects.

It was suggested in Ref. [9] that nontrivial topology of space should display itself in the multivalued nature of all observable fields, i.e. the number of fields should be a dynamical variable. The argument is that in the case of general position an arbitrary quantum state mixes different topologies of space. From the other side, any measurement of such a state should be carried out by a detector which obeys classical laws and, therefore, the detector introduces a background space of a particular topology (of course, on the classical level the topology is always defined and does not change). This means that the topology of space must not be a direct observable, and the only chance to keep the information on the topology is to allow all the fields (which are specified on the background space) to be multivalued.

The corresponding extension of the standard quantum field theory was developed in Ref. [9] and it was proposed there that effects related to the multivalued nature of the physical fields could reveal themselves at large spatial scales. In the present paper we show that the logarithmic behavior of the gravitational potential at large distances may indeed appear as a result of nontrivial properties of the vacuum state in MOFT. In fact, analogous modifications hold for all fields and, in particular, for the Coulomb potential. In this sense at large scales we should observe not only dark matter, but dark charges of all sorts as well.

We show that in the case when the number of fields is conserved MOFT reduces to the standard field theory in which interaction constants (e.g., the gravitational and the fine structure constants) undergo an additional renormalization and, as a consequence, may acquire a dependence on spatial scales (observational limits on the scale-dependence of the gravitational constant have been already considered, e.g., in Ref.[10]). From the formal standpoint such a renormalization looks as if particles lose their point-like character and acquire an additional distribution in space, i.e., each particle turns out to be surrounded with a "dark halo".

The density distribution of the halo follows properties of the vacuum state which formed during the quantum period in the evolution of the Universe. At the moment, we do not have an exact model describing the formation of properties of the vacuum and, therefore, our consideration of the vacuum structure has a phenomenological character. Namely, we assume that upon the quantum period of the Universe the matter was thermalized with a very high temperature. Then, as the temperature dropped during the early stage of the evolution of the Universe, the topological structure of the space has tempered and the subsequent evolution resulted only in the cosmological shift of the physical scales. We show that in MOFT such kind of assumptions leads almost immediately to the logarithmic growth of the gravitational potential in some range of scales.

Let M be a background basic space. Let us specify an arbitrary field φ on it. We suppose that the action for the field can be presented in the form (for the sake of simplicity we consider the case of linear perturbations only)

$$S = \int_M d^4x \left(-\frac{1}{2}\varphi \widehat{L}\varphi + \alpha J \varphi \right), \tag{2}$$

where $\widehat{L} = \widehat{L}(\partial)$ is a differential operator (e.g., in the case of a massive scalar field $\widehat{L}(\partial) = \partial^2 + m^2$), J is an external current, which is produced by a set of point sources ($J = \sum J_k \delta(x - x_k(s))$, where $x_k(s)$ is a trajectory of a source), and α is the value of the elementary charge for sources. Thus, the field φ obeys the equation of motion

$$-\widehat{L}\varphi + \alpha J = 0. \tag{3}$$

We note that such a structure is valid for perturbations in gauge theories ($\varphi = \delta A_\mu$, where α is the gauge charge) and in gravity ($\varphi = l_{pl}\delta g_{\mu\nu}$, where $\alpha = l_{pl}$ is the Planck length).

In the Modified Field Theory we admit that the number of fields is a variable, therefore we replace the field φ with a set of fields φ^a, $a = 0, 1, ...N(x)$. In this manner, we introduce an additional variable $N(x)$ which in MOFT plays the role of an operator of the number of fields. Thus, the total action assumes the structure

$$S = \int_M d^4x \sum_{a=0}^{N(x)} \left(-\frac{1}{2}\varphi^a \widehat{L}\varphi^a + \alpha J \varphi^a \right), \tag{4}$$

where the number of fields $N(x)$, in general, depends on the position in the background space M and, therefore, the sum stands inside the integral over M. Fields φ^a are supposed to obey the identity principle and, therefore, they equally interact with the external current. We also note that, unlike the field φ, trajectories of sources $x_a(s)$ have a single-valued character, while the sum over sources automatically accounts for possible variations in their number, which may appear in processes of topology changes.

