Institut für Angewandte Analysis und Stochastik

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International Symposium
"Operator Equations and Numerical Analysis"
September 28 – October 2, 1992
Gosen (nearby Berlin)

S. Prößdorf (ed.)

submitted: 25th November 1992

Scientific Communication & Conference Center Gosen
Eichwalder Straße 100, D–O-1251 Gosen, Germany

Organizing Committee:	S. Prößdorf (Berlin), B. Silbermann (Chemnitz)
Conference Manager:	Mrs. M. Teuchert

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Preface

This Symposium was held at the Scientific Communication and Conference Center Berlin, Eichwalder Straße 100, O – 1251 Gosen (Germany). It can be considered as a continuation of a long tradition of workshops on special topics of the theory and numerical analysis of integral equations held during the last decade in Auerbach, Binz, Biesenthal and Einsiedel. The 23 foreign guests of the Symposium came from Austria (1), CIS (3), Estonia (2), Finland (2), Georgia (3), Israel (1), Italy (3), Poland (2), Portugal (3), Sweden (2) and the USA (1). 37 participants were Germans.

The main intention of this conference was to bring together researchers from both the areas of Operator Equations and Numerical Analysis to discuss problems and to stimulate the transfer of results, methods and applications between these fields. 17 survey lectures (45 minutes) and 29 short communications (30 minutes) were devoted to the following topics:

(1) Approximation methods for integral, pseudodifferential and operator equations.
(2) Theory and Numerical Analysis for boundary integral equations on non-smooth surfaces and boundary–value problems.
(3) Banach algebra techniques in Operator Theory and Numerical Analysis.
(4) Applications of integral and pseudodifferential equations.

The lectures and the numerous discussions have demonstrated the close connections and the interplay between these areas. In particular, the conference has shown the increasing enormous influence of the local principles and Banach algebra techniques to the Numerical Analysis. The large number of participating top–experts has contributed essentially to the success of the Symposium.

I express my gratitude to all participants, in particular, to the lecturers, chairmen and participants in the discussions. Especially, I would like to thank Mrs. M. Teuchert and Mrs. A. Giese for the excellent organization and the German Research Council (DFG) for sponsoring the Symposium.

The great interest of the participants suggests to organize a subsequent conference on this subject.

Berlin, 15th October 1992

S. Prößdorf

Titles of the Lectures

D. Berthold (Chemnitz):
A fast algorithm for solving the airfoil equation

V. Didenko (Odessa):
On the approximative solution of some operator equations with conjugation

R. Duduchava (Tiflis):
Pseudodifferential operators on compact manifolds with Lipschitz boundary

M. Efendiev (Stuttgart):
Nonlinear Riemann–Hilbert problems for multiply connected domains

J. Elschner (Berlin):
Exponential convergence of spline approximation methods for Wiener–Hopf equations

R. Gorenflo (Berlin):
Regularized differentiation of arbitrary positive (not necessarily integer) order

K. Gürlebeck (Chemnitz):
On hypercomplex solution methods for interface problems

A. Hommel (Chemnitz):
Discrete fundamental solutions for canonical problems in the plane

L. Jentsch (Chemnitz):
On boundary value problems with interface corners

Y. Karlovich (Odessa):
Algebras of convolution type operators with shifts

V. Kozlov (Linköping):
Parabolic boundary–value problems in domains with conical points

R. Kress (Göttingen):
On a quadrature method for a logarithmic integral equation of the first kind

N. Krupnik (Ramat–Gan):
Extensions of Fredholm symbols

G. Mastroianni (Naples):
Some new results on Lagrange interpolation

V. Maz'ya (Linköping):
Asymptotic behaviour of solutions of ordinary differential equations with variable operator coefficients

E. Meister (Darmstadt):
Some interior and exterior boundary–value problems for the Helmholtz equation in a quadrant

D. Mirschinka (Chemnitz):
Integral operators with fixed singularities on the half plane with applications

G. Monegato (Turin):
Nyström interpolants based on zeros of Legendre polynomials for non compact one–dimensional integral equations

