A Metric Theory of Gravity with Condensed Matter Interpretation

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Abstract

We define a metric theory of gravity with preferred Newtonian frame $(X^i(x), T(x))$ by

$$L = L_{GR} + \Xi g^{\mu\nu}\delta_{ij}X^i_{,\mu}X^j_{,\nu} - \Upsilon g^{\mu\nu}T_{,\mu}T_{,\nu}$$

It allows a condensed matter interpretation which generalizes LET to gravity.

The Ξ-term influences the age of the universe. $\Upsilon > 0$ allows to avoid big bang singularity and black hole horizon formation. This solves the horizon problem without inflation. An atomic hypothesis solves the ultraviolet problem by explicit regularization. We give a prediction for cutoff length.

1 Introduction

The theory we propose here is a metric theory of gravity with a predefined Newtonian background frame. Variables are the metric tensor field $g_{\mu\nu}(x)$, matter fields $\psi^m(x)$, and the Galilean coordinates $X^i(x), T(x)$ of the preferred frame. Compared with GR, the Lagrangian of the theory contains additional terms which depend on these preferred coordinates:

$$L = R + \Lambda + L_{matter}(g_{\mu\nu}, \psi^m) + \Xi g^{\mu\nu}\delta_{ij}X^i_{,\mu}X^j_{,\nu} - \Upsilon g^{\mu\nu}T_{,\mu}T_{,\nu}$$

Ξ and Υ are additional "cosmological constants" which have to be defined by observation, δ_{ij} is a predefined Euclidean metric in the Newtonian background space. For the preferred coordinates we obtain the harmonic condition:

$$\Box X^i = \Box T = 0$$

The additional terms in the Lagrangian compared with GR lead to additional terms in the Einstein equations. This distinguishes the theory from attempts to combine unmodified Einstein equations with harmonic coordinates, as GR with harmonic gauge (cf. [6],[9]) or theories where the preferred frame remains hidden (cf. [10], [12]).

The additional terms lead to observable effects. The predefined Newtonian frame explains the flatness of the universe. A positive value of Ξ increases the age of the universe. A positive value of Υ avoids the big bang singularity and leads to time-symmetric solutions with a big crash before the big bang. This solves the horizon problem of relativistic cosmology without inflation theory.

Similar to "Planck ether" concepts [8],[13] ultraviolet quantization problems are solved by explicit, physical regularization related based on an "atomic hypothesis" for the condensed matter interpretation. This hypothesis predicts a cutoff different from Planck length, which seems to increase together with the universe.

2 Motivation

To motivate the Lagrange density, it is sufficient to motivate the harmonic equations for X^i, T. This special requirement about coordinate dependence allows to justify the Lagrangian of our theory in a similar way as the assumption of independence from X^i, T justifies the Lagrangian of GR.

The theory allows a condensed matter interpretation, which explains these harmonic equations as conservation laws for condensed matter. But we do not have to rely on the condensed matter interpretation. We consider here a new axiom for quasi-classical quantum gravity and EPR-realism as independent motivations for our theory. But there is also a sufficient number of other problems of GR which disappear in our theory: local energy and momentum density for the gravitational field, definition of vacuum state and Fock space in semi-classical gravity, the problem of time and topological foam in quantum gravity – all these problems are closely related with the non-existence of a Newtonian framework in GR which is available in our theory.

2.1 Lagrange formalism

If we require the harmonic condition as the equation for the preferred coordinates, the general form of the Lagrangian is a simple consequence.

Indeed, once we handle the preferred coordinates as independent fields, we can require covariance of the equations without restricting generality. Thus, we have to find a Lagrangian

$$L = L(g_{\mu\nu}, \psi^m, X^i, T)$$

for a covariant set of equation which contains the harmonic equations $\square X^i = \square T = 0$. The simplest way to obtain covariant equations is a covariant Lagrangian. The simplest way to obtain the harmonic equations is to use standard scalar Lagrangians for X^i, T and to assume that the remaining part does not depend on X^i, T. But that means that the requirements for the remaining part are the same as the standard requirements for a general-relativistic Lagrangian – thus, de-facto we have obtained our Lagrangian

$$L = L_{GR}(g_{\mu\nu}, \psi) + \Xi g^{\mu\nu}\delta_{ij}X^i_{,\mu}X^j_{,\nu} - \Upsilon g^{\mu\nu}T_{,\mu}T_{,\nu}$$

This is not a strong derivation – we have preferred the simplest possibilities instead of considering the general case. But this seems justified by Occam's razor, and is sufficient to explain the Lagrangian. Thus, to explain the equations of our theory completely it is sufficient to explain the harmonic equation for X^i, T.

