Lognormal random field approximations to LIBOR market models

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Abstract

We study several approximations for the LIBOR market models presented in [1, 2, 5]. Special attention is payed to log-normal approximations and their simulation by using direct simulation methods for log-normal random fields. In contrast to the conventional numerical solution of SDE's this approach simulates the solution directly at the desired point and is therefore much more efficient. We carry out a path-wise comparison of the approximations and give applications to the valuation of the swaption and the trigger swap.

1 Introduction

By far the most important class of traded interest rate derivatives is constituted by derivatives which are specified in terms of LIBOR rates. The LIBOR[1] rate L is the annualized effective interest rate over a forward period $[T_1, T_2]$ and can be expressed in terms of two zero-coupon bonds B_1 and B_2 with face value \$1, maturing at T_1 and T_2, respectively,

$$L(t; T_1, T_2) := \frac{\frac{B_1(t)}{B_2(t)} - 1}{T_2 - T_1}, \tag{1}$$

where as usual T_2 is the settlement date for the accrual LIBOR period. Brace, Gatarek and Musiela [1], as well as Jamshidian [2], constructed an arbitrage free model for the LIBOR rate process in order to price LIBOR derivatives such as caps, swaptions and more complicated types in a direct way. In [1] the dynamics of the continuous family of processes $\{L(t, T, T + \delta) \mid T \geq 0,\ 0 \leq t \leq T\}$ is studied for a fixed $\delta > 0$, whereas Jamshidian [2] considered for a discrete set of tenors $\{T_1, \ldots, T_n\}$ the processes $\{L_i(t) := L(t, T_i, T_{i+1}) \mid t \leq T_i,$ $i = 1, \ldots, n-1\}$. In both papers [1, 2] special attention is payed to so called *LIBOR market models* which are models where for every settlement date the LIBOR process has deterministic volatility. In a market model, each LIBOR is a *log-normal* martingale under the numeraire measure given by the bond which terminates at the LIBOR's settlement date.

In this sequel we concentrate on a LIBOR market model for a discrete set of tenors given by a stochastic differential equation (SDE) in the terminal bond measure as developed in Jamshidian [2], equipped with a special correlation structure proposed by Schoenmakers and Coffey [5]. In this model we will test the valuation of several LIBOR derivatives such as the 'plain vanilla' swaption and the more 'exotic' trigger swap. For a detailed analysis of the trigger swap and the valuation of exotic LIBOR derivatives in general we refer to Schoenmakers and Coffey [5]. For the LIBOR process, as being a solution of the SDE, we have constructed different path-wise approximations and in particular log-normal approximations and carried out implementations. The results are subjected to mutual comparison and a ranking between the different approximations is thus obtained. The main advantage of the log-normal approximations is that their distributions can be simulated very fast by a Gaussian random field of log-LIBORs with a drift and correlation structure determined by the specific approximation. As the valuation of a LIBOR derivative generally comes down to the computation of the expected value of some functional of the LIBOR process, a large class of derivatives can be valuated quite fast by random field simulation.

Several approximations are derived in section (2) where a mutual comparison is studied. In section (3) we construct a log-normal random field simulation algorithm and in section (4) we

[1] LIBOR stands for London Inter Bank Offer Rate.

consider the valuation of swaptions and triggerswaps and compare the results for different simulation algorithms and different correlation structures. In particular, in section (4) it is observed that swaption prices depend on the input correlation parameters of the LIBOR model under consideration in a numerical stable way, in contrast to correlation parameters in low-factor LIBOR models which tend to unstable behaviour under calibration to swaption prices. See, for a more detailed discussion of this issue, Schoenmakers and Coffey[5].

2 Different approximations, log-normal approximations

For a given tenor structure $0 < T_1 < T_2 < \ldots < T_n$ we consider a Jamshidian LIBOR market model [2] for the forward LIBOR processes L_i in the terminal bond numeraire $\mathbb{P}_n$,

$$dL_i = -\sum_{j=i+1}^{n-1} \frac{\delta_j L_i L_j \, \gamma_i \cdot \gamma_j}{(1+\delta_j L_j)} \, dt + L_i \, \gamma_i \cdot dW^{(n)}, \tag{2}$$

where, for $i = 1, \ldots, n-1$, the L_i are defined in the intervals $[t_0, T_i]$, $\delta_i = T_{i+1} - T_i$ and $\gamma_i = (\gamma_{i,1}, \ldots, \gamma_{i,n-1})$ are given deterministic functions, called factor loadings, defined in $[t_0, T_i]$, respectively. In (2), $(W^{(n)}(t) \mid t_0 \le t \le T_{n-1})$ is a standard $n-1$-dimensional Wiener process under $\mathbb{P}_n$. It is convenient to deal with the following integral form of (2):

$$\ln \frac{L_i(t)}{L_i(t_0)} = -\int_{t_0}^{t} \sum_{j=i+1}^{n-1} \frac{\delta_j L_j |\gamma_i||\gamma_j| \rho_{ij}}{1+\delta_j L_j} \, ds - \frac{1}{2}\int_{t_0}^{t} |\gamma_i|^2 ds + \int_{t_0}^{t} \gamma_i \cdot dW^{(n)}, \tag{3}$$

where $\rho_{ij} = \gamma_i \cdot \gamma_j \,/ |\gamma_i||\gamma_j|$. In practice, we may define the vectors $\gamma_i/|\gamma_i|$ through the matrix (ρ_{ij}) by applying a Cholesky decomposition.

Note that only the first term in the right hand side of (3) is generally non-Gaussian. Let us consider the contribution of the non-Gaussian term where we assume for simplicity that the functions γ_i are constants. We introduce the notations: $\rho_i = \sum_{j=i+1}^{n-1} |\rho_{ij}|$, $\rho = \max_i \rho_i$, $\delta = \max_i \delta_i$, and $\gamma = \max_i |\gamma_i|$. Let us denote by $\tilde{L}$ the maximum value of the L_i, i.e., $\tilde{L} = \max_i \sup_{t_0 \le t \le T_i} L_i(t)$. Then, we may write (3) as

$$\ln \frac{L_i(t)}{L_i(t_0)} = \varepsilon_i - \frac{1}{2}|\gamma_i|^2 (t-t_0) + |\gamma_i| \sqrt{t-t_0}\, Z_i(t),$$

where $Z_i(t)$ is a standard normal distributed random variable and ε_i can be estimated by $|\varepsilon_i| \le (t-t_0)\delta \tilde{L} \gamma^2 \rho_i$. So, by neglecting ε_i we cause in L_i only a small *relative* error of order of ε_i when

$$|\varepsilon_i| \le (t-t_0)\delta \tilde{L} \gamma^2 \rho_i \le (t-t_0)\delta \tilde{L} \gamma^2 \rho << 1. \tag{4}$$

Note that e.g. for typical values, $\delta = 0.25$, $\gamma = 0.4$, $\tilde{L} = 0.07$, $t - t_0 = 5$ this relative error is about $1.4\rho\%$. However, dependent on ρ and the length of the tenor structure this error can become rather large in practice.

