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HÖLDER CONTINUITY OF THE HOLONOMY MAPS FOR HYPERBOLIC BASIC SETS II

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1. Introduction

In this paper we want to show that the Hölder exponent κ_0 defined in [1] is generically the best one in an essential set of diffeomorphisms. Namely, we construct an (according to the C^1–topology) open subset $\mathcal{V}$ of C^1–diffeomorphisms of the three-dimensional sphere S^3 in which each diffeomorphism f has a one–dimensional hyperbolic attractor Λ_f (a solenoid). Moreover, for $\kappa > \kappa_0$ there exists a residual subset $\mathcal{G}$ in $\mathcal{V}$ with the property: For $g \in \mathcal{G}$ we can find two local stable manifolds $W^s_{loc}(p)$ and $W^s_{loc}(q)$ of Λ_g such that the holonomy mapping between $W^s_{loc}(p) \cap \Lambda$ and $W^s_{loc}(q) \cap \Lambda$ is *not* Hölder continuous with the exponent κ.

In particular, this shows that we can't get better estimates of the variation of the local transverse Hausdorff dimension using only the Hölder continuity of the holonomy mapping, even if we exclude exceptions (i.e. diffeomorphisms in the complement of a residual subset).

For definitions and additional literature see [1].

2. The set $\mathcal{B}$

Let $V = S^1 \times \mathbb{D}^2$ be the solid torus. Points p in V have the coordinates (t, z_1, z_2) $(t \in S^1, (z_1, z_2) \in \mathbb{D}^2)$.

In V we use the usual Riemannian metric.

Let $f_0 : S^3 \to S^3$ be a C^1–diffeomorphism of the three–dimensional sphere S^3 having a C^1–coordinate system such that for some solid torus V the restriction of f_0 to V maps V into itself and has the form

$$f_0(t, z_1, z_2) = (\emptyset(t), \psi_1(t, z_1), \psi_2(t, z_2)) \tag{1}$$

where the mappings $\emptyset : S^1 \to S^1$, $\psi_i : S^1 \times \mathbb{I} \to \mathbb{I}$ $(i = 1, 2)$ ($\mathbb{I}$ is the interval $[-1, 1]$) fulfil the inequalities:

$$0 < \mu^- < \widetilde{\mu}(t, z_2) < \mu^+ < \lambda^- < \widetilde{\lambda}(t, z_2) < \lambda^+ < \lambda^+\eta^+ < 1 \tag{2}$$
$$1 < \eta^- < \widetilde{\eta}(t) < \eta^+ < \infty$$

$$\frac{\lambda^-}{\mu^+} > \eta^+, \quad \text{and} \quad \lambda^+ < \frac{1}{2} \tag{3}$$

with $(t, z_1, z_2) \in V$ and $\mu^-, \mu^+, \lambda^-, \lambda^+, \eta^-, \eta^+ \in \mathbb{R}^+$ and

$$\begin{aligned} \widetilde{\mu}(t, z_2) &= \frac{\partial}{\partial x}\psi_2(t, x)\,|_{x=z_2} \\ \widetilde{\lambda}(t, z_1) &= \frac{\partial}{\partial x}\psi_1(t, x)\,|_{x=z_1} \\ \widetilde{\eta}(t) &= \frac{d}{dt}\emptyset(s)\,|_{s=t} \ . \end{aligned} \tag{4}$$

Remark 2.1. The stable foliation $\mathcal{W}^s$ is always C^1 for one–dimensional hyperbolic attractors and depends continuously on f. From now on we consider the restriction of diffeomorphisms of S^3 to V.

Using the techniques from stable manifold theory ([2], [3]) we can see that in a small C^1–neighborhood $\mathcal{V}$ of f_0 we have a strong stable foliation $\mathcal{W}^{ss}$ of class C^1 and a weak stable foliation $\mathcal{W}^{ws}$ of class C^0 both with C^1–leaves (the second one is not unique!) and these foliations depend continuously of f in the C^1– and C^0–topology, respectively. For this we consider the map $\widetilde{f}^{-1} : \mathcal{L} \to \mathcal{L}$ of the complete metric space of all one–dimensional C^1–subfoliations $\mathcal{U}^{ss}$ of $\mathcal{W}^s_{loc}$ $(U(x) \subset W^s_{loc})$ with angles to the z_2–axis in the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$ and with a metric generating the C^1–topology, the map $\widetilde{f}^{-1}$ being defined by

$$\widetilde{f}^{-1}(\mathcal{U}^{ss}) = f^{-1} \circ \ e\,|_{f(V)}(\mathcal{U}^{ss}) \tag{5}$$

where $e|_{f(V)}(\mathcal{U}^{ss})$ denotes the restriction of $\mathcal{U}^{ss}$ to $f(V)$. By simple calculations we get that this map is a contraction with a factor less than $\frac{\mu^+}{\lambda^-}\eta^+ < 1$. So we get by the Banach fixed point theorem a C^1–foliation $\mathcal{W}^{ss}$. The leaves of $\mathcal{W}^{ss}$ can be characterized as follows

$$\begin{aligned} W^{ss}(p) &= \left\{ q \in W^s_{loc}(p) | \forall r \in W^s_{loc}(p) : \lim_{k\to\infty} \frac{d(f^k p, f^k q)}{d(f^k p, f^k r)} < \infty \right\} \\ &= \left\{ q \in W^s_{loc}(p) | \exists c_1 > 0 : c_1^{-1} < \frac{\|(d_p f^k\,|_{E^s})^{-1}\|^{-1}}{d(f^k p, f^k q)} < c_1 \right\} \ . \end{aligned} \tag{6}$$

In order to construct $\mathcal{W}^{ws}$ we choose a C^1–foliation $\mathcal{F}$ of $Cl(V \setminus f(V))$ with the properties:

(1) The tanged spaces E^{ws} of $\mathcal{F}$ have an angle to the z_1–axis in the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$.

(2) For p from the boundary ∂V of V holds $df(E_p^{ws}) = E_{fp}^{ws}$. ($E_q^{ws}$ denote the tangent space to the leaf $W^{ws}(q)$ through q at the point q)

(3) This foliation on $Cl(V \setminus f(V))$ varies continuously in the C^1–topology with f.