2.2 Condensed Matter Interpretation

The theory allows a reformulation in terms of condensed matter theory. Instead of the gravitational field $g_{\mu\nu}$, we introduce classical condensed matter variables – density $\rho(x,t)$, velocity $v^i(x,t)$, and stress tensor $\sigma^{ij}(x,t)$ – by the following formulas:

$$\hat{g}^{00} = g^{00}\sqrt{-g} = \rho \tag{1}$$
$$\hat{g}^{i0} = g^{i0}\sqrt{-g} = \rho v^i \tag{2}$$
$$\hat{g}^{ij} = g^{ij}\sqrt{-g} = \rho v^i v^j - \sigma^{ij} \tag{3}$$

These condensed matter variables are Galilean covariant. If $\rho(x,t) > 0$ and $\sigma^{ij}(x,t)$ is positive definite they define a Lorentz metric. Moreover, the harmonic equation transforms into classical conservation laws:

$$\partial_t\rho + \partial_i(\rho v^i) = 0 \tag{4}$$
$$\partial_t(\rho v^j) + \partial_i(\rho v^i v^j - \sigma^{ij}) = 0 \tag{5}$$

Note also the very natural expression for the additional terms of the Lagrangian:

$$L\sqrt{-g} = L_{GR}\sqrt{-g} + \Xi(\rho v^2 - \sigma^{ii}) - \Upsilon\rho$$

Note that the conservation laws remain unchanged even if there are other "matter fields" $\psi^m(x)$. That means, these fields do not describe external matter, but inner steps of freedom of the condensed matter itself. Thus, the condensed matter is described by ρ, v^i, σ^{ij} and "inner steps of freedom" ψ^m. That's why the momentum related with inner steps of freedom is already taken into account.

In some sense, this interpretation of "matter fields" in this condensed matter interpretation unifies gravity with usual matter fields. More important is that it explains the harmonic equations for X^i, T in the presence of matter fields, and therefore the whole theory.

The non-gravity limit of the condensed matter interpretation is Lorentz ether theory. Thus, this interpretation may be considered as a generalization of Lorentz ether theory to gravity. This suggests to name this interpretation *general ether theory.*

2.3 Quantum gravity motivation

It is straightforward that the introduction of a Newtonian framework solves the most serious conceptual problems of GR quantization: the problem of time [7], topological foam, the information loss problem. The problems with local energy and momentum density of the gravitational field and the uncertainty of the definition of Fock space and vacuum state in semi-classical QFT, which are also connected with the absence of a preferred frame in GR, may be mentioned too. But to introduce a Newtonian framework to solve these problems is often criticized as an ad-hoc simplification. That's why I prefer to present a quantum gravity motivation of different type, related with quasi-classical quantum gravity (superpositions of semi-classical solutions).

If we consider superpositions of gravitational fields $g_{\mu\nu}(x)$, the classical notion of covariance may be generalized in two ways: c-covariance denotes covariance if we use the same diffeomorphism for all fields, q-covariance allows different diffeomorphisms for different fields [1]. The Einstein equations are q-covariant. Canonical GR quantization also defines a q-covariant theory. Instead, our theory is c-covariant. To motivate our theory it would be sufficient to motivate the existence of c-covariant objects.

For this purpose, let's consider the probability that a super-positional state $|g^1\rangle + |g^2\rangle$ of gravitational fields switches into $|g^1\rangle - |g^2\rangle$ because of gravitational interaction with a test particle φ. Let's consider non-relativistic quantum gravity – two-particle Schrödinger theory with Newtonian potential, with particles g and φ. Here gravitational interaction transforms the initial state into the state $|g^1, \varphi^1\rangle + |g^2, \varphi^2\rangle$. We ignore the test particle and compute the resulting one-particle state for g, which is in general a mixed state. The transition probability $|g^1\rangle + |g^2\rangle \to |g^1\rangle - |g^2\rangle$ is $\frac{1}{2}(1 - \langle\varphi^1|\varphi^2\rangle)$, thus, depends on the scalar product $\langle\varphi^1|\varphi^2\rangle$.

The natural generalization to semi-classical theory for $\varphi^{1/2}(x)$ is the solution for the test particle on the background $g^{1/2}_{\mu\nu}(x)$ created by the particle $g^{1/2}$. The scalar product between these functions is only c-covariant, not q-covariant.

