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Temporal dissipative solitons in a delayed model of a ring semiconductor laser

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Abstract

Temporal cavity solitons are short pulses observed in periodic time traces of the electric field envelope in active and passive optical cavities. They sit on a stable background so that their trajectory comes close to a stable CW solution between the pulses. A common approach to predict and study these solitons theoretically is based on the use of Ginzburg-Landau-type partial differential equations, which, however, cannot adequately describe the dynamics of many realistic laser systems. Here for the first time we demonstrate formation of temporal cavity soliton solutions in a time-delay model of a ring semiconductor cavity with coherent optical injection, operating in anomalous dispersion regime, and perform bifurcation analysis of these solutions.

1 Introduction

Temporal localised structures (TLS) of light propagating along the axial direction in nonlinear cavities attracted significant theoretical and experimental attention in the last decade due to their potential applications for optical data storage and transmission [1–4]. Similarly to the solitons of nonlinear Schrödinger equation [5], dissipative optical TLS known also as temporal cavity solitons are localized in time and can be studied with the help of complex Ginzburg-Landau-type equations in the co-moving reference frame as stationary solutions of a properly constructed ordinary differential equations [6]. Although this approach allows a detailed bifurcation analysis of TLS solutions, complex Ginzburg-Landau models are hardly applicable to account accurately for certain important physical effects in realistic laser devices, such as those containing intracavity semiconductor medium [7]. This is why travelling wave-type models [8, 9] are commonly used to model the dynamics of semiconductor devices. However, since the traveling wave models are rather complicated and their analysis is usually limited to direct numerical simulations, an alternative and more simple approach to the analysis of multimode semiconductor lasers was proposed in [10–12] based on the use of delay differential equations (DDEs). DDE laser models can be derived from the travelling wave equations under certain non-restrictive simplifying physical assumptions and they proved to be a viable alternative to the standard models based on partial differential equations. In addition to asymptotic stability analysis [12, 13] the DDE approach allows for numerical study [10–16] of CW and periodic intensity regimes using well-developed Floquet theory and software packages such as DDE-BIFTOOL [17].

In this paper, using a DDE model we investigate cavity solitons first predicted theoretically in the anomalous dispersion regime in the Lugiato-Lefever equation (LLE) [18], which is equivalent to the driven damped nonlinear Schrödinger equation . This equation describes qualitatively the dynamics of the electric field envelope in a passive optical cavity subject to weak coherent optical injection, when the injection frequency is close to a resonant frequency of the cavity. However, far enough from the resonance one can observe a bistability between two branches of dissipative solitons corresponding to different longitudinal cavity modes. This phenomenon is missing in the LLE and can be studied

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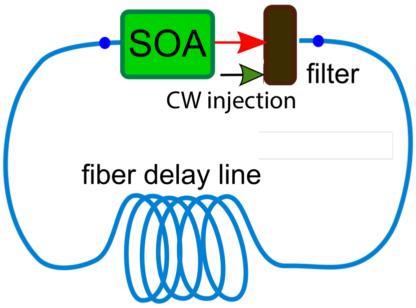


Figure 1: Schematic representation of an optically injected ring laser consisting of a SOA as an amplifying medium, a spectral filter, and dispersive long fiber delay line.

using a travelling-wave-type equation [19]. Similarly to the traveling wave equation DDE models account fully for the multimode nature of the optical cavities, and, in addition, the anomalous dispersion of the fiber waveguide can be described by including a distributed delay term into model equations [7]. Here, we develop a DDE model to study dissipative soliton in an optically injected ring cavity laser containing semiconductor optical amplifier (SOA), long dispersive fiber delay line, and a narrow bandpass spectral filter [13] . We perform stability analysis of the injection-locked steady states in the limit of large delay [20] and demonstrate analytically the appearance of modulational instability and cavity solitons. Finally, we reduce full distributed DDE model to a simplified DDE model that preserves the effect of the chromatic dispersion on the dynamics of the ring laser, and perform numerical continuation and stability analysis of the periodic cavity soliton solutions in this model using the software package DDE-BIFTOOL.