Now we apply f to the foliation $\mathcal{F}$ and get a foliation $f(\mathcal{F})$ on $Cl(f(V) \setminus f^2(V))$. By repeating this process we get the desired weak stable foliation $\mathcal{W}^{ws}$. From the construction we can derive that for $p \in V$, $q \in W^{ws}(p)$ a positive constant c_2 exists such that

$$c_2^{-1} < \frac{\|d_p f^k\big|_{E_p^s}\|}{\|d_q f^k\big|_{E_q^{ws}}\|} < c_2 \qquad k = 1, 2, \ldots \qquad \text{and}$$

$$c_2^{-1} < \frac{\|d_p f^k\big|_{E_p^s}\|}{d(f^k p, f^k q)} < c_2 \qquad k = 1, 2, \ldots . \tag{7}$$

We can choose $\mathcal{V}$ to be an arcwise connected neighborhood of f in $Emb^1(V, V)$ such that for $g \in \mathcal{V}$ holds:

(1) g has an extension to S^3

(2) $g(V) \subset V$

(3) $\bigcap_{n \in \mathbb{N}} g^n(V) = \Lambda_g$ is a hyperbolic attractor

(4) there exist the strong stable and weak stable foliations $\mathcal{W}_g^{ss}$ and $\mathcal{W}_g^{ws}$ as described above.

Moreover, for $g \in V$ we can choose a C^1–cordinate system $\chi_g : S^1 \times \mathbb{I} \times \mathbb{I} \to V$ – depending continuously on g – such that

$$g(t, z_1, z_2) = (\emptyset^{(g)}(t), \psi_1^{(g)}(t, z_1), \psi_2^{(g)}(t, z_1, z_2))$$

with C^1–mappings $\emptyset^{(g)} : S^1 \to S^1$, $\psi_1^{(g)} : S^1 \times \mathbb{I} \to \mathbb{I}$, $\psi_2^{(g)} : S^1 \times \mathbb{I} \times \mathbb{I} \to \mathbb{I}$.

Remark 2.2. The intrinsic distances d^u, d^{ws}, d^{ss} of $\mathcal{W}_{loc}^u$, $\mathcal{W}^{ws}$, $\mathcal{W}^{ss}$, respectively, are equivalent to the Riemannian. Furthermore, if two points p, q from the same local stable manifold have non–empty intersection $W^{ws}(p) \cap W^{ss}(q) = r$ then their Riemannian distance is equaivalent to the distance $\max\{d^{ws}(p, r), d^{ss}(r, q)\}$.

Now we define for $f \in \mathcal{V}$ and $p \in V$

$$\begin{aligned} \mu(p) &= \mu_f(p) = \|d_p f\big|_{E_p^{ss}}\| \\ \lambda(p) &= \lambda_f(p) = \|d_p f\big|_{E_p^{ws}}\| \\ \eta(p) &= \eta_f(p) = \|d_p f\big|_{E_p^u}\| \end{aligned} \tag{8}$$

These functions are continuous and depend continuously on f. With the help of these notations and the mean value theorem we can rewrite (6) and (7):

$$0 \ < \ c_3^{-1} < \frac{\|(d_p f^k \big|_{E_p^s})^{-1}\|^{-1}}{\Pi_{i=0}^{k-1}\mu(f^i p)} < c_3 \tag{9}$$

$$0 \ < \ c_4^{-1} < \frac{\|d_p f^k \big|_{E_p^s}\|}{\Pi_{i=0}^{k-1}\lambda(f^i p)} < c_4 \ . \tag{10}$$

After these considerations the exponent κ_0 can be defined as (for the original definition see [1]):

$$\kappa_0 = \kappa_0(f) = \liminf_{n\to\infty} \inf_{p\in\Lambda} \frac{\sum_{i=0}^{n}[ln\ \lambda(f^i p) - ln\ \eta(f^i p)]}{\sum_{i=0}^{n} ln\ \mu(f^i p)} \ . \tag{11}$$

In the following we need the projection P along the leaves of $\mathcal{W}^{ss}$ to the strip $z_2 = 0$. This projection may not be well–defined on the whole torus V but if we consider a small compact neighborhood 0 of Λ in V and a diffeomorphism g sufficiently near to f_0 the leaves $\mathcal{W}_g^{ss}(p)$ $(p \in 0)$ of the strong stable foliation $\mathcal{W}_g^{ss}$ corresponding to g are almost horicontal and therefore intersect the strip $z_2 = 0$ in a unique point. That enables us to define the projection

$P : 0 \subset V \to S^1 \times \mathbb{I}$. From now on let $\mathcal{V}$ be small enough to define the projection P. Additionally we use the notations:

$Q : V \to S^1$ is the projection along the leaves of $\mathcal{W}^s$ to the circle $z_1 = z_2 = 0$

$\emptyset = Q \circ f \circ Q^{-1} : S^1 \to S^1$

$D_t = W^s_{loc}(p) \qquad t \in S^1, p \in Q^{-1}(t)$.

Remark 2.3. If for two points q, p from the same local stable manifold the distance in the strip $z_2 = 0$ is not to large then the intersection $W^{ws}(p) \cap W^{ss}(q)$ is non–empty. For points r, s from the same local unstable manifold the distance $|Q(r) - Q(s)|$ in S^1 is equivalent to the distance $d^u(r, s)$ and hence to their Riemannian distance.

The holonomy mapping

$$\pi^t_{t'} : D_t \cap \Lambda \to D_{t'} \cap \Lambda \qquad \left(t, t' \in S^1, |t - t'| < \frac{1}{2}\right)$$

is defined by

$$\pi^t_{t'}(p) = W^u_{loc}(p) \cap D_{t'} \ .$$

Note that there is a unique point in the intersection on the right–hand side.

Now we can state the theorem:

Theorem 2.1. *There exists a residual subset $\mathcal{G}$ in $\mathcal{V}$ with the property: For $f \in \mathcal{G}$ and $\kappa > \kappa_0$ points t and t' exist in S^1, $|t - t'| < \frac{1}{2}$ such that the holonomy mapping $\pi^t_{t'}$ is* ***not*** *Hölder continious with the exponent κ.*

3. Symbolic dynamics for S^1

Because $\emptyset : S^1 \to S^1$ is expanding we can construct a Markov partition consisting of closed arcs with $m \times m$ – matrix $A = A_f = (a_{ij})$ and the corresponding subshift of finite type:

$$\Sigma_A = \left\{\underline{x} = (x_1, x_2, \dots) \in \{0, 1, \dots, m-1\}^{\mathbb{I}^+} | a_{x_i x_{i+1}} = 1 \quad i = 1, 2, \dots \right\} .$$

Then there exists a continuous finite–to–one surjection $\rho = \rho_f : \Sigma_A \to S^1$ making the diagram

$$\begin{tikzcd} \Sigma_A \arrow[r, "\sigma"] \arrow[d, "\rho"'] & \Sigma_A \arrow[d, "\rho"] \\ \Lambda \arrow[r, "\phi"'] & \Lambda \end{tikzcd}$$

commutative (σ is the shift operator on Σ_A).