The approximation by neglecting the non-Gaussian terms ε_i in (3) will be called $(0)-$approximation to (2) which satisfies

$$dL_i^{(0)} = L_i^{(0)} \, \gamma_i \cdot dW^{(n)} \;, \tag{5}$$

and is given by the explicit solution

$$L_i^{(0)}(t) = L_i(t_0) \exp \left\{ -\frac{1}{2} \int_{t_0}^{t} \gamma_i^2(s) ds + \int_{t_0}^{t} \gamma_i(s) \cdot dW^{(n)}(s) \right\}. \tag{6}$$

Below we show for illustration (see Figs. 1,2) some typical samples of $L_i(t)$ and $L_i^{(0)}(t)$, where we chose $n = 21$, $|\gamma_1| = \ldots = |\gamma_{n-1}| = 0.4$, and

$$\rho_{ij} = \frac{b_i \wedge b_j}{b_i \vee b_j} , \qquad b_i = \exp\{\beta i^\alpha\}.$$

The correlations are thus defined via two parameters, β and α, see also [5]. In our simulations, presented in the figures below, we took $\alpha = 0.8$, $\beta = 0.1$ and $\alpha = 0.8$, $\beta = 0.3$, respectively. Further we chose $t_0 = 0$ and a uniform tenor structure $T_i = i\delta$ with $\delta = 0.25$, $i = 1, \ldots, 21$. The initial L values were taken to be $L_i(0) = 0.061$.

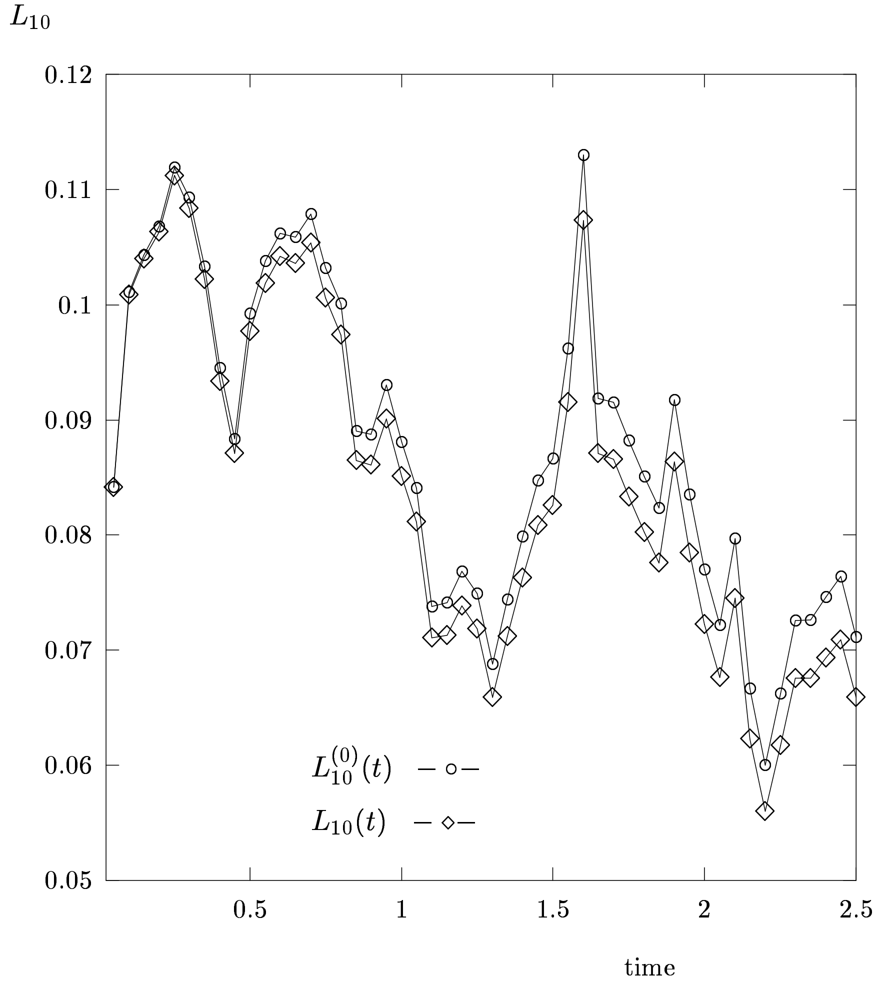

Fig.1 A sample of $L_{10}(t)$ and $L_{10}^{(0)}(t)$, for $\alpha = 0.8$ and $\beta = 0.1$.

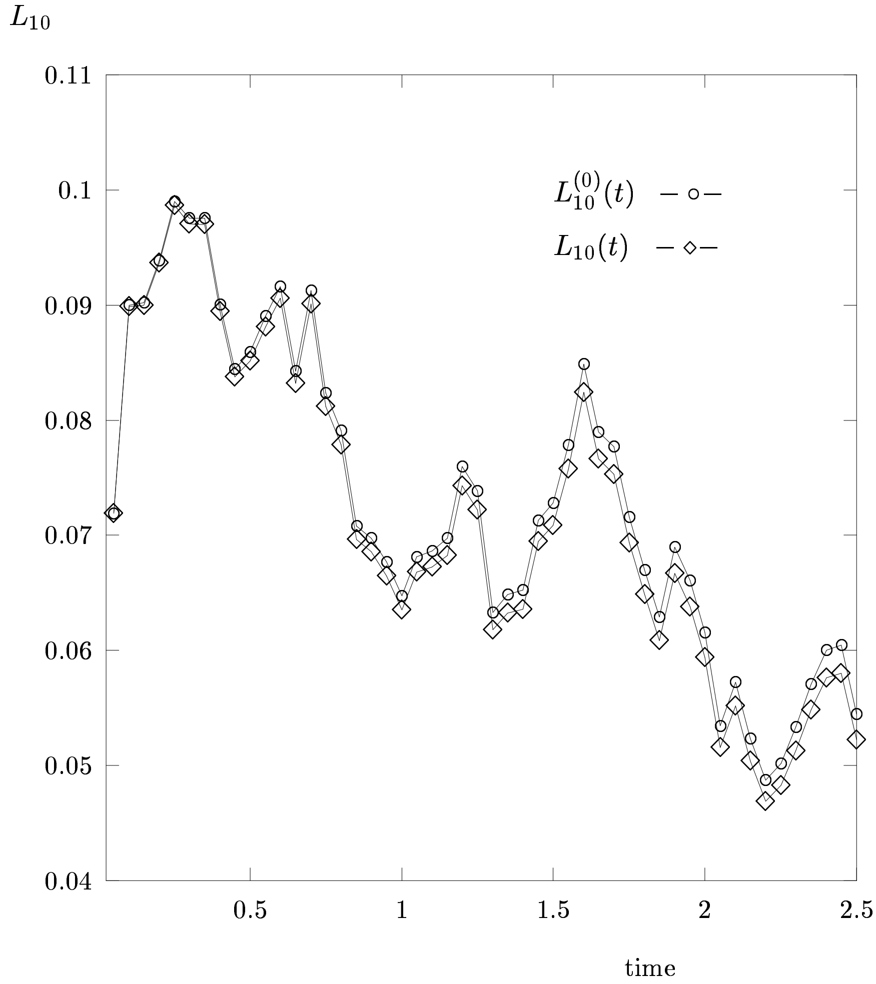

Fig.2 A sample of $L_{10}(t)$ and $L_{10}^{(0)}(t)$, for $\alpha = 0.8$ and $\beta = 0.3$.

From the trajectories presented in Figs.1-2 it is seen that on the initial time interval, the function $L_{10}^{(0)}$ approximates the function L_{10} very good. For increasing time, however, the discrepancy increases. Note that the larger β, the lesser the correlation time ρ and by (4) the lesser the discrepancy. This is confirmed by our observation presented above.

From the pictures in Figs.1-2 we see that the (0)$-$approximation is good for small times, whereas from (4) we see that for large i the (0)-approximation is also good because ρ_i decreases with i (e.g., ρ_{n-1} vanishes). More details about the (0) and other approximations are presented in Tables 1-5.

