Spectral properties of coupled wave equations

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Abstract

Essential features of two-section DFB semiconductor lasers can be described by a boundary value problem for the so-called coupled wave equations, a linear hyperbolic system of first order partial differential equations with piecewise constant coefficients. In this paper we investigate spectral properties of an operator H defined by this boundary value problem. We prove that H generates a C_0-group of bounded operators in a suitable Hilbert space $\mathcal{U}$, that all but finitely many eigenvalues of H are simple and have negative real parts and that there exists a basis in $\mathcal{U}$ consisting of root functions of H, where all but finitely many of these root functions are eigenfunctions.

1 Introduction

Distributed feedback (DFB) semiconductor lasers are promising optical devices for telecommunication. They can be used to obtain selfsustained oscillations with high frequency [2], to regenerate signals in shape and frequency and to have the properties of a switch [5].

The following mathematical model can be used to explain several pulsation mechanisms of DFB-lasers, such as dispersive self Q-switching, mode beating [7] and spatial hole burning [6]. It consists of a boundary value problem for a linear hyperbolic system of first order complex-valued partial differential equations with piecewise constant coefficients, the so-called coupled wave equations, see [1] - [8]. For the special case of two section lasers these equations can be rewritten in the form

$$\left.\begin{array}{rcl} \partial_t u_1(t,x) &=& v_g\Big(-\partial_x u_1(t,x) + c(x)u_1(t,x) + d_1 u_2(t,x)\Big) \\ \partial_t u_2(t,x) &=& v_g\Big(\partial_x u_2(t,x) + d_2 u_1(t,x) + c(x)u_2(t,x)\Big) \end{array}\right\} - l_1 < x < l_2,\ t > 0, \quad (1.1)$$

where $u_1(t,x)$ and $u_2(t,x)$ describe the slowly varying complex amplitudes of the forward and backward traveling waves (after averaging over the transverse plane and separating terms varying rapidly in space and time) of the electric field, $l_1 > 0$ and $l_2 > 0$ are the lengths of the two laser sections, d_1 and d_2 are complex coupling coefficients, v_g is the group velocity (cf. [7]), and c is a propagation coefficient which is assumed to be constant in the laser sections, i.e.

$$c(x) = \begin{cases} c_1 & \text{for} \quad -l_1 < x < 0, \\ c_2 & \text{for} \quad 0 < x < l_2 \end{cases} \quad (1.2)$$

is a given piecewise constant function, where the complex coefficients c_1 and c_2 are the so-called propagation coefficients in the left and the right laser section, respectively.

The reflection properties at the facets of the laser are described by the boundary conditions

$$\begin{array}{rcl} u_1(t,-l_1) &=& r_1\, u_2(t,-l_1), \\ u_2(t,l_2) &=& r_2\, u_1(t,l_2), \end{array} \quad (1.3)$$

where r_1 and r_2 are given complex coefficients satisfying

$$0 < |r_j| < 1 \quad \text{for} \quad j = 1, 2. \tag{1.4}$$

The boundary value problem (1.1), (1.3) can be formulated as an abstract linear evolution equation

$$\frac{du}{dt} = v_g H u$$

in a suitable Hilbert space. For $d_1 = d_2 = 0$, (1.1) describes the so-called Fabry-Perot laser, in that case the operator H is denoted by H_0.

Recently (cf.[8]), certain spectral properties of the unbounded linear operator H have been determined. In the present paper we continue these investigations. Especially, we will study the dependence of the spectrum of H on the coupling coefficients d_1 and d_2 and on the reflection coefficients r_1 and r_2. The obtained results are useful for establishing the existence of integral manifolds for some nonlinear evolution system which appears if the system (1.1), (1.3) is nonlinearly coupled with balance equations for the carrier densities in the laser, and for the description of rotating and modulated wave solutions and of forced frequency locking properties of such systems (cf. [9]).