Here we want to remark that the Markow partition can be chosen in the way that A_f is locally constant as a function of f and the mapping ρ_f depends continuously on f.

On Σ_A we define the following conitinuous functions:

$$\begin{aligned} \alpha(\underline{x}) &= \alpha(\underline{x}, f) = ln\, \frac{\min\{\lambda(p) | p \in Q^{-1}(\rho\underline{x})\}}{\max\{\eta(p) | p \in Q^{-1}(\rho\underline{x})\}} \\ \beta(\underline{x}) &= \beta(\underline{x}, f) = ln\, \min\{\mu(p) | p \in Q^{-1}(\rho\underline{x})\} \end{aligned} \tag{12}$$

and the partial orbit means:

$$S_n(\underline{x}) = S_n(\underline{x}, f) = \frac{\sum_{i=0}^n \alpha(\sigma^i \underline{x})}{\sum_{i=0}^n \beta(\sigma^i \underline{x})} . \tag{13}$$

Remark 3.1. *The functions α and β depend continuously on f.*

We shall show that two points whose orbits stay for a much longer time close to each other than they stay far away have the same asymptotic behavior of their partial orbit means.

Definition 3.1. Given two sequences of natural numbers $\{a_k\}_{k=1}^{\infty}$, $\{b_k\}_{k=1}^{\infty}$ and a sequence of points $\{\underline{x}^{(k)}\}_{k=1}^{\infty}$ from Σ_A we say a point $\underline{y}$ from Σ_A is $\{a_k\}$, $\{b_k\}$ – close to $\{\underline{x}^{(k)}\}$ if for all $k \in \mathbb{N}^+$ the coordinates $x_i^{(k)}$ and $\underline{y}_{a_k+i}$ $(i = 1, 2, \ldots, b_k)$ coincide.

Remark 3.2. This definition implies that $\rho\sigma^{a_k+i}(\underline{y})$ and $\rho\sigma^i(\underline{x}^{(k)})$ are in the same elements of the Markov partition for $i = 1, \ldots, b_k$; $k = 1, 2, \ldots$.

We denote the set of all to $\{\underline{x}^{(k)}\}$ $\{a_k\}, \{b_k\}$ – close points by $E(\{a_k\}, \{b_k\}, \{\underline{x}^{(k)}\})$. Our aim is to show next lemma:

Lemma 3.1. *Let $\{a_k\}$ and $\{b_k\}$ be two sequences of natural numbers fulfilling*

$$\limsup \frac{a_k}{b_k} = 0 \tag{14}$$

Then for all sequences $\{\underline{x}^{(k)}\}$ from Σ_A the limit

$$\lim_{k\to\infty} \sup_{\underline{y}\in E(\{a_k\},\{b_k\},\{\underline{x}^{(k)}\})} \left| S_{a_k+b_k}(\underline{y}) - S_{b_k}(\underline{x}^{(k)}) \right| \tag{15}$$

exists and is equal to zero provided $E(\{a_k\}, \{b_k\}, \{\underline{x}^{(k)}\})$ is not empty.

Proof. First of all we fix $\{\underline{x}^{(k)}\}$ and two sequences $\{a_k\}$ and $\{b_k\}$ with non–empty $E(\{a_k\}, \{b_k\}, \{\underline{x}^{(k)}\})$. The continuity of α and β forces for all positive reals γ the existence of a natural number K such that for all $\underline{u} \in \Sigma_A$ the inequalities

$$\begin{aligned}(1-\gamma)\alpha(\underline{u}) &> \inf_{\underline{w}\in E_K(\underline{u})} \alpha(\underline{w}) > \sup_{\underline{w}\in E_K(\underline{u})} \alpha(\underline{w}) > (1+\gamma)\alpha(\underline{u})\\ (1-\gamma)\beta(\underline{u}) &> \inf_{\underline{w}\in E_K(\underline{u})} \beta(\underline{w}) > \sup_{\underline{w}\in E_K(\underline{u})} \beta(\underline{w}) > (1+\gamma)\beta(\underline{u})\end{aligned} \tag{16}$$

hold, where $E_K(\underline{u}) = \{\underline{w} \in \Sigma_A | w_1 = u\ w_2 \ldots w_K = u_K\}$ is the cylinder set of length K of $\underline{u}$ (Note: The set of all cylinder sets forms a basis of the topology of Σ_A.).
Now we can conclude:

$$\begin{aligned}
&\lim_{k\to\infty} \sup_{\underline{y}\in E(\{a_k\},\{b_k\},\{\underline{x}^{(k)}\})} \left| S_{a_k+b_k}(\underline{y}) - S_{b_k}(\underline{x}^{(k)}) \right| = \\
&= \lim_{k\to\infty} \sup_{\underline{y}\in E(\{a_k\},\{b_k\},\{\underline{x}^{(k)}\})} \left| \frac{\sum_{i=0}^{a_k-1} \alpha(\sigma^i \underline{y}) + \sum_{i=a_k}^{a_k+b_k} \alpha(\sigma^i \underline{y})}{\sum_{i=0}^{a_k-1} \beta(\sigma^i \underline{y}) + \sum_{i=a_k}^{a_k+b_k} \beta(\sigma^i \underline{y})} - \frac{\sum_{i=0}^{b_k} \alpha(\sigma^i \underline{y})}{\sum_{i=0}^{b_k} \alpha(\sigma^i \underline{y})} \right| \\
&\le \lim_{k\to\infty} \sup_{\underline{y}\in E(\{a_k\},\{b_k\},\{\underline{x}^{(k)}\})} \left| \frac{a_k\alpha^- + (1+\gamma)\sum_{i=0}^{b_k} \alpha(\sigma^i \underline{y}) + K\alpha^-}{a_k\beta^+ + (1-\gamma)\sum_{i=0}^{b_k} \beta(\sigma^i \underline{y}) + K\beta^+} - \frac{\sum_{i=0}^{b_k-K} \alpha(\sigma^i \underline{y}) + K\alpha^-}{\sum_{i=0}^{b_k-K} \beta(\sigma^i \underline{y}) + K\beta^+} \right| \\
&\le \frac{2\gamma}{1-\gamma} \lim_{k\to\infty} S_{b_k}(\underline{x}^{(k)}) \le \frac{2\gamma}{1-\gamma}\, \frac{\alpha^-}{\beta^+}
\end{aligned}$$

with $\alpha^- = \min\ \alpha(\underline{x})$, $\beta^+ = \max\ \beta(\underline{x})$.

