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Travelling Wave Equations for Semiconductor Lasers with Gain Dispersion

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Abstract

This paper modifies the coupled mode model for semiconductor lasers, taking into account the gain dispersion of the optical waveguide. Fitting the true gain curve by a Lorentzian, we obtain a correction for the dielectric function of the waveguide. A review of the derivation of the coupled mode model from the Maxwell Equations, including the corrected dielectric function, leads to an extended set of model equations. This extended model consists of the modified coupled mode equations and additional polarization equations and reflects spectral selectivity due to the geometry (waveguide dispersion) as well as the material properties (material dispersion). Although it is mathematically more complex, it does not increase the computational effort for the dynamical simulation essentially and, thus, it should replace the original model at least for numerical calculations.

1 Introduction

A standard model for Distributed Feedback (DFB) Lasers consists of two coupled travelling-wave-equations for the amplitudes of two counter-propagating optical waves and a rate equation for the longitudinally averaged carrier density [Marcenac 93, Bandelow94]. It includes the spectral selectivity effects due to the corrugation of the active zone in DFB lasers by a coupled mode formalism. However, the gain dispersion effects due to material properties are completely neglected which leads to a wrong behaviour of the model in the limit of Fabry-Perot lasers or in case of multi-mode operation.

The objective of this paper is to establish a more realistic model approximating the true gain curve of the material by a Lorentzian shape function. [Ning, Indik, Moloney 97] use a similar technique to model high-power bulk lasers by a set of lateral-longitudinal effective Bloch equations. Moreover, the models used by [Marcenac 93] included gain dispersion effects by filtering techniques.

In order to obtain the new model, we follow the derivation of the model in [Bandelow94] from the classical Maxwell equations. That derivation based on the assumption, that the dielectric function $\varepsilon(\omega)$ varies linearly with the frequency ω in the spectral interval of interest. To incorporate the nonlinear dispersion of the optical gain, we supplement this linear ε by a Lorentzian. Whereas the linear contribution is treated as in [Bandelow94], the additional term is regarded as an independent polarization, for which an equation of motion is derived.

Then, the basic steps and simplifications of the derivation in [Bandelow94] are applied to the Maxwell equations and the polarization equation:

1. Choose a longitudinally homogeneous reference waveguide and a central reference frequency and compute theoretically the transversal modes for this waveguide and frequency.

2. Expand the solution for the real waveguide in terms of these modes.

3. Confine to one leading mode and establish a set of local equations for the coefficients of this mode by averaging over the transversal cross-section.

4. Assume a periodic corrugation of the waveguide, predict the oscillation in the scale of the Bragg grating and focus on the spatially slowly varying fields to obtain equations reflecting only the global behaviour of the solution (coupled mode formalism).

We go through this process starting from the classical Maxwell equations and the polarization equation obtained from the Lorentzian fitting of the gain curve. The result are two coupled wave equations for two amplitudes $\Psi_\pm$ corrected by polarization terms $\pi_\pm$:

$$\mp i\partial_z\Psi_\pm - iv_g^{-1}\partial_t\Psi_\pm + \delta\beta\Psi_\pm + \kappa_\pm\Psi_\mp + \pi_\pm = 0$$

with the group velocity v_g, the carrier density dependent detuning of the propagation constant $\delta\beta$ and the coupling coefficients $\kappa_\pm$. The polarization terms $\pi_\pm$ are computed by two polarization equations

$$-i\partial_t\pi_\pm = (\delta + i\Gamma)\cdot\pi_\pm + A\Psi_\pm$$

with the parameters δ, Γ and A, obtained from the gain curve fitting. These equations, together with the correspondingly corrected carrier rate equations, model the side mode suppression due to material and grating in a more realistic way than the model without gain dispersion in [Bandelow94].

Section 2 introduces the basic denotations used in this paper and discusses the Lorentzian type polarization modelling dispersive gain. The following sections are devoted to the steps 1 through 4, mentioned above.

A few remarks about the validity of the simplifications:

The computation and expansion in terms of transversal modes is in principle possible for general waveguide geometries. In Section 5, however, we assume the waveguide to be a small perturbation of the homogeneous reference waveguide. The coupled mode formalism in Section 6 is specific to periodically corrugated waveguides.

2 The Basic Equations for Optical Waveguides

Our starting point are the Maxwell equations for the classical electric field $\vec{\mathcal{E}}$ and the magnetic field $\vec{\mathcal{H}}$

$$\nabla\times\vec{\mathcal{E}} + \mu_0\partial_t\vec{\mathcal{H}} = 0 \tag{1}$$

$$\nabla\times\vec{\mathcal{H}} - \partial_t\vec{\mathcal{D}} = j_{\mathrm{sp}} \tag{2}$$

where $\vec{\mathcal{D}}$ denotes the electric displacement, including a linear frequency dependent part and a nonlinear frequency dependent part of the polarization. The fields $\vec{\mathcal{E}}$, $\vec{\mathcal{H}}$, $\vec{\mathcal{D}}$ depend on three spatial variables ($\vec{r}$) and time.

The waveguide structures of semiconductor laser diodes have typically a length of several hundred μm, a width of some μm and a height of several hundred nm. The length is referred to as longitudinal z-dimension, the height and width as transversal dimension $(x, y) = \vec{r}_{tr}$ with tr being the symbol for "transversal".

j_{sp} is a stochastic current density modelling the spontaneous emission. It is supposed to have a small influence once the device is lasing above the threshold and to have the mean value 0 [Bandelow94].

The magnetic field constant is denoted by μ_0 and the electric field constant by ε_0.

The Fourier transformed variables are indicated by a tilde.