In Fig.3 we show a sample for the Bond price $B_{31}(T_i)$ and its (0)-approximation $B_{31}^{(0)}(T_i)$, $i = 0, \ldots, 31$. In contrast to the results presented in Figs.1-2, the maximum descrepancy happens around the middle of the time interval $(0, T_{31})$. The reason is that the Bond price $B_{31}(T_i)$ involves the product of all libor rates L_j, $j = i, \ldots, 30$ by $B_{31}(T_i) = \prod_{j=i}^{30}(1 + \delta L_j(T_i))^{-1}$. Indeed, either when i is close to zero or when i is close to 30 where the drift terms become small, the approximations $L_j^{(0)}(T_i)$, $j = i, \ldots, 30$ are close to $L_j(T_i)$ and so $B_{31}^{(0)}(T_i)$ is close to $B_{31}(T_i)$.

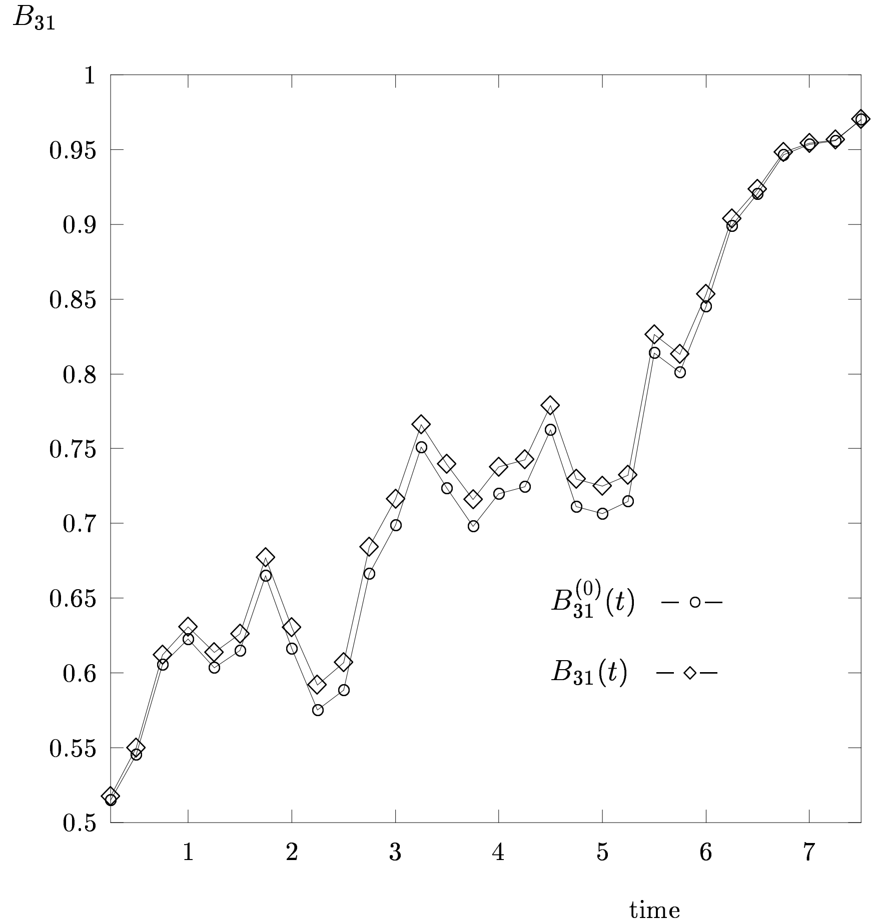

Fig.3 A sample of Bond prices $B_{31}(t)$ and its (0)-approximation, for $\alpha = 0.8$, $\beta = 0.1$, $n = 31$.

It is of interest to consider more refined approximations to L and in particular to look for lognormal approximations improving $L^{(0)}$. By replacing L_j in the right-hand side of (2) with $L_j^{(0)}$ we come to what we call the (1)-approximation:

$$dL_i^{(1)} = -\sum_{j=i+1}^{n-1} \frac{\delta_j L_i^{(1)} L_j^{(0)} \ \gamma_i \cdot \gamma_j}{1 + \delta_j L_j^{(0)}} dt + L_i^{(1)} \gamma_i \cdot dW^{(n)}, \tag{7}$$

The solution to (7) is given explicitly by:

$$\ln \frac{L_i^{(1)}(t)}{L_i(t_0)} = -\sum_{j=i+1}^{n-1} \int_{t_0}^{t} \frac{\delta_j L_j^{(0)}(s) \gamma_i \cdot \gamma_j \ (s)}{1 + \delta_j L_j^{(0)}(s)} ds - \frac{1}{2} \int_{t_0}^{t} \gamma_i^2(s) ds + \int_{t_0}^{t} \gamma_i(s) \cdot dW^{(n)}(s). \tag{8}$$

It turns out that this approach improves very much the (0)-approximation indeed, and the results presented in Tables 1-5 below confirm this conclusion. It should be noted, however, that the (1)-approximation is unfortunately non-lognormal, in contrast to the (0)-approximation. Therefore, for each j we approximate the process

$$Z_j(t) := \frac{\delta_j L_j^{(0)}}{1 + \delta_j L_j^{(0)}}$$

with a Gaussian process in (8) as follows. Let the function f be defined as $f(x) := x/(1+x)$, so $f^{(-1)}(x) = x/(-x+1)$ and $Z_j = f(\delta_j L_j^{(0)})$. Hence Z_j satisies the SDE

$$\begin{aligned} dZ_j &= f'(\delta_j L_j^{(0)})\delta_j L_j^{(0)}\gamma_j \cdot dW^{(n)} + \frac{1}{2}f''(\delta_j L_j^{(0)})[\delta_j L_j^{(0)}|\gamma_j|]^2 dt \\ &= f' \circ f^{(-1)}(Z_j)\, f^{(-1)}(Z_j)\,\gamma_j \cdot dW^{(n)} + \frac{1}{2}f'' \circ f^{(-1)}(Z_j)\,[f^{(-1)}(Z_j)\,|\gamma_j|]^2 dt \\ &= : a(Z_j,t)dt + b(Z_j,t)\cdot dW^{(n)}, \end{aligned}$$

with initial condition $Z_j(t_0) = f(\delta_j L_j(t_0))$. The Picard$-0$ and Picard-1 iteration for the solution of this SDE are respectively

$$Z_j^{(0)}(t) :\equiv Z_j(t_0) = \tfrac{\delta_j L_j(t_0)}{1+\delta_j L_j(t_0)} \qquad \text{and}$$
$$Z_j^{(1)}(t) = Z_j(t_0) + \textstyle\int_{t_0}^t [a(Z_j(t_0),s)ds + b(Z_j(t_0),s)\cdot dW^{(n)}(s)] =$$
$$f(\delta_j L_j(t_0)) + \tfrac{1}{2}f''(\delta_j L_j(t_0))\delta_j^2 L_j^2(t_0)\textstyle\int_{t_0}^t |\gamma_j|^2 ds + f'(\delta_j L_j(t_0))\delta_j L_j(t_0)\int_{t_0}^t \gamma_j \cdot dW^{(n)}(s) \quad ,$$

which are clearly both Gaussian. The next Picard iteration, however, will be non-Gaussian in general. By using $Z_j^{(0)}$ in (8) we find a lognormal approximation which we call the $(g)-$ approximation,

$$\ln \frac{L_i^{(g)}(t)}{L_i(t_0)} = \int_{t_0}^t \frac{-|\gamma_i(s)|^2}{2}ds + \int_{t_0}^t \gamma_i(s)\cdot dW^n(s) - \sum_{j=i+1}^{n-1}\int_{t_0}^t \frac{\delta_j L_j(t_0)\gamma_i\cdot\gamma_j\ (s)}{1+\delta_j L_j(t_0)}ds \tag{9}$$

which turns out to be a considerable path-wise improvement of the $(0)-$approximation and is suggested in [1, 5]. By expanding f, f' and f'' as $f(x) = x - x^2 + \mathcal{O}(x^3)$, $f'(x) = 1 - 2x + \mathcal{O}(x^2)$ and $f''(x) = -2 + \mathcal{O}(x)$ respectively, $x = \delta_j L_j(t_0)$ and denoting identity modulo terms of order $\mathcal{O}(x^2)$ and $\mathcal{O}(x^3)$ by $\simeq$ and $\cong$ respectively, we have

$$\begin{aligned} Z_j^{(1)}(t) &\cong Z_j^{(0)} - \delta_j^2 L_j^2(t_0)\int_{t_0}^t |\gamma_j|^2 ds + (\delta_j L_j(t_0) - 2\delta_j^2 L_j^2(t_0))\int_{t_0}^t \gamma_j \cdot dW^{(n)}(s) \\ &\simeq Z_j^{(0)} + \delta_j L_j(t_0)\int_{t_0}^t \gamma_j\cdot dW_s^{(n)} \simeq \delta_j L_j(t_0)(1 + \int_{t_0}^t \gamma_j \cdot dW_s^{(n)}), \end{aligned}$$

