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An approach to bond pricing

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An Approach to Bond Pricing

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Abstract. The paper proposes a simple arbitrage free approach for modelling bond prices. A natural structure of the volatility and expected return premium of bond price processes is directly obtained.

Keywords

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1. Introduction.

During the last years one observed an explosion in markets for interest rate options, futures, forwards and related securities. Therefore interest rate derivative security pricing has come to be an important area of investigation. Until now it seems that there is no consistent and reliable basis for valuing interest rate dependent derivative securities as it is provided by the Black and Scholes (1973) approach for stock options. As the market practice and literature indicates there is a deep need for a realistic and computationally tractable model.

Any meaningful valuation model has first of all to give a consistent term structure representation. In particular in such a model a discount bond has to reach its deterministic face value at maturity and should have a nonstationary volatility decreasing to zero at maturity. Furthermore, the bond prices should not violate an arbitrage free valuation principle reflecting the internal market dynamics.

As an extension of the Black and Scholes (1973) model several authors developed models which obtain option prices as functions of bond prices by the condition of no arbitrage opportunities for the options with respect to a continuously instantaneous interest paying savings account. This includes e.g. Merton (1973), Vasicek (1977), Richard (1978), Brennan and Schwartz (1979), Langetieg (1980), Courtadon (1982) and Ball and Torus (1983).

Another important approach starts from equilibrium models for the considered economy and is developed in Dothan (1978), Cox, Ingersoll and Ross (1985), Longstaff (1989) and others. The Cox, Ingersoll and Ross equilibrium model requires rather strong assumptions on production opportunities and risk preferences etc. It seems to be difficult to verify such assumptions which are needed to obtain reliable results.

During the last decades it turned out that martingale methods provide a systematic way to exclude arbitrage opportunities in stock market models. Along this line one finds more recent papers on interest rate derivatives by Ho and Lee (1986), Jamishidian (1988), Morton (1988), Heath, Jarrow and Morton (1989), Black, Derman and Toy (1990), Hull and White (1990), Sandmann and Sondermann (1991), El Karoui, Myneni and Vishwanathan (1992). The authors of the last paper model the term structure and choose a reference price process together with certain change of propability measure to prevent arbitrage opportunities between the bonds and the reference price process. In their approach each bond price process discounted by a reference price process represents a martingale in the sense of Harrison and Kreps (1979) and Harrison and Pliska (1981) under certain measure transformations. This methodology leads to a relatively complicated analysis involving forward rate processes which are indexed on a two dimensional time domain. Consequently one is facing considerable practical difficulties in computing actual bond prices and other derivatives.

The access to bond pricing which we are going to propose keeps the model simple and uses purely stochastic analytic tools to deduce the bond price dynamics.

We are basing our model on the assumption of a given spot rate process $r = \{r_t, 0 \leq t < \infty\}$, the instantaneous interest rate process. Furthermore, we assume that there is no arbitrage opportunity in bonds in the following sense. If we invest at time t the amount $P(t,T)$, the bond price, into a continuously instantaneous interest r paying savings account, then the expectation of the value of this account at maturity T is exactly the face value of the bond that is one monetary unit. This property shall hold under the given fixed probability measure for each bond and any time before its maturity.

In other words the expectation of a zero coupon discount bond invested in a savings acccount which is paying continuously the spot interest rate equals at maturity the face value of the bond. In this approach there is no direct assumption about the existence of an equilibrium in the considered economy. Also the risk preferences of the agents expressed as "market price of risk" are not involved in the modelling. Further, the bond price does not represent the expectation of the inverse of the savings account; instead we will see that it is the inverse of the expectation of the savings account.

We will observe in our bond price dynamics a naturally occuring return premium being equal to the square of the volatility of the bond price process itself. The aim of this paper is to present a straightforward and general but also simple framework which also allows one in practice to conveniently evaluate bonds and interest rate dependent derivatives.