It is easy to see that the main effect of the introduction of the number of identical fields is the renormalization of the charge (the constant α). For example, let us consider the simplest case when $N(x)$ is a constant: $N(x) = N_0 = const$. We introduce a new set of fields as follows

$$\varphi^a = \frac{\widetilde{\varphi}(x)}{\sqrt{N_0}} + \delta\varphi^a, \quad \sum_a \delta\varphi^a = 0 \tag{5}$$

where $\widetilde{\varphi}$ is the effective field [9]

$$\widetilde{\varphi}(x) = \frac{1}{\sqrt{N_0}} \sum_{a=0}^{N_0} \varphi^a(x). \tag{6}$$

Then the action splits into two parts

$$S=\int\limits_M d^4x\left(-\frac{1}{2}\sum_a \delta\varphi^a\widehat{L}\delta\varphi^a\right)+\int_M d^4x\left(-\frac{1}{2}\widetilde{\varphi}\widehat{L}\widetilde{\varphi}+\widetilde{\alpha}J\widetilde{\varphi}\right). \tag{7}$$

The first part represents a set of free fields $\delta\varphi^a$ which are not involved into interactions between particles and, therefore, cannot be directly observed. The second part represents the standard action for the effective field $\widetilde{\varphi}$ with a new value for the charge $\widetilde{\alpha}=\sqrt{N_0}\alpha$.

In general case $N(x)$ is an operator-valued function and so will be the charge. However, the effective field and the transformation (5) can be introduced in this case as well, provided we are working with Fourier transforms, i.e. in the momentum representation, where the states of the fields can be classified in terms of free particles. It is, of course, quite usual in the quantum field theory, and we show in the next section that carrying out this approach in the framework of MOFT leads indeed to a physically meaningful theory.

In the Fourier representation action (4) takes the form

$$S=\int dt d^3k\sum_{a=0}^{N(k)}\left(-\frac{1}{2}\varphi_k^{a*}\widehat{L}(\partial_t,-ik)\varphi_k^a+\alpha J_k^*\varphi_k\right), \tag{8}$$

where $\varphi_k^a=1/(2\pi)^{3/2}\int\varphi^a(x)\exp(-ikx)\,d^3x$, $(a=0,1,...N(k))$, and $N(k)$ is the operator of the number of fields in the momentum space (it is, of course, not the Fourier transform of $N(x)$).

When $N(k)$ conserves, making the same kind of transformation as in (5) we bring the action to the form

$$S=\int dt d^3k\left(-\frac{1}{2}\sum_a\delta\varphi_k^{a*}\widehat{L}\delta\varphi_k^a-\frac{1}{2}\widetilde{\varphi}_k^*\widehat{L}\widetilde{\varphi}_k+\widetilde{\alpha}(k)\,J_k^*\widetilde{\varphi}_k\right). \tag{9}$$

We see that the action for the effective field $\widetilde{\varphi}$ coincides with that in the standard theory, but the charge $\widetilde{\alpha}(k)=\sqrt{N(k)}\alpha$ becomes now scale-dependent.

We recall that $N(k)$ is an operator and we, strictly speaking, should consider an average value for the charge

$$\langle\widetilde{\alpha}(k)\rangle=\left\langle\sqrt{N(k)}\right\rangle\alpha. \tag{10}$$

The homogeneity and isotropy of the Universe allows $\langle N(k)\rangle=N_k(t)$ to be an arbitrary function of $|k|$.

In a sense, the function N_k characterizes the structure of the momentum space M^* (note that the structure of the basic space M itself is not specified here). If we assume (and we do so) that processes with topology transformations have stopped after the quantum period in the evolution of the Universe, then the structure of the momentum space conserves indeed and the function $\langle N(k)\rangle$ depends on time via only the cosmological shift of scales, i.e., $\langle N(k)\rangle=N_{k(t)}$, where $k(t)\sim 1/a(t)$ and $a(t)$ is the scale factor. In this manner, function N_k represents some new universal characteristic of the physical space.

In this section we describe the structure of the vacuum state for effective fields. For simplicity, we consider a real scalar field, while generalization to the case of spin one and spin two particles is obvious. Consider the expansion of the field operator φ in plane waves,

$$\varphi(x) = \sum_k \left(2\omega_k L^3\right)^{-1/2} \left(c_k e^{ikx} + c_k^+ e^{-ikx}\right), \tag{11}$$

where $\omega_k = \sqrt{k^2 + m^2}$, and $k = 2\pi n/L$, with $n = (n_x, n_y, n_z)$; for the sake of convenience, we introduce periodic boundary conditions with a period length L (when it is necessary sums can be replaced with integrals, as $L \to \infty$, via the usual prescription: $\sum \to \int (L/2\pi)^3 \, d^3k$). In the case of free particles the expression for the Hamiltonian is