The electric displacement in the frequency domain (ω) is supposed to contain the following two terms:

$$\tilde{\vec{\mathcal{D}}}(\omega) = \varepsilon_0\varepsilon(\omega)\cdot\tilde{\vec{\mathcal{E}}}(\omega) + \tilde{\vec{\mathcal{P}}}(\omega) \tag{3}$$

Only the fist term is taken into account by the travelling wave equations as used in [Bandelow94]. Additionally, we assume that ε depends only weakly on ω and can be approximated by the relation

$$\omega\varepsilon(\omega) \quad = \quad \omega_0\varepsilon(\omega_0) + \left.\frac{\partial\omega\varepsilon}{\partial\omega}\right|_{\omega=\omega_0} \cdot(\omega-\omega_0). \tag{4}$$

We keep these assumptions and suppose that all effects of gain dispersion are contained in the polarization term

$$\tilde{\vec{\mathcal{P}}}(\omega) = \varepsilon_0\chi(\omega)\tilde{\vec{\mathcal{E}}}(\omega). \tag{5}$$

For simplicity, we model the nonlinear frequency dependence $\chi(\omega)$ by one Lorentzian response function:

$$\chi(\vec{r},\omega) = \frac{A(\vec{r})}{\omega-\omega_0-\Omega_r(\vec{r})}. \tag{6}$$

Similarly, [Ning, Indik, Moloney 97] used a sum of Lorentzians to include gain dispersion into the effective Bloch equations for high power bulk lasers. The approximation has to be valid only in a small range around ω_0 corresponding to the frequency range of optical transitions in the active material.
The addition of this nonlinear $\vec{\mathcal{P}}$ term in (3) is the crucial modification compared to [Bandelow94]. The complex frequency

$$\Omega_r := \delta + i\Gamma/2$$

is the relative resonance frequency which has the difference δ to the reference frequency ω_0 and the Full Width at Half Maximum Γ of the Lorentzian. $2A/\Gamma$ is the resonance maximum $\max\{\mathrm{Im}(\chi(\omega))\}$. A, δ and Γ are treated as known functions. They have to be fitted by comparison with experimental data or may be obtained from microscopic theory [Ning, Indik, Moloney 97]. Later we will use also the transversally averaged quantities instead of the fully space dependent functions. Supposing a negligibly slow time dependence of the coefficients A and Ω_r, relation (5) reads in the time domain

$$\partial_t\vec{\mathcal{P}} = i(\omega_0+\Omega_r(\vec{r}))\cdot\vec{\mathcal{P}} + i\varepsilon_0 A(\vec{r})\cdot\vec{\mathcal{E}}. \tag{7}$$

The ε and the $\frac{\partial\omega\varepsilon}{\partial\omega}$, taken at ω_0, in the right-hand side of (4) may depend on $\vec{r}$ but not on the frequency. We allow ε to depend only on ω and $\vec{r}$ but not on $\vec{\mathcal{E}}$ itself. Thus, nonlinear optical effects are not included in our considerations.

3 The Transversal Modes

The waveguides under consideration have a longitudinally varying ε. E.g., DFB lasers have a corrugation of small amplitude and a spatial period of several hundred nm. Thus, we want to split the Maxwell equations into a transversal system and a dynamic longitudinal system. This has been carried out in [Snyder, Love 91], but only for the case of harmonically varying fields leading to a time-independent longitudinal system. At first, we obtain a system of wave solutions of system (1), (2) for a longitudinally homogeneous

(i. e. averaged) reference waveguide and, secondly, we expand the solution for the real waveguide in terms of these wave solutions. To be more precise:

We choose a longitudinally homogeneous reference waveguide $\bar{\varepsilon}(\vec{r}_{tr})$ and a reference frequency ω_0 and seek for a wave number $\bar{\beta}$ and solutions $\vec{\bar{E}}$ and $\vec{\bar{H}}$ having the form of a longitudinal wave:

$$\vec{\bar{E}} = \vec{E}(\vec{r}_{tr})e^{i\cdot(\omega_0 t-\bar{\beta}z)} \quad (8)$$
$$\vec{\bar{H}} = \vec{H}(\vec{r}_{tr})e^{i\cdot(\omega_0 t-\bar{\beta}z)}. \quad (9)$$

Replacing $\varepsilon(\vec{r})$ and $\frac{\partial\omega\varepsilon}{\partial\omega}$ by the longitudinally averaged values $\bar{\varepsilon}(\vec{r}_{tr})$, $\frac{\partial\omega\bar{\varepsilon}}{\partial\omega}$ and omitting the nonlinear polarization effects, we obtain the equation for $\vec{\bar{D}}$

$$\partial_t\vec{\bar{D}} = i\omega_0\varepsilon_0\bar{\varepsilon}(\vec{r}_{tr})e^{i\cdot(\omega_0 t-\bar{\beta}z)}\cdot\vec{E}(\vec{r}_{tr}) \quad (10)$$

from (3).

Inserting (8), (9) and (10) in the Maxwell equations (1) and (2), multiplying by $e^{-i\cdot(\omega_0 t-\bar{\beta}z)}$ and neglecting j_{sp} yields the “homogeneous Maxwell equations”

$$\nabla_{tr}\times\vec{E}(\vec{r}_{tr}) - i\bar{\beta}e_z\times\vec{E}(\vec{r}_{tr}) + i\omega_0\mu_0\vec{H}(\vec{r}_{tr}) = 0 \quad (11)$$
$$\nabla_{tr}\times\vec{H}(\vec{r}_{tr}) - i\bar{\beta}e_z\times\vec{H}(\vec{r}_{tr}) - i\omega_0\varepsilon_0\bar{\varepsilon}\vec{E}(\vec{r}_{tr}) = 0 \quad (12)$$

being independent of z. We denoted $\nabla_{tr} := (\partial_x, \partial_y, 0)^T$ and $e_z := (0,0,1)^T$ in the equations above. This is an eigenvalue problem for $\bar{\beta}$ and $(\vec{E},\vec{H})$ being quadratic in $\bar{\beta}$. It is supposed to contain a discrete spectrum of guided modes. Particularly, if $((E_x, E_y, E_z), (H_x, H_y, H_z))$ is a mode with wave number $\bar{\beta}$, $((E_x, E_y, -E_z), (-H_x, -H_y, H_z))$ is a mode with wave number $-\bar{\beta}$. It corresponds to field components propagating in the opposite direction.

The eigenpairs of the reference waveguide are treated as known constants $(\bar{\beta}_{\pm\nu})$ and functions $\vec{E}_{\pm\nu}(\vec{r}_{tr})$ and $\vec{H}_{\pm\nu}(\vec{r}_{tr})$ in the following paragraphs.

Relations between the transversal modes

The Integral Theorem of Gauss states

$$\int_{tr} d\vec{r}_{tr}\nabla\times\vec{v}\cdot\vec{w} - \nabla\times\vec{w}\cdot\vec{v} = \oint_{\partial tr}(\vec{v}\times\vec{w} - \vec{w}\times\vec{v})\cdot\vec{\nu}$$

with arbitrary vector fields $\vec{v}$ and $\vec{w}$ on a transversal cross-section tr and the unit normal $\vec{\nu}$ at the boundary ∂tr. Assuming the fields $\vec{E}_{\pm\nu}$ and $\vec{H}_{\pm\nu}$ to be negligibly small at the transversal boundary of the waveguide, we obtain several relations (ref. [Bandelow94]) for the integrals over the transversal plane for each mode pair ν being helpful for later simplifications. Integration over the transversal plane is denoted by $\int_{tr} d\vec{r}_{tr}$.