(Note that (12) implies $\lim_{k\to\infty} b_k = +\infty$.) The lemma follows now from the arbitrary choice of γ. $\square$

4. A RESIDUAL SUBSET G IN Σ_A

In this section we fix $\kappa > \kappa_0$.

Noting that for $\underline{x} \in \Sigma_A$

$$\lim_{k\to\infty} \sup_{p\in Q^{-1}(\rho\underline{x})} \frac{1}{k} \left| \alpha(\underline{x}, f^k) - \sum_{j=0}^{k-1} ln\ \frac{\lambda(f^j p)}{\eta(f^j p)} \right| = 0$$

and

$$\lim_{k\to\infty} \sup_{p\in Q^{-1}(\rho\underline{x})} \frac{1}{k} \left| \beta(\underline{x}, f^k) - \sum_{j=0}^{k-1} ln\ \mu(f^j p) \right| = 0 \tag{17}$$

hold we get by the definition of κ_0 the existence of a natural number L, a sequence of points $\{\underline{x}^{(k)}\}$ in Σ_A and an increasing sequence of natural numbers $\{n_k\}$ fitting the inequality

$$\lim_{k\to\infty} S_{n_k}(\underline{x}^{(k)}, f^L) = \kappa' < \kappa\ . \tag{18}$$

Let in the following f be replaced by f^L (This won't change Λ). Without loss of generality we can assume that $[ln\ n_k] > T$, where T is the number $T = \min\{t \in \mathbb{N}^+ | A^t > 0\}$ and $[a]$ denotes the greatest integer part of a real number a.

For $k \in \mathbb{N}^+$ and

$$M = M_f = \left[\frac{ln\ \frac{\lambda^+}{\mu^-}}{ln\ \eta^-} \right] + 1 \tag{19}$$

we define

$$\begin{aligned} \widetilde{G}_k = \Big\{ \underline{y} \in \Sigma_A | y_i \ &= \ y_{[ln\ n_k]+n_k+i} = \cdots = y_{M([ln\ n_k]+n_k)+i} \\ y_{[ln\ n_k]+j} \ &= x_j^{(k)} \\ i \ &= \ 1, \ldots, [ln\ n_k] + n_k \quad j = 1, \ldots, n_k \Big\}\ . \end{aligned} \tag{20}$$

Thus the set $\widetilde{G}_k$ consists of all points of Σ_A whose first $[ln\ n_k] + n_k$ coordinates recur M–times and whose coordinates from $[ln\ n_k] + 1$ to $n_k + [ln\ n_k]$ coincide with the first n_k–ones of $\underline{x}^{(k)}$. Hence the set $\widetilde{G}_k$ is open and meets all non–empty cylinder sets of length $[ln\ n_k] - T$. This implies the density of the open sets

$$G_j = \bigcup_{k=j}^{\infty} \widetilde{G}_k \tag{21}$$

in Σ_A.

Remark 4.1. If the diffeomorphism g is close enough to f we can achieve an analoguous inequality to (16) *for g using the same sequences $\{n_k\}$ and $\{\underline{x}^{(k)}\}$. Also $M_g = M_f$ holds. Therefore, we can use the set G_j for all diffeomorphisms in a sufficiently small neighborhood of f.*

Now the set

$$G = \bigcap_{j=1}^{\infty} G_j \tag{22}$$

is residual in Σ_A. From the properties of ρ we can deduce that the set $\rho(G)$ is residual in S^1 (Cylinder sets are mapped onto closed arcs.).

Analyzing the construction of G we see that all points in G are $\{[ln\ l_k]\}$, $\{l_k\}$–close to $\{\underline{y}^{(k)}\}$ for some subsequences $\{l_k\}$ of $\{n_k\}$ and $\{\underline{y}^{(k)}\}$ of $\{\underline{x}^{(k)}\}$ where $\underline{y}^{(k)} = \underline{x}^{(j)}$ for $n_j = l_k$. By the use of the lemma we then get for $\underline{y} \in G$:

$$\lim_{k\to\infty} |S_{[ln\ l_k]+l_k}(\underline{y}) - S_{l_k}(\underline{y}^{(k)})| = 0 \tag{23}$$

(remark: $\lim_{k\to\infty} S_{l_k}(\underline{y}^{(k)}) = \kappa'$!).

To $\underline{y} \in G$ and $k \in \mathbb{N}^+$ we define $\underline{u}^{(k)} = \underline{u}^{(k)}(\underline{y})$ as the only pre–image of $\underline{y}$ under $\sigma^{[ln\ l_k]+l_k}$ that lies in the cylinder set $E_{[ln\ l_k]+l_k}(\underline{y})$. So it is the point

$$\begin{aligned} \underline{u}^{(k)} = \{\underline{u} \in \Sigma_A | u_1 = y_1 \ldots u_{[ln\ l_k]+l_k} &= y_{[ln\ l_k]+l_k} \\ u_{[ln\ l_k]+l_k+j} &= y_j \qquad j = 1, 2, \ldots\} . \end{aligned} \tag{24}$$

We set

$$m_k = l_k + [ln\ l_k] . \tag{25}$$

5. Transversal overcrossings over G

Definition 5.1. We say Λ has a *transversal overcrossing* over the points (p, q) $(q, p \in \Lambda)$ iff:

(1) $P(p) = P(q)$
(2) $P(W^u_{loc}(p))$ is transversal to $P(W^u_{loc}(q))$

Sometimes we will say that Λ has a transversal overcrossing over $t = Q(p) = Q(q)$.

Let us now assume that Λ has a transversal overcrossing over some t from $\rho(G)$.

Because of the transversality there exists $\varepsilon_1 > 0$, $c_5 > 0$ and $c_6 > 0$ such that for $t' \in U_{\varepsilon_1}(t) \subset S^1$

$$c_5|t'-t| \le d(P(W^u_{loc}(p) \cap D_{t'}, P(W^u_{loc}(q) \cap D_{t'})) \le c_6|t-t'| \ . \tag{26}$$

Moreover, the distance

$$d(W^u_{loc}(p) \cap D_{t'}, W^u_{loc}(q) \cap D_{t'}) \tag{27}$$

is greater than some positive constant c_7. Let us assume that ε_1 is chosen so small that for all $t' \in U_{\varepsilon_1}(t)$ the intersection of $W^{ws}(W^u_{loc}(p) \cap D_{t'})$ and $W^{ss}(W^u_{loc}(q) \cap D_{t'})$ is non–empty. We fix now some t' in $U_{\varepsilon_1}(t)$ and denote the distance $|t'-t|$ by δ. Define $t_k = \rho(\underline{u}^{(k)}(\rho^{-1}(t)))$ $(k \in \mathbb{N}^+)$. Although $\rho^{-1}(t)$ may consist of more than one point the definition of t_k is correct. Then

$$\emptyset^{m_k}(t_k) = t \tag{28}$$

and by (17)

$$|t_k - t| < (\eta^-)^{-M\ m_k} \ . \tag{29}$$

Let t'_k denote the only pre–image of t' under $\emptyset^{m_k}$ such that

$$\emptyset^{m_k}\Big|_{[t'_k, t_k]} : [t'_k, t_k] \to [t', t]$$

is a diffeomorphism.