$$H_0 = \sum_k \omega_k c_k^+ c_k \ . \tag{12}$$

When the number of fields is variable, the set of annihilation/creation operators $\left\{c_k, c_k^+\right\}$ is replaced by the expanded set $\left\{c_{a,k}, c_{a,k}^+\right\}$, where $a \in [1, \ldots N_k]$, and N_k is the number of fields for a given wave number k. For a free field the energy is an additive quantity, so it can be written as

$$H_0 = \sum_k \sum_{a=1}^{N_k} \omega_k c_{a,k}^+ c_{a,k} \ . \tag{13}$$

In MOFT it is supposed that the fields are identical and obey the Fermi statistics (for motivations and more details, see Ref. [9]). Thus, the eigenvalues of the Hamiltonian can be written straightforwardly

$$\widehat{H}_0 = \sum_{n,k} n\omega_k N_{n,k} \, , \tag{14}$$

where $N_{n,k}$ is the number of field modes with the given wave number k and number of scalar particles n; since we assume Fermi statistics, we should set $N_{n,k} = 0$ or 1. Assuming that upon the quantum period of the evolution of the Universe topology transformations are suppressed, we should require that the number of fields conserves $\sum_n N_{n,k} = N_k = const$ in every mode. Thus, we find that the field ground state Φ_0 is characterized by occupation numbers

$$N_{n,k} = \theta\left(\mu_k - n\omega_k\right), \tag{15}$$

where $\theta(x)$ is the Heaviside step function and μ_k is the chemical potential. For the spectral number of fields we get

$$N_k = \sum_{n=0}^{\infty} \theta\left(\mu_k - n\omega_k\right) = 1 + \left[\frac{\mu_k}{\omega_k}\right] \ , \tag{16}$$

where $[x]$ denotes the integral part of x. We interpret the function N_k as a geometric characteristic of the momentum space, so it should be common for all types of Bose fields. Then assigning a specific value for the function N_k, the expression (16) defines the value of the respective chemical potential μ_k for a given field.

The creation and annihilation operators for the effective field are introduced as follows [9]

$$\widetilde{c}_k = \frac{1}{\sqrt{N_k}} \sum_{a=1}^{N_k} c_{a,k}, \quad \widetilde{c}_k^{+} = \frac{1}{\sqrt{N_k}} \sum_{a=1}^{N_k} c_{a,k}^{+}, \tag{17}$$

while the interaction term in (9) takes the form

$$S_{int} = \int dt \sum_k \widetilde{\alpha}(k) \left(\widetilde{c}_k J_k + \widetilde{c}_k^{+} J_k^{+} \right) \tag{18}$$

with $\widetilde{\alpha}(k) = \sqrt{N_k}\alpha$.

4. Origin of the spectral number of fields

As it was already mentioned above, the function N_k forms during the quantum stage of the evolution of the Universe, when processes involving topology changes took place. It is well known that near the singularity the evolution of the Universe is governed by a scalar field (responsible for a subsequent inflationary phase). For the sake of simplicity we neglect the presence of all other sorts of particles during the quantum stage and suppose that the Universe was filled with massive scalar particles only.

Upon the quantum period the Universe is supposed to be described by the homogeneous metric of the form

$$ds^2 = dt^2 - a^2(t)\, dl^2, \tag{19}$$

where $a(t)$ is the scale factor, and dl^2 is the spatial interval. It is natural to expect (at least it is the simplest possibility) that the state of the scalar field was thermalized with a very high temperature $T > T_{Pl}$ where T_{Pl} is the Planck temperature. The state of the field was characterized by the thermal density matrix with $\mu = 0$ (for the number of fields varies) and with mean values for occupation numbers $\langle N_{k,n} \rangle = \left(\exp\left(\frac{n\omega_k}{T}\right) + 1 \right)^{-1}$. On the early stage $m \ll T$, and the temperature and the energy of particles depend on time as $T = \widetilde{T}/a(t)$, $k = \widetilde{k}/a(t)$. When the temperature drops below a critical value T_* , which corresponds to the moment $t_* \sim t_{pl}$, topological structure (and the number of fields) tempers. This generates the value of the chemical potential for scalar particles $\mu \sim T_*$.