1. Inserting $(\vec{E}_{+\nu}, \vec{H}_{+\nu})$ into the equations (11) and (12) and multiplying them by $\vec{H}_{+\nu}$ and $\vec{E}_{+\nu}$ yields an expression for $\bar{\beta}_\nu$:

$$i\bar{\beta}_\nu = i\omega_0\frac{\int_{tr}(\varepsilon_0\bar{\varepsilon}\vec{E}_{+\nu}^2 + \mu_0\vec{H}_{+\nu}^2)d\vec{r}_{tr}}{\int_{tr} e_z\cdot\vec{E}_{+\nu}\times\vec{H}_{+\nu}d\vec{r}_{tr}}. \quad (13)$$

2. Equation (11) for $(\vec{E}_{+\nu}, \vec{H}_{+\nu})$ multiplied by $\vec{H}_{-\nu}$ and equation (12) for $(\vec{E}_{-\nu}, \vec{H}_{-\nu})$ multiplied by $\vec{E}_{+\nu}$ relate $\int_{tr} \vec{H}_{+\nu}\vec{H}_{-\nu} d\vec{r}_{tr}$ and $\int_{tr} \vec{E}_{+\nu}\vec{E}_{-\nu} d\vec{r}_{tr}$:

$$-\mu_0 \int_{tr} \vec{H}_{+\nu}\vec{H}_{-\nu} d\vec{r}_{tr} = \varepsilon_0 \int_{tr} \bar{\varepsilon}\vec{E}_{+\nu}\vec{E}_{-\nu} d\vec{r}_{tr}. \tag{14}$$

3. Similarly, differentiating the equations (11) and (12) for $(\vec{E}_{-\nu}, \vec{H}_{-\nu})$ with respect to ω_0 and using (14) we obtain a relation for the dependence $\frac{\partial \bar{\beta}_\nu}{\partial \omega}$ of the eigenvalue β on the reference frequency ω_0:

$$\left.\frac{\partial \bar{\beta}_\nu}{\partial \omega}\right|_{\omega=\omega_0} = \frac{\varepsilon_0 \int_{tr} (\frac{\partial \omega \bar{\varepsilon}}{\partial \omega} + \bar{\varepsilon})\vec{E}_{+\nu}\vec{E}_{-\nu} d\vec{r}_{tr}}{\int_{tr} e_z \cdot \vec{E}_{+\nu} \times \vec{H}_{+\nu} d\vec{r}_{tr}}. \tag{15}$$

4. The transversal modes satisfy an orthogonality relation (ref. [Snyder, Love 91]): We have

$$\int_{tr} e_z \cdot \vec{E}_{\pm\nu} \times \vec{H}_{\pm\tilde{\nu}} d\vec{r}_{tr} = 0$$

for $\nu \neq \tilde{\nu}$.

4 The Expansion in the Transversal Modes

We consider the real waveguide to be a perturbation of the homogeneous reference waveguide and expand the fields $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$ of the real waveguide as a sum of the transversal modes $\vec{E}_{\pm\nu}$ and $\vec{H}_{\pm\nu}$ and a part $(\vec{\mathcal{E}}_{rad}, \vec{\mathcal{H}}_{rad})$ corresponding to the non-guided modes:

$$\vec{\mathcal{E}} = \left(\sum_{\pm\nu} a_{\pm\nu}(z,t)\vec{E}_{\pm\nu}(\vec{r}_{tr}) + \vec{\mathcal{E}}_{rad} \right) \cdot e^{i\omega_0 t} \tag{16}$$

$$\vec{\mathcal{H}} = \left(\sum_{\pm\nu} a_{\pm\nu}(z,t)\vec{H}_{\pm\nu}(\vec{r}_{tr}) + \vec{\mathcal{H}}_{rad} \right) \cdot e^{i\omega_0 t}. \tag{17}$$

The relation between $\vec{\mathcal{D}}$ and $\vec{\mathcal{E}}$ is (see (3), and (4))

$$i\omega\tilde{\vec{\mathcal{D}}} = \varepsilon_0 \left(i\varepsilon\omega_0 + i\frac{\partial \omega\varepsilon}{\partial \omega}(\omega - \omega_0) \right) \tilde{\vec{\mathcal{E}}} + i\omega_0\tilde{\vec{\mathcal{P}}}. \tag{18}$$

The formula has to be a valid approximation only in a small range around ω_0 corresponding to the frequency range of optical transitions in the active material. Thus, we replaced $i\omega\tilde{\vec{\mathcal{P}}}$ by $i\omega_0\tilde{\vec{\mathcal{P}}}$ in (18) for simplicity.
The inverse Fourier Transform of relation (18) yields $\partial_t\vec{\mathcal{D}}$ (ref. [Landau, Lifshitz 63])

$$\partial_t\vec{\mathcal{D}} = \varepsilon_0 \left(\sum_{\pm\nu} (i\omega_0\varepsilon + \frac{\partial \omega\varepsilon}{\partial \omega}\partial_t) a_{\pm\nu}(z,t)\vec{E}_{\pm\nu}(\vec{r}_{tr}) + \frac{1}{\varepsilon_0}\partial_t\vec{\mathcal{D}}_{rad} \right) \cdot e^{i\omega_0 t} + i\omega_0\vec{\mathcal{P}}. \tag{19}$$

The nonlinear dispersion part $\vec{\mathcal{P}}$ of (19) is expanded with the same basis as the electric field $\vec{\mathcal{E}}$:

$$\vec{\mathcal{P}} = \varepsilon_0 \left(\sum_{\pm\nu} p_{\pm\nu}(z,t)\vec{E}_{\pm\nu}(\vec{r}_{tr}) + \vec{\mathcal{P}}_{rad} \right) \cdot e^{i\omega_0 t}. \tag{20}$$

Using these expansions for $\vec{\mathcal{E}}$, $\vec{\mathcal{H}}$, $\vec{\mathcal{D}}$ and $\vec{\mathcal{P}}$, we will obtain equations for the coefficients from the Maxwell equations in the next steps.

