Weierstraß–Institut
für Angewandte Analysis und Stochastik

im Forschungsverbund Berlin e.V.

Preprint ISSN 0946 – 8633

Numerical Simulation for Lossy Microwave Transmission Lines Including PML

Georg Hebermehl [1], Friedrich-Karl Hübner [1], Rainer Schlundt[1],

Thorsten Tischler [2], Horst Zscheile [2], Wolfgang Heinrich[2]

submitted: 20th October 2000

[1] Weierstrass-Institute for
Applied Analysis and Stochastics,
Mohrenstraße 39,
D - 10117 Berlin, Germany,
E-Mail: hebermehl@wias-berlin.de,
huebner@wias-berlin.de,
schlundt@wias-berlin.de

[2] Ferdinand-Braun-Institut
für Höchstfrequenztechnik,
Albert-Einstein-Str. 11,
D-12489 Berlin, Germany,
E-Mail: tischler@fbh-berlin.de,
zscheile@fbh-berlin.de,
w.heinrich@ieee.org

Preprint No. 612
Berlin 2000

1991 *Mathematics Subject Classification.* 35Q60, 65F15, 65N22..

Key words and phrases. Microwave device simulation, Maxwell's equations, PML boundary condition, Finite integration technique, Eigenvalue problem.

Edited by
Weierstraß–Institut für Angewandte Analysis und Stochastik (WIAS)
Mohrenstraße 39
D — 10117 Berlin
Germany

Fax:	+ 49 30 2044975
E-Mail (X.400):	c=de;a=d400-gw;p=WIAS-BERLIN;s=preprint
E-Mail (Internet):	preprint@wias-berlin.de
World Wide Web:	http://www.wias-berlin.de/

Abstract

Finite-difference analysis of transmission lines including lossy materials and radiation effects leads to a complex eigenvalue problem. A method is presented which preserves sparseness and delivers only the small number of interesting modes out of the complete spectrum. The propagation constants are found solving a sequence of eigenvalue problems of modified matrices with the aid of the shift-and-invert mode of the Arnoldi method. In an additional step non physical Perfectly Matched Layer modes are eliminated.

Contents

1 Introduction **1**

2 Boundary Value Problem **3**

3 Eigen Value Problem **4**

4 Detecting PML Modes **6**

5 Numerical Example **6**

List of Figures

1 γ-plane 8
2 k_z-plane 8
3 Lateral radiation of a coplanar waveguide 9
4 Propagating modes of a coplanar waveguide 9

1 Introduction

Microwave circuits are used in mobile communications, radio links, sensors, and automotive systems. The commercial applications cover the microwave and lower millimeter-wave range, i.e., the frequencies between 1 GHz and about 80 GHz. For special applications in radioastronomy and optoelectronic devices also higher frequencies up to 1 THz and at 300 THz are used, respectively.

Basic elements of microwave integrated circuits are their transmission lines, whose propagation behavior has to be determined accurately. The propagation behavior of the transmission lines can be calculated by applying Maxwell's equations to the infinitely long homogeneous transmission line structure and solving an eigenvalue problem [1].

For numerical treatment, the computational domain has to be truncated by electric or magnetic walls or by a so-called absorbing boundary condition simulating open space. A very efficient formulation for the latter case is the Perfectly Matched Layer (PML). These layers consist of an artificial material with complex anisotropic material properties [2]. The PML provide absorbing properties for any frequency, polarization and angle of incidence.

In the presence of losses or absorbing boundary conditions the matrix of the eigenvalue problem becomes complex. The system matrix is sparse and of high order. This requires efficient solvers that preserve sparseness and deliver only the small number of interesting modes out of the complete spectrum.

In earlier time- and memory-consuming methods [1] and [3] the complete set of eigenvalues and of corresponding propagation constants was computed and sorted in order to select the desired propagation constants. In the previous papers [4] (lossless case) and [5] (lossy case including PML) the authors presented a method which avoids the computation of all eigenvalues to find the few required propagation constants. The sparse-storage technique is applied. The computing times increase with the frequency and using PML. A feasible computation is possible for frequencies up to 600 GHz. In order to reduce the numerical effort and to simulate circuits for higher frequencies a new method is presented here.