Using $Z_j^{(1)}$ while neglecting second order terms leads to another lognormal approximation, the $(g1)-$approximation:

$$\ln\frac{L_i^{(g1)}(t)}{L_i(t_0)} = \int_{t_0}^t \frac{-|\gamma_i|^2}{2}ds + \int_{t_0}^t \gamma_i(s)\cdot dW^{(n)}(s) - \sum_{j=i+1}^{n-1}\int_{t_0}^t \delta_j L_j(t_0)(1+\int_{t_0}^s \gamma_j(u)\cdot dW_u^{(n)})\gamma_i\cdot\gamma_j\ (s)ds. \tag{10}$$

The $(g1)-$approximation in its turn improves the $(g)-$approximation significantly as will appear from a comparative analysis below. Similarly, we may include also the second order terms and thus define a lognormal $(g1')-$approximation which, however, is only slightly better than the $(g1)$ and is, in fact, subordinate to a final lognormal approximation which we construct below.

Instead of $L_j^{(0)}$ we now plug in $L_j^{(g)}$ in the right-hand side of (2) and we arrive at the $(2)-$ approximation which is given explicitly by,

$$\ln\frac{L_i^{(2)}(t)}{L_i(t_0)} = -\sum_{j=i+1}^{n-1}\int_{t_0}^t \frac{\delta_j L_j^{(g)}(s)\gamma_i\cdot\gamma_j\ (s)}{1+\delta_j L_j^{(g)}(s)}ds - \frac{1}{2}\int_{t_0}^t \gamma_i^2(s)ds + \int_{t_0}^t \gamma_i(s)\cdot dW^{(n)}(s). \tag{11}$$

Now construct a Gaussian approximation for $f(L_j^{(g)})$ as above. We redefine

$$Z_j := f(\delta_j L_j^{(g)}) = \frac{\delta_j L_j^{(g)}}{1+\delta_j L_j^{(g)}}$$

Hence Z now satisfies the SDE

$$\begin{aligned} dZ_j &= f'(\delta_j L_j^{(g)})\delta_j L_j^{(g)}\gamma_j \cdot dW^{(n)} + \frac{1}{2}f''(\delta_j L_j^{(g)})[\delta_j L_j^{(g)}|\gamma_j|]^2 dt + \\ & \quad f'(\delta_j L_j^{(g)})\left\{-\delta_j \sum_{k=j+1}^{n-1} \frac{\delta_k L_k(t_0)\gamma_k \cdot \gamma_j}{1+\delta_k L_k(t_0)} L_j^{(g)}\right\} dt \end{aligned}$$

with initial condition $Z_j(t_0) = f(\delta_j L_j(t_0))$. Obviously, replacing Z_j by the Picard-0 iteration $Z_j^{(0)} \equiv f(\delta_j L_j(t_0))$ gives the $(g)-$approximation again, whereas the Picard-1 iteration now leads to

$$\begin{aligned} Z_j^{(1)}(t) = f(\delta_j L_j(t_0)) + f'(\delta_j L_j(t_0))\delta_j L_j(t_0) \textstyle\int_{t_0}^t \gamma_j \cdot dW^{(n)} + \frac{1}{2}f''(\delta_j L_j(t_0))\delta_j^2 L_j^2(t_0) \int_{t_0}^t |\gamma_j|^2 ds \\ + f'(\delta_j L_j(t_0))\left\{-\delta_j \textstyle\sum_{k=j+1}^{n-1} \frac{\delta_k L_k(t_0) L_j(t_0)}{1+\delta_k L_k(t_0)}\right\} \int_{t_0}^t \gamma_j \cdot \gamma_k dt \end{aligned} \tag{12}$$

It should be noted that when instead of $L_j^{(g)}$ we plug in $L_j^{(1)}$ or $L_j^{(2)}$ in the right hand side of (2), although we get better and better explicit non-lognormal approximations, the Gaussian Picard-1 approximation for $Z_j = f(\delta_j L_j^{(1)})$ and $Z_j = f(\delta_j L_j^{(2)})$, respectively, is the same as in (12). So we do not get better Gaussian approximations in this way. In fact, we may derive (12) directly from (2). Clearly by neglecting second order terms in (12) we get the $(g1)-$approximation again, whereas by keeping second order terms we get a new log-normal approximation, $(g2)$ say, given by

$$\ln \frac{L_i^{(g2)}(t)}{L_i(t_0)} = -\sum_{j=i+1}^{n-1} \int_{t_0}^{t} \tilde{Z}(s)\gamma_i \cdot \gamma_j \ (s)ds - \frac{1}{2}\int_{t_0}^{t} \gamma_i^2(s)ds + \int_{t_0}^{t} \gamma_i(s) \cdot dW^{(n)}(s), \tag{13}$$

where

$$\begin{aligned} \tilde{Z}(t) \cong f(\delta_j L_j(t_0)) - \delta_j^2 L_j^2(t_0) \textstyle\int_{t_0}^t |\gamma_j|^2 ds + \\ \left\{-\delta_j \textstyle\sum_{k=j+1}^{n-1} \frac{\delta_k L_k(t_0) L_j(t_0)}{1+\delta_k L_k(t_0)}\right\} \int_{t_0}^t \gamma_j \cdot \gamma_k dt + (1-2\delta_j L_j(t_0))\delta_j L_j(t_0) \int_{t_0}^t \gamma_j \cdot dW^{(n)} \end{aligned}$$

Here we note that the $(g1')-$approximation differs from $(g2)$ in that the term with the sum is missing.

It is now interesting to carry out a comparative numerical analysis of the different approximations presented. The numerical solution of the relevant stochastic differential equations are solved by the Euler scheme.

For a correct path-wise comparison, it is necessary to construct all the approximations in one common probability space. In the numerical schemes, it is easily achieved by using one and the same Wiener increments for all approximations.

In the next tables we show how often the relative error (in percents) of the corresponding approximation to L_5, L_{10}, and L_{20} lies in the relevant percentage intervals (first columns). For instance, the relative error between $\hat{L}_i$, the numerical solution to the original equation (2) and $\hat{L}_i^{(1)}$, the numerical solution to the equation (7) is defined as

$$\epsilon_i = \max_{1\le j\le i} \frac{|\hat{L}_i(T_j) - \hat{L}_i^{(1)}(T_j)|}{\hat{L}_i(T_j)},$$

and corresponds, e.g., in the tables to the third column. The relative errors to other approximations are defined analogously.

In all tables we chose uniformly $|\gamma_1| = \ldots = |\gamma_{n-1}| = 0.4$, $L_1(0) = \ldots = L_{n-1}(0) = 0.061$ and $\alpha = 0.8$.