D. Oestreich (Freiberg):
Numerical solution of a nonlinear integro–differential equation of Prandtl's type

N. Ortner (Innsbruck):
Operational formulae for the construction of fundamental solutions

V. Pilidi (Rostov a.D.):
On the uniform invertibility of regular approximations of singular integral operators

J. Prestin (Rostock):
Marcinkiewicz–Zygmund–type inequalities and interpolation

S. Prößdorf (Berlin):
Convergence theory of wavelet approximation methods for pseudodifferential equations

D. Przeworska–Rolewicz (Warsaw):
Smooth solutions to linear equations with right invertible operators

A. Rathsfeld (Berlin):
Numerical solution of the double layer potential equation over polyhedral boundaries

S. Roch (Chemnitz):
Spline approximation methods cutting off singularities

S. Rolewicz (Warsaw):
On Φ–Subdifferentiability and Φ–Differentiability

K. Ruotsalainen (Oulu):
The collocation method for nonlinear boundary integral equations

A.–M. Sändig (Rostock):
Calculation of singularities for inclusions with conical points

A. dos Santos (Lisbon):
Convolution equations on a union of two intervals

J. Saranen (Oulu):
Quadrature methods for boundary integral equations on curves

U. Schmid (Munich):
A collocation method for singular integral equations on Hölder spaces

G. Schmidt (Berlin):
Approximation of Poincaré–Steklov operators with boundary elements

R. Schneider (Darmstadt):
Wavelet approximation methods for periodic pseudodifferential operators: matrix compression and fast solution

E. Shargorodsky (Tiflis):
Boundary value problems for elliptic pseudodifferential operators and some of their applications

B. Silbermann (Chemnitz):
Non–strongly converging approximating methods

W. Spann (Munich):
Error estimates for the approximation of semi–coercive variational inequalities

F.–O. Speck (Lisbon):
Meromorphic factorization, partial index estimates and elastodynamic diffraction problems

E.P. Stephan (Hanover):
h-p version of the boundary element method for two– and threedimensional problems

F. Teixeira (Lisbon):
Wiener–Hopf–Hankel operators and diffraction by wedges

G.M. Vainikko (Tartu):
Higher order collocation methods for multidimensional weakly singular integral equations

E. Venturino (Iowa):
Stability and convergence of a hyperbolic tangent method for singular integral equations

E. Wegert (Freiberg):
Discrete nonlinear Riemann–Hilbert problems

W. Wendland (Stuttgart):
Substructuring for boundary integral equations

L. v. Wolfersdorf (Freiberg):
On the theory of nonlinear singular integral equations

D. Zarnadze (Tiflis):
On generalization of the least squares method for the operator equations in some Frechet spaces

Scientific Programme

Monday, September 28, 1992

9.00	Opening
9.15 – 10.00	V. Maz'ya (Linköping): Asymptotic behaviour of solutions of ordinary differential equations with variable operator coefficients
10.00 – 10.20	*Coffee break*
10.20 – 10.50	A.–M. Sändig (Rostock): Calculation of singularities for inclusions with conical points
10.50 – 11.20	A. Rathsfeld (Berlin): Numerical solution of the double layer potential equation over polyhedral boundaries
11.20 – 11.50	K. Gürlebeck (Chemnitz): On hypercomplex solution methods for interface problems
11.50 – 12.20	A. Hommel (Chemnitz): Discrete fundamental solutions for canonical problems in the plane
12.30 – 14.30	*Lunch*
14.30 – 15.15	E. Meister (Darmstadt): Some interior and exterior boundary–value problems for the Helmholtz equation in a quadrant
15.15 – 16.00	L. v. Wolfersdorf (Freiberg): On the theory of nonlinear singular integral equations
16.00 – 16.20	*Coffee break*
16.20 – 16.50	R. Kress (Göttingen): On a quadrature method for a logarithmic integral equation of the first kind
16.50 – 17.20	K. Ruotsalainen (Oulu): The collocation method for nonlinear boundary integral equations
17.20 – 17.50	D. Oestreich (Freiberg): Numerical solution of a nonlinear integro–differential equation of Prandtl's type
17.50 – 18.20	D. Berthold (Chemnitz): A fast algorithm for solving the airfoil equation
18.30 – 19.30	*Dinner*