The physically relevant fields have to be real. Thus, we have to insert the real parts $\mathrm{Re}\,\vec{\mathcal{E}} = \frac{1}{2}(\vec{\mathcal{E}} + \vec{\mathcal{E}}^*)$, $\mathrm{Re}\,\vec{\mathcal{H}} = \frac{1}{2}(\vec{\mathcal{H}} + \vec{\mathcal{H}}^*)$, $\mathrm{Re}\,\vec{\mathcal{D}} = \frac{1}{2}(\vec{\mathcal{D}} + \vec{\mathcal{D}}^*)$ and $\mathrm{Re}\,\vec{\mathcal{P}} = \frac{1}{2}(\vec{\mathcal{P}} + \vec{\mathcal{P}}^*)$ into the Maxwell equations (1) and (2).

To obtain equations for the coefficients $a_{\pm\nu}$ and $p_{\pm\nu}$, we multiply (1) and (2) by $2 \cdot e^{-i\omega_0 t}$ and omit the fast oscillating complex conjugate parts. This is called the "Rotating Wave Approximation" (ref. [Gardiner91]). Furthermore, the rapidly decaying "radiation terms" $\vec{\mathcal{E}}_{rad}$, $\vec{\mathcal{H}}_{rad}$, $\vec{\mathcal{D}}_{rad}$ and $\vec{\mathcal{P}}_{rad}$ are neglected.

Reduction to the fundamental TE mode of the reference waveguide

We confine ourselves to one pair of guided modes as is done in e. g. [Agrawal80] in the following sections. This will be a good approximation if the longitudinal perturbation of ε is small, e. g. in DFB lasers. The mode pair is denoted by $\pm\beta$, $\vec{E}_\pm$ and $\vec{H}_\pm$ and its coefficients are denoted by $a_\pm$ and $p_\pm$, respectively.

To keep the calculations simple, this leading mode is assumed to be a TE mode, i. e. the relations $\vec{E}_+ = \vec{E}_- = \vec{E}_{tr} =: \vec{E}$, $E_z = 0$, $\vec{H}_{tr+} = -\vec{H}_{tr-}$ and $H_{z+} = H_{z-}$ are fulfilled. This is a good approximation for the situation in a quasi-planar waveguide.

Particularly, the relations

$$E_x = E_z = H_y = 0 \qquad H_x = \frac{\beta}{\omega_0 \mu_0} E_y \qquad H_z = \frac{i}{\omega_0 \mu_0} \frac{\partial E_y}{\partial x}$$

hold for $E_{tr} = E_y$ in the TE case. These relations simplify the homogeneous Maxwell equations to a scalar Helmholtz eigenvalue problem for β and E_y. All other field components are now determined by β and E_y.

The polarization is supposed to have a transversal structure similar to the electric field in the following paragraphs:

$$\vec{\mathcal{P}} = \varepsilon_0 p(z,t) \vec{E}(\vec{r}_{tr}) e^{i\omega_0 t}. \tag{21}$$

The confinement of the polarization within the active zone can be treated by setting $A(\vec{r}) = 0$ outside. The assumptions on the concrete form of the reference waveguide as well as of the transversal mode are not necessary to obtain the resulting equations. Using other types of modes or reference waveguides leads us to other formulas for the coefficients. Despite the modes are treated as known, the coefficients in the final model equations are fitted by experiments anyway. The only restriction of importance is, that $\vec{\mathcal{E}}$, $\vec{\mathcal{H}}$ and $\vec{\mathcal{P}}$ are transversally *single moded.* This is a reasonable assumption for modern laser structures [ESA 13, 89] and was also made by e. g. [Agrawal80].

5 The Local Amplitude Equations with Nonlinear Gain Dispersion

The differential equation for the nonlinear polarization $\vec{\mathcal{P}}$ (7) leads by (21) and (16) to a linear differential equation for $p = p_+ + p_-$. The common factor $\vec{E}$ is nonzero. Thus,

$$\partial_t p(z,t) = i\Omega_r(\vec{r}) \cdot p(z,t) + iA(\vec{r}) \cdot (a_+(z,t) + a_-(z,t)) \tag{22}$$

is valid for the coefficients p and $a_\pm$.
The Maxwell equations (1) and (2) with the expansion of Paragraph 4 and its simplifications read as

$$(a_+ + a_-)\nabla_{tr} \times \vec{E} + e_z \times \partial_z (a_+ + a_-)\vec{E} + i\omega_0\mu_0(a_+\vec{H}_+ + a_-\vec{H}_-) + \mu_0 \cdot (\partial_t a_+ \vec{H}_+ + \partial_t a_- \vec{H}_-) = 0 \quad (23)$$

$$a_+\nabla_{tr} \times \vec{H}_+ + a_-\nabla_{tr} \times \vec{H}_- + e_z \times \partial_z(a_+\vec{H}_+ + a_-\vec{H}_-) - \varepsilon_0 \frac{\partial\omega\varepsilon}{\partial\omega}\partial_t(a_+ + a_-)\vec{E} - i\omega_0\varepsilon_0(\varepsilon(a_+ + a_-) + p)\vec{E} = 2\jmath_{\mathrm{sp}} e^{-i\omega_0 t} \quad (24)$$

The eigenvalue equations (11) and (12) for $\vec{E}$ and $\vec{H}_+$ and $\vec{H}_-$ provide replacements for the terms $\nabla_{tr}\times\vec{E}$ and $\nabla_{tr}\times\vec{H}_\pm$ in the TE case. The insertion yields (since $e_z\times\vec{H}_- = -e_z\times\vec{H}_+$)

$$(a_+ + a_-) \cdot (i\bar{\beta}e_z \times \vec{E} - i\omega_0\mu_0\vec{H}_+) + \partial_z(a_+ + a_-)e_z \times \vec{E} + i\omega_0\mu_0(a_+\vec{H}_+ + a_-\vec{H}_-) + \mu_0 \cdot (\partial_t a_+\vec{H}_+ + \partial_t a_-\vec{H}_-) = 0 \quad (25)$$

$$(a_+ + a_-) \cdot (i\bar{\beta}e_z \times \vec{H}_+ + i\varepsilon_0\omega_0\bar{\varepsilon}\vec{E}) + \partial_z(a_+ - a_-)e_z \times \vec{H}_+ - \varepsilon_0\frac{\partial\omega\varepsilon}{\partial\omega}\partial_t(a_+ + a_-)\vec{E} - i\omega_0\varepsilon_0(\varepsilon(a_+ + a_-) + p)\vec{E} = 2\jmath_{\mathrm{sp}} e^{-i\omega_0 t} \quad (26)$$