For instance, the numbers in the columns 2 - 7 of table 1 show the fraction of 700000 samples for which the event shown in the first column happens. From these results we see that among all the path-wise approximations, the best one is the (2)-approximation, which is however non-lognormal. Among the lognormal approximations, the $(g2)-$approximation shows the best results. Also we conclude that the approximations are better when the LIBORs are more de-correlated. Indeed, de-correlation diminishes the drifts in (2). Note that the fact that a path-wise approximation is not good enough (e.g., see the (0)-approximation in column 6) does not imply that the statistical characteristics will be approximated not good as well. We will illustrate this in the case of swap and trigger swap, section (4).

$100 \cdot \epsilon_5$	(2)	(1)	(g2)	(g1)	(g)	(0)
$\leq 0.25\,\%$	0.9	0.561	0.0159	0.8059E-02	0.7529E-03	0.
$\leq 0.5\,\%$	0.954	0.804	0.0966	0.0591	0.0122	0.
$\leq 0.75\,\%$	0.974	0.895	0.208	0.136	0.0398	0.
$\leq 1\,\%$	0.984	0.937	0.324	0.219	0.0781	0.
$\leq 1.5\,\%$	0.993	0.973	0.574	0.395	0.166	0.
$\leq 2\,\%$	0.996	0.986	0.835	0.606	0.255	0.3857E-04
$\leq 2.5\,\%$	0.998	0.992	0.89	0.856	0.338	0.5471E-03
$\leq 3\,\%$	0.999	0.995	0.913	0.93	0.417	0.2987E-02
$\leq 3.5\,\%$		0.997	0.930	0.944	0.494	0.01
$\leq 4\,\%$		0.998	0.944	0.955	0.569	0.0246
$\leq 4.5\,\%$		0.9987	0.955	0.964	0.64	0.0472
$\leq 5\,\%$		0.999	0.964	0.971	0.707	0.0785
$\leq 6\,\%$			0.976	0.981	0.819	0.1597
$\leq 7\,\%$			0.984	0.987	0.891	0.2564
$\leq 8\,\%$			0.989	0.991	0.921	0.356
$\leq 9\,\%$			0.992	0.993	0.935	0.450
$\leq 10\,\%$			0.994	0.995	0.945	0.535
$\leq 12\,\%$			0.996	0.997	0.961	0.672
$\leq 14\,\%$			0.998	0.998	0.972	0.769
$\leq 16\,\%$			0.998	0.999	0.979	0.837
$\leq 18\,\%$			0.999		0.985	0.884
$\leq 20\,\%$					0.988	0.917

Table 1. The cumulative distribution of the relative error ϵ_{20}, for different approximations; $\beta = 0.1$, $N = 700000$, $T_1 = 1$, $n = 31$.

$100 \cdot \epsilon_5$	(2)	(1)	(g2)	(g1)	(g)	(0)
$\leq 0.25\,\%$	0.9478	0.5046	0.1198	0.0491	0.0054	0.
$\leq 0.5\,\%$	0.9859	0.8420	0.3710	0.1844	0.0466	0.
$\leq 0.75\,\%$	0.9953	0.9417	0.6321	0.3335	0.1115	0.
$\leq 1\,\%$	0.9980	0.9751	0.8551	0.5064	0.1817	0.
$\leq 1.5\,\%$	0.9996	0.9946	0.9179	0.9262	0.3126	0.
$\leq 2\,\%$	0.9998	0.9985	0.9487	0.9660	0.4297	0.
$\leq 2.5\,\%$	0.9999	0.9995	0.9672	0.9788	0.5400	0.
$\leq 3\,\%$		0.9997	0.9801	0.9871	0.6382	0.7500E-04
$\leq 3.5\,\%$		0.9999	0.9878	0.9920	0.7252	0.7250E-03
$\leq 4\,\%$			0.9920	0.9948	0.8	0.3975E-02
$\leq 4.5\,\%$			0.9949	0.9964	0.8584	0.1258E-01
$\leq 5\,\%$			0.9963	0.9976	0.9008	0.3023E-01
$\leq 6\,\%$			0.9984	0.9989	0.9444	0.9710E-01
$\leq 7\,\%$			0.9992	0.9994	0.9617	0.2052
$\leq 8\,\%$			0.9995	0.9996	0.9720	0.3341
$\leq 9\,\%$			0.9997	0.9997	0.9804	0.4633
$\leq 10\,\%$			0.9997	0.9998	0.9859	0.5813
$\leq 12\,\%$			0.9998	0.9999	0.9927	0.7584
$\leq 14\,\%$			0.9999	0.9999	0.9962	0.8668
$\leq 16\,\%$					0.9979	0.9270
$\leq 18\,\%$					0.9990	0.9607
$\leq 20\,\%$					0.9994	0.9785

Table 2. The cumulative distribution of the relative error ϵ_{10}, for different approximations; $\beta = 0.1$, $N = 40000$, $T_1 = 1$, $n = 31$.

$100 \cdot \epsilon_5$	(2)	(1)	(g2)	(g1)	(g)	(0)
$\leq 0.25\,\%$	0.9970	0.8810	0.5064	0.2157	0.5102	0.
$\leq 0.5\,\%$	0.9999	0.9929	0.8975	0.5917	0.1799	0.
$\leq 0.75\,\%$		0.9991	0.9425	0.9670	0.3079	0.
$\leq 1\,\%$		0.9999	0.9670	0.9812	0.4208	0.
$\leq 1.5\,\%$			0.9889	0.9936	0.6111	0.
$\leq 2\,\%$			0.9963	0.9979	0.7659	0.
$\leq 2.5\,\%$			0.9985	0.9990	0.8728	0.5000E-04
$\leq 3\,\%$			0.9992	0.9995	0.9320	0.8500E-03
$\leq 3.5\,\%$			0.9996	0.9999	0.9603	0.7400E-02
$\leq 4\,\%$			0.9999	0.9999	0.9743	0.2947E-01
$\leq 4.5\,\%$			0.9999		0.9827	0.8088E-01
$\leq 5\,\%$					0.9879	0.1643
$\leq 6\,\%$					0.9947	0.3950
$\leq 7\,\%$					0.9974	0.6185
$\leq 8\,\%$					0.9988	0.7864
$\leq 9\,\%$					0.9992	0.8846
$\leq 10\,\%$					0.9997	0.9412
$\leq 12\,\%$						0.9856
$\leq 14\,\%$						0.9966
$\leq 16\,\%$						0.9991

Table 3. The cumulative distribution of the relative error ϵ_5, for different approximations; $\beta = 0.1$, $N = 40000$, $T_1 = 1$, $n = 31$.