Tuesday, September 29, 1992

8.30 – 9.15	N. Ortner (Innsbruck): Operational formulae for the construction of fundamental solutions
9.15 – 10.00	V. Kozlov (Linköping): Parabolic boundary–value problems in domains with conical points
10.00 – 10.20	*Coffee break*
10.20 – 10.50	L. Jentsch (Chemnitz): On boundary value problems with interface corners
10.50 – 11.20	D. Mirschinka (Chemnitz): Integral operators with fixed singularities on the half plane with applications
11.20 – 11.50	R. Duduchava (Tiflis): Pseudodifferential operators on compact manifolds with Lipschitz boundary
11.50 – 12.20	E. Shargorodsky (Tiflis): Boundary value problems for elliptic pseudodifferential operators and some of their applications
12.30 – 14.30	*Lunch*
14.30 – 15.15	D. Przeworska–Rolewicz (Warsaw): Smooth solutions to linear equations with right invertible operators
15.15 – 16.00	E.P. Stephan (Hanover): *h-p* version of the boundary element method for two– and threedimensional problems
16.00 – 16.20	*Coffee break*
16.20 – 16.50	G. Schmidt (Berlin): Approximation of Poincaré–Steklov operators with boundary elements
16.50 – 17.20	M. Efendiev (Stuttgart): Nonlinear Riemann–Hilbert problems for multiply connected domains
17.20 – 17.50	E. Wegert (Freiberg): Discrete nonlinear Riemann–Hilbert problems
17.50 – 18.20	S. Rolewicz (Warsaw): On Φ–Subdifferentiability and Φ–Differentiability
18.30 – 19.30	*Dinner*

Wednesday, September 30, 1992

8.30 – 9.15	G. Monegato (Turin): Nyström interpolants based on zeros of Legendre polynomials for non compact one–dimensional integral equations
9.15 – 10.00	S. Prößdorf (Berlin): Convergence theory of wavelet approximation methods for pseudo-differential equations
10.00 – 10.20	*Coffee break*
10.20 – 10.50	V. Didenko (Odessa): On the approximative solution of some operator equations with conjugation
10.50 – 11.20	U. Schmid (Munich): A collocation method for singular integral equations on Hölder spaces
11.20 – 11.50	V. Pilidi (Rostov a.D.): On the uniform invertibility of regular approximations of singular integral operators
11.50 – 12.20	E. Venturino (Iowa): Stability and convergence of a hyperbolic tangent method for singular integral equations
12.30 – 14.30	*Lunch*

Excursion

Thursday, October 1, 1992

8.30 – 9.15	J. Saranen (Oulu): Quadrature methods for boundary integral equations on curves
9.15 – 10.00	W. Wendland (Stuttgart): Substructuring for boundary integral equations
10.00 – 10.20	*Coffee break*
10.20 – 10.50	D. Zarnadze (Tiflis): On generalization of the least squares method for the operator equations in some Frechet spaces
10.50 – 11.20	W. Spann (Munich): Error estimates for the approximation of semi–coercive variational inequalities
11.20 – 11.30	*Break*
11.30 – 12.00	J. Prestin (Rostock): Marcinkiewicz–Zygmund–type inequalities and interpolation
12.00 – 12.30	G. Mastroianni (Naples): Some new results on Lagrange interpolation
12.30 – 14.30	*Lunch*
14.30 – 15.15	R. Schneider (Darmstadt): Wavelet approximation methods for periodic pseudodifferential operators: matrix compression and fast solution
15.15 – 16.00	F. Teixeira (Lisbon): Wiener–Hopf–Hankel operators and diffraction by wedges
16.00 – 16.20	*Coffee break*
16.20 – 16.50	A. dos Santos (Lisbon): Convolution equations on a union of two intervals
16.50 – 17.20	F.–O. Speck (Lisbon): Meromorphic factorization, partial index estimates and elastodynamic diffraction problems
17.20 – 17.30	*Break*
17.30 – 18.00	B. Silbermann (Chemnitz): Non–strongly converging approximating methods
18.00 – 18.30	J. Elschner (Berlin): Exponential convergence of spline approximation methods for Wiener–Hopf equations
18.30 – 19.30	*Dinner*

Friday, October 2, 1992

8.30 – 9.15	N. Krupnik (Ramat–Gan): Extensions of Fredholm symbols
9.15 – 10.00	Y. Karlovich (Odessa): Algebras of convolution type operators with shifts
10.00 – 10.20	*Coffee break*
10.20 – 11.05	S. Roch (Chemnitz): Spline approximation methods cutting off singularities
11.05 – 11.35	R. Gorenflo (Berlin): Regularized differentiation of arbitrary positive (not necessarily integer) order
11.35 – 11.45	*Break*
11.45 – 12.30	G.M. Vainikko (Tartu): Higher order collocation methods for multidimensional weakly singular integral equations