2 Preliminaries. Results

Let $\mathcal{U}$ be the complex Hilbert space $L^2((-l_1, l_2); C^2)$, i.e the elements of $\mathcal{U}$ are pairs of complex valued L^2-functions on the interval $(-l_1, l_2)$. The space $\mathcal{U}$ is equipped with the usual scalar product

$$\langle u, v \rangle := \int_{-l_1}^{l_2} (u_1(x)\overline{v_1(x)} + u_2(x)\overline{v_2(x)})dx.$$

Let H and H_0 be the unbounded linear operators mapping $\mathcal{D} \subset \mathcal{U}$ into $\mathcal{U}$, where $\mathcal{D}$ is given by

$$\mathcal{D} := \{u \in W^{1,2}((-l_1, l_2); C^2) : u_1(-l_1) = r_1 u_2(-l_1), u_2(l_2) = r_2 u_1(l_2)\} \tag{2.1}$$

and which are defined by

$$\begin{aligned} Hu &:= (-u_1' + c(x)u_1 + d_1 u_2, u_2' + d_2 u_1 + c(x)u_2), \\ H_0 u &:= (-u_1' + c(x)u_1, u_2' + c(x)u_2). \end{aligned} \tag{2.2}$$

Here $W^{1,2}((-l_1, l_2); C^2)$ is the usual Sobolev space, and, hence, $\mathcal{D}$ is dense in $\mathcal{U}$, and by u_j' we denote the derivative of u_j with respect to the space variable x.

Concerning the spectrum of H and of H_0 (denoted by spec H and spec H_0, respectively) the following result has been proven in [8]:

Theorem 1 (i) *The spectrum of H consists of countably many isolated eigenvalues λ_j $(j = 1, 2, \ldots)$. All these eigenvalues are geometrically simple, i.e.* $\dim \ker(H - \lambda_j I) = 1$, *and have finite algebraic multiplicity, i.e.*

$$\dim\ \mathcal{U}_j < \infty \quad \textit{with} \quad \mathcal{U}_j := \bigcup_{k=1}^{\infty} \ker(H - \lambda_j I)^k.$$

Moreover, in each $\mathcal{U}_j$ there exists a basis $\mathcal{B}_j$, consisting of root functions of H, such that $\cup_{j=1}^{\infty} \mathcal{B}_j$ is a basis in $\mathcal{U}$ (with respect to the L^2-norm) and in $\mathcal{D}$ (with respect to the $W^{1,2}$-norm).

(ii) *It holds*

$$\text{spec } H_0 = \left\{ \lambda \in C : \lambda = \frac{1}{l_1 + l_2} \Big(\frac{1}{2} \ln(r_1 r_2) + c_1 l_1 + c_2 l_2 + k\pi i \Big),\ k \in Z \right\}, \tag{2.3}$$

and all elements of spec H_0 *are simple eigenvalues.*

In (2.3) we denote by $\ln(r_1 r_2)$ the complex number z_0 satisfying $e^{z_0} = r_1 r_2$ and $0 \leq \arg z_0 \leq \arg z$ for all $z \in C$ with $e^z = r_1 r_2$ and $0 \leq \arg z$.

In the present paper we will prove the following results:

Theorem 2 *For all $\lambda \in$ spec H it holds*

$$\text{dist}(\lambda, \text{spec } H_0) \leq \max \left\{ \left| \frac{d_1}{r_1} \right|,\ \left| \frac{d_2}{r_2} \right| \right\}.$$

Here we use the standard notation $\text{dist}\,(\lambda, \text{spec } H_0) := \inf \{ |\lambda - \mu| : \mu \in \text{spec } H_0 \}$.

Theorem 3 *The operator H generates a C_0-group of bounded operators in $\mathcal{U}$.*

Theorem 4 *For all $\varepsilon > 0$ and $c_* > 0$ there exists a $\lambda_* > 0$ such that the following holds: If $|c_j| < c_*$ and $|d_j| < c_*$ for $j = 1, 2$ and if $\lambda \in$ spec H satisfies $|\lambda| > \lambda_*$, then* $\text{dist}\,(\lambda, \text{spec } H_0) < \varepsilon$.

Theorem 4 means that all but finitely many modes of the two section DFB laser, described by H, are damped, if all modes of the corresponding Fabry-Perot laser are damped.