2. Bond Pricing Model.

We start from a probability space $(\Omega, \underline{\mathcal{F}}, \mathcal{P})$ with filtration $\mathcal{F} = (\mathcal{F}_t)_{t\geq 0}$ fulfilling the usual conditions. We assume that the instantaneous interest rate process $r = \{r_t, 0 \leq t < \infty\}$ is given by the Ito stochastic differential equation

$$dr_t = a\,(t, r_t)dt + b\,(t, r_t)dW_t, \tag{1}$$

$t \geq 0$, with $\mathcal{F}_0$-measurable initial value, $r_0 \in \Re^1$ where $W = \{W_t, 0 \leq t < \infty\}$ denotes an $\mathcal{F}$-adapted Wiener process under the probability measure $\mathcal{P}$. We assume that the drift and diffusion coefficients are such that a unique solution of (1) exists, e.g. Lipschitz continuity and linear growth rate for a

and b. Finally, we assume for all $0 \leq t \leq u \leq T < \infty$ that

$$0 < E(\exp\left\{\int_t^u r_s ds\right\} \Big| \mathcal{F}_t) < \infty. \tag{2}$$

Let us denote by $P(t) = P(r_t, t, T)$ the price of a zero coupon discount bond at time t with face value $P(T) = 1$ at maturity T. We deliberately sometimes write the bond price $P(r_t, t, T)$ as a function depending on the spot rate r_t. Our main objective is now to describe the dynamics of the bond price process $P = \{P(t), 0 \leq t \leq T\}$ with respect to the underlying term structure of the instantaneous interest rate process $r = \{r_t, 0 \leq t < \infty\}$.

In the extensive literature on derivative securities pricing often one finds the assumption of a bond dynamics corresponding to a linear growth equation without any noise term. But as one realizes from practical observations, this assumption is violated in reality. First one notes that a bond price process has unbounded variation and is not differentiable, so there should be some stochastic differential in the growth equation. Very close to maturity the volatility of bond prices tends towards zero which should be also reflected in any good model. Finally, from the view point of interest rate dependent security derivative pricing it would be desireable to have the savings account process discounted by any bond as a martingale. As we mentioned in the introduction there is a large amount of work done in the literature to improve this situation.

The main difficulty consists in the problem of defining a simple practically relevant and mathematically tractable criterion for excluding arbitrage opportunities in bond pricing. If we look at the traditional deterministic linear growth model with r known, then one could rewrite it for all time instants $t \in [0, T]$ in the form

$$P(t) \exp\left\{\int_t^T r_s \, ds\right\} = 1,$$

which means that the bond invested in a continuously interest rate r paying savings account returns the face value of the bond at maturity. Obviously this cannot hold in our stochastic context.

More general and also mathematically convenient is the following similar assumption on the bond price dynamics:

We say that a bond pricing system has *no arbitrage* if for all $0 \leq t \leq T < \infty$,

$$E\left(P(t) \exp\left\{\int_t^T r_s \, ds\right\} \Big| \mathcal{F}_t\right) = 1. \tag{3}$$

Immediately we obtain the fundamental relation

$$P(t) = \left(E \left(\exp \left\{ \int_t^T r_s ds \right\} \Big| \mathcal{F}_t \right) \right)^{-1} \tag{4}$$

expressing the bond price. We will see in section 4 that a savings account discounted by any bond is a martingale. Thus a bond can be also interpreted as the price of a forward contract on the savings account paying one monetary unit at maturity.

Let us introduce notations for partial derivatives of the bond price $P = P(t) = P(r_t, t, T)$ in the form

$$P_t = \frac{\partial}{\partial t} P, \quad P_r = \frac{\partial}{\partial r} P \quad \text{and} \quad P_{rr} = \frac{\partial^2}{\partial r^2} P$$

to formulate the following result.