We consider the following sequences of points

$$\begin{aligned} \tilde{p}_k &= W^u_{\varepsilon_1}(p) \cap D_{t_k} & \tilde{q}_k &= W^u_{\varepsilon_1}(q) \cap D_{t_k} \\ \tilde{p}'_k &= W^u_{\varepsilon_1}(p) \cap D_{t'_k} & \tilde{q}'_k &= W^u_{\varepsilon_1}(q) \cap D_{t'_k} \\ \tilde{r}_k &= W^{ws}(\tilde{p}_k) \cap W^{ss}(\tilde{q}_k) & \tilde{r}'_k &= W^{ws}(\tilde{p}'_k) \cap W^{ss}(\tilde{q}'_k) \end{aligned} \tag{30}$$

$$\begin{aligned} p_k &= f^{m_k}(\tilde{p}_k) & q_k &= f^{m_k}(\tilde{q}_k) \\ p'_k &= f^{m_k}(\tilde{p}'_k) & q'_k &= f^{m_k}(\tilde{q}'_k) \end{aligned}$$

$$\begin{aligned} r_k &= W^{ws}(p_k) \cap W^{ss}(q_k) & r'_k &= W^{ws}(p'_k) \cap W^{ss}(q'_k) \\ &= f^{m_k}(\tilde{r}_k) & &= f^{m_k}(\tilde{r}'_k) \ . \end{aligned}$$

Then we deduce from

$$\pi^{t_k}_{t'_k}(\tilde{p}_k) = \tilde{p}'_k \quad \text{and} \quad \pi^{t_k}_{t'_k}(\tilde{q}_k) = \tilde{q}'_k :$$

$$\text{the relation} \qquad p'_k = \pi^t_{t'}(p_k) \quad \text{and} \quad q'_k = \pi^t_{t'}(q_k) \ . \tag{31}$$

Using the properties of the above construction and the mean value theorem we have for some positive constants and

$$
\begin{aligned}
k(\varepsilon) = \min\Big\{k \in \mathbb{N}^+ | \forall t \in S^1, \forall p,q \in D_t &\ :\ \frac{\lambda(f^k p)}{\lambda(f^k q)} < 1+\varepsilon \text{ and} \\
\forall p \in V, \forall q \in W^u(p) &\ :\ \frac{\lambda(f^{-k} p)}{\lambda(f^{-k} q)} < 1+\varepsilon\Big\} \\
m(\varepsilon) = \min\Big\{k \in \mathbb{N}^+ | \forall t \in S^1, \forall p,q \in D_t &\ :\ \frac{\eta(f^k p)}{\eta(f^k q)} < 1+\varepsilon \text{ and} \\
\forall p \in V, \forall q \in W^u(p) &\ :\ \frac{\eta(f^{-k} p)}{\eta(f^{-k} q)} < 1+\varepsilon\Big\} \\
n(\varepsilon) = \min\Big\{k \in \mathbb{N}^+ | \forall t \in S^1, \forall p,q \in D_t &\ :\ \frac{\mu(f^k p)}{\mu(f^k q)} < 1+\varepsilon \text{ and} \\
\forall p \in V, \forall q \in W^u(p) &\ :\ \frac{\mu(f^{-k} p)}{\mu(f^{-k} q)} < 1+\varepsilon\Big\}
\end{aligned}
$$

$$
\begin{aligned}
& d(p'_k, q'_k) \geq c_8\ d(p'_k, r'_k) \geq \\
\geq\ & c_9\ d(\widetilde{p}'_k, \widetilde{r}'_k) \left(\frac{\lambda^-}{\lambda^+}\right)^{k(\varepsilon)} \Pi_{i=0}^{m_k-1} \lambda(f^i \widetilde{p}'_k)(1+\varepsilon)^{-1} \geq \\
\geq\ & c_{10}\left\{\left(\frac{\lambda^-}{\lambda^+}\right)^{k(\varepsilon)} \Pi_{i=0}^{m_k-1} \lambda(f^i \widetilde{p}'_k)(1+\varepsilon)^{-1}\right\} c_5 |t'_k - t| \geq \\
\geq\ & c_{10}\left\{\left(\frac{\lambda^-}{\lambda^+}\right)^{k(\varepsilon)} \Pi_{i=0}^{m_k-1} \lambda(f^i \widetilde{p}'_k)(1+\varepsilon)^{-1}\right\} c_5 \{|t'_k - t_k| - |t - t_k|\} \geq \\
\geq\ & c_{10}\left\{\left(\frac{\lambda^-}{\lambda^+}\right)^{k(\varepsilon)} \Pi_{i=0}^{m_k-1} \lambda(f^i \widetilde{p}'_k)(1+\varepsilon)^{-1}\right\} \times \\
& \times \left\{c_{11}\delta \left(\frac{\eta^-}{\eta^+}\right)^{m(\varepsilon)} \Pi_{i=0}^{m_k-1} \eta(f^i \widetilde{p}'_k)^{-1}(1+\varepsilon)^{-1} - c_5(\eta^-)^{-Mm_k}\right\} \geq \\
\geq\ & c_{12}\left(\frac{\eta^-}{\eta^+}\right)^{2k(\varepsilon)} \left(\frac{\eta^-}{\eta^+}\right)^{2m(\varepsilon)} \Pi_{i=0}^{m_k-1} \lambda(f^i \widetilde{p}_k)\left[\eta(f^i \widetilde{p}_k)^{-1}\right](1+\varepsilon)^{-4} \geq \\
\geq\ & C_1(\varepsilon) \Pi_{i=0}^{m_k-1} \lambda(f^i \widetilde{p}_k)\left[\eta(f^i \widetilde{p}_k)^{-1}\right](1+\varepsilon)^{-4} \qquad (32)
\end{aligned}
$$