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ABSTRACTS

A Fast Algorithm for Solving the Airfoil Equation

D. Berthold (Chemnitz)

We consider the singular integral equation

$$\frac{1}{\pi}\int_{-1}^{1}\frac{u(y)\sigma(y)}{y-x}\,dy-\frac{\nu}{\pi}\int_{-1}^{1} ln|x-y|u(y)\sigma(y)dy+ \\ +\int_{-1}^{1}k(x,y)u(y)\sigma(y)dy=f(x)\,, \qquad (1)$$

$x\in(-1,1)$, where $\sigma(y)=(1-y)^{-1/2}(1+y)^{1/2}$, $\nu\neq 0$ is a given complex number, and f and k are given continuous functions defined on $[-1,1]$ and $[-1,1]^2$, resp. In the following, we propose a fully discretized, optimal convergent $0(n\log n)$–algorithm (being based on the quadrature method) for calculating an approximate solution u_n of (1) in form of a polynomial of degree at most $n-1$.

For $\varrho=\sigma$ and $\varrho=\mu:=\sigma^{-1}$ let $\{p_i^\varrho,\ i=0,1,...,\}$ be an orthogonal system of polynomials with respect to the scalar product $(u,v)_\varrho:=\pi^{-1}\int_{-1}^{1}u(y)\overline{v(y)}\varrho(y)dy$. Furthermore, choose a natural number m satisfying the two conditions:
(i) $(2n+1)/(2m+1)$ is an integer and (ii) $m^2\approx n$. Our algorithm calculates the Fourier coefficients $\alpha_{n,i}:=(u_n,p_i^\sigma)_\sigma$, $i=0,...,n-1$, of the approximate solution u_n and consists of two steps (cp. [1]).

Step 1: Calculate $\alpha_{n,i}$, $i=m,...,n-1$, by $\alpha_{n,i}:=(v_n,p_i^\sigma)_\sigma$, where v_n is a polynomial of degree at most $n-1$, which is the solution of the collocation method for equation (1) with $k(x,y)\equiv 0$. Here the collocation points are the zeros of p_n^μ. The complexity of Step 1 is only $0(n\log n)$ (see [3]), [4]).

Step 2: Calculate $\alpha_{n,i}$, $i=0,...,m-1$, by $\alpha_{n,i}:=(w_m,p_i^\sigma)_\sigma$, where w_m is a polynomial of degree at most $m-1$, which is the solution of quadrature method for equation (1) with a special right–hand side. This right–hand side is formed in a certain way using the function f and the already computed Fourier coefficients $\alpha_{n,i}$, $i=m,...,n-1$.

Using multiple grid ideas for the quadrature method ([2]) we need $0(m^2+n\log n)=0(n\log n)$ operation for Step 2, too ([3], [4]).

In order to investigate the convergence we define a scale of Sobolev–like subspaces $L^2_{\varrho,s}$ of the weighted space L^2_ϱ by

$$L^2_{\varrho,s}:=\left\{u\in L^2_\varrho:\|u\|^2_{\varrho,s}:=\sum_{i=0}(1+i)^{2s}|(u,p_i^\varrho)_\varrho|^2<\infty\right\}\,,\ s\geq 0\,,$$

$\varrho=\sigma$ and $\varrho=\mu$, resp. Now, if we suppose that $f\in L^2_{\mu,s}$, $k(\cdot,y)\in L^2_{\mu,s+\delta}$, $k(x,\cdot)\in L^2_{\sigma,s+\delta}$, $s>\frac{1}{2}$, $\delta>0$, then we have the optimal convergence rate (see [3])

$$\|u-u_n\|_{\sigma,t}\leq cn^{t-s}\|u\|_{\sigma,s}\quad\text{for all } t>\frac{1}{2}\quad\text{with } s-\delta\leq t\leq s\,.$$

References

1. Amosov, B.: On the approximate solution of elliptic pseudodifferential equations on a smooth closed curve. Z. Anal. Anw. 9 (1990), 545–563.
2. Berthold, D., W. Hoppe, B. Silbermann: The numerical solution of the generalized airfoil equation. J. Int. Eq. Appl. 4 (2) 1992, to appear.
3. Berthold, D., W. Hoppe, B. Silbermann: A fast algorithm for solving the generalized airfoil equation. To appear in: J. Comp. Appl. Math.
4. Steidl, G., M. Tasche: Fast algorithms for discrete orthogonal polynomial transforms. To appear in: Morgana, C./Kreuzburg, R. (eds.), Spectral technique. Theory and Application.