THEOREM 1. *If we additionally assume that a and b are continuously differentiable with respect to t and twice continuously differentiable with respect to r, then the price P of a bond with maturity $T \in [0, \infty)$ is the solution of the nonlinear partial differential equation.*

$$P_t + a\, P_r + \frac{1}{2}\, (b\, P)^2\, \frac{\partial}{\partial r}\, \left(P^{-2}\, P_r\right) - r\, P = 0 \tag{5}$$

for all $t \in [0, T)$ and $r \in \Re^1$ with terminal condition

$$P(r_T, T, T) = 1, \tag{6}$$

where a and b are the drift and diffusion coefficients of the instantaneous interest rate process given in (1).

The proof of this theorem is provided at the end of this section.

We could rewrite equation (5) in a form similar to that for the bond price proposed in Cox, Ingersoll and Ross (1985) and obtain

$$P_t + \left(a - \frac{b^2 P_r}{P} \right) P_r + \frac{1}{2} b^2 P_{rr} - r P = 0. \tag{7}$$

We note that in their framework a non-constant market risk parameter

$$\lambda = \frac{b^2}{r} \frac{P_r}{P}, \tag{8}$$

would reflect our bond price dynamics.

Let us again rewrite the partial differential equation (5), but now in the form

$$P_t + aP_r + \frac{1}{2}\, b^2 \frac{\partial^2}{\partial r^2}\, P - \left(r + \left(b \frac{P_r}{P} \right)^2 \right) P = 0. \tag{9}$$

This indicates that the expected instantaneous return premium of our bond price is $\left(b\frac{P_r}{P}\right)^2$, which we will observe also in the corresponding stochastic differential equation for the bond price process in the next section. We remark that in many contributions to bond pricing theory the existence of a return or risk premium is neglected. Here we obtain it in a very natural way.

PROOF OF THEOREM 1. From (4) we get for $0 \leq t \leq T < \infty$

$$P(r_t, t, T) = \frac{1}{u(r_t, t, T)} \tag{10}$$

with

$$u(r_t, t, T) = E\left(\exp\left\{ \int_t^T r_s ds \right\} \Big| \mathcal{F}_t \right). \tag{11}$$

Applying the Feynman-Kac formula (see e.g. p. 153 in Kloeden and Platen (1992)) one obtains $u = u(r, t, T)$ as solution of the linear partial differential equation

$$u_t + au_r + \frac{1}{2}\, b^2 u_{rr} + ru = 0 \tag{12}$$

with terminal condition

$$u(r, T, T) = 1 \tag{13}$$

for $t \in [0, T)$ and $r \in \Re^1$. Let us remark that we obtain from (10) the following relations between partial derivatives of P and u:

$$u_t = -u^2 P_t, \quad u_r = -u^2 P_r \tag{14}$$

$$\begin{aligned} u_{rr} &= -u^2 \left(P_{rr} - 2u (P_r)^2 \right) \\ &= -\frac{\partial}{\partial r} \left(P^{-2} P_r \right). \end{aligned}$$

Substituting the expressions (14) into (12) we obtain

$$-u^2 \left(P_t + aP_r + \frac{1}{2}\, b^2 u^{-2} \frac{\partial}{\partial r} \left(P^{-2} P_r \right) - ru^{-1} \right) = 0. \tag{15}$$

Now, using (10) and (2) we end up with the equation

$$-u^2\left(P_t + aP_r + \frac{1}{2}b^2P^2\frac{\partial}{\partial r}\left(P^{-2}P_r\right) - rP\right) = 0, \tag{16}$$

which proves (5). The terminal condition (6) follows directly from (11) and (10). □

We remark that even in cases when the partial differential equation (5) may make no sense, then still the bond price is well defined by (4). In this paper we have chosen this simple and straightforward way to derive the corresponding bond price, e.g. similar to Cox, Ingersoll and Ross (1985). A much more general result will be proved in a forthcoming paper.