and

$$
\begin{aligned}
& d(p_k, q_k) \leq c_{13} \max\{d(r_k, p_k), d(r_k, q_k)\} \leq \\
\leq \;& c_{13} \max\left\{(\lambda^+)^{m_k} d(\widetilde{r}_k, \widetilde{p}_k), d(r_k, q_k)\right\} \leq \\
\leq \;& c_{13} \max\left\{c_5(\lambda^+)^{m_k}(\eta^-)^{-Mm_k}, d(r_k, q_k)\right\} \leq \\
\leq \;& c_{13} \max\left\{c_5(\mu^-)^{m_k}, d(r_k, q_k)\right\} \leq \\
\leq \;& c_{13} \max\left\{c_5(\mu^-)^{m_k}, c_{14}\left(\frac{\mu^-}{\mu^+}\right)^{n(\varepsilon)} \left[\Pi_{i=0}^{m_k-1}\mu(f^i\widetilde{q}_k)(1+\varepsilon)\right] d(\widetilde{r}_k, \widetilde{q}_k) \leq \right. \\
\leq \;& c_{15}\left(\frac{\mu^-}{\mu_+}\right)^{2n(\varepsilon)} \Pi_{i=0}^{m_k-1}\mu(f^i\widetilde{q}_k)(1+\varepsilon)^2 \\
\leq \;& C_2(\varepsilon)\Pi_{i=0}^{m_k-1}\mu(f^i\widetilde{q}_k)(1+\varepsilon)^2 \;. \qquad (33)
\end{aligned}
$$

Now we choose ε so small that

$$\frac{\frac{1}{n}\sum_{i=0}^{n}[ln\ \lambda(f^i\widetilde{p}_k) - \ln\ \eta(f^i\widetilde{p}_k)] - 4\ ln\ (1+\varepsilon)}{\frac{1}{n}\sum_{i=0}^{n} ln\ \mu(f^i\widetilde{p}_k) + 2\ ln\ (1+\varepsilon)} < \kappa\ .$$

This is possible because of (21), (22). Therefore we conclude

$$\lim_{k\to\infty} \frac{d(p_k', q_k')}{d(p_k, q_k)^\kappa} \geq \lim_{k\to\infty} \frac{C_1(\varepsilon)\Pi_{i=0}^{m_k-1}\lambda(f^i\widetilde{p}_k)[\eta(f^i\widetilde{p}_k)^{-1}]}{C_2(\varepsilon)\Pi_{i=0}^{m_k-1}\mu(f^i\widetilde{p}_k)^\kappa} = \infty \qquad (34)$$

i.e. $\pi_{t'}^t$ is not Hölder continuous with exponent κ.

So it remains to show that we can arrange a transversal overcrossing over $\rho(G)$ generically. Thus we have to show that the set

$$\mathcal{G}\{f \in \mathcal{V} | \Lambda_f \text{ has a transversal overcrossing over } \rho_f(G)\} \qquad (35)$$

is residual in $\mathcal{V}$.

First of all we want to fix some overcrossing and supervise it under small pertubations of the diffeomorphism $f \in V$.

From [2] we know that the local unstable manifolds vary continuously in the C^1–topology under perturbations of the diffeomorphism f. Moreover, the strong stable foliation also varies continuously in the C^1–topology. Hence, the sets

$$
\begin{aligned}
\mathcal{V}_{a,(p,q)}(f) \;=\; & \Big\{ g \in \mathcal{V} | \exists \text{ a continuous family of diffeomorphisms} \\
& f_\tau : V \to V\ (\tau \in [0,1]), f_0 = f,\ f_1 = g \text{ and two curves } p_\tau, \\
& q_\tau \text{ in } V \text{ s.t. } p_0 = q,\ q_0 = q,\ p_\tau \in \Lambda_{f_\tau},\ q_\tau \in \Lambda_{f_\tau} \text{ and} \\
& \Lambda_{f_\tau} = \bigcap_{n\in\mathbb{N}} f_\tau^n(V) \text{ has the transversal overcrossing} \\
& (p_\tau, q_\tau) \text{ with angle grater than } a. \Big\} \qquad (36)
\end{aligned}
$$

are open for all $f \in \mathcal{V}$, $a > 0$, $p, q \in \Lambda_f$.
For every natural number k we define

(37) $\mathcal{V}^k_{a,(p,q)}(f) = \{g \in \mathcal{V}_{a,(p,q)}(f) \mid$ The overcrossing (p_1, q_1) lies over $\rho_g(G_k)\}$.

The openess of $\mathcal{V}^k_{a,(p,q)}(f)$ is proved by remarking the continuous dependence of f of the mapping ρ_f and the continuous dependence in the C^1–topology of the local stable and the local unstable manifolds ([2]). Moreover, the fixed overcrossing for $g \in \bigcap_{k=1}^{\infty} \mathcal{V}^k_{a,(p,q)}(f)$ lies in $\rho_g(G_k)$ for all natural k and therefore in $\rho_g(G)$.
Let us write f according to the foliations $\mathcal{W}^{ss}$ and $\mathcal{W}^{ws}$ as

$$f(t, z_1, z_2) = (\emptyset(t), \psi_1(t, z_1), \psi_2(t, z_1, z_2))$$

with C^1–mappings $\emptyset : S^1 \to S^1$, $\psi_1 : S^1 x \mathbb{I} \to \mathbb{I}$ and $\psi_2 : S^1 x \mathbb{I} x \mathbb{I} \to \mathbb{I}$. Our next step is to consider, for $\varepsilon > 0$, special perturbations of the mapping f, where the perturbed mappings lie in the set

$$Z_\varepsilon(f) = \{g = (\emptyset(t), \psi_1(t, z_1) + \psi_1(t, z_1, z_2)) \in \mathcal{V} \mid \delta \in C^1(S^1, \mathbb{I}), |\delta| \leq \varepsilon\} .$$

The advantage of these perturbations is that they preserve the foliations $\mathcal{W}^{ss}$ and $\mathcal{W}^{ws}$ and the mapping $\emptyset$ and, therefore, they don't change the projections P and Q. So we can define for $g \in Z_\varepsilon(f)$ a conjugating homeomorphism $h_g : \Lambda_f \to \Lambda_g$ via the formula

$$(38) \qquad h_g(p) = \lim_{n\to\infty} g^n(f^{-n}(p)) = \bigcap -n \in \mathbb{N}\ g^n(D_{Q(f^{-n}(p))}) \quad (p \in \Lambda_f)$$