Using an estimation for the maximum value of the real part and a preset maximum value of the imaginary part the region containing the interesting propagation constants is defined as a rectangle $\hat{F}$. Using conformal mapping relations between the plane of propagation constants and the plane of eigenvalues the rectangle $\hat{F}$ is mapped into an area F which is bounded by parabolas. A method is presented which finds the eigenvalues of this area solving a sequence of eigenvalue problems of modified matrices with the aid of the shift-and-invert mode of the Arnoldi method [6].

Special attention is paid to the so-called Perfectly Matched Layer boundary conditions, which influence the mode spectrum in a significant way [7]. Particularly, the problem of detecting the desired modes out of the mode spectrum is treated.

2 Boundary Value Problem

The structures under investigation can be described as an interconnection of infinitely long transmission lines, which have to be longitudinally homogeneous. The junction, the so-called discontinuity, may have an arbitrary structure. Cross-sectional planes, the so-called ports, are defined on the transmission lines. The whole structure may be surrounded with an enclosure. A three-dimensional boundary value problem can be formulated using the integral form of Maxwell's equations in the frequency domain

$$\oint_{\partial\Omega} \vec{H} \cdot d\vec{s} = \int_{\Omega} \jmath\omega[\epsilon]\vec{E} \cdot d\vec{\Omega}, \qquad \oint_{\cup\Omega} ([\epsilon]\vec{E}) \cdot d\vec{\Omega} = 0, \tag{1}$$

$$\oint_{\partial\Omega} \vec{E} \cdot d\vec{s} = -\int_{\Omega} \jmath\omega[\mu]\vec{H} \cdot d\vec{\Omega}, \qquad \oint_{\cup\Omega} ([\mu]\vec{H}) \cdot d\vec{\Omega} = 0, \tag{2}$$

$$\vec{D} = [\epsilon]\vec{E}, \;\; \vec{B} = [\mu]\vec{H}, \quad [\epsilon] = \mathrm{diag}\left(\epsilon_x, \epsilon_y, \epsilon_z\right), \;\; [\mu] = \mathrm{diag}\left(\mu_x, \mu_y, \mu_z\right), \tag{3}$$

in order to compute the electromagnetic field. The PML's are filled with an artificial material with complex anisotropic material properties. Therefore, the quantities are diagonal complex tensors. At the ports p the transverse electric field $\vec{E}_t(z_p)$ is given by superposing transmission line modes $\vec{E}_{t,l}(z_p)$. On all other parts of the computation domains surface the tangential electric or magnetic field is assumed to be equal zero.

$$\vec{E}_t(z_p) = \sum_{l=1}^{m^{(p)}} w_l(z_p)\vec{E}_{t,l}(z_p), \quad \vec{E} \times \vec{n} = 0 \quad \text{or} \quad \vec{H} \times \vec{n} = 0. \tag{4}$$

The transverse electric mode fields are the solutions of an eigenvalue problem for the transmission lines. Starting from the Maxwellian equations (1), (2) each of these equations is solved on a three-dimensional grid. Using the Finite Integration Technique [8] in the frequency domain (FDFD for Finite-Difference method in the Frequency Domain) [9] Equations (1), (2) are transformed into a set of Maxwellian grid equations

$$A^T D_{s/\mu}\vec{b} = \jmath\omega\epsilon_0\mu_0 D_{A_\epsilon}\vec{e}, \qquad B D_{A_\epsilon}\vec{e} = 0, \tag{5}$$

$$A D_s \vec{e} = -\jmath\omega D_{A_\mu}\vec{b}, \qquad \tilde{B} D_{A_\mu}\vec{b} = 0. \tag{6}$$

The vectors $\vec{e}$ and $\vec{b}$ contain the components of the electric field intensity and the magnetic flux density of the elementary cells, respectively. The diagonal matrices $D_{s/\mu}$, D_{A_ϵ}, D_s, and D_{A_μ} contain the information on cell dimension and material. A, B, and $\tilde{B}$ are sparse and contain the values 1, -1, and 0 only. Eliminating the components of the magnetic flux density from the two equations of the left-hand side of (5), (6) we get the system of linear algebraic equations

$$(A^T D_{s/\mu} D_{A_\mu}^{-1} A D_s - k_0^2 D_{A_\epsilon})\vec{e} = 0, \quad k_0 = \omega\sqrt{\epsilon_0\mu_0}, \tag{7}$$

which have to be solved using the boundary conditions. k_0 is the wavenumber in vacuum.