Theorem 1 and Theorem 4 yield the following corollary which gives a partial answer to an open problem stated in [8]:

Corollary *If $\lambda \in$ spec H and if $|\lambda|$ is sufficiently large, then λ is a simple eigenvalue of the operator H.*

3 Proofs of the Results

3.1 Proof of Theorem 2

Following an idea of J. Rehberg from [8] we introduce an isomorphism T from $\mathcal{U}$ onto $\mathcal{U}$ such that TH_0T^{-1} is normal. From $THT^{-1} = TH_0T^{-1} + T(H - H_0)T^{-1}$ and from the property that the operator $T(H - H_0)T^{-1}$ is bounded (cf. (2.2)), we get that THT^{-1} can be viewed as a bounded perturbations of a normal operator. According to the spectral theory of such operators (cf., e.g., [10, Chapter V.3]) we have

$$\text{spec } H \subseteq \{\lambda \in C : \text{dist}(\lambda, \text{spec } H_0) \leq \|T(H - H_0)T^{-1}\|\}. \tag{3.1}$$

The isomorphism T we are working with is defined by

$$Tu := \left(r_1^{-1} e^{\alpha(x+l_1)} u_1, e^{-\alpha(x+l_1)} u_2\right) \quad \text{with} \quad \alpha := \frac{\text{Re} \ln(r_1 r_2)}{2(l_1 + l_2)}.$$

It is easy to verify that T maps $\mathcal{D}$ (cf. (2.1)) onto

$$\tilde{\mathcal{D}} := \{u \in W^{1,2}([-l_1, l_2]; C^2) : u_1(-l_1) = u_2(-l_1), u_1(l_2) = u_2(l_2)\}.$$

Hence, $\tilde{\mathcal{D}}$ is the domain of definition of THT^{-1} and TH_0T^{-1}. Furthermore, from (2.2) it follows

$$\begin{aligned} TH_0T^{-1}u &= (-u_1' + (c(x) + \alpha)u_1, u_2' + (c(x) + \alpha)u_2) \quad \text{for} \quad u \in \tilde{\mathcal{D}}, \\ T(H - H_0)T^{-1}u &= (r_1^{-1} d_1 e^{2\alpha(x+l_1)} u_2, r_1 d_2 e^{-2\alpha(x+l_1)} u_1). \end{aligned} \tag{3.2}$$

Straightforward calculations show that TH_0T^{-1} is normal. Furthermore, (1.4) yields that $\alpha < 0$. Hence, for all $u \in \mathcal{U}$ we have

$$\|T(H - H_0)T^{-1}u\|^2 = \int_{-l_1}^{l_2} (|r_1^{-1} d_1 e^{2\alpha(x+l_1)} u_2|^2 + |r_1 d_2 e^{-2\alpha(x+l_1)} u_1|^2) dx \leq$$

$$\leq \left|\frac{d_1}{r_1}\right|^2 \int_{-l_1}^{l_2} |u_2|^2 dx + |r_1 d_2 e^{-2\alpha(l_1+l_2)}|^2 \int_{-l_1}^{l_2} |u_1|^2 dx \leq \max\left\{\left|\frac{d_1}{r_1}\right|^2, \left|\frac{d_2}{r_2}\right|^2\right\} \|u\|^2.$$

Therefore, the validity of Theorem 2 follows from (3.1).

3.2 Proof of Theorem 3

We consider the densely defined unbounded linear operator $A : \tilde{\mathcal{D}} \subseteq \mathcal{U} \to \mathcal{U}$ defined by $Au := (-u_1', u_2')$. It is easy to see that iA is self-adjoint. Hence, by Stone's Theorem, A is a generator of a C_0-group of unitary operators in $\mathcal{U}$ (cf. [11, Theorem 1.10.8]). But $THT^{-1} - A$ is a bounded operator on $\mathcal{U}$ (cf. (3.2)). Therefore, THT^{-1} is a generator of a C_0-group in $\mathcal{U}$, too (cf. [11, Theorem 3.1.1]). Hence, H is a generator of a C_0-group in $\mathcal{U}$, q.e.d.

3.3 Proof of Theorem 4

We introduce the 2×2-matrices

$$J := \begin{bmatrix} -\,-1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } D := \begin{bmatrix} 0 & d_1 \\ d_2 & 0 \end{bmatrix}. \tag{3.3}$$