(This is the homeomorphism used in the Ω–stability theorem ([2]).). It has the following useful property:

$$(39) \qquad Q \circ h_g = Q \ .$$

In what follows we need the notations:
If p and q lie in the set D_t $(t \in S^1)$ we write

$$(40) \qquad t'_i = Q(f^{-i}(p)) = Q(g^{-i}(p_g))$$

$$(41) \qquad t''_i = Q(f^{-i}(q)) = Q(g^{-i}(q_g)) \quad i = 1, 2, \dots .$$

For the next step let us assume that f has a (not nessecarily transverse) overcrossing (p, q) over the points p and q in different components of the set $D_t \cap f(V)$.
We claim:

Lemma 5.1. *For $\varepsilon > 0$ there exists a neighborhood U of t in S^1 such that for every $s \in U$ there is a mapping in $Z_\varepsilon(f)$ which has an overcrossing over s.*

Proof. Let $\varepsilon > 0$ be fixed. Because p and q lie in different components of $D_t \cap f(V)$ the points t_1' and t_1'' are different. Therefore we can choose a neighborhood U of t such that for all r from U the points $r_1' = r_1'(p^{(r)})$ and $r_1'' = r_1''(q^{(r)})$ have a distance at least $\frac{|t_1'-t_1''|}{2}$, where $p^{(r)} = W^u_{loc}(p) \cap D_r$ and $q^{(r)} = W^u_{loc}(q) \cap D_r$. Moreover, we can choose U so small that the points $P(p^{(r)})$ and $P(q^{(r)})$ are closer than $\frac{1-2\lambda^+}{1-\lambda^+}\ \frac{\varepsilon}{2}\ |t_1' - t_1''|$.
Now we take some s from U. If $P(p^{(s)}) = P(q^{(s)})$ then Λ_f has already the desired overcrossing over s. So let us assume than $P(p^{(s)})$ lies above $P(q^{(r)})$ (i.e. $P(p^{(s)}) = P(p^{(s)})$). The way we have chosen U ensures the existence of a mapping $\delta_1 \in C^1(S^1, \mathbb{I})$ with the properties:

$$\delta_1(s_1') = -\tfrac{\varepsilon}{4}|t_1' - t_1''|,\ \delta_1(s_1') = \tfrac{\varepsilon}{4}|t_1' - t_1''| \tag{1}$$

$$|\delta(t)| \le \tfrac{\varepsilon}{4}|t_1' - t_1''| \qquad t \in S^1 \tag{2}$$

$$g = (\emptyset, \psi_1 + \delta_1, \psi_2) \in Z_\varepsilon(f)\ . \tag{3}$$

Calculating the difference of the z_1–coordinates $z_1(p_g^{(s)})$ and $z_1(q_g^{(s)})$ we get

$$\begin{aligned}
&z_1(p_g^{(s)}) - z_1(q_g^{(s)}) = \\
&= \lim_{n\to\infty}[\psi_1(s_1', \psi_1(s_2', \dots \psi_1(s_n', 0) + \delta_1(s_n') \dots) + \delta_1(s_2')) + \delta_1(s_1') - \\
&\quad -\psi_1(s_1'', \psi_1(s_2'', \dots \psi_1(s_n'', 0) + \delta_2(s_n'') \dots) + \delta_1(s_2'')) + \delta_1(s_1'')] < \\
&< \lim_{n\to\infty}[\psi_1(s_1', \psi_1(s_2', \dots \psi_1(s_n', 0) \dots)) - \psi_2(s_1'', \psi_1(s_2'', \dots \psi_1(s_n'', 0) \dots) \dots))] - \\
&\quad -\frac{\varepsilon}{2}|t_1' - t_1''| + \frac{\lambda^+}{1-\lambda^+}\frac{\varepsilon}{2}|t_1' - t_1''| = \\
&= z_1(p_g^{(s)}) - z_1(q^{(s)})\frac{1-2\lambda^+}{1-\lambda^+}\ \frac{\varepsilon}{2}\ |t_1' - t_1''| < 0
\end{aligned} \tag{42}$$

and see that after this perturbation $P(p_g^{(s)})$ lies below $P(q_g^{(s)})$. The continuous dependence of $p_g^{(s)}$ and $q_g^{(s)}$ on the perturbation δ_1 implies that in the family $g_\tau = (\emptyset, \psi_1 + \tau\delta_1, \psi_2) \in Z_\varepsilon(f)$ $(\tau \in [0,1])$ there exists a diffeomorphism which has an overcrossing over s. $\square$

Because a transversal overcrossing is still transversal after small perturbations we get the

Corollary 5.1. *$\mathcal{V}^k_{a,(p,q)}(f)$ is dense in $\mathcal{V}_{a,(p,q)}(f)$ and consequently, $\mathcal{G} \cap \mathcal{V}_{a,(p,q)}(f)$ is residual in $\mathcal{V}_{a,(p,q)}(f)$.*

The next lemma deals with special (not nessecarily transverse) overcrossings.

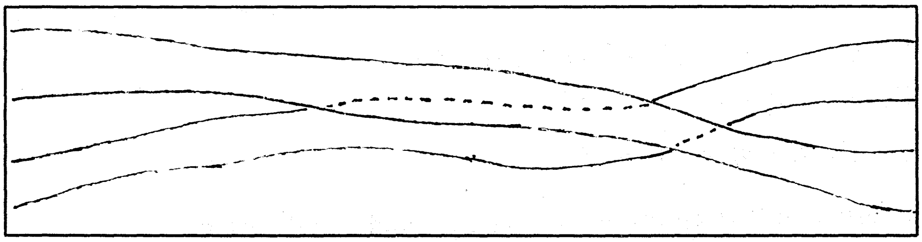
Lemma 5.2. *In every neighborhood of f there is a diffeomorphism $g \in \mathcal{V}$ with the properties:*

There are two different points p_g and q_g in Λ_g with $P(p_g) = P(q_g)$, $Q(p_g) = Q(p_g) = s$ and a neighborhood U of s'_1 in S^1 such that

(1) $s'_1 \notin U \quad i = 2, 3, \ldots$ and

(2) $s''_i \notin U \quad i = 1, 2, \ldots$.

Proof. No matter how $f(V)$ is embedded into V , $P(f(V))$ must have an overcrossing as in the figure.



This means we have two disjoint arcs T_1 and T_2 in S^1 with endpoints e_1, e'_1 and e_2, e'_2, respectively such that

(1) $\emptyset(T_1) = \emptyset(T_2)$

(2) $\emptyset(e_1) = \emptyset(e_2), \quad \emptyset(e'_1) = \emptyset(e'_2)$

(3) $P(f(D_{e_1}))$ lies below $P(f(D_{e'_2}))$ and $P(f(D_{e'_1}))$ lies below $P(f(D_{e'_2}))$.