The ansatz with three coefficients $a_\pm$ and p is too restrictive to satisfy the system (25), (26) and (22). Only a transversally averaged version of ((22), (25), (26)) can be fulfilled. The averaged equations are the scalar products with the mode integrated over the transversal plane (denoted by $\int_{tr} d\vec{r}_{tr}$). We introduce the abbreviations

$$\begin{array}{lll}
N & := & 2\int_{tr} e_z \cdot \vec{E} \times \vec{H}_+ d\vec{r}_{tr} = -2\int_{tr} \vec{E} \times \vec{H}_- \cdot e_z d\vec{r}_{tr} \\
h^2 & := & \int_{tr} \vec{H}_+ \cdot \vec{H}_+ d\vec{r}_{tr} = \int_{tr} \vec{H}_- \cdot \vec{H}_- d\vec{r}_{tr} \\
h^2_\pm & := & \int_{tr} \vec{H}_- \cdot \vec{H}_+ d\vec{r}_{tr} \\
e^2 & := & \int_{tr} \vec{E} \cdot \vec{E} d\vec{r}_{tr} \\
J_{\mathrm{sp}} & := & \int_{tr} 2\jmath_{\mathrm{sp}} e^{-i\omega_0 t} \cdot \vec{E} d\vec{r}_{tr} \\
\bar{\varepsilon}_{tr} & := & \int_{tr} \bar{\varepsilon}\vec{E} \cdot \vec{E} d\vec{r}_{tr}/e^2.
\end{array}
\qquad
\begin{array}{lll}
\varepsilon_{tr} & := & \int_{tr} \varepsilon\vec{E} \cdot \vec{E} d\vec{r}_{tr}/e^2 \\
\frac{\partial\omega\varepsilon}{\partial\omega}_{tr} & := & \int_{tr} \frac{\partial\omega\varepsilon}{\partial\omega}\vec{E} \cdot \vec{E} d\vec{r}_{tr}/e^2 \\
A_{tr} & := & \int_{tr} A\vec{E} \cdot \vec{E} d\vec{r}_{tr}/e^2 \\
\frac{\partial\omega\bar{\varepsilon}}{\partial\omega}_{tr} & := & \int_{tr} \frac{\partial\omega\bar{\varepsilon}}{\partial\omega}\vec{E} \cdot \vec{E} d\vec{r}_{tr}/e^2 \\
\Omega_{r,tr} & := & \int_{tr} \Omega_r\vec{E} \cdot \vec{E} d\vec{r}_{tr}/e^2
\end{array}$$

N, e^2, h^2, $h^2_\pm$, $\bar{\varepsilon}_{tr}$ are pure constants, ε_{tr}, A_{tr}, Ω_{tr} depend only on z. The index $_{tr}$ at the quantities $\bar{\varepsilon}$, ε, $\frac{\partial\omega\varepsilon}{\partial\omega}$, $\frac{\partial\omega\bar{\varepsilon}}{\partial\omega}$ A and Ω_r indicating the transversal averaging will be omitted for brevity.
Using these abbreviations the relations (13), (14) and (15) can be adapted for TE modes to

$$i\beta N = i\omega_0(\varepsilon_0\bar{\varepsilon}e^2 + \mu_0 h^2) \quad (27)$$

$$-\mu_0 h^2_\pm = \varepsilon_0\bar{\varepsilon}e^2 \quad (28)$$

$$\left.\frac{\partial\beta}{\partial\omega}\right|_{\omega=\omega_0} = \frac{1}{N}\varepsilon_0(\frac{\partial\omega\bar{\varepsilon}}{\partial\omega} + \bar{\varepsilon})e^2 \quad (29)$$

Here $\frac{\partial\beta}{\partial\omega}$ corresponds to the inverse of the group velocity.

The projection of (22), $\int_{tr} d\vec{r}_{tr}(22)\cdot\vec{E}\cdot\vec{E}$, yields an ordinary differential equation for p:

$$\partial_t p = i\Omega_r p + iA(a_+ + a_-). \tag{30}$$

The averaged amplitude equations $\int_{tr} d\vec{r}_{tr}((25)\cdot\vec{H}_+ + (26)\cdot\vec{E})$ and $-\int_{tr} d\vec{r}_{tr}((25)\cdot\vec{H}_- + (26)\cdot\vec{E})$ are scalar linear equations for $a_\pm$:

$$N\partial_z a_- - \left(i\beta N + \frac{\beta N}{\omega_0}\partial_t\right) a_- + \partial_t(a_+ + a_-)\cdot\left[\frac{\beta N}{\omega_0} - \varepsilon_0\left(\bar{\varepsilon} + \frac{\partial\omega\varepsilon}{\partial\omega}\right)e^2\right] + i\omega_0\varepsilon_0 e^2\cdot\left[(\bar{\varepsilon}-\varepsilon)(a_+ + a_-) - ip\right] = J_{\mathrm{sp}} \tag{31}$$

$$N\partial_z a_+ + \left(i\beta N + \frac{\beta N}{\omega_0}\partial_t\right) a_+ - \partial_t(a_+ + a_-)\cdot\left[\frac{\beta N}{\omega_0} - \varepsilon_0\left(\bar{\varepsilon} + \frac{\partial\omega\varepsilon}{\partial\omega}\right)e^2\right] - i\omega_0\varepsilon_0 e^2\cdot\left[(\bar{\varepsilon}-\varepsilon)(a_+ + a_-) - ip\right] = -J_{\mathrm{sp}} \tag{32}$$

The equations (32) and (31) are scaled by N and $-N$, respectively.
The resulting *local equations* for the amplitudes read

$$\partial_z a_+ + (i\beta + i\Delta_\varepsilon)a_+ + \left(\frac{\partial\beta}{\partial\omega}\right)_{\mathrm{loc}}\partial_t a_+ + \left[\left(\frac{\partial\beta}{\partial\omega}\right)_{\mathrm{loc}} - \frac{\beta}{\omega_0}\right]\partial_t a_- + i\Delta_\varepsilon a_- + iC_p p = -F \tag{33}$$

$$-\partial_z a_- + (i\beta + i\Delta_\varepsilon)a_- + \left(\frac{\partial\beta}{\partial\omega}\right)_{\mathrm{loc}}\partial_t a_- + \left[\left(\frac{\partial\beta}{\partial\omega}\right)_{\mathrm{loc}} - \frac{\beta}{\omega_0}\right]\partial_t a_+ + i\Delta_\varepsilon a_+ + iC_p p = F. \tag{34}$$