3. Stochastic Differential Equation for the Bond Price Process.

The bond price process $P = \{P(t) = P(r_t, t, T), 0 \leq t \leq T\}$ which we usually observe at the market was until now interpreted only as a function of time t, maturity T and fluctuating instantaneous interest rate r_t in our model. Even if $P(r, t, T)$ is differentiable with respect to r and t we obtain $P(t) = P(t, r_s, T)$ as a functional of r_t which forms a process and fluctuates similarly to the spot rate r_t itself. This dynamics can be obtained by the Ito formula and is described in the following stochastic differential equation.

COROLLARY 1. *Under the assumptions of Theorem 1 we obtain for the bond price process* $P = \{P(t) = P(r_t, t, T), 0 \leq t \leq T\}$ *the Ito stochastic differential equation*

$$\frac{dP(t)}{P(t)} = \left[\Theta(r_t, t, T)^2 + r_t\right] dt - \Theta(r_t, t, T)\, dW_t \tag{17}$$

with volatility

$$\Theta(r_t, t, T) = -b(t, r_t)\,\frac{P_r(r_t, t, T)}{P(r_t, t, T)}. \tag{18}$$

We will give the proof of this corollary at the end of this section.

The expected return premium Θ^2 reflects the price for the risk covered by the bank to meet the face value of the bond. It is easy to realize from (17) that the expected return premium Θ^2 for a bond is represented just by the square of the volatility of the bond price itself as we have seen already from (9).

We note that under our no arbitrage criterion the normalized expected return excess rate is one, thus it is independent on time, maturity, spot rate etc. To be more specific we obtain from (17) for the return excess

$$\varrho_\Delta(t,T) = \frac{P(r_{t+\Delta}, t+\Delta, T) - P(r_t, t, T)}{P(r_t, t, T)} - r_t\,\Delta,$$

$\Delta > 0$, $t \leq T$, the normalized expected return excess rate

$$\lim_{\Delta \to 0} \frac{E\left(\varrho_\Delta(t,T) \middle| \mathcal{F}_t\right)}{\mathrm{Var}(\varrho_\Delta(t,T))},$$

which is scaled by the variance

$$\mathrm{Var}(\varrho_\Delta(t,T)) = E\left(\left(\varrho_\Delta(t,T) - E\left(\varrho_\Delta(t,T) \middle| \mathcal{F}_t\right)\right)^2 \middle| \mathcal{F}_t\right)$$

measuring the increase of the risk. In other words the local gain of any bond minus that of a savings account is in the average proportional to the squared volatility of the bond.

PROOF OF COROLLARY 1. Applying the Ito formula to $P(r_t, t, T)$ as a function of the spot rate r_t and time t we get

$$dP = \left(P_t + aP_r + \frac{1}{2}b^2 P_{rr}\right) dt + bP_r dW. \tag{19}$$

Now, using the partial differential equation (7) and the notation (18) it follows that

$$\begin{aligned} dP &= \left(rP + \frac{b^2 P_r^2}{P}\right) dt + b\frac{P_r}{P}\, P\, dW, \\ &= \left(r + \Theta^2\right) P\, dt - \Theta P\, dW, \end{aligned} \tag{20}$$

which proves (17). □

4. Discounting by Bonds.

Let us introduce a continuously instantaneous interest paying savings account $B = \{B_t, 0 \leq t < \infty\}$ characterized by the linear growth equation

$$dB_t = r_t\, B_t\, dt, \tag{21}$$

$0 \leq t < \infty$, $B_0 = 1$.