Let us neglect the temperature corrections, which are essential only at $t \sim t_*$ and whose role is in smoothing the real distribution N_k . Then at the moment $t \sim t_*$ the ground state of the field will be described by (15) with $\mu_k = \mu = const \sim T_*$. During the subsequent evolution the physical scales are subjected to the cosmological shift, however the form of this distribution in the commoving frame must remain the same. Thus, on the later stages $t \geq t_*$, we find

$$N_k = 1 + \left[\frac{\widetilde{k}_1}{\Omega_k(t)} \right], \tag{20}$$

where $\Omega_k(t) = \sqrt{a^2(t)\, k^2 + \widetilde{k}_2^2}$, $\widetilde{k}_1 \sim a_0 \mu$, and $\widetilde{k}_2 \sim a_0 m$ $(a_0 = a(t_*))$.

Consider now properties of the function N_k. There is a finite interval of wave numbers $k \in [k_{\min}(t), k_{\max}(t)]$ on which the number of fields N_k changes its value from $N_k = 1$ (at the point $k_{\max}$) to the maximal value $N_{\max} = 1 + \left[\widetilde{k}_1/\widetilde{k}_2\right]$ (at the point $k_{\min}$). This causes the variation of the charge values from $\alpha_{\min} = \alpha$, to $\alpha_{\max} = \sqrt{N_{\max}}\alpha$. The boundary points of the interval of k depend on time and are expressed via the free phenomenological parameters $\widetilde{k}_1$ and $\widetilde{k}_2$ as follows

$$k_{\max} = \frac{1}{a(t)}\sqrt{\widetilde{k}_1^2 - \widetilde{k}_2^2}, \quad k_{\min} = \frac{1}{a(t)}\sqrt{\widetilde{k}_1^2/(N_{\max}-1)^2 - \widetilde{k}_2^2}.$$

And in the wave number range $k \leq k_{\min}(t)$ the number of fields remains constant $N_k = N_{\max}$.

During the later stages of the evolution of the Universe ($t \gg t_*$) the contribution from all other fields becomes essential. However processes involving topology transformations are suppressed and the structure of the momentum space is described by the distribution (20). We note that the real distribution can be different from (20), which depends on the specific picture of topology transformations in the early Universe and requires the construction of the exact theory (in particular, thermal corrections smoothen the step-like distribution (20)). However we believe that the general features of N_k will remain the same.

5. The Law of gravity

The dependence of charge values upon wave numbers leads to the fact that the standard expressions for the Newton's and Coulomb's energy of interaction between point particles break down. In this section we consider corrections to the Newton's law of gravity (corrections to the Coulomb's law are identical). The interaction constant $\alpha \sim m\sqrt{G}$ (where m is the mass of a particle), and MOFT gives $G \to G(k) = N_k G$. To make estimates, we note that at the moment $t \sim t_*$ the mass of scalar particles should be small as compared with the chemical potential (which has the order of the Planck energy), which gives $\widetilde{k}_1 \gg \widetilde{k}_2$. Then in the range $k_{\max}(t) \geq k \gg k_{\min}(t)$ the function N_k can be approximated by

$$N_k \sim 1 + [k_{\max}(t)/k].$$

Consider two rest point particles with masses m_1 and m_2. Then the Fourier transform for the energy of the gravitational interaction between particles is given by the expression

$$V(\mathbf{k}) = -\frac{4\pi G m_1 m_2}{|\mathbf{k}|^2} N_k, \tag{21}$$

which in the coordinate representation is given by the integral

$$V(r) = \frac{1}{2\pi^2}\int_0^\infty \left(V(\omega)\,\omega^3\right)\frac{\sin(\omega r)}{\omega r}\frac{d\omega}{\omega}. \tag{22}$$

From (16) and (20) we find that this integral can be presented in the form

(23)
$$V\left(r\right)=-\frac{2Gm_{1}m_{2}}{\pi}\sum_{n=0}^{N_{\max}-1}\int_{0}^{k_{n}}\frac{\sin\left(\omega r\right)}{\omega r}d\omega=-\frac{Gm_{1}m_{2}}{r}\left(1+\sum_{n=1}^{N_{\max}-1}\frac{2Si\left(k_{n}r\right)}{\pi}\right)$$

where $k_n=\frac{1}{a(t)n}\sqrt{\widetilde{k}_1^2-n^2\widetilde{k}_2^2}$. The first term $n=0$ of the sum in (23) gives the standard expression for the Newton's law of gravity, while the terms with $n>1$ describe corrections. From (23) we find that in the range $k_1 r=k_{\max}r\ll 1$, $Si\left(k_n r\right)\sim k_n r$ and corrections to the Newton's potential give the constant