Let $\lambda \in \operatorname{spec} H$. Then, according to Theorem 1, (1.2), (2.2) and (3.3), there exists an $u \in \mathcal{D}$ such that

$$\begin{aligned} u'(x) &= ((\lambda - c_1)J - JD)\,u(x) \quad \text{for } -\,-l_1 < x < 0, \\ u'(x) &= ((\lambda - c_2)J - JD)\,u(x) \quad \text{for } 0 < x < l_2, \end{aligned} \tag{3.4}$$

$$u_1(-l_1) = r_1,\ u_2(-l_1) = 1,\ u_2(l_2) = r_2 u_1(l_2). \tag{3.5}$$

The Sobolev embedding theorem implies $\mathcal{D} \subseteq C([-l_1, l_2]; C)$. Thus, u is continuous in $x = 0$, and we get from (3.4),(3.5)

$$u(l_2) = \exp(l_2(\lambda - c_2)J - l_2 JD)\exp(l_1(\lambda - c_1)J - l_1 JD)\begin{bmatrix} r_1 \\ 1 \end{bmatrix},$$

where exp is the usual exponential map for matrices. By means of the matrices

$$B_j := \exp(l_j(\lambda - c_j)J - l_j JD) - \exp l_j(\lambda - c_j)J, \quad j = 1, 2 \tag{3.6}$$

we can represent $u(l_2)$ in the form

$$\begin{aligned} u(l_2) &= [\exp(l_1(\lambda - c_1) + l_2(\lambda - c_2))J + B_2 \exp(l_1(\lambda - c_1)J) + \\ &\quad + \exp(l_2(l - c_2)J)B_1 + B_2 B_1]\begin{bmatrix} r_1 \\ 1 \end{bmatrix}. \end{aligned} \tag{3.7}$$

The matrices B_j depend on c_1, c_2, d_1, d_2 and λ. In what follows we prove

$$\|B_j\| \to 0 \text{ as } \mathrm{Im}\lambda \to \infty \text{ locally uniformly with respect to } c_1, c_2, d_1, d_2 \text{ and } \mathrm{Re}\lambda. \tag{3.8}$$

This means that for all $\epsilon > 0$ and $c_* > 0$ there exists a $\lambda_* > 0$ such that $\|B_j\| < \epsilon$ if $|\mathrm{Im}\lambda| > \lambda_*$ and if $|c_1|, |c_2|, |d_1|, |d_2|$, and $|\mathrm{Re}\lambda|$ are smaller than c_*.

Let us prove (3.8). From (3.3) we obtain $J^2 = I,\ JD = -DJ,\ (JD)^2 = -\,-d_1d_2I$, and $(\mu J + D)^2 = (\mu^2 - d_1d_2)I$. Therefore, for any complex μ such that $\mu^2 \neq d_1d_2$ we have

$$\begin{aligned} \exp(\mu J + JD) &= \sum_{k=0}^{\infty} \frac{1}{k!}(\mu J + JD)^k = \sum_{k=0}^{\infty} (\mu^2 - d_1d_2)^k \left(\frac{I}{(2k)!} + \frac{\mu J + JD}{(2k+1)!} \right) \\ &= I \sum_{k=0}^{\infty} \frac{(\sqrt{\mu^2 - d_1d_2})^{2k}}{(2k)!} + \frac{\mu J + JD}{\sqrt{\mu^2 - d_1d_2}} \sum_{k=0}^{\infty} \frac{(\sqrt{\mu^2 - d_1d_2})^{2k+1}}{(2k+1)!} \\ &= I \cosh\sqrt{\mu^2 - d_1d_2} + (\mu J + JD)\frac{\sinh\sqrt{\mu^2 - d_1d_2}}{\sqrt{\mu^2 - d_1d_2}}. \end{aligned} \tag{3.9}$$

Remark that (3.9) is valid for both values of the square root. Moreover, it can be easily proved that

$$\begin{aligned} \cosh\sqrt{\mu^2-d_1d_2}-\cosh\mu\to 0 \quad &\text{as} \quad \operatorname{Im}\mu\to\infty,\\ \frac{\mu\sinh\sqrt{\mu^2-d_1d_2}}{\sqrt{\mu^2-d_1d_2}}-\sinh\mu\to 0 \quad &\text{as} \quad \operatorname{Im}\mu\to\infty,\\ \frac{\mu\sinh\sqrt{\mu^2-d_1d_2}}{\sqrt{\mu^2-d_1d_2}}\to 0 \quad &\text{as} \quad \operatorname{Im}\mu\to\infty \end{aligned}$$

locally uniformly with respect to d_1, d_2 and $\mathrm{Re}\mu$. By means of these relations we obtain from (3.6) and (3.9) the validity of (3.8).