3 Eigen Value Problem

Because the transmission lines are longitudinally homogeneous any field can be expanded into a sum of so-called modal fields which vary exponentially in the longitudinal direction

$$\vec{E}(x, y, z) = \underline{\vec{E}}(x, y) e^{\mp \jmath k_z z}. \tag{8}$$

A substitution of ansatz (8) into the system of linear algebraic equations (7) and the elimination of the longitudinal electric field intensity components by means of the electric-field divergence equation $BD_{A_\epsilon}\vec{e} = 0$ (see (5)) gives an eigenvalue problem

$$C\underline{\vec{e}} = \gamma \underline{\vec{e}}, \quad \gamma = -4\sin^2(hk_z) \approx -4(hk_z)^2 = u + \jmath v. \tag{9}$$

$\underline{\vec{e}}$ consists of components of the discretized eigenfunctions $\underline{\vec{E}}$. $2h$ is the length of an elementary cell in z-direction. The sparse matrix C is in general nonsymmetric complex. We can use the approximation $\sin(x) \approx x$ in (9) if we choose h to be small enough, which is anyway necessary to get small discretization errors. The relation between the propagation constants k_z and the eigenvalues γ is nonlinear, and can be expressed as

$$k_z = \frac{\jmath}{2h} \ln\left(\frac{\gamma}{2} + 1 + \sqrt{\frac{\gamma}{2}\left(\frac{\gamma}{2} + 2\right)}\right) = \beta - \jmath\alpha. \tag{10}$$

A propagation constant k_z and its corresponding eigenfunction is called a mode. We are interested only in the modes with the smallest magnitude of imaginary part, but possibly with large real part of their propagation constant.

For our method we must limit the search for propagation constants by a maximum value k_f of their real part. A reasonable estimation of this maximum value is derived for the lossy case including PML for inhomogeneously filled waveguides in [5]

$$\Re(k_z) \leq k_f = \omega \Re(\sqrt{\epsilon_m \mu_m}). \tag{11}$$

ϵ_m and μ_m are properties of the material that yields the largest value of the right-hand side of Equation (11). Using a maximum value k_f of the real part and a preset maximum value α_m of the imaginary part of the propagation constant the region containing the interesting propagation constants is defined as a rectangle $\hat{F}$ (see Fig. (2)) bounded by the lines

$$\beta = \pm k_f \quad \text{and} \quad \alpha = \pm \alpha_m. \tag{12}$$

We consider the plane of eigenvalues (γ-plane) and the plane of propagation constants (k_z-plane). The conformal mapping relations between the γ- and the k_z-plane are (see (9) and (10))

$$u = -4h^2(\beta^2 - \alpha^2), \quad v = 8h^2\alpha\beta. \tag{13}$$

Using the inverse of the mapping (13) the lines (12) of the k_z-plane are transformed into the parabolas (see Fig. (1), (2))

$$v = \pm 4hk_f\sqrt{u + 4h^2{k_f}^2} \quad \text{and} \quad v = \pm 4h\alpha_m\sqrt{-u + 4h^2{\alpha_m}^2}, \tag{14}$$

respectively. That means, we have to find all eigenvalues of the region bounded by the parabolas (14).

The eigenvalues are obtained as follows. q points

$$\hat{P}_k(\beta_k, \alpha_m), \quad k = 1(1)q, \quad \beta_1 > 0, \quad \beta_q = k_f, \tag{15}$$

are defined on the interval $[0, k_f]$ of the line $\alpha = \alpha_m$. The distance between the points have not to be equidistant. The points $\hat{P}_k$ are transformed into the points P_k of the γ-plane. They are located on the parabola ((14), right formula). The q circles C_k of the γ-plane

$$(u + m_k)^2 + v^2 = {r_k}^2, \;\; r_k = \sqrt{(\Im(P_k))^2 + (m_k - \Re(P_k))^2}, \quad k = 1(1)q, \tag{16}$$

with

$$m_1 = 0, \;\; m_k = \frac{(\Re(P_{k+1}))^2 - (\Re(P_k))^2 + (\Im(P_{k+1}))^2 - (\Im(P_k))^2}{2|\Re(P_{k+1}) - \Re(P_k)|}, \tag{17}$$

centered on the u-axis cover the region bounded by the parabolas (14).