$$\delta V\sim-\frac{2Gm_{1}m_{2}}{\pi}\sum_{n=1}^{N_{\max}-1}k_{n}. \tag{24}$$

Thus, in this range we have the standard Newton's force. In the range $k_{\min}r\gg 1$, we get $\frac{2}{\pi}Si\left(k_n r\right)\sim 1$ and for the energy (23) we find

$$V\left(r\right)\sim-\frac{G'm_{1}m_{2}}{r},$$

where $G'=GN_{\max}$. Thus, on scales $r\gg 1/k_{\min}$ the Newton's law is restored, however the gravitational constant increases in $N_{\max}$ times. In the intermediate range $1/k_{\min}\gg r\gg 1/k_{\max}$ the corrections can be approximated as

$$\delta V\left(r\right)\sim\frac{2Gm_{1}m_{2}}{\pi}\frac{\widetilde{k}_{1}}{a\left(t\right)}\ln\left(\frac{\widetilde{k}_{2}}{a\left(t\right)}r\right), \tag{25}$$

i.e., they have a logarithmic behavior.

We note, that from the dynamical point of view the modification of the Newton's law of gravity can be interpreted as if point sources lose their point-like character and acquire an additional distribution in space. Indeed, let m_1 be a test particle which moves in the gravitational field created by a point source m_2. Then, assuming the Newton's law is unchanged, from (23) we conclude that the source m_2 is distributed in space with the density

(26)
$$\rho\left(r\right)=\frac{m_{2}}{2\pi^{2}}\int_{0}^{\infty}\left(N_{k}k^{3}\right)\frac{\sin\left(kr\right)}{kr}\frac{dk}{k}=m_{2}\left(\delta\left(\vec{r}\right)+\frac{1}{2\pi^{2}}\sum_{n=1}^{N_{\max}-1}\frac{\sin\left(k_{n}r\right)-k_{n}r\cos\left(k_{n}r\right)}{r^{3}}\right).$$

The total mass contained within a radius r is

$$M\left(r\right)=4\pi\int_{0}^{r}s^{2}\rho\left(s\right)ds=m_{2}\left(1+\frac{2}{\pi}\sum_{n=1}^{N_{\max}-1}\left(Si\left(k_{n}r\right)-\sin\left(k_{n}r\right)\right)\right). \tag{27}$$

Thus, in the range $r\ll 1/k_{\max}$ we find $M\left(r\right)\sim m_2$, i.e., one may conclude that the gravitational field is created by a point source with the mass m_2. However in the range $1/k_{\min}>r>1/k_{\max}$ the mass increases $M\left(r\right)\sim m_2 k_{\max}r$, and for $r\gg 1/k_{\min}$ the mass reaches the value $M\left(r\right)\sim m_2 k_{\max}/k_{\min}$.

We see that in MOFT the distributions of the dark matter and the actual matter are strongly correlated (by the rule (26)), and the resulting behavior of the dynamically determined mass $M(r)$ seems to agree with the observations. We stress that the theoretical scheme of MOFT was not invented to fit the dark matter distribution. On the contrary, the logarithmic behavior of the effective field potentials simply appears in the thermodynamically equilibrium state at the low temperature, as a by-product of a non-trivial structure of MOFT vacuum.

6. Conclusions

In this manner we have shown that in the case when the number of fields is conserved MOFT reduces to the standard field theory in which interaction constants undergo a renormalization and, in general, acquire a dependence on spatial scales. From the dynamical standpoint such a renormalization looks as if particles lose their point-like character and acquire an additional distribution in space, i.e., each point source is surrounded with a halo of dark matter. This halo carries charges of all sorts and its distribution around a point source follows properties of the vacuum. The latter forms during the quantum period in the evolution of the Universe, and the rigorous consideration of vacuum properties requires constructing an exact theory.