Then we obtain for the savings account discounted by a bond with maturity T from (21) and (17), by the Ito formula, the dynamics

$$(22)\quad d\left(\frac{B_t}{P(t)}\right) = \left[\frac{1}{P(t)} r_t B_t - \frac{B_t}{P(t)^2} P(t) \left[\Theta(r_t,t,T)^2 + r_t\right] + \frac{B_t}{P(t)^3}\Theta(r_t,t,T)^2 P(t)^2\right] dt + \frac{B_t}{P(t)^2}\Theta(r_t,t,T)P(t)dW_t = \Theta(r_t,t,T)\left(\frac{B_t}{P(t)}\right) dW_t$$

for $t \in [0,T]$, which shows that $Z = \left\{Z_t = \frac{B_t}{P(r_t,t,T)}, 0 \le t \le T\right\}$ represents a $(\mathcal{P},\mathcal{F})$-martingale. This indicates that our measure $\mathcal{P}$ is similar to that proposed in El Karoui, Myneni and Vishwanathan (1992) who used forward rates and maturity dependent measure transformations. Here we dealt until now with one probability measure $\mathcal{P}$ under which the martingale property for Z holds for any maturity $T \ge t$. Moreover, it is an easy exercise to show that any asset X with dynamics of the form

$$(23)\qquad dX_t = r_t X_t\, dt + dM_t,$$

where M is a $(\mathcal{P},\mathcal{F})$-continuous martingale orthogonal to W, provides a $(\mathcal{P},\mathcal{F})$-martingale after discounting by a bond. This is a very appealing property which allows straightforward derivative security pricing. More precisely, if we form a portfolio

$$V_t = \xi_t X_t + \eta_t P(t) = P(t)\, E(f(X_T)\,|\,\mathcal{F}_t)$$

holding at time $t \in [0,T]$ the amount ξ_t in the asset X_t and the amount η_t in the T-maturity bond $P(t)$ to replicate a contingent claim $H = f(X_T)$, then it is appropriate to consider the discounted value process

$$\bar{V}_t = \frac{V_t}{P(t)} = \xi_t \bar{X}_t + \eta_t = E(f(X_T)\,|\,\mathcal{F}_t)$$

involving the discounted asset

$$\bar{X}_t = \frac{X_t}{P(t)} = \bar{X}_0 + \int_0^t \frac{1}{P(s)} dM_s + \int_0^t \Theta(r_s,s,T)\,\bar{X}_s\, dW_s,$$

which represents a $(\mathcal{P},\mathcal{F})$-continuous martingale. Applying the Ito formula and the Kolmogorov backward equation we obtain under the assumption that our market is complete (e.g. W and M are the only martingales influencing $\bar{X}$) that

$$f(X_T) = \bar{V}_T = \bar{V}_t + \int_0^t \frac{\partial}{\partial \bar{X}} \bar{V}_s \, d\bar{X}_s.$$

Thus the contingent claim is replicated choosing the hedge ratio as $\xi_t = \frac{\partial}{\partial \bar{X}} \bar{V}_t$.

As we will now see a transformation of the probability measure will be necessary to obtain the pricing measure of an asset which involves a component of W in its noise. To make this more precise let us consider a bond P with maturity T which we will discount by another bond P^* having maturity $T^* \leq T$. Then we introduce by the Radon-Nikodym derivative

$$\text{(24)} \quad \frac{d\tilde{\mathcal{P}}}{d\mathcal{P}} = \xi_{t,T} = \exp\left\{ - \int_t^T \frac{1}{2} \Theta(r_s, s, T)^2 ds + \int_t^T \Theta(r_s, s, T) dW_s \right\}$$

a new probability measure $\tilde{\mathcal{P}}$ under which the process

$$\text{(25)} \quad \tilde{W}_s = W_s - \int_t^s \Theta(r_u, u, T) du$$

is a Wiener process, that is a $(\tilde{\mathcal{P}}, \mathcal{F})$-martingale. Applying the Ito formula, (17) and (25) we obtain

$$\begin{aligned}
\text{(26)} \quad d\left(\frac{P(s)}{P^*(s)}\right) &= \frac{P(s)}{P^*(s)} \left[\Theta(r_s, s, T) \; (\Theta(r_s, s, T) - \Theta(r_s, s, T^*))\right] ds \\
&\quad + \frac{P(s)}{P^*(s)} \left(\Theta(r_s, s, T^*) - \Theta(r_s, s, T)\right) dW_s \\
&= \frac{P(s)}{P^*(s)} \left(\Theta(r_s, s, T^*) - \Theta(r_s, s, T)\right) d\tilde{W}_s,
\end{aligned}$$