Using the notation

$$\begin{aligned} &(v_1(\lambda,c_1,c_2,d_1,d_2), v_2(\lambda,c_1,c_2,d_1,d_2)) :=\\ &:= \left(B_2\begin{bmatrix} e^{-l_1(\lambda-c_1)} & 0\\ 0 & e^{l_1(\lambda-c_1)}\end{bmatrix}+\begin{bmatrix} e^{-l_2(\lambda-c_2)} & 0\\ 0 & e^{l_2(\lambda-c_2)}\end{bmatrix}B_1+B_2B_1\right)\begin{bmatrix} r_1\\ 1\end{bmatrix}, \end{aligned} \tag{3.10}$$

we get from (3.7)

$$\begin{aligned} u_1(l_2) &= e^{-l_1(\lambda-c_1)-\lambda_2(\lambda-c_2)}r_1+v_1(\lambda,c_1,c_2,d_1,d_2),\\ u_2(l_2) &= e^{l_1(\lambda-c_1)+l_2(\lambda-c_2)}+v_2(\lambda,c_1,c_2,d_1,d_2). \end{aligned}$$

Hence, according to (3.5) the following matrix vanishes:

$$e^{l_1(\lambda-c_1)+l_2(1-c_2)}-r_1r_2e^{-l_1(\lambda-c_1)-l_2(\lambda-c_2)}+v_2(\lambda,c_1,c_2,d_1,d_2)-r_2v_1(\lambda,c_1,c_2,d_1,d_2). \tag{3.11}$$

Now, suppose that Theorem 4 is not true. Then there exist $\varepsilon>0, c_*>0$ and complex sequences $\lambda^{(k)}, c_1^{(k)}, c_2^{(k)}, d_1^{(k)}$ and $d_2^{(k)}$ such that $|\mathrm{Im}\lambda^{(k)}|\to\infty$ for $k\to\infty$ and that for all k we have $|\mathrm{Re}\lambda^{(k)}|<c_*,\ |c_1^{(k)}|<c_*,\ |c_2^{(k)}|<c_*,\ |d_1^{(k)}|<c_*,\ |d_2^{(k)}|<c_*,$

$$\text{dist }(\lambda^{(k)}, \text{spec } H_0^{(k)})\ge\varepsilon \tag{3.12}$$

and (cf. 3.11)

$$\begin{aligned} 0 \;=\; & e^{l_1(\lambda^{(k)}-c_1^{(k)})+l_2(\lambda^{(k)}-c_2^{(k)})}-r_1r_2e^{-l_1(\lambda^{(k)}-c_1^{(k)})-l_2(\lambda^{(k)}-c_2^{(k)})}+\\ & +v_2(\lambda^{(k)},c_1^{(k)},c_2^{(k)},d_1^{(k)},d_2^{(k)})-r_2v_1(\lambda^{(k)},c_1^{(k)},c_2^{(k)},d_1^{(k)},d_2^{(k)}). \end{aligned} \tag{3.13}$$

Here $H_0^{(k)}$ is the operator H_0, defined by (2.2) with coefficients $c_1^{(k)}$ and $c_2^{(k)}$ (cf. (1.2)). But (3.8), (3.10) and (3.13) imply

$$e^{2(l_1(\lambda^{(k)}-c_1^{(k)})+l_2(\lambda^{(k)}-c_2^{(k)}))}\to r_1r_2 \text{ for } k\to\infty,$$

and this contradicts to (2.3) and (3.12).

3.4 Proof of the Corollary

For $0 \le \varepsilon \le 1$ and $k \in Z$ let us introduce the notation $H_\varepsilon := H_0 + \varepsilon(H - H_0)$ (cf. (2.2)) and

$$\sigma_\varepsilon^{(k)} := \left\{ \lambda \in \text{spec } H_\varepsilon : \left| \lambda - \frac{1}{l_1 + l_2} \left(\frac{1}{2} \ln(r_1 r_2) + c_1 l_1 + c_2 l_2 + k\pi i \right) \right| \le \frac{\pi}{2(l_1 + l_2)} \right\}.$$

According to Theorem 1 (ii) there exists a $k_0 \in N$ such that for all $k \in Z$ with $|k| > k_0$ and for all $\varepsilon \in [0, 1]$ the sets $\sigma_\varepsilon^{(k)}$ are spectral sets of H_ε, i.e. they are open and closed in the spectrum of the operator H_ε. Moreover, there exists a $\lambda_* > 0$ such that for all $\varepsilon \in [0, 1]$ and for all $\lambda \in \text{spec}H_\varepsilon$ with $|\text{Im}\lambda| > \lambda_*$ we have $\lambda \in \sigma_\varepsilon^{(k)}$ for a certain k. Hence, it remains to show that, for large k, the spectral sets $\sigma_1^{(k)}$ consist of exactly one simple eigenvalue.