On the Approximative Solution of Some Operator Equations with Conjugation

V. Didenko (Odessa)

We consider the following singular integral equations with conjugation

$$
\begin{aligned}
(K\varphi)(t) \equiv a(t)\varphi(t) + \frac{b(t)}{\pi i}\int_\Gamma \frac{\varphi(\tau)d\tau}{\tau - t} + c(t)\overline{\varphi(t)} + \frac{d(t)}{\pi i}\int_\Gamma \frac{\overline{\varphi(\tau)}d\tau}{\tau - t} - \\
\frac{p(t)}{\pi i}\overline{\int_\Gamma \frac{\varphi(\tau)d\tau}{\tau - t}} - \frac{h(t)}{\pi i}\overline{\int_\Gamma \frac{\overline{\varphi(\tau)}d\tau}{\tau - t}} = f(t)\,,\; t \in \Gamma\,,
\end{aligned}
\tag{1}
$$

where $a, b, c, d, p, h \in C(\Gamma)$ and Γ is a simple closed curve with corners. We present a stability analysis of quadrature, spline collocation and qualocation methods for equation (1). All these methods are stable if and only if the operator $K \in \mathcal{B}_{add}(L^2(\Gamma))$ and some operators $A_\tau \in \mathcal{B}_{add}(\tilde{l}^2)$, $\tau \in \Gamma$, which depend on the approximation method, are invertible. Furthermore we investigate the Fredholmness of the operators A_τ, $\tau \in \Gamma$, and point out that the local operators have some interesting properties. For example, their indices are independent of the parameters of the considered approximation methods, but they depend on the values of coefficients a, b, c, d, p, h at the point $\tau \in \Gamma$. In the special case $p(\tau) = h(\tau) = 0$ the index of A_τ can be equal to -1, 0 or $+1$ only.

Pseudodifferential Operators on Compact Manifolds with Lipschitz Boundary

R. Duduchava (Tiflis) and F.–O. Speck (Lisbon)

In the manuscript, written in 1985 and published only in 1990, a Bessel potential operator (BPO) for a two-dimensional angle was constructed. In the meantime two papers of R. Schneider appeared which were based on this manuscript and succeeded in constructing BPO's for a Lipschitz domain $\Omega \subset \mathbb{R}_n$. Two kinds of BPO's were defined: with non-smooth and with $S^r_{1,0}(\mathbb{R}_n)$-symbol (of the Hörmander class). These operators are called order reduction operators and help to lift a pseudodifferential operator $A : \tilde{H}^s_p(\Omega) \to H^{s-r}_p(\Omega)$ to the singular integral operator $B : L_q(\Omega) \to L_p(\Omega)$ equivalently, where Ω is any special or general (including compact) Lipschitz domain in $\mathbb{R}_n$. Earlier such results where known only for Ω with the smooth boundary.

The results of R. Schneider for BPO's with non-smooth symbols are extended here from the case $p = 2$ to the general one $1 < p < \infty$. Then ΨDO's with non C^∞-smooth symbols on manifolds Ω with Lipschitz boundary are defined on Bessel potential spaces $\tilde{H}^s_p(\Omega) \to H^{s-r}_p(\Omega)$ using operators of local type; the following results are obtained for them: a Fredholm criteria in terms of the local representatives, the fredholmity of ΨDO's with locally sectorial matrix symbols and triviality of their index. Here similar results of F.-O. Speck, M. Costabel - E. Stephan, R. Schneider and some others are generalized while the proof gets more transparent.

ΨDO's and BPO's in Hölder–Zygmund spaces $\tilde{Z}^\alpha_p(\Omega)$, $Z^\alpha_p(\Omega)$ $(\alpha > 0,\ 1 \leq p \leq \infty)$ on a manifold with a Lipschitz boundary are treated as well.