2 Delayed model of a dispersive semiconductor ring laser

Let us consider a ring laser shown schematically in Fig. 1. The laser is subject to a single-mode optical injection and contains three main elements: SOA acting as an amplifying medium, spectral filter, and dispersive fiber delay line. To describe the chromatic dispersion of the delay line we use the approach of Ref. [7], where it was assumed that the dispersion is caused by a single Lorentzian absorption line with the full-width at half-maximum Γ and the central frequency Ω strongly detuned with respect to the frequency of the lasing transition. Normal dispersion regime in this case corresponds to $\Omega > 0$ and the anomalous dispersion – to $\Omega < 0$. To model the dynamics of the injected laser we use the following set of DDEs for the complex envelope of the electric field $A(t)$ at the entrance of the SOA, material polarisation $P(t)$, and the saturable gain of the SOA $G(t)$ [7]:

$$\frac{dA}{dt} + (\gamma - iw)A = \gamma\sqrt{\kappa}e^{(1-i\alpha)G/2+i\varphi}\left[A_T + P_T\right] + \eta e^{iw_0 t}, \tag{1}$$

$$\frac{dG}{dt} = \gamma_g\left[g_0 - G - (e^G - 1)\left|A_T + P_T\right|^2\right], \tag{2}$$

$$P(t) = -\sigma L\int_{-\infty}^{t} e^{-(\Gamma+i\Omega)(t-s)}\frac{J_1\left[\sqrt{4\sigma(t-s)}\right]}{\sqrt{\sigma(t-s)}}A(s)ds, \tag{3}$$

where $A_T = A(t-T)$, $P_T = P(t-T)$, T is the cavity round trip time, γ and w describe the spectral width and the central frequency of the filter, η and w_0 are the strength and the frequency of the optical injection, σ is the total dispersion strength proportional to the delay line length. The parameters κ and

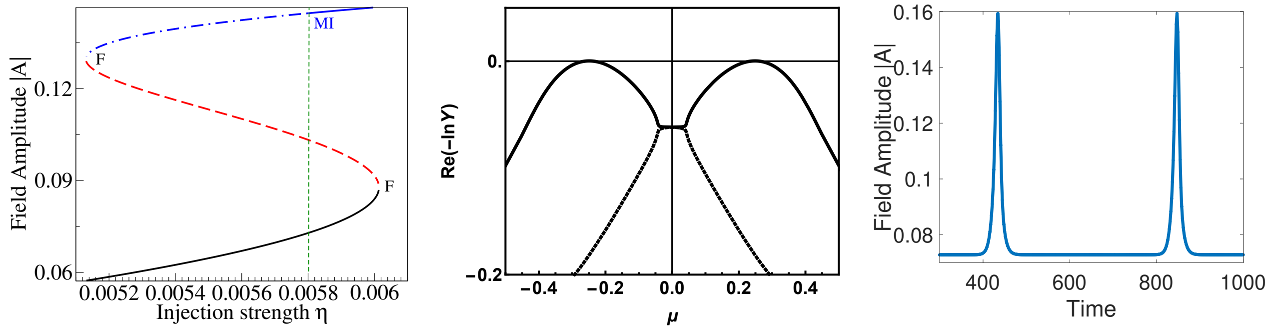


Figure 2: Left: S-shaped branch of injection-locked CW states of Eqs. (1)-(3) obtained by varying optical injection rate η for $\sigma \approx 2000$. Center: branches of pseudo-continuous spectrum illustrating the destabilization of the upper part of CW branch via modulational instability (MI) at $\eta \approx 0.0058$. Right: temporal cavity solitons with the repetition period close to $T = 400$ (right). Other parameters are: $\Omega = -13$, $\Gamma = 0.001$, $\alpha = 5$,$\kappa = 0.3$, $\gamma = \gamma_g = 1$, $g_0 = 1.19$, $\eta = 0.0058$, $w = w_0 = 0$ and $\phi = -0.2 + \frac{\sigma(\alpha\Gamma-\Omega)}{\Gamma^2+\Omega^2} - \frac{\alpha \log \kappa}{2}$.

φ, describe, respectively, linear attenuation and phase shift per cavity round trip, α is the linewidth enhancement factor, γ_g is the carrier relaxation rate, and g_0 is the pump parameter.