$100 \cdot \epsilon_5$	(2)	(1)	(g2)	(g1)	(g)	(0)
$\leq 0.25\,\%$	0.8953	0.5452	0.0113	0.0066	0.2500E-03	0.
$\leq 0.5\,\%$	0.9514	0.7923	0.0806	0.0502	0.9325E-02	0.
$\leq 0.75\,\%$	0.9724	0.8879	0.1802	0.1198	0.3142E-01	0.
$\leq 1\,\%$	0.9828	0.9330	0.2889	0.1978	0.6515E-01	0.
$\leq 1.5\,\%$	0.9915	0.9703	0.5142	0.3571	0.1451	0.
$\leq 2\,\%$	0.9956	0.9846	0.7954	0.5410	0.2279	0.
$\leq 2.5\,\%$	0.9974	0.9910	0.8803	0.7905	0.3043	0.1750E-03
$\leq 3\,\%$	0.9985	0.9946	0.9017	0.9192	0.3743	0.1375E-02
$\leq 3.5\,\%$	0.9988	0.9966	0.9193	0.9356	0.4478	0.5475E-02
$\leq 4\,\%$	0.9992	0.9978	0.9355	0.9473	0.5192	0.1398E-01
$\leq 4.5\,\%$	0.9994	0.9986	0.9469	0.9566	0.5880	0.2850E-01
$\leq 5\,\%$	0.9996	0.9989	0.9565	0.9641	0.6526	0.5120E-01
$\leq 6\,\%$	0.9998	0.9993	0.9698	0.9746	0.7695	0.1122
$\leq 7\,\%$	0.9998	0.9997	0.9785	0.9826	0.8533	0.1950
$\leq 8\,\%$	0.9999	0.9998	0.9848	0.9869	0.9029	0.2857
$\leq 9\,\%$	0.9999	0.9998	0.9887	0.9903	0.9233	0.3774
$\leq 10\,\%$	0.9999	0.9999	0.9916	0.9931	0.9354	0.4598
$\leq 12\,\%$	0.9999	0.9999	0.9952	0.9961	0.9524	0.6015
$\leq 14\,\%$		0.9999	0.9969	0.9974	0.9645	0.7090
$\leq 16\,\%$			0.9979	0.9983	0.9735	0.7879
$\leq 18\,\%$			0.9985	0.9986	0.9799	0.8453
$\leq 20\,\%$			0.9988	0.9988	0.9845	0.8857

Table 4. The cumulative distribution of the relative error ϵ_{20}, for different approximations; $\beta = 0,\ N = 40000$, $T_1 = 0.25$, $n = 31$.

$100 \cdot \epsilon_5$	(2)	(1)	(g2)	(g1)	(g)	(0)
$\leq 0.25\,\%$	0.9887	0.9268	0.2079	0.1371	0.0269	0.
$\leq 0.5\,\%$	0.9979	0.9873	0.5301	0.3830	0.1300	0.
$\leq 0.75\,\%$	0.9992	0.9965	0.8094	0.6410	0.2448	0.
$\leq 1\,\%$	0.9997	0.9986	0.9075	0.8799	0.3492	0.2500E-04
$\leq 1.5\,\%$	0.9999	0.9997	0.9516	0.9612	0.5365	0.7750E-03
$\leq 2\,\%$		0.9999	0.9734	0.9789	0.7033	0.1385E-01
$\leq 2.5\,\%$			0.9844	0.9873	0.8310	0.5765E-01
$\leq 3\,\%$			0.9909	0.9925	0.9054	0.1432
$\leq 3.5\,\%$			0.9944	0.9951	0.9386	0.2579
$\leq 4\,\%$			0.9959	0.9967	0.9521	0.3784
$\leq 4.5\,\%$			0.9972	0.9976	0.9620	0.4933
$\leq 5\,\%$			0.9979	0.9982	0.9696	0.5953
$\leq 6\,\%$			0.9987	0.9989	0.9806	0.7493
$\leq 7\,\%$			0.9993	0.9995	0.9880	0.8462
$\leq 8\,\%$			0.9995	0.9996	0.9921	0.9068
$\leq 9\,\%$			0.9997	0.9998	0.9948	0.9436
$\leq 10\,\%$			0.9998	0.9999	0.9965	0.9631
$\leq 12\,\%$			0.9999	0.9999	0.9983	0.9849
$\leq 14\,\%$					0.9992	0.9934
$\leq 16\,\%$					0.9996	0.9968

Table 5. The cumulative distribution of the relative error ϵ_{20}, for different approximations; $\beta = 0.5,\ N = 40000$, $T_1 = 1$, $n = 31$.

3 Simulation of a log-normal random field (DST)

The results of section 2 listed in Tables 1-5 clearly show that lognormal models $(g2)$, $(g1)$, (g) and (0) are good approximations to the solution of SDE (2). This suggests the following direct simulation technique (DST): construct lognormal random field models whose first two statistical moments are consistent with those of the approximations $(g2)$, $(g1)$, (g), (0).

The motivation of DST is clear: in contrast to numerical solution of stochastic differential equations there is no need for taking small time steps; in DST, it is possible to construct the solution directly at the desired points, e.g., at the points of the given tenor structure $0 < T_1 < T_2 < \dots T_n$. Therefore, DNT takes generally much less computer time.

To be more specific, let us construct the direct simulation algorithm consistent with the (g)-approximation.

We thus have to construct a lognormal random field

$$\mathcal{L}^{(g)}(i,t) = \exp\{\xi^{(g)}(i,t)\} \tag{14}$$

with gaussian $\xi^{(g)}(i,t)$, $i = 1,\dots,n-1$, $t_0 \le t \le T_i$, whose mean and covariation structure coincide with that of $\ln(L_i^{(g)}(t)/L_i(t_0))$, $t_0 \le t \le T_i$, $i = 1,\dots n-1$, in the $\mathbb{P}_n-$ measure:

$$\langle \xi^{(g)}(i,t)\rangle = \langle \ln\Big(\frac{L_i^{(g)}(t)}{L_i(t_0)}\Big)\rangle, \tag{15}$$

$$\langle \xi^{(g)}(i_1,t_1), \xi^{(g)}(i_2,t_2)\rangle = \langle \ln\Big(\frac{L_{i_1}^{(g)}(t_1)}{L_{i_1}(t_0)}\Big), \ln\Big(\frac{L_{i_2}^{(g)}(t_2)}{L_{i_2}(t_0)}\Big)\rangle. \tag{16}$$

From (9) we see that

$$\langle \xi^{(g)}(i,t)\rangle \equiv \mu^{(g)}(i;t_0,t) = -\sum_{j=i+1}^{n-1} \frac{\delta_j L_j(t_0)}{1+\delta_j L_j(t_0)} \int_{t_0}^{t} \gamma_i \cdot \gamma_j(s)ds - \frac{1}{2}\int_{t_0}^{t} |\gamma_i|^2(s)ds, \tag{17}$$

$$\langle \xi^{(g)}(i_1,t_1), \xi^{(g)}(i_2,t_2)\rangle \equiv cov^{(g)}(i_1,i_2;t_0,t_1\wedge t_2) + \mu^{(g)}(i_1;t_0,t_1)\mu^{(g)}(i_2;t_0,t_2), \tag{18}$$

where

$$cov^{(g)}(i_1,i_2;t_0,t) = \int_{t_0}^{t} \gamma_i(s)\gamma_j(s)ds.$$

In practice, one usually evaluates LIBOR derivatives which depend on the values $L_i(T_j)$, $i = 1,\dots,n-1$, $j = 1,\dots,i$. Therefore, we have to construct numerically the desired random field $\mathcal{L}^{(g)}(i,T_j)$, $i = 1,\dots,n-1$; $j = 1,\dots i$. To do this, we could simulate the gaussian vector with the given covariance structure by a conventional simulation technique. However the specific time correlation suggests a different simulation algorithm, [3]. Indeed, in the first step, we simulate a $n-1$-dimensional gaussian vector $(\xi^{(g)}(1,T_1),\dots,\xi^{(g)}(n-1,T_1))$ as

$$\xi^{(g)}(i,T_1) = \mu^{(g)}(i;t_0,T_1) + \sum_{k=1}^{K_1} h_{ik}^{(1)}\eta_k^{(1)}, \quad i = 1,\dots,n-1 \tag{19}$$

where the positive integer number K_1 and the entries $h_{ik}^{(1)}$ are chosen so that

$$\sum_{k=1}^{K_1} h_{ik}^{(1)} h_{jk}^{(1)} = cov^{(g)}(i,j;t_0,T_1), \qquad i,j = 1,\dots n-1,$$

and $\{\eta_k^{(1)}\}_{k=1}^{K_1}$ is a set of independent standard gaussian random numbers.