Let $m_\varepsilon^{(k)}$ be the sum of the algebraic multiplicities of all eigenvalues in $\sigma_\varepsilon^{(k)}$. Perturbation results for spectral sets consisting of finitely many eigenvalues (cf. [10, Chapter IV.5]) yield that, for $|k| > k_0$, each $m_\varepsilon^{(k)}$ depends continuously on ε. But from Theorem 1 (ii) it follows $m_0^{(k)} = 1$. Hence, $m_1^{(k)} = 1$ for $|k| > k_0$.

Remark For $|k| > k_0$ we denote by $\lambda^{(k)}$ the unique element of $\sigma_1^{(k)}$, i.e. $\lambda^{(k)}$ is an eigenvalue of the operator H. Then Theorem 4 yields

$$\lambda^{(k)} = \frac{1}{l_1 + l_2} \left(\frac{1}{2} \ln(r_1 r_2) + c_1 l_1 + c_2 l_2 + k\pi i \right) + o(1) \quad \text{as} \quad k \to \infty.$$

Moreover, it is easy to calculate the following, more precise asymptotic expansion

$$\lambda^{(k)} = \frac{1}{l_1 + l_2} \left(\frac{1}{2} \ln(r_1 r_2) + c_1 l_1 + c_2 l_2 + k\pi i + \frac{i}{2k} \left(\frac{d_1 l_1}{r_1} + \frac{d_2 l_2}{r_2} - d_1 l_2 r_2 - d_2 l_1 r_1 \right) \right) + o\Big(\frac{1}{k}\Big)$$

as $k \to \infty$.

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References

[1] H. Kogelnik and C.V. Shank, *Coupled wave theory of distributed feedback lasers*, J. Appl. Phys. **43** (1972), 2327-2335.

[2] U. Feiste, M. Möhrle, B. Sartorius, J. Hörer and R. Löffler, *12 GHz to 64 GHz continuous frequency tuning in selfpulsating 1.55 μ quantum well DFB lasers.* IEEE J. Selected Topics in Quantum Electr. **1** (1995), 535-538.

[3] P. Vankelwinkelberge, G. Morthier and R. Baets, *GLADIS - a longitutinal multimode model for the analysis of the static, dynamic and stochastic behavior of diode lasers with distributed feedback*, IEEE J. Quantum Electr. **26** (1990), 1728-1741.

[4] D.D. Marcenac and J.E. Caroll, *Distinction between multimoded and single-moded self-pulsations in DFB lasers*, Electr. Lett. **30** (1994), 1137-1138.

[5] M. Möhrle, B. Sartorius, R. Steingrüber and P. Wolfram, *Electrically switchable self-pulsations in integrable multisection DFB-lasers*, IEEE Photonics Technol. Lett. **8** (1996), 28-31.

[6] R. Schatz, *Longitudinal spatial instability in symmetric semiconductor lasers due to spatial hole burning*, IEEE J. Quantum Electr. **28** (1992), 1443-1449.

[7] H. Wenzel, H.-J. Wünsche, U. Bandelow and J. Rehberg, *Mechanism of self-pulsations in two-section DFB lasers*, IEEE J. Quantum Electr. **32** (1996), 69-78.

[8] J. Rehberg, H.-J. Wünsche, U. Bandelow and H. Wenzel, *Spectral properties of a system describing fast pulsating DFB lasers*, ZAMM **76** (1996), 75-77.

[9] D. Peterhof, L. Recke and B. Sandstede, *On frequency locking of self-pulsating two-section DFB lasers*, in: *Self-Organization in Activator-Inhibitor Systems: Semiconductors, Gas-Discharge and Chemical Active Media*, Contributions to the 157$^{\text{th}}$ WE-Heraeus-Seminar, ed. by H. Engel et al., Wissenschaft & Technik Verlag, Berlin, 1996, 218-222.

[10] T. Kato, *Perturbation Theory for Linear Operators*, Springer-Verlag, Berlin, 1966.

[11] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer-Verlag, Berlin, 1983.