As a new axiom for quantum gravity we propose that this scalar product is well-defined and observable. This axiom does not have the fault of being an ad-hoc simplification to avoid topological problems, but is a natural generalization of an observable of pre-relativistic quantum gravity. Thus, it is sufficiently motivated. Moreover, it has some beauty: it is a typical quantum observable with global character, it does not depend on questionable assumptions about local measurements.

Once we accept scalar products, a preferred system of coordinates is a very natural object. It is natural to assume that the scalar products define an isomorphism between the related L^2-spaces. Such an isomorphism allows to transfer the projective measure related with position measurement on a simple fixed state (the "vacuum") to other gravitational fields, thus, to define common coordinates on all gravitational fields, with a common topology as a consequence.

Independent of the last argument, the axiom requires to reject canonical GR quantization because of its q-covariance, while canonical quantization of our theory allows to make c-covariant predictions for such scalar products.

2.4 Realistic motivation

As shown by the proof of Bell's inequality [3] and their experimental falsification by Aspect [2] there is a contradiction between Einstein causality and the EPR criterion of reality [5]. This is usually interpreted as an experimental falsification of EPR-realism. But this is incorrect – only if we accept Einstein causality as an axiom, Aspect's experiment falsifies EPR-realism.

We can as well turn the argument against Einstein causality. We simply use EPR-realism and causality as axioms. With these axioms, Aspect's experiment falsifies Einstein causality and allows to prove the existence of a preferred foliation. Moreover, the existence of a preferred foliation allows to use Bohmian mechanics [4] instead of quantum theory.

The condensed matter interpretation of our theory defines such a preferred foliation.

3 Predictions

For a metric theory of gravity with Einstein equations in the limit $\Xi, \Upsilon \to 0$ it is not problematic to fit existing observation as well as GR fits observation. Instead, it is a non-trivial problem to distinguish the theory from GR by observation. Nonetheless, especially if $\Upsilon > 0$, this seems possible.

3.1 Ξ as a dark matter candidate

Let's consider at first the homogeneous universe solutions of the theory. Because of the Newtonian background frame, only a flat universe may be homogeneous. Thus, we make the ansatz:

$$ds^2 = d\tau^2 - a^2(\tau)(dx^2 + dy^2 + dz^2)$$

Note that in this ansatz the universe does not really expand, the observable expansion is an effect of shrinking rulers. Below we nonetheless use standard relativistic language. Using some matter with $p = k\varepsilon$ we obtain the equations ($8\pi G = c = 1$):

$$\begin{aligned} 3(\dot{a}/a)^2 &= -\Upsilon/a^6 + 3\Xi/a^2 + \Lambda + \varepsilon \\ 2(\ddot{a}/a) + (\dot{a}/a)^2 &= +\Upsilon/a^6 + \Xi/a^2 + \Lambda - k\varepsilon \end{aligned}$$

The influence of the Ξ-term on the age of the universe is easy to understand. For $\Xi > 0$ it behaves like homogeneously distributed dark matter with $p = -(1/3)\varepsilon$. It influences the age of the universe. A similar influence on the age of the universe has a non-zero curvature in GR cosmology. It seems not unreasonable to hope that a non-zero value for Ξ may be part of the solution of the dark matter problem.

3.2 $\Upsilon > 0$ solves the horizon problem without inflation

Instead, Υ influences the early universe, its influence on later universe may be ignored. But, if we assume $\Upsilon > 0$, the qualitative behaviour of the early universe changes in a remarkable way. We obtain a lower bound a_0 for $a(\tau)$ defined by

$$\Upsilon/a_0^6 = 3\Xi/a_0^2 + \Lambda + \varepsilon$$

The solution becomes symmetric in time, with a big crash followed by a big bang. For example, if $\varepsilon = \Xi = 0, \Upsilon > 0, \Lambda > 0$ we have the solution

$$a(\tau) = a_0 \cosh^{1/3}(\sqrt{3\Lambda}\tau)$$

Now, in such a time-symmetric universe the horizon is, if not infinite, at least big enough to solve the cosmological horizon problem (cf. [11]) without inflation. Because the flatness of the universe does not need explanation too, there is no necessity for inflation theory. This qualitative property remains valid for arbitrary small values $\Upsilon > 0$. The evidence for a hot state of the universe gives upper bounds for Υ.

3.3 $\Upsilon > 0$ stops gravitational collapse before horizon formation

Now, cosmological observation gives upper limits for Ξ, Υ. For computations in the solar system, it is possible to use the "GR approximation" $\Upsilon, \Xi \to 0$. But for strong gravitational fields they may become important again. Let's describe how to detect the domain of application of this GR approximation.