In the l-th step $(2 \le l \le n-1)$ we have:

$$\xi^{(g)}(i,T_l) = \xi^{(g)}(i,T_{l-1}) + \mu^{(g)}(i;t_0,T_l) - \mu^{(g)}(i;t_0,T_{l-1}) + \sum_{k=1}^{K_l} h_{ik}^{(l)}\eta_k^{(l)}, \quad i = l,\dots,n-1. \quad (20)$$

The positive integer number K_l and the entries $h_{ik}^{(l)}$ are chosen so that

$$\sum_{k=1}^{K_l} h_{ik}^{(l)} h_{jk}^{(l)} = cov^{(g)}(i,j;T_{l-1},T_l), \qquad i,j = l,\dots n-1, \quad (21)$$

where $\eta_l^{(l)}$, $\eta_{l+1}^{(l)}$, $\dots$, $\eta_{n-1}^{(l)}$ is a set of independent standard gaussian random numbers.

Thus after $n-1$ steps we find

$$L_i^{(g)}(T_j) = L_i(t_0)\mathcal{L}^{(g)}(i,T_j) = L_i(t_0)\exp\{\xi^{(g)}(i,T_j)\}, \qquad i = 1,\dots,n-1;\ j = 1,\dots,i. \quad (22)$$

Here we presented simulation of a lognormal random field consistent with the g-approximation. Analogously, the same could be easily done for the lognormal approximations (0). Indeed, the simulation formulae (19)- (22) remain the same, but the functions $\mu^{(g)}$ and $cov^{(g)}$ should be replaced with

$$\mu^{(0)}(i;t_0,t_1) = 0, \quad \text{and} \quad cov^{(0)}(i_1,i_2;t_0,t) = cov^{(g)}(i_1,i_2;t_0,t),$$

for the $(0)-$approximation. For the $g1$-approximation we may define,

$$\mu^{(g1)}(i;t_0,t) = -\sum_{j=i+1}^{n-1} \delta_j L_j(t_0) \int_{t_0}^{t} \gamma_i \cdot \gamma_j(s)ds - \frac{1}{2}\int_{t_0}^{t} |\gamma_i|^2(s)ds.$$

From (10) we derive

$$\ln\frac{L_i^{(g1)}(t)}{L_i(t_0)} = \mu^{(g1)}(i;t_0,t) + \int_{t_0}^{t}\left[1 - \sum_{j=i+1}^{n-1} \delta_j L_j(t_0)\int_s^t \gamma_j\cdot\gamma_i(u)du\right]\gamma_i(s)\cdot dW_s$$

and thus find

$$Cov[\xi^{(g1)}(i_1,t_1),\xi^{(g1)}(i_2,t_2)] = \int_{t_0}^{t_1\wedge t_2}\gamma_{i_1}\cdot\gamma_{i_2}(s)\left[1-\sum_{j=i_1+1}^{n-1}\delta_j L_j(t_0)\int_s^{t_1}\gamma_j\cdot\gamma_{i_1}(u)du\right]\times \left[1-\sum_{k=i_2+1}^{n-1}\delta_k L_k(t_0)\int_s^{t_2}\gamma_k\cdot\gamma_{i_2}(u)du\right]ds$$

and similar expressions for the $(g2)$-approximation can be derived from (13). However, unfortunately the covariance functions of $(g1,2)$ have not the special structure as in the case of the (0) and $(g)-$ approximation, so the simulation of the corresponding random fields might be slower.

Remark Note that the cost of the simulation algorithm used for the $(g)-$approximation has the order $O(n^4)$ since in the l-th step, we apply the Cholesky decomposition (21) whose cost has the order $O(n^3)$. The conventional direct method would take about $O(n^6)$ operations. It should be noted also that if the factor loadings functions γ_i do not depend on time, then the cost of our algorithm is $O(n^3)$, since we apply the Cholesky decomposition only once, at the first step.

There is one interesting feature of our algorithm which is to be stressed: in practice, one often specifies the model not by the factor loadings γ_i, but through the quantities

$$Cov(i,j,t) \equiv \int_{t_0}^{t}\gamma_i\cdot\gamma_j(s)ds$$

which can be determined from the Cap/Swaption markets, see also [5]. In our random field approximations just these quantities are only relevant and can thus be plugged in directly, whereas in case the approximations are obtained by numerical solution of the relevant SDE, it is needed to calculate the factor loadings by Cholesky decomposition of the time derivatives of $Cov(i,j,t)$, generally, in each integration step. This can be very time consuming, especially when the factor loadings are time dependent.

4 Valuation of swaptions and trigger swaps

We now present some test results on the valuation of two typical LIBOR derivatives: the swaption and the trigger swap. For a derivation of the several valuation formulas, see e.g. [5]

The value of a swaption, an option to swap LIBOR against a fixed coupon κ at the settlement dates $T_2, \ldots, T_n$, can be represented in the $\mathbb{P}_n$ measure by

$$Swpn(t) = \sum_{j=1}^{n-1} B_n(t)\mathbb{E}_n \left[\frac{B_{j+1}(T_1)}{B_n(T_1)} 1_A (L_j(T_1) - \kappa)\delta_j \ |\mathcal{F}_t \right], \tag{23}$$

In (23), A denotes the $\mathcal{F}_{T_1}$ measurable event $\{S(T_1) > \kappa\}$, where the swaprate $S(T_1)$ is given by

$$S(T_1) := \frac{1 - B_n(T_1)}{\sum_{k=1}^{n-1} \delta_k B_{k+1}(T_1)} = \frac{-1 + \prod_{k=1}^{n-1}(1 + \delta_k L_k(T_1))}{\sum_{k=1}^{n-1} \delta_k \prod_{i=k+1}^{n-1}(1 + \delta_i L_i(T_1))}$$

and $B_{j+1}(T_1)/B_n(T_1)$ can be expressed in the LIBORs by

$$\frac{B_{j+1}(T_1)}{B_n(T_1)} = \prod_{i=j+1}^{n-1} (1 + \delta_i L_i(T_1)).$$

In a trigger swap contract with specified trigger levels $K_1, \ldots, K_n$, as soon as $L_i(T_i) > K_i$ one has to swap LIBOR against a fixed coupon κ for the remaining period $[T_i, T_n]$ with settlement dates $T_{i+1}, \ldots, T_n$.

The value of the trigger swap in the $\mathbb{P}_n$ measure can be expressed by

$$Trswp(t) = \sum_{p=1}^{n-1} B_n(t)\mathbb{E}_n \left[1_{[\tau=p]} \frac{1}{B_n(T_p)} \left(1 - B_n(T_p) - \kappa \sum_{j=p}^{n-1} B_{j+1}(T_p)\delta_j \right) \mid \mathcal{F}_t \right], \tag{24}$$

where τ, the trigger index, is given by $\tau := \min_{1 \le p < n}\{p \mid L_p(T_p) > K_p\}$, see [5]. In (24) the expression inside the expectation can be expressed in LIBORS only and we thus have

$$Trswp(t) = \sum_{p=1}^{n-1} B_n(t)\mathbb{E}_n \left[1_{[\tau=p]} \left(-1 + \prod_{i=p}^{n-1}(1 + \delta_i L_i(T_p)) - \kappa \sum_{j=p}^{n-1} \delta_j \prod_{i=j+1}^{n-1} (1 + \delta_i L_i(T_p)) \right) \mid \mathcal{F}_t \right]. \tag{25}$$