The term $\left(\frac{\partial\beta}{\partial\omega}\right)_{\mathrm{loc}}$ indicates the appearance of $\frac{\partial\omega\varepsilon}{\partial\omega}$ of the real waveguide instead of the $\frac{\partial\omega\bar{\varepsilon}}{\partial\omega}$ of the reference waveguide. The coefficients can be specialized in the case of a TE mode for a planar waveguide with $k_0 = \frac{\omega_0}{c}$ to

$$\begin{aligned}
\frac{\partial\beta}{\partial\omega} &= \frac{1}{N}\varepsilon_0\left(\frac{\partial\omega\bar{\varepsilon}}{\partial\omega} + \bar{\varepsilon}\right)e^2 = \frac{k_0^2}{2\beta\omega_0}\cdot\frac{\int_{tr}\left(\bar{\varepsilon} + \frac{\partial\omega\bar{\varepsilon}}{\partial\omega}\right)E_y^2 d\vec{r}_{tr}}{\int_{tr} E_y^2 d\vec{r}_{tr}} \\
\left(\frac{\partial\beta}{\partial\omega}\right)_{\mathrm{loc}} &= \frac{\partial\beta}{\partial\omega} + \varepsilon_0\left(\frac{\partial\omega\varepsilon}{\partial\omega} - \frac{\partial\omega\bar{\varepsilon}}{\partial\omega}\right)\frac{e^2}{N} = \frac{k_0^2}{2\beta\omega_0}\cdot\frac{\int_{tr}\left(\bar{\varepsilon} + \frac{\partial\omega\varepsilon}{\partial\omega}\right)E_y^2 d\vec{r}_{tr}}{\int_{tr} E_y^2 d\vec{r}_{tr}} \\
\Delta_\varepsilon &= \frac{\omega_0\varepsilon_0 e^2\cdot(\varepsilon - \bar{\varepsilon})}{N} = \frac{k_0^2}{2\beta}\cdot\frac{\int_{tr}(\varepsilon - \bar{\varepsilon})E_y^2 d\vec{r}_{tr}}{\int_{tr} E_y^2 d\vec{r}_{tr}} \\
C_p &= \frac{\omega_0\varepsilon_0 e^2}{N} = \frac{k_0^2}{2\beta} \\
F &= \frac{J_{\mathrm{sp}}}{N} = \frac{\mu_0\omega_0}{2\beta}e^{-i\omega_0 t}\cdot\frac{\int_{tr} j_{\mathrm{sp}_y} E_y d\vec{r}_{tr}}{\int_{tr} E_y^2 d\vec{r}_{tr}}.
\end{aligned}$$

The terms $\frac{\partial\beta}{\partial\omega} - \frac{\beta}{\omega_0}$ and $\left(\frac{\partial\beta}{\partial\omega}\right)_{\mathrm{loc}} - \frac{\beta}{\omega_0}$ are quite small as long as the effective refractive index remains almost constant with respect to frequency, i. e. β/k_0 is independent of the choice of ω_0.

6 The Case of Periodically Modulated Waveguides — the Coupled Mode Formalism

The local equations obtained in Section 5 are an approximation for arbitrary small perturbations of homogeneous quasi-planar waveguides. We are particularly interested in devices with periodic corrugations of ε of spatial period length Λ (Bragg grating in DFB lasers). Then, the equations contain terms, jumping in each period of the Bragg grating (i. e. ε, $\frac{\partial \omega\varepsilon}{\partial\omega}$, A or Ω). These terms are hidden in Δ_ε or $\left(\frac{\partial\beta}{\partial\omega}\right)_{\rm loc}$ in (33) and (34). Using the coupled mode formalism we want to focus on the large scale behaviour of the solutions.

We confine ourselves to the first Fourier component of the Bragg diffraction. This leads to a solution $a_\pm$ spatially oscillating with a frequency near to the Bragg frequency π/Λ and non-differentiable at each jump of the coefficients. In order to focus on the large spatial scale in comparison with the Bragg period, we apply the coupled mode formalism.

Consequently, we introduce spatially slowly varying amplitudes amplitudes $b_\pm(z,t)$ and $p_\pm(z,t)$ and extract the first order short-scale oscillation as a factor $e^{\mp i\pi z/\Lambda}$:

$$a_\pm(z,t) \;=\; b_\pm(z,t)e^{\mp i\pi z/\Lambda} \quad \text{and} \quad p(z,t) \;=\; p_+(z,t)e^{-i\pi z/\Lambda} + p_-(z,t)e^{i\pi z/\Lambda}\;. \qquad (35)$$

The substitution of (35) into (33) (multiplied by $e^{i\pi z/\Lambda}$) yields an extra factor $e^{2i\pi z/\Lambda}$ in front of b_- and p_-:

$$\partial_z b_+ + (i\beta - \frac{i\pi}{\Lambda} + i\Delta_\varepsilon)b_+ + \left(\frac{\partial\beta}{\partial\omega}\right)_{\rm loc}\partial_t b_+ + iC_p p_+ + \\ e^{2i\pi z/\Lambda}\cdot\left[\left(\left(\frac{\partial\beta}{\partial\omega}\right)_{\rm loc} - \frac{\beta}{\omega_0}\right)\partial_t b_- + i\Delta_\varepsilon b_- + iC_p p_-\right] = -Fe^{i\pi z/\Lambda}. \qquad (36)$$

Similarly, we multiply (34) by $e^{-i\pi z/\Lambda}$ after the substitution of (35):

$$-\partial_z b_- + (i\beta - \frac{i\pi}{\Lambda} + i\Delta_\varepsilon)b_- + \left(\frac{\partial\beta}{\partial\omega}\right)_{\rm loc}\partial_t b_- + iC_p\pi_- + \\ e^{-2i\pi z/\Lambda}\cdot\left[\left(\left(\frac{\partial\beta}{\partial\omega}\right)_{\rm loc} - \frac{\beta}{\omega_0}\right)\partial_t b_+ + i\Delta_\varepsilon b_+ + iC_p p_+\right] = Fe^{-i\pi z/\Lambda}. \qquad (37)$$

The equations (36), (37),(40) and (41) are linear in the spatially slowly varying $b_\pm$ and $p_\pm$. Their coefficients depend on z and still oscillate within the Bragg period. Consequently, after averaging of the coefficients along one grating period ($\frac{1}{\Lambda}\int_z^{z+\Lambda} dz$), the solutions represent only the large scale behaviour of $b_\pm$. The averages of the coefficients are their 0th and 1st spatial Fourier components:

$$\bar{\Delta}_\varepsilon(z) = \frac{1}{\Lambda}\int_z^{z+\Lambda}\Delta_\varepsilon(\zeta)d\zeta \qquad\qquad \Delta_\varepsilon^\pm(z) = \frac{1}{\Lambda}\int_z^{z+\Lambda}\Delta_\varepsilon(\zeta)e^{\pm 2i\pi\zeta/\Lambda}d\zeta.$$