Our approach to ΨDO's is based on the local principle, which is much refined, but enables to involve under the investigation ΨDO's with symbols out of the Hörmander class $S_{1,0}(\Omega \times \mathbb{R}^n)$. The method provides a more simple approach to the investigation of the solvability of equations appearing in mechanics and mathematical physics. One of such applications is demonstrated in the concluding section, where the crack problem for isotropic elastic media with steady oscillation is considered.

The paper will appear in "Mathematische Nachrichten".

Nonlinear Riemann–Hilbert Problems for Multiply Connected Domains

M.A. Efendiev and W.L. Wendland (Stuttgart)

In this paper we consider the global existence of solutions to nonlinear Riemann–Hilbert problems (abbr. RHP) for a q–connected domain with $q \geq 2$. Let G_q be a bounded q–connected domain with boundary $\Gamma = \partial G_q = \cup_{k=1}^{q} \Gamma_k$, where Γ_k are Jordan curves with parametric representations $t_k = t_k(s_k)$; $0 \leq s_k < 2\pi$ and $q \geq 1$. The nonlinear RHP can be formulated as follows: Find a holomorphic function $\phi(z) = u(z) + iv(z)$ in G_q which is continuous in the closure $\overline{G}_q$ satisfying the boundary condition

$$f(\zeta, u(\zeta), v(\zeta)) = 0 \quad \text{for } \zeta \in \partial G_q \,. \quad (H_q)$$

Clearly (H_q) generalizes two well–known classical problems of analytic function theory; namely:

1. The Riemann conformal mapping problem.
2. The linear Riemann–Hilbert problem.

During the last twenty years, many results have been obtained for the nonlinear RHP for a simply connected domain; in particular, results based only on topological conditions for the function $f(\zeta, u, v)$. It was A.I. Snirelmann with his fundamental work [2] who initiated the new geometrical approach to this problem generalizing the conformal mapping problem for $q = 1$. However, his analysis did not cover the linear RHPs and problems where the family of curves $f(\zeta, u, v) = 0$ in the u-v–plane with parameter $\zeta \in \partial G_1$ consists of non–closed curves going off to infinity which come up in hydrodynamical applications. This case of non–closed curves was treated with two different geometric methods by M.A. Efendiev [1] and by E. Wegert [3], respectively. In the case of nonlinear as well as of linear RHPs for multiply connected domains, i.e. $q \geq 2$, arise new fundamental difficulties. Here, we present for these cases new global existence theorems based on the introduction of $(q - 2)$ additional degrees of freedom for the curves $f(\zeta, u, v; \vec{w}) = 0$, where $\vec{w} \in R^{q-2}$. The modified nonlinear RHP now reads as:

Find a pair $(\vec{w}, \Phi(z)) \in R^{q-2} \times \mathcal{A}$ where $\mathcal{A}$ denotes the analytic functions in G_q which are continuous in $\overline{G}_q$, which satisfies the boundary conditions

$$f_j(t_j(s_j), u(t_j(s_j)), v(t_j(s_j)); \vec{w}) = 0 \quad \text{on } \Gamma_j;\ j = 1, ..., q \,. \quad (H_q)$$

The existence proof is based on a degree theory of the so–called Fredholm quasilinear mappings in Banach spaces. Under sufficient geometrical conditions for the q families of curves $f_j = 0$ we obtain existence.

References

1. Efendiev, M.A.: On a property of the conjugate integral and a nonlinear Hilbert problem. English transl.: in Soviet Math. Dokl., vol. 35 (1987), N3, 535–539.
2. Snirelmann, A.I.: The degree of a quasi–linearlike mapping and the nonlinear Hilbert problem. English transl.: Math. USSR Sbornik 18 (1974), 373–396.
3. Wegert, E.: Topological methods for strongly nonlinear Riemann–Hilbert problems for holomorphic functions. Math. Nachr. 137 (1988), 144–157.