We now simulate the prices of swaptions and trigger swaps for the LIBOR trajectories simulated in the tables 1,2 and 4, where the strike is taken to be the initial swaprate; $\kappa = 0.06045$ and all trigger levels equal to the strike; $K_p = \kappa$ for every p. The Monte Carlo errors are based on three standard deviations. Note that the discrepancy between an option value simulated with L and a value simulated with some approximation should be interpreted as a systematic error caused by the approximation since the trajectories of L and the approximation are constructed with one and the same Wiener increments.

simulation	*swaption*	*M.C. error*	*trig. swap*	*M.C. error*
L	0.4400E-01	0.3390E-03	0.4747E-01	0.4242E-03
$L^{(2)}$	0.4396E-01	0.3367E-03	0.4745E-01	0.4240E-03
$L^{(1)}$	0.4387E-01	0.3356E-03	0.4739E-01	0.4237E-03
$L^{(g2)}$	0.4432E-01	0.3772E-03	0.4749E-01	0.4273E-03
$L^{(g1)}$	0.4414E-01	0.3749E-03	0.4735E-01	0.4265E-03
$L^{(g)}$	0.4579E-01	0.4519E-03	0.4940E-01	0.4355E-03
$L^{(0)}$	0.5188E-01	0.5441E-03	0.5450E-01	0.4521E-03

Table 6. Swaption and trigger swap values for different approximations; $\beta = 0.1$, $N = 700000$, $T_1 = 1$, $n = 31$.

simulation	*swaption*	*M.C. error*	*trig. swap*	*M.C. error*
L	0.4389E-01	0.1441E-02	0.4799E-01	0.1810E-02
$L^{(2)}$	0.4384E-01	0.1432E-02	0.4796E-01	0.1808E-02
$L^{(1)}$	0.4376E-01	0.1427E-02	0.4792E-01	0.1807E-02
$L^{(g2)}$	0.4421E-01	0.1561E-02	0.4802E-01	0.1832E-02
$L^{(g1)}$	0.4404E-01	0.1553E-02	0.4791E-01	0.1828E-02
$L^{(g)}$	0.4568E-01	0.1770E-02	0.5002E-01	0.1853E-02
$L^{(0)}$	0.5176E-01	0.2038E-02	0.5533E-01	0.1927E-02

Table 7. Swaption and trigger swap values for different approximations; $\beta = 0.1$, $N = 40000$, $T_1 = 1$, $n = 31$.

simulation	*swaption*	*M.C. error*	*trig. swap*	*M.C. error*
L	0.2830E-01	0.7791E-03	0.4487E-01	0.7704E-03
$L^{(2)}$	0.2828E-01	0.7764E-03	0.4486E-01	0.7704E-03
$L^{(1)}$	0.2826E-01	0.7753E-03	0.4488E-01	0.7701E-03
$L^{(g2)}$	0.2844E-01	0.8171E-03	0.4488E-01	0.7705E-03
$L^{(g1)}$	0.2838E-01	0.8147E-03	0.4494E-01	0.7705E-03
$L^{(g)}$	0.2892E-01	0.8843E-03	0.4517E-01	0.7753E-03
$L^{(0)}$	0.3139E-01	0.9783E-03	0.4508E-01	0.7972E-03

Table 8. Swaption and trigger swap values for different approximations; $\beta = 0$, $N = 40000$, $T_1 = 0.25$, $n = 31$.

simulation	*swaption*	*M.C. error*	*trig. swap*	*M.C. error*
L	0.2691E-01	0.7269E-03	0.1995E-01	0.1287E-02
$L^{(2)}$	0.2691E-01	0.7267E-03	0.1994E-01	0.1286E-02
$L^{(1)}$	0.2690E-01	0.7265E-03	0.1992E-01	0.1286E-02
$L^{(g2)}$	0.2693E-01	0.7289E-03	0.1998E-01	0.1295E-02
$L^{(g1)}$	0.2689E-01	0.7282E-03	0.1991E-01	0.1294E-02
$L^{(g)}$	0.2715E-01	0.7366E-03	0.2048E-01	0.1307E-02
$L^{(0)}$	0.2880E-01	0.7638E-03	0.2365E-01	0.1338E-02

Table 9. Swaption and trigger swap values for different approximations; $\beta = 0.5$, $N = 40000$, $T_1 = 1$, $n = 31$.

$\rho_{i,j}$	1	4	7	10	13	16	19	22	25	28
1	1.00									
4	0.82	1.00								
7	0.69	0.84	1.00							
10	0.59	0.72	0.86	1.00						
13	0.51	0.62	0.74	0.86	1.00					
16	0.44	0.54	0.64	0.75	0.87	1.00				
19	0.39	0.47	0.56	0.65	0.76	0.87	1.00			
22	0.34	0.41	0.49	0.57	0.67	0.77	0.88	1.00		
25	0.30	0.36	0.43	0.51	0.59	0.67	0.77	0.88	1.00	
28	0.26	0.32	0.38	0.45	0.52	0.60	0.68	0.78	0.88	1.00

Table 9. Forward log LIBOR correlations $\rho(\ln L_i(T_1), \ln L_j(T_1))$, for $\alpha = 0.8$ and $\beta = 0.1$.

$\rho_{i,j}$	1	4	7	10	13	16	19	22	25	28
1	1.00									
4	0.36	1.00								
7	0.15	0.42	1.00							
10	0.07	0.19	0.46	1.00						
13	0.03	0.09	0.22	0.48	1.00					
16	0.02	0.05	0.11	0.24	0.49	1.00				
19	0.01	0.02	0.06	0.12	0.25	0.51	1.00			
22	0.00	0.01	0.03	0.06	0.13	0.26	0.52	1.00		
25	0.00	0.01	0.02	0.03	0.07	0.14	0.27	0.53	1.00	
28	0.00	0.00	0.01	0.02	0.04	0.07	0.15	0.28	0.54	1.00

Table 10. Forward log LIBOR correlations $\rho(\ln L_i(T_1), \ln L_j(T_1))$, for $\alpha = 0.8$ and $\beta = 0.5$.

5 Conclusion

For practical relevance, simulation prices of derivatives should be well within so called bid-ask spreads: A bid-ask spread can be estimated roughly by the change of the claim price due to an overall LIBOR-volatility movement of 5% up and 5% down. By experiment we found out that for the examples above this comes down to desire a relative accuracy of about 5%, both for the swaption and the triggerswap. So, from table 6, where the Monte Carlo error (defined as 3 standard deviations) is much smaller than the spread we may conclude that the $(g1)-$approximation performs excellent whereas the $(g)-$approximation performs tolerable. The $(0)-$approximation, however, produces relative errors of more than 6%. From tables 6 and 7 we see also that 40000 payoff simulations are sufficient to reach a Monte Carlo error below 5% and when simulated with the DSM method applied to the $(g)-$ approximation this takes a few seconds for the swaption and a few minutes for the triggerswap respectively. However, simulation of these prices by solving the SDE for L, $L^{(1)}$ or $L^{(2)}$ by using small time steps takes much longer, for instance, a few hours for the triggerswap.

References

[1] Brace, A., Gatarek, D., Musiela, M.: The Market Model of Interest Rate Dynamics. *Mathematical Finance,* 7 (2), 127-155, (1997).

[2] Jamshidan, F.: LIBOR and swap market models and measures. *Finance and Stochastics,* (1), 293-330, (1997).

[3] Sabelfeld, K.K.: Monte Carlo methods in boundary value problems. Springer-Verlag, (1991).

[4] Schoenmakers, J.G.M., Heemink, A.W.: Fast Valuation of Financial Derivatives. *Journal of Computational Finance*, first issue, (1997).

[5] Schoenmakers, J.G.M., Coffey, B.: LIBOR rate models, related derivatives and model calibration. *Preprint no. 480, Weierstrass Institute Berlin*, (1999).