Analogously, we obtain $\left(\overline{\frac{\partial\beta}{\partial\omega}}\right)_{\rm loc}$ and $\left(\frac{\partial\beta}{\partial\omega}\right)^\pm_{\rm loc}$ as Fourier components of $\left(\frac{\partial\beta}{\partial\omega}\right)_{\rm loc}$. The coefficient C_p is constant as well as $\frac{\beta}{\omega_0}$, thus, $\bar{C}_p = C_p$ and $C_p^\pm = 0$. The averages of the right-hand-side fluctuation are denoted by

$$F_\pm(z) = \mp\frac{1}{\Lambda}\int_z^{z+\Lambda} Fe^{\pm i\pi\zeta/\Lambda}d\zeta.$$

The equations with the averaged coefficients and the slowly varying and *smooth* solutions $\Psi_\pm$ and $\pi_\pm$ read

$$\partial_z \Psi_+ + (i\beta - \frac{i\pi}{\Lambda} + i\bar{\Delta}_\varepsilon)\Psi_+ + \overline{\left(\frac{\partial\beta}{\partial\omega}\right)}_{\text{loc}} \partial_t \Psi_+ + \left(\frac{\partial\beta}{\partial\omega}\right)^+_{\text{loc}} \partial_t \Psi_- + i\Delta^+_\varepsilon \Psi_- + iC_p \pi_+ = F_+ \tag{38}$$

$$-\partial_z \Psi_- + (i\beta - \frac{i\pi}{\Lambda} + i\bar{\Delta}_\varepsilon)\Psi_- + \overline{\left(\frac{\partial\beta}{\partial\omega}\right)}_{\text{loc}} \partial_t \Psi_- + \left(\frac{\partial\beta}{\partial\omega}\right)^-_{\text{loc}} \partial_t \Psi_+ + i\Delta^-_\varepsilon \Psi_+ + iC_p \pi_- = F_-. \tag{39}$$

The coupled polarization equations can be obtained similarly. The substitution by (35) for p into the polarization equation and multiplication by $e^{i\pi z/\Lambda}$ respectively $e^{-i\pi z/\Lambda}$ yields

$$\partial_t p_+ + \partial_t p_- e^{2i\pi z/\Lambda} = i\Omega_r p_+ + iAb_+ + e^{2i\pi z/\Lambda}[i\Omega_r p_- + iAb_-] \tag{40}$$

$$\partial_t p_- + \partial_t p_+ e^{-2i\pi z/\Lambda} = i\Omega_r p_- + iAb_- + e^{-2i\pi z/\Lambda}[i\Omega_r p_+ + iAb_+] \tag{41}$$

Again, we average the coefficients by $\Lambda^{-1}\int_{z_l}^{z_l+\Lambda} dz$ along one grating period. If we denote the averages of the coefficients by $\bar{A}$, and $\bar{\Omega}_r$ and the first Fourier components by $A^\pm$ and $\Omega^\pm$ we obtain two differential equations with smooth solutions by π_+ and π_-:

$$\partial_t \pi_+ = i\bar{\Omega}_r \pi_+ + i\bar{A}\Psi_+ + i\Omega_r^+ \pi_- + iA^+\Psi_- \tag{42}$$

$$\partial_t \pi_- = i\bar{\Omega}_r \pi_- + i\bar{A}\Psi_- + i\Omega_r^- \pi_+ + iA^-\Psi_+ \tag{43}$$

We give a few remarks about the accuracy and the range of validity of the coupled mode formalism:

- We included only the effects of the Bragg grating up to the first order. A Fourier component of spatial frequency $n\pi/\Lambda$ leads to a set of coupled mode equations describing the nth order Bragg diffraction. However, only the component with $n = 1$ is situated in the frequency range of optical transitions in the active material. (ref. [Bandelow94])

- The coupled mode equations are globally exact in the stationary case $\partial_t b_\pm = \partial_t p_\pm = 0$ in the following sense (ref. [Bandelow94]): The smooth global-scale amplitudes $\Psi_\pm$ and $\pi_\pm$ intersect their locally detailed counterparts $b_\pm$ and $p_\pm$ in each Bragg period. This means, the solutions of (38), (39) and (42), (43) simply hide the local details of $b_\pm$ and $p_\pm$.

- The solutions $\Psi_\pm$ and $\pi_\pm$ will be a good approximation to $b_\pm$ and $p_\pm$ if the corrugation is small, i. e. in terms of coupling coefficients $\kappa^\pm = \Delta^\pm_\varepsilon$: $|\kappa^\pm\Lambda| \ll 1$ [Bandelow94]. E. g., this means a change of the effective index of about 1%.

- Another consideration justifying the coupled mode formalism is:

  We can consider the continuous equations (38), (39) and (42), (43) as a $O(\Lambda)$ approximation of a spatially discrete finite difference equation for piecewise linear continuous $b_\pm$ and a spatially discrete equation for piecewise constant $\pi_\pm$. This approximation will be good if the grating period length Λ ($\approx$ 200nm) is sufficiently small compared to the typical variation of some μm of Ψ and π.

7 Results and Conclusions

Let β, $\vec{E}(\vec{r}_{tr})$, $\vec{H}_+(\vec{r}_{tr})$ be the first TE mode of a longitudinally homogeneous reference waveguide, ω_0 a reference frequency and Λ the length of the grating period in the Bragg grating.

$\left(\frac{\partial\beta}{\partial\omega}\right)_{\text{loc}}$ simplifies to $\frac{\partial\beta}{\partial\omega}$ and $\bar{\Delta}_\varepsilon$ vanishes for a suitably chosen reference waveguide. The $\Delta_\varepsilon^\pm$ appear as the coupling coefficients $\kappa^\pm$. The non-diagonal coefficients $\left(\frac{\partial\beta}{\partial\omega}\right)^\pm_{\text{loc}}$ of the time derivatives become small compared with $\frac{\partial\beta}{\partial\omega}$ and, thus, are omitted.