l points Q_i are defined on the periphery of C_k in order to find all eigenvalues, located in the circle C_k. The matrix C is extended by the diagonal matrix Q (see (19)) which consists of the set $\mathcal{E}$ with the l complex elements Q_i. The q eigenvalue problems

$$(\bar{C} - m_k I)\underline{\vec{e}} = (\gamma - m_k)\underline{\vec{e}}, \quad k = 1(1)q, \tag{18}$$

$$\text{with} \quad \bar{C} = \begin{pmatrix} Q & \\ & C \end{pmatrix}, \quad Q = diag(Q_1, ..., Q_l), \tag{19}$$

are solved with the aid of the implicitly restarted Arnoldi method. The number m of eigenvalues to be computed must be l on the first call to the Arnoldi procedure. m is raised by l for so long until at least one value $Q_i \in \mathcal{E}$ was found. Separating the new values on each eigenvalue problem k, we are sure to have found all eigenvalues which are located in the corresponding circles C_k. Applying (13) the circles C_k are transformed into Cassini curves $\hat{C}_k$

$$(\beta^2 + \alpha^2)^2 - \frac{m_k}{2h^2}(\beta^2 - \alpha^2) = \frac{{r_k}^2}{16h^4} - \frac{{m_k}^2}{16h^4}, \tag{20}$$

which cover the rectangle $\hat{F}$ containing all desired propagation constants.

In general the Arnoldi method does not converge using the regular mode for our eigenvalue problem. Thus, the shift-and-invert mode is applied with the solution of systems of linear algebraic equations looking for eigenvalues of largest magnitude.

We use the combined unifrontal/multifrontal method [10] for the solution of the partly ill-conditioned nonsymmetric complex linear algebraic equations.

We note, that the assumption $\sin(x) \approx x$ in (9) yields the map (see (13)) between the circles (16) of the γ-plane and the well-known Cassini curves (20) in the k_z-plane. But, if we abandon this assumption, we receive a corresponding mapping between the circles and curves which can be interpreted as modified Cassini curves.

4 Detecting PML Modes

We use the PML in order to calculate the eigen modes of open waveguide structures. Without the PML the finite difference computational domain has to be surrounded by either electric or magnetic walls. In this case we would have a huge number of box modes, which are not an intrinsic property of the waveguide. Since the total number of modes is constant, the PML cannot remove any mode but only shift them into other parts of the eigenvalue spectrum. We want to distinguish these PML-modes from the desired ones. The PML-modes are characterized by their high power concentration in the PML area. Thus, to eliminate the PML-modes we calculate the magnitude of the power flow of each computed mode in the PML ($P^{(PML)}$), in the waveguide region ($P^{(WG)}$), and in the total computational domain (P):

$$\begin{aligned} P &= P^{(PML)} + P^{(WG)} \\ &= \int\limits_{\Omega^{(PML)}} \left(\vec{E}_t \times \vec{H}^*_{t,m}\right) \cdot d\vec{\Omega} + \int\limits_{\Omega^{(WG)}} \left(\vec{E}_t \times \vec{H}^*_{t,m}\right) \cdot d\vec{\Omega}\,. \end{aligned} \tag{21}$$

A mode is specified as PML-mode if

$$r^{(PML)} = \frac{P^{(PML)}}{P} > \xi\,, \tag{22}$$

with values $\xi = 0.2, \ldots, 0.6$, found empirically. This criterion works for a wide frequency range. In the case of very high frequencies, where the physical modes are strongly radiating, the relation $r^{(PML)}$ of the physical modes and the PML modes approach each other and the criterion may accept even some PML modes. In this case, one has to consult additional information, e. g. field density plots, in order to decide which eigenvalues belong to the desired guided modes.