In applying to cosmological problems it is convenient to suppose that the Newton's law of gravity remains intact, while the variation of the gravitational constant is phenomenologically described by the presence of a dark matter. In the simplest case properties of the vacuum and that of dark matter can be described by two phenomenological parameters which represent the two characteristic scales. They are the minimal scale $r_{\min} = 2\pi/k_{\max}$ on which the dark matter starts to show up (and on which the law of gravity (23) starts to deviate from the Newton's law) and the maximal scale $r_{\max} = 2\pi/k_{\min}$ which defines the fraction of the dark matter or the total increase of interaction constants $G_{\max} \approx G r_{\max}/r_{\min}$ (and after which the Newton's law restores). The minimal scale $r_{\min}$ can be easily estimated (e.g., see Ref.[8]) and constitutes a few *kpc*. To get analogous estimate for the maximal scale $r_{\max}$ is not so easy. This requires the exact knowledge of the total matter density Ω_{tot} for the homogeneous background and the knowledge of the baryon fraction Ω_b which gives $r_{\max}/r_{\min} \sim \Omega_{tot}/\Omega_b$ (where $\Omega = \rho/\rho_{cr}$ and ρ_{cr} is the critical density).

If we accept the value $\Omega_{tot} \sim 1$ (which is predicted by inflationary scenarios) and take the upper value for baryons $\Omega_b \leq 0.03$ (which comes from the primordial nuclearsynthesis), we find $r_{\max}/r_{\min} \geq 30$. Another estimate can be found from restrictions on parameters of inflationary scenarios. Indeed, in inflationary models correct values for density perturbations give the upper boundary for the mass of the scalar field $m \leq 10^{-5} m_{Pl}$ which gives $r_{\max}/r_{\min} \geq 10^5 T_*/m_{pl}$, where T_* is the critical temperature at which topology has been tempered.

From our point of view, the most interesting picture of the Universe appears in the case when the maximal scale is absent ($r_{\max} \to \infty$, or at least $r_{\max} \gg R_H$, where R_H is the Hubble radius). In this case a uniform distribution of baryons in space is consistent with closed cosmological models only. Indeed, the number of baryons contained within a radius r in the case of a uniform distribution with a density n_b is given by $N_b(r) \sim n_b r^3$ and the mass of every baryon increases according to (27) as $m_b(r) \sim m_p r/r_{\min}$ (m_p is the proton mass). Thus, for the total mass (baryons plus dark matter) contained within the radius r we get $M_{tot}(r) = m_b(r) N_b(r) + \delta M(r) \geq \rho_b r^4 / r_{\min}$ (where $\rho_b = m_p n_b$ and $\delta M(r)$ accounts for the contribution of baryons from the outer region which does exist according to (26)). This means that the lower limit for the total density increases with the radius $\rho_{tot} \sim M_{tot}/r^3 \geq \rho_b r / r_{\min}$ and for sufficiently large $r \sim r_{cr}$ it will reach the value $\rho_b r_{cr} / r_{\min} = \rho_{cr}$, i.e., $\Omega_{tot} > 1$ and such a Universe must correspond to a closed cosmological model. Then $r_{\max}$ coincides with the radius of the Universe $r_{\max} = R$ and for the mean value of the matter density we will find $\Omega_{tot} = \Omega_b R / r_{\min}$.

We note that this does not mean that open cosmological models are forbidden at all. This, however, means that in open models luminous matter has a specific nonuniform distribution in space $\rho(x)$ (the total density nevertheless is uniformly distributed, i.e., the dark matter compensates exactly the inhomogeneity of the luminous matter). Indeed, let Ω_{tot} be a constant in space and let it be of order 1. Then the total mass $M_{tot}(r) = m_b(r) N_b(r) + \delta M(r)$ contained within a radius r behaves as $\sim r^3 \rho_{tot}$ and, therefore, the number of baryons should follow the law $N_b(r) \leq r^2 r_{\min} \rho_{tot} / m_p$. Such a law for the number of baryons can be achieved when the luminous matter is located on thin two-dimensional surfaces separated by empty regions of ever growing size (i.e., baryons have a kind of a fractal distribution in space). This, in fact, is consistent with the observed picture of the Universe at large scales $r \geq 100 Mpc$.

In this manner in open models the mean density of baryons depends on the scale of averaging out $\rho_b(r) = \rho_{tot} r_{\min} / r$ and at the Hubble scale $r \sim R_H$ we find $\Omega_b / \Omega_{tot} \sim r_{\min} / R_H$. If we consider larger scales we find $\rho_b \to 0$ as $r \to \infty$, i.e., the expansion of the Universe is governed by dark matter alone.

In conclusion we point out that all the interaction constants (the gravitational constant G the electron charge e, gauge charges) depend on time via the cosmological shift of scales $r_{\min}(t)$ and $r_{\max}(t)$ which may give rise to a number of interesting processes in the early Universe. We do not discuss here all the possibilities but just point out to them which, in general, may be used to rule out or, better, to confirm the theory suggested.

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