3 Stability analysis in the limit of large delay, modulational instability, and cavity solitons

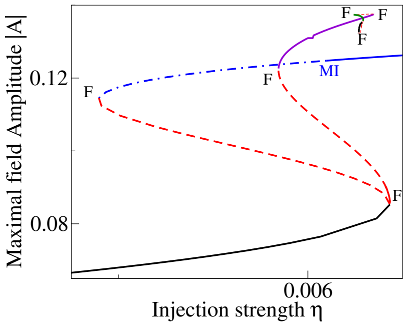
By choosing the injection frequency as a reference frequency we can set $w_0 = 0$ in Eqs. (1)-(3). Then, the injection-locked CW solution of these equations takes the form $A(t) = A_0 e^{i\varphi_0}$ and $G(t) = G_0$, where $P(t)$ can be expressed as $P(t) = P_0 = \left(e^{-\sigma/[\Gamma+i\Omega]} - 1\right) A_0 e^{i\phi_0}$. Similarly to the case of LLE [6], we are interested in the situation, where the CW branch shown in the left panel of Fig. 2 exhibits a bistable behaviour due to the presence of strong nonlinear phase-amplitude coupling introduced by the linewidth enchancement factor α.

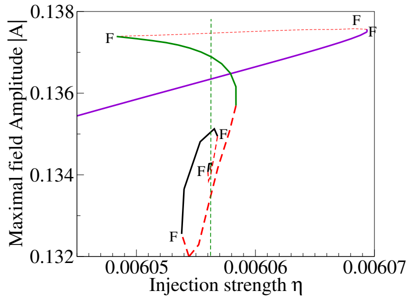
We look for cavity solitons in the vicinity of the bistability curve when the upper CW state is destabilised at large enough dispersion strength σ via a modulational instability in the anomalous dispersion regime ($\Omega < 0$). Linearizing Eqs. (1)-(3) near the injection locked CW solution, assuming that linear perturbations evolve exponentially in time $\delta A, \delta P, \delta G \propto e^{\lambda t}$, where λ is the eigenvalue, evaluating the integral (3) as $\delta P \propto e^{\lambda t}\left(e^{-\sigma/[\Gamma+\lambda+i\Omega]} - 1\right)$, and taking determinant of the Jacobian of the resulting system we obtain a transcendental characteristic equation in the following form

$$c_1(\lambda)Y^2 + c_2(\lambda)Y + c_3(\lambda) = 0, \tag{4}$$

where $Y = e^{-\lambda T}$ is the exponential term that appears from the delayed variables A_T, P_T. We look for instability of the CW solution in the limit of large delay $T \gg 1$, when the eigenvalues λ with infinitesimally small real parts λ_1, $\lambda = i\mu + (\lambda_1 + i\lambda_2)/T + \mathcal{O}(1/T^2)$, belonging to the so-called pseudo-continuous spectrum [20] cross the imaginary axis. To this end substitute $\lambda \approx i\mu$ in the coefficients c_1, c_2, and c_3 in the characteristic equation 4 and solve this equation to express λ_1 as a function of μ

$$\lambda_1 = -\operatorname{Re}\log Y_k(\mu),$$





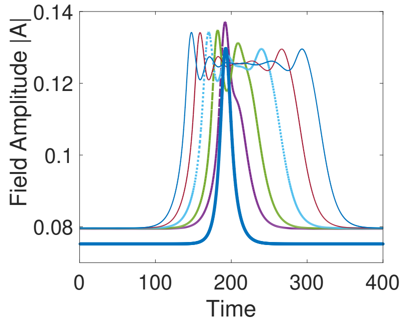


Figure 3: Left: Bifurcation diagram illustrating branches of CW and periodic cavity soliton solutions of (6)-(8) obtained by varying injection rate η. Center: a magnified region where the soliton branch shows a spiralling behaviour leading to a multistability between cavity solitons with different widths. Right: Profiles of different cavity solitons on the spiralling branch for $\eta = 0.006$ (thick solid) and $\eta \approx 0.00606$ (thin lines). Here $\sigma = 9, \Omega = -2, \Gamma = 0, \kappa = 0.25, g_0 = 1.33, \phi = -0.252 - \frac{\alpha \log \kappa}{2}$, and other parameters are as in Fig. 2.