$t \leq s \leq T^*$, which shows that $\frac{P}{P^*}$ is a $(\tilde{\mathcal{P}}, \mathcal{F})$-martingale. We can now price a contingent claim $H = f(P(T^*))$ in a similar way as above but using as discounted value process $\bar{V}_t = \tilde{E}\left(f(P(T^*)) \,|\, \mathcal{F}_t\right)$, which is the conditional expectation under $\tilde{\mathcal{P}}$.

As a final comment, we remark that the bond P discounted by the savings account B that is $\frac{P}{B}$ also forms a $(\tilde{\mathcal{P}}, \mathcal{F})$-martingale. This indicates that

$$\text{(27)} \quad P(t) = \tilde{E}\left(\exp\left\{ - \int_t^T r_s ds \right\} \Big| \mathcal{F}_t \right),$$

where $\tilde{E}$ denotes the expectation with respect to $\tilde{\mathcal{P}}$. The price (27) is similar to that proposed in a wide range of papers but using instead our measure $\mathcal{P}$. For comparison let us denote this price by

$$\hat{P}(t) = E\left(\exp\left\{-\int_t^T r_s ds\right\}\Big|\mathcal{F}_t\right). \tag{28}$$

It is easy to show from (4) by Jensen's inequality that

$$P(t) \leq \hat{P}(t). \tag{29}$$

A price $\hat{P}$ would overprice a bond P in our sense by neglecting any return premium in the dynamics. In some degree this is better achieved in the Cox, Ingersoll and Ross approach.

5. Bond Price for the Vasicek Interest Rate.

It is possible to explicitely compute some bond prices if a specific term structure for the spot rate process is given. As an example we illustrate this for the Vasicek (1977) instantaneous interest rate process given by the Ito stochastic differential equation.

$$dr_t = A\,(R - r_t)dt + AC\,dW_t, \tag{30}$$

where A denotes the back driving force, R is the average interest rate and C represents a noise parameter. It is straightforward to check that the nonlinear partial differential equation (5) with $a = A(R - r)$ and $b = AC$ is solved by the bond price

$$P(r,t,T) = \exp\{-y(T-t)\} \tag{31}$$

with yield

$$y = R + \frac{d}{T-t}(r - R) + C^2\left(\frac{1}{2} - \frac{d}{T-t} + \frac{1}{2}f\right) \tag{32}$$

duration

$$d = \frac{1}{A}\left(1 - \exp\{-A(T-t)\}\right) \tag{33}$$

and

$$f = \frac{1}{2A(T-t)}\left(1 - \exp\{-2A(T-t)\}\right). \tag{34}$$

We note from (18) that the volatility of the bond price process here has the form

$$\Theta = CAd \tag{35}$$

and therefore we have the expected return premium $\Theta^2 = C^2A^2d^2$. It is easily seen from (35) and (33) that the volatility Θ becomes zero at maturity and asymptotically approaches the parameter C far from maturity.

The duration d is approximately linear with $d \approx T - t$ for t close to maturity and asymptotically constant with $d \approx \frac{1}{A}$ far from maturity. The yield y in (32) represents nearly the spot rate r for t close to maturity and is asymptotically $y \approx R + \frac{C^2}{2}$ far from maturity.

It is possible for important other term structures of interest rates to obtain corresponding explicit expressions for bond prices. In other cases efficient stochastic numerical methods of the type described in Kloeden and Platen (1992) and Hofmann, Platen and Schweizer (1992) can be applied which allow one to accurately and efficiently compute bond prices and other security derivatives.

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