The dynamic coupled mode equations including nonlinear polarization effects with these simplifications read as follows:

They consist of a Travelling Wave part for the slowly varying amplitude $\Psi_\pm$

$$\begin{aligned}
\partial_z\Psi_+ + (i\beta - \frac{i\pi}{\Lambda})\Psi_+ + \frac{\partial\beta}{\partial\omega}\partial_t\Psi_+ + i\kappa^+\Psi_- + iC_p\pi_+ &= F_+ && (44)\\
-\partial_z\Psi_- + (i\beta - \frac{i\pi}{\Lambda})\Psi_- + \frac{\partial\beta}{\partial\omega}\partial_t\Psi_- + i\kappa^-\Psi_+ + iC_p\pi_- &= F_- && (45)
\end{aligned}$$

and equations for the slowly varying polarization $\pi_\pm$:

$$\begin{aligned}
\partial_t\pi_+ &= i\bar{\Omega}_r\pi_+ + i\bar{A}\Psi_+ + i\Omega_r^+\pi_- + iA^+\Psi_- && (46)\\
\partial_t\pi_- &= i\bar{\Omega}_r\pi_- + i\bar{A}\Psi_- + i\Omega_r^-\pi_+ + iA^-\Psi_+. && (47)
\end{aligned}$$

The coefficients have the following meaning: The inverse of $\frac{\partial\beta}{\partial\omega}$ is the group velocity v_g and is considered to be constant in time.

The first longitudinal Fourier component of ε leads to the coupling coefficients $\kappa_\pm$ between the forward and the backward travelling wave:

$$\kappa^\pm = \varepsilon_0\omega_0 \frac{\int_z^{z+\Lambda} e^{\pm 2i\pi z/\Lambda}\int_{tr}\varepsilon(\vec{r}))\vec{E}(\vec{r}_{tr})\cdot\vec{E}(\vec{r}_{tr})d\vec{r}_{tr}dz}{2\Lambda\int_{tr} e_z\times\vec{E}(\vec{r}_{tr})\cdot\vec{H}_+(\vec{r}_{tr})d\vec{r}_{tr}} = \frac{k_0^2}{2\beta}\cdot\frac{\int_z^{z+\Lambda} e^{\pm 2i\pi z/\Lambda}\int_{tr}\varepsilon(\vec{r})E_y^2 d\vec{r}_{tr}dz}{\Lambda\int_{tr}E_y^2 d\vec{r}_{tr}}$$

They disappear for homogeneous waveguides. A rectangular tooth shape leads to a maximum coupling in the first Fourier components (ref. [Agrawal80, Hardy84]). The coupling factor for the inclusion of the nonlinear polarization C_p can be scaled, but the product of $\bar{A}$ as well as $A^\pm$ in the polarization equation and C_p has to be kept constant. Another choice of C_p corresponds to another scaling of π in relation to Ψ.

$$C_p = \varepsilon_0\omega_0\frac{\int_{tr}\vec{E}(\vec{r}_{tr})\cdot\vec{E}(\vec{r}_{tr})d\vec{r}_{tr}}{2\int_{tr} e_z\times\vec{E}(\vec{r}_{tr})\cdot\vec{H}_+(\vec{r}_{tr})d\vec{r}_{tr}} = \frac{k_0^2}{2\beta}$$

The coefficients in the polarization equations can be obtained experimentally by fitting of the gain curve with the Lorentzian $\chi(\omega) = \frac{A}{\omega-\delta-i\Gamma}$. Their longitudinal Fourier components

$$\begin{aligned}
\bar{A} &= \frac{\int_z^{z+\Lambda}\int_{tr}A(\vec{r})\vec{E}(\vec{r}_{tr})\cdot\vec{E}(\vec{r}_{tr})d\vec{r}_{tr}dz}{\Lambda\int_{tr}\vec{E}(\vec{r}_{tr})\cdot\vec{E}(\vec{r}_{tr})d\vec{r}_{tr}} = \frac{\int_z^{z+\Lambda}\int_{tr}A(\vec{r})E_y^2 d\vec{r}_{tr}dz}{\Lambda\int_{tr}E_y^2 d\vec{r}_{tr}}\\
A^\pm &= \frac{\int_z^{z+\Lambda} e^{\pm 2i\pi z/\Lambda}\int_{tr}A(\vec{r})\vec{E}(\vec{r}_{tr})\cdot\vec{E}(\vec{r}_{tr})d\vec{r}_{tr}dz}{\Lambda\int_{tr}\vec{E}(\vec{r}_{tr})\cdot\vec{E}(\vec{r}_{tr})d\vec{r}_{tr}} = \frac{\int_z^{z+\Lambda} e^{\pm 2i\pi z/\Lambda}\int_{tr}A(\vec{r})E_y^2 d\vec{r}_{tr}dz}{\Lambda\int_{tr}E_y^2 d\vec{r}_{tr}},
\end{aligned}$$

and, similarly, $\bar{\Omega}_r$ and $\Omega_r^{\pm}$ are the cross-section averages weighted by the transversal mode of the fully space dependent quantities.

The right hand side F models the spontaneous emission (ref. [Marcenac 93, Bandelow94] for remarks and more detailed references). It is supposed to have a small influence above the threshold. Thus, after the initialization of the lasing process, it will be either set to zero or considered as stochastic.

$$F_{\pm} = \mp \frac{\int_z^{z+\Lambda} \int_{tr} j_{\mathrm{sp}} e^{-i\omega_0 t \pm i\pi z/\Lambda} \vec{E}(\vec{r}_{tr}) d\vec{r}_{tr} dz}{\int_{tr} e_z \times \vec{E}(\vec{r}_{tr}) \cdot \vec{H}_{+}(\vec{r}_{tr}) d\vec{r}_{tr}} = \mp \frac{\int_z^{z+\Lambda} e^{\pm i\pi z/\Lambda} \int_{tr} j_{\mathrm{sp}} E_y d\vec{r}_{tr} dz}{\int_{tr} E_y^2 d\vec{r}_{tr}} \cdot \frac{\mu_0 \omega_0}{2\beta} e^{-i\omega_0 t}$$

The system (44), (45), (46), (47) is only the optical part of the laser model. They have to be complemented by a model for β, its dependence on the carrier density and a rate equation for the evolution of the carrier density. The laser model used in [Bandelow94] for multi-section lasers neglects spatial hole burning effects and considers β as spatially constant in each section of the laser. These equations can be used together with the new optical model as well, only the stimulated emission/recombination terms have to be adjusted appropriately.

The set of the modified optical travelling-wave-equations, the polarization equations and the carrier rate equations establish a model, that takes the side-mode suppression due to material gain dispersion into account and leads, consequently, to more realistic simulation results for a wide range of devices. Although the model is mathematically more complex, it does not increase the computational effort for the dynamic simulation essentially. It reproduces the qualitatively accurate results of the original model for DFB lasers.

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