5 Numerical Example

As an example we have calculated the modes of a coplanar waveguide with finite substrate height, and finite ground metallization according to Figure (3 a), for the frequencies $f = 10(10)800$ GHz. As the waveguide is symmetrical, only its right-hand half must be discretized. The computational domain consists of the cross section of the waveguide, the ground metallization and the PML regions. The PML's are bounded with electric walls, while the symmetry plan is formed as a magnetic wall. The structure is subdivided into $n_{xy} = n_x n_y$ elementary cells. A graded mesh of $n_x = 89$, $n_y = 109$ cells, including 12-cell PML regions is used in the given example, so the dimension of the eigenvalue problem is $2n_x n_y - n_b = 17955$. n_b is determined by the number of cells filled with perfectly conducting material.

Figures (1), (2) refer to the frequency $f = 300$ GHz. Three modes were identified as guided wave modes, according to (22) with $\xi = 0.35$. The corresponding propagation constants are characterized by a +. To go into detail these modes are the parallel plate line mode (PPL), the coplanar waveguide mode (CPW), and a higher order mode (HM10) that is caused by the finite substrate height. In Figure (3 b) a field pattern plot of the CPW mode is given, which illustrates the leakage due to lateral radiation of this mode. Figure (4) shows the effective permittivity of the different physical modes over the whole frequeny range.

It should be mentioned, that the PML modes can be well separated for frequencies up to 500 GHz using criterion (22). For higher frequencies some PML modes with $r^{(PML)} < 0.35$ arise and one has to separate them by checking the field distribution.

Concerning the sequencing technique for the eigenvalue search, one can say that, especially for high frequencies, the numerical effort could be reduced considerably in comparison to the method presented in [5].