Now we fix two periodic points p' and q'. The density of their unstable manifolds in Λ yields existence of an arc in $W^u(p')$ passing through $f(D_{e_1})$ and an arc in $W^u(q')$ passing through $f(D_{e_2})$. Hence, we can find two points $p \in W^u(p')$ and $q \in W^u(q')$ in different components of $D_t \cap f(V)$ such that $P(p) = P(q)$, $Q(p) = Q(p) = s$.

Using the fact there are only countably many periodic or pre–periodic points and lemma 2 we can find a diffeomorphism g arbitraily close to f such that there two periodic points p'_g and q'_g and two points $p_g \in W^u(p'_g)$ and $q_g \in W^u(q'_g)$ in different components of $D_s \cap f(V)$ with $P(p_g) = P(q_g)$, $Q(p_g) = Q(p_g) = s$ and s is neither periodic nor pre–periodic.

Our construction implies that $s'_1 \neq s''_1$ (The component of $f(V) \cap D_s$ containing p_s id disjoint to the component containing q_g.). Moreover, s'_i and s''_i converge to the orbits $\{Q(f^{-n}(p'_g))\}$ and $\{Q(f^{-n}(q'_g))\}$, respectively and all the points s'_i, s''_j are different $(i, j = 1, 2, \ldots)$. Since the orbits of $Q(f(p'_g))$ and $Q(f(p'_g))$ have a positive distance from s, this ensures the existence of the desired neighborhood U of s_1'.

This lemma shows that the set of diffeomorphism in $\mathcal{V}$ which have an overcrossing (p_g, q_g) with the properties mentioned in lemma 3 is dense in $\mathcal{V}$. Consequently, if we can make these overcrossings transversal by arbitrarily small perturbations we get a dense set of diffeomorphisms in $\mathcal{V}$ each of which has a transversal overcrossing.
So let us assume that

$$d_{p_g} P(T_{p_g} W^u(p_g)) - d_{q_g} P(T_{q_g} W^u(q_g)) = 0 \tag{43}$$

and consider a mapping $\delta_2 \in C^1(S^1, \mathbb{I})$ with the properties

(1) $|\frac{d}{dt}\delta_2(t)| \leq 1$ $\quad t \in S^1$

(2) $\delta_2(t) = 0$ $\quad t \notin U(s_1')$

(3) $\delta_2(s_1') = 0$

(4) $\frac{d}{dt}\delta_2(t)\Big|_{t=s_1'} = 1$

where $U(s_1')$ is the neighborhood given by lemma 3.
Lemma 3 implies:

$$\begin{aligned} \delta_2(s_1') = \delta_2(s_1'') = \frac{d}{dt}\delta_2(t)\Big|_{t=s_1''} &= 0 \qquad i = 1, 2, \ldots \qquad \text{and} \\ \frac{d}{dt}\delta_2(t)\Big|_{t=s_1'} &= 0 \qquad i = 2, 3, \ldots \; . \end{aligned} \tag{44}$$

Let $\varepsilon > 0$ be sufficiently small.
Then the perturbations we use have the form

$$g = (\emptyset, \widetilde{\psi}_1, \psi_2) \to (\emptyset, \widetilde{\psi}_1 + \delta_2, \psi_2) = e \in Z_\varepsilon(g) \; .$$

We claim that after these perturbations the overcrossing is transversal. First we calculate the difference of the z_1–coordinates of p_e and q_e:

$$z_1(p_e) - z_1(q_e) = \tag{45}$$

$$= \lim_{n\to\infty} \Big[\widetilde{\psi}_1(s_1', \widetilde{\psi}_1(s_2', \ldots \widetilde{\psi}_1(s_n', 0) + \varepsilon\delta_2(s_n') \ldots) + \varepsilon\delta_2 s')) + \varepsilon\delta_2(s_1') - \tag{46}$$

$$-\widetilde{\psi}_1(s_1'', \widetilde{\psi}_1(s_2'', \ldots \widetilde{\psi}_1(s_n'', 0) + \varepsilon\delta_2(s_n'') \ldots) + \varepsilon\delta_2(s_2'')) + \varepsilon_2\delta(s_1'')\Big] = \tag{47}$$

$$= z_1(p_g) - z_1(q_g) = 0 \; . \tag{48}$$

This means that we have indeed an overcrossing over s after these perturbations. The calculation of the difference of the slopes of the projected local unstable manifolds $W^u_{loc}(p_e)$

and $W^u_{loc}(q_e)$ at s shows that this overcrossing is transversal:

$$d_{P_e}P(T_{p_e}W^u(p_e)) - d_{q_e}P(T_{q_e}W^u(q_e)) = \tag{49}$$

$$= \lim_{n\to\infty} \frac{\partial}{\partial t}\Big[\widetilde{\psi}_1(s_1, \widetilde{\psi}_1(s_2', \ldots \widetilde{\psi}_1(s_n', 0) + \varepsilon\delta_2(s_n')\ldots) + \varepsilon\delta_2 s_2')) + \varepsilon\delta_2(s_1') - \tag{50}$$

$$-\widetilde{\psi}_1(s_1'', \widetilde{\psi}_1(s_2'', \ldots \widetilde{\psi}_1(s_n'', 0) + \varepsilon\delta_2(s_n'')\ldots) + \varepsilon\delta_2(s_2'')) + \varepsilon_2\delta(s_1'')\Big] = \tag{51}$$

$$= d_{p_g}P(T_{p_g}W^u(p_g)) - d_{q_g}P(T_{q_g}W^u(q_g)) + \varepsilon > 0\ . \tag{52}$$

Therefore, this overcrossing can be made transversal under arbitrary small perturbations. This proves that

$$\bigcup_{\substack{f\in\mathcal{V}, a>0\\ (p,q)\in\Lambda_f\times\Lambda_f}} \mathcal{V}_{a,(p,q)}(f)$$

is dense in $\mathcal{G}$. Hence, $\mathcal{G}$ is residual in $\mathcal{V}$.

This was all we had to prove. □

References

[1] J. Schmeling, Ra. Siegmund–Schultze; Hölder continuity of the holonomy maps for hyperbolic sets I; *to appear*

[2] M.W. Hirsch, C.C. Pugh; Stable manifolds and hyperbolic sets; *Proc. Symp. Pure Math.*, Vol. XIV (1970), 133–163

[3] M.W. Hirsch, C.C. Pugh, M. Shub; Invariant manifolds; *Lect. Notes Math.* 583 (1977)

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