Let's assume we have a GR solution. First, we have to find the correct Galilean coordinates. For this purpose we have to define appropriate initial and boundary conditions for these coordinates. They may be obtained from gluing with the global universe solution, or from symmetry considerations. For example, for a spherically symmetric stable star we use harmonic coordinates which make the solution spherically symmetric and stable:

$$ds^2 = (1 - \frac{mm'}{r})(\frac{r-m}{r+m}dt^2 - \frac{r+m}{r-m}dr^2) - (r+m)^2 d\Omega^2$$

(the function m(r) with $0 < m < r, m' > 0$ defines the mass inside the sphere in appropriate units). For a collapsing star, these coordinates may be used as initial values. Once we have found the preferred Galilean coordinates, we have to prove if $g^{\mu\nu}$ remains small enough. Else, the GR approximation becomes invalid.

For example, for the Schwarzschild solution this happens near the horizon. The ansatz $m(r) = (1 - \Delta)r$ defines a stable solution for $p = \varepsilon$:

$$\begin{aligned} ds^2 &= \Delta^2 dt^2 - (2-\Delta)^2(dr^2 + r^2 d\Omega^2) \\ 0 &= -\Upsilon\Delta^{-2} + 3\Xi(2-\Delta)^{-2} + \Lambda + \varepsilon \\ 0 &= +\Upsilon\Delta^{-2} + \Xi(2-\Delta)^{-2} + \Lambda - \varepsilon \end{aligned}$$

Even if $\Upsilon, \Xi, \Lambda \approx 0$, for $\Delta \ll 1$ we can ignore only the terms with Ξ, Λ, but not the Υ-term. We obtain a time-independent solution $\varepsilon = \Upsilon\Delta^{-2} > 0$ for the inner part of a star, with time dilation $\Delta^{-1} = \sqrt{\varepsilon/\Upsilon}$. Once no horizon exists, the old notion "frozen star" seems more appropriate than "black hole". Frozen stars remain visible, but highly redshifted for small Υ.

If we interpret for example quasars as frozen stars, this leads to a relation between redshift and mass: $\Delta \sim M$.

3.4 The cutoff length in quantum gravity

Quantization of a condensed matter theory in a classical Newtonian framework is essentially simpler compared with GR quantization. The preferred Newtonian framework avoids most conceptual problems (problem of time [7], topological foam, information loss problem), allows to define uniquely local energy and momentum density for the gravitational field as well as the Fock space and vacuum state in semi-classical theory.

What remains are the ultraviolet problems. But they may be cured by explicit, physical regularization if we accept an "atomic hypothesis" in our condensed matter interpretation. Unlike in renormalized QFT, the relationship between bare and renormalized parameters obtains a physical meaning.

Similar ideas are quite old and in some aspects commonly accepted among particle physicists [8]. Usually it is expected that the critical cutoff length is of order of the Planck length $a_P \approx 10^{-33} cm$ [8],[13]. But an atomic hypothesis for our condensed matter interpretation predicts a different cutoff: Once we interpret ρ as the number of "atoms" per volume, we obtain the prediction

$$\rho(x) V_{cutoff} = \text{cons.}$$

Considering this prediction for the homogeneous universe, we find that the cutoff length seems to expand together with the universe. More accurate, our rulers shrink compared with the cutoff length.

References

[1] J. Anandan, Phys. Rev. D53, nr. 2, p.779-786, 1996

[2] A. Aspect et.al., Phys. Rev.Lett. 49, 1801-1807, 1982

[3] J.S. Bell, Speakable and unspeakable in quantum mechanics, Cambridge University Press, Cambridge, 1987

[4] D. Bohm, Phys. Rev. 85, 166-193, 1952

[5] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, 777-780, 1935

[6] V. Fock, Theorie von Raum, Zeit und Gravitation, Akademie-Verlag Berlin, 1960

[7] C. Isham, gr-qc/9210011

[8] F. Jegerlehner, hep-th/9803021

[9] K.V. Kuchar, C.G. Torre, Physical Review D, vol. 44, nr. 10, pp. 3116-3123, 1991

[10] A.A. Logunov et.al., Theor.Math.Phys. 69 Nr 3, 1986
A.A. Logunov, Theor.Math.Phys. 70 Nr 1, 1987

[11] J.R. Primack, astro-ph/9707285

[12] I. Schmelzer, gr-qc/9605013

[13] G.E. Volovik, cond-mat/9806010