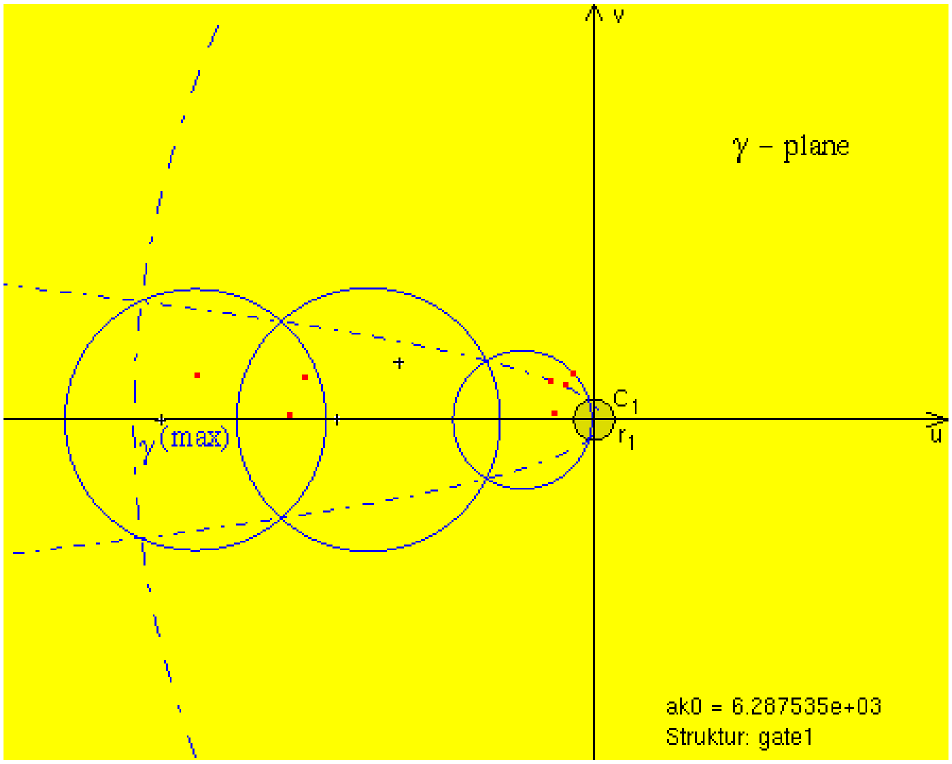


Figure 1: γ-plane

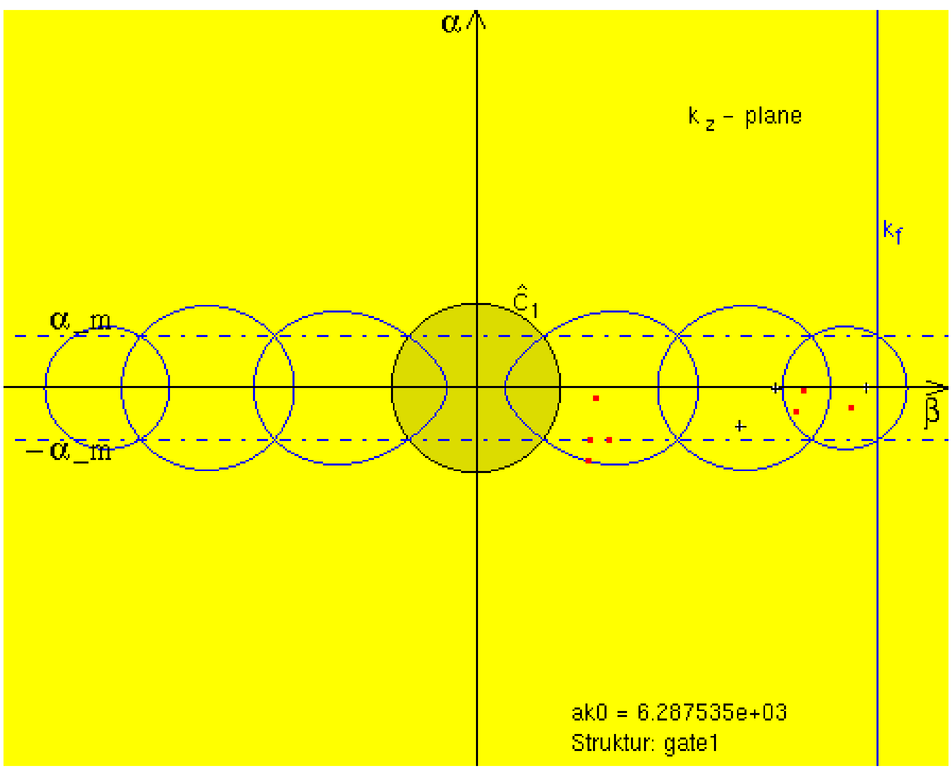
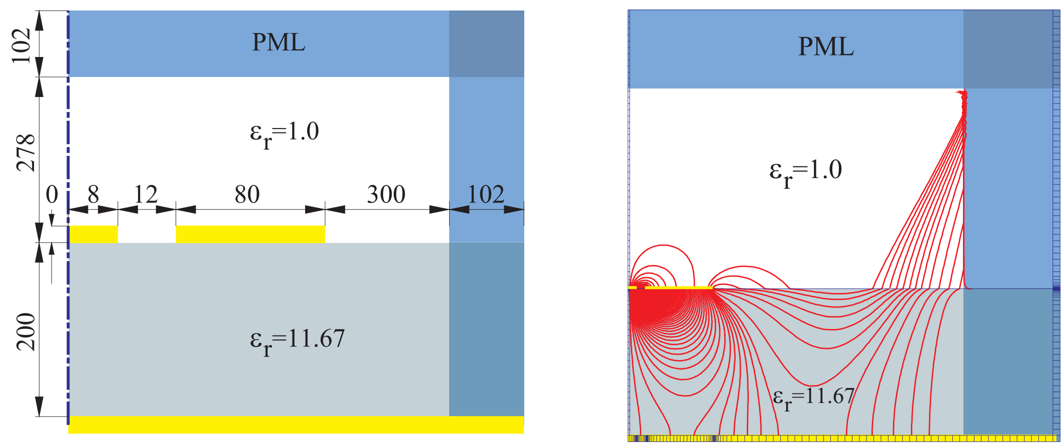


Figure 2: k_z-plane



(a) Geometry of the CPW (all parameters in μm) (b) *E*-Field pattern of the CPW- Mode at $f = 300$ GHz