where Y_1 and Y_2 represent two roots of quadratic equation (4) corresponding to the two curves of pseudo-continuous spectrum shown in the cental panel of Fig. 2. We note that the above-described algorithm of pseudo-continuous specrum calculation in the large delay limit was developed for conventional DDEs [20], but it is also valid when the distributed delay term (3) is present. In the absence optical injection, $\eta = 0$, the necessary analytical condition for the appearance in the anomalous dispersion regime of the modulational instability of the CW state with the rotation frequency ν was derived in [7]:

$$\alpha D_2 < -\frac{1}{\gamma^2}, \tag{5}$$

where second-order dispersion coefficient is given by $D_2 = \mathrm{Im}\,\frac{\mathrm{d}^2}{\mathrm{d}\nu^2}\left(\frac{-\sigma}{\Gamma + i(\Omega+\nu)}\right)$. We have used this condition to locate modulational instability of the injection-locked steady state at $\sigma = 2000$ for $\Omega = -13$, see central panel of Fig. 2, where the branches of pseudo-continuous spectrum are shown in the criticality. Right panel of Fig. 2 shows a stable cavity soliton obtained with the help of direct numerical integration of (1)-(3).

4 Reduced DDEs

Using Padé approximant of the integrand in (3) in the frequency domain we can simplify the distributed delay system (1)-(3) to obtain an approximate DDE model with a single fixed delay that is still capable of describing the formation of cavity solitons in the anomalous dispersion regime

$$\frac{dA}{dt} + (\gamma - iw)A = \frac{\gamma\sqrt{\kappa}e^{(1-i\alpha)G/2+i\varphi}}{1 + \frac{i\sigma}{2(\Omega - i\Gamma)}}\left[\left(1 - \frac{i\sigma}{2(\Omega - i\Gamma)}\right)A_T + P_T\right] + \eta e^{iw_0 t}, \tag{6}$$

$$\frac{dG}{dt} = \gamma_g\left[g_0 - G - (e^G - 1)\frac{\left|\left(1 - \frac{i\sigma}{2(\Omega - i\Gamma)}\right)A_T + P_T\right|^2}{1 + \frac{\sigma(\sigma - 4\Gamma)}{4(\Gamma^2 + \Omega^2)}}\right], \tag{7}$$

$$\frac{dP}{dt} = -\left[i(\Omega - i\Gamma) + \frac{\sigma}{2\left(1 + \frac{i\sigma}{2(\Omega - i\Gamma)}\right)}\right] P - \frac{\sigma}{1 + \frac{i\sigma}{2(\Omega - i\Gamma)}} A. \tag{8}$$

Using the algorithm described in the previous subsection, we have performed stability analysis of the injection locked states of Eqs. (6)-(8) in the large delay limit and found the parameter values where the the upper part of the CW branch exhibits a modulational instability in the anomalous dispersion regime. We used the software package DDE-BIFTOOL [17] to perform a continuation and stability analysis of the cavity soliton branch, see Fig. 3, and demonstrated a very good qualitative agreement between the simplified DDE model and the original distributed delay model (1)-(3). Unlike to that of the LLE model [6], the cavity soliton branch shown in Fig. 3 demonstrates a spiralling behavior, which bares similarities to the spatial cavity soliton branches calculated earlier in the models of semiconductor devices [21].

5 Conclusion

In this short communication, we have studied with the help of distributed delay model the effect of chromatic dispersion on the dynamics of an optically injected ring semiconductor laser. In the limit of large delay we have performed analytical stability analysis of injection-locked CW states, and in the anomalous dispersion regime found the modulational instability point of the upper part of bistable CW branch. We demonstrated numerically the formation of temporal cavity solitons and proposed a simplified DDE model with a single fixed delay for their description. Bifurcation analysis of cavity solitons of the simplified model has beep performed with the help of DDE-BIFTOOL [17] software package. The results of this analysis we found to be quite similar to those obtained earlier with the LLE equation [6].

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