Figure 3: Lateral radiation of a coplanar waveguide

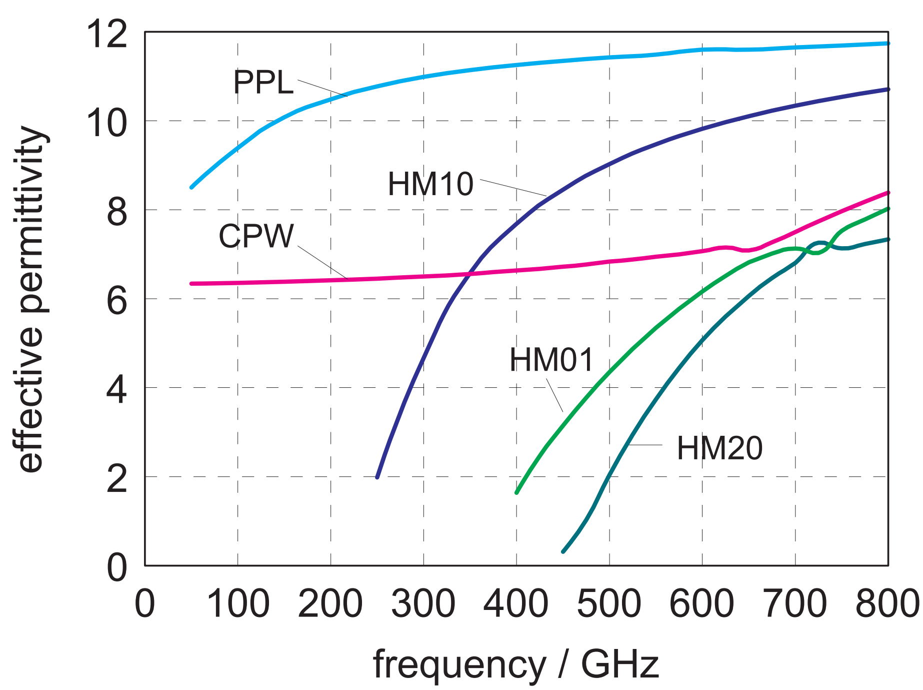


Figure 4: Propagating modes of a coplanar waveguide

References

[1] Christ, A., Hartnagel, H. L.: Three-Dimensional Finite-Difference Method for the Analysis of Microwave-Device Embedding. IEEE Transactions on Microwave Theory and Techniques **35** (1987) 688–696

[2] Sacks, Z. S., Kingsland, D. M., Lee, R., Lee, J.-F.: A Perfectly Matched Anisotropic Absorber for Use as an Absorbing Boundary Condition. IEEE Transactions on Antennas and Propagation **43** (1995) 1460–1463

[3] Klingbeil, H., Beilenhoff, K., Hartnagel, H. L.: FDFD Full-Wave Analysis and Modeling of Dielectric and Metallic Losses of CPW Short Circuits. IEEE Transactions on Microwave Theory and Techniques **44** (1996) 485–487

[4] Hebermehl, G., Schlundt, R., Zscheile, H., Heinrich, W.: Eigen Mode Solver for Microwave Transmission Lines. The International Journal for Computation and Mathematics in Electrical and Electronic Engineering **16**, (1997) 108–122

[5] Hebermehl, G., Hübner, F.-K., Schlundt, R., Tischler, T., Zscheile, H., Heinrich, W.: On the Computation of Eigen Modes for Lossy Microwave Transmission Lines Including Perfectly Matched Layer Boundary Conditions. The International Journal for Computation and Mathematics in Electrical and Electronic Engineering **20** (2000) (to appear)

[6] Lehoucq, R. B.: Analysis and Implementation of an Implicitly Restarted Arnoldi Iteration. Rice University, Department of Computational and Applied Mathematics, Technical Report **13** (1995) 1-135

[7] Tischler, T., Heinrich, W.: The Perfectly Matched Layer as Lateral Boundary in Finite-Difference Transmission-Line Analysis. 2000 Int. Microwave Symp. Digest **1** (2000) 121-124

[8] Weiland, T.: A Discretization Method for the Solution of Maxwell's Equations for Six-Component Fields. Electronics and Communication (AEÜ) **31** (1977) 116–120

[9] Beilenhoff, K., Heinrich, W., Hartnagel, H. L.: Improved Finite-Difference Formulation in Frequency Domain for Three-Dimensional Scattering Problems. IEEE Transactions on Microwave Theory and Techniques **40** (1992) 540-546

[10] Davis, T. A., Duff, I. S.: A Combined Unifrontal/Multifrontal Method for Unsymmetric Sparse Matrices. University of Florida, Technical Report **16** (1997) 1–18