Exponential Convergence of Spline Approximation Methods for Wiener–Hopf Equations

J. Elschner (Berlin)

We consider the numerical solution of the Wiener–Hopf integral equation

$$u(x) - \int_0^\infty k(x-y)u(y)dy = f(x)\,,\ x \in (0,\infty)\,, \tag{1}$$

by approximation methods based on piecewise polynomials. Such equations arise in a number of applications in radiative equilibrium and transfer, in refraction of electromagnetic waves, and in crack problems in linear elasticity. In particular, in the problem of a pressurised crack in the form of a cross, the kernel function

$$k(x) = -\pi^{-1}\mathrm{sech}^2(x)$$

arises. Using piecewise polynomials of degree $[\mu n]$, $\mu > 0$, subordinate to the partitions $\{0, 1, ..., n-1\}$, we obtain results on stability and exponential convergence in the L_q norm, $1 \leq q \leq \infty$, of Galerkin, collocation and Nyström quadrature methods for the approximate solution of Eq. (1). A main ingredient of the convergence analysis is a smoothness result with respect to a certain scale of countable normed spaces of real–analytic functions with exponential decay at infinity.

Regularized Differentiation of Arbitrary Positive (not necessarily integer) Order

R. Gorenflo (Berlin)

For $\alpha > 0$, $1 \leq p \leq \infty$, consider the operator $J^\alpha : L_p(0,b) \to L_p(0,b)$, given by

$$(J^\alpha u)(t) := \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} u(s)\, ds . \tag{1}$$

For $0 < b < \infty$ this operator is compact, its inverse unbounded. Hence the integral equation

$$J^\alpha u = g\,,\ g \text{ given, } u \text{ unknown}\,, \tag{2}$$

is *ill–posed.* If instead of g we have available only a function f with

$$||g - f||_p \leq \varepsilon \tag{3}$$

we require some *additional information* on (the smoothness of) u if we want to find a good estimate or approximation. In cooperation with Vu Kim Tuan in Hanoi/Vietnam the following result has been obtained.

Assume that the zero extension $\tilde{u}$ of u (coinciding with u on $[0,b]$, vanishing outside this interval) has the Hölder property

$$||\tilde{u}(. - h) - \tilde{u}(.)||_p \leq E h^\beta \tag{4}$$

with numbers $E > 0$, $\beta \in (0,1]$. Then by a fractional difference technique with appropriately chosen steplength h one can determine a function u_h on $\mathbb{R}$ such that

$$||u_h - \tilde{u}||_p \leq K(\alpha) E^{\frac{\alpha}{\alpha+\beta}}\, \varepsilon^{\frac{\beta}{\alpha+\beta}} \tag{5}$$

with a constant $K(\alpha)$ depending only on α.

On Hypercomplex Solution Methods for Interface Problems

K. Gürlebeck (Chemnitz)

It is considered a transmission problem in the case of a domain G with an inclusion G_1. In a first step an explicit representation for the solution of a boundary value problem in G_1 is given using known results of hypercomplex function theory and its applications to elliptic boundary value problems. In a second part a hypercomplex method for the solution of boundary value problems in a ring-shaped domain is demonstrated. Again an explicit description of the solution is possible. Therefore it is necessary to study $\mathbb{H}$-regular functions in multiple connected domains. We are interested in the behaviour of these functions near the different parts of the boundary and in the description of the orthogonal complement of the subspace of $\mathbb{H}$-regular functions in $L_2(G\backslash G_1)$. Collecting the representation formulas in G_1 and $G\backslash G_1$ we derive a hypercomplex integral equation on $\Gamma_1 = \partial G_1$ by the help of the transmission conditions. The uniqueness of the solution is proved using some new results concerning the boundary values of $\mathbb{H}$-regular functions in multiple connected domains. The solvability of the integral equation is investigated in a constructive manner. An equivalent formulation of the problem is given in form of a generalized Riemann–Hilbert–Problem for $\mathbb{H}$-regular functions. This problem can be solved explicitly. Using this solution the representation formula for the solution of the transmission problem can be simplified. The result is an explicit integral representation which depends only on the data on Γ_1 and on the solution of a boundary value problem in the whole domain G. The representation formula holds in G without any change by crossing the inner boundary Γ_1.

Discrete Fundamental Solutions for Canonical Problems in the Plane

A. Hommel (Chemnitz)

We start with a general theorem about boundary projection operators (see Ryabenkij: "Method of difference potentials for any tasks in continuous mechanics", Moscow, "Nauka", 1987, in russian). In this sense the integral representation for functions of the class $C^{(2)}$ is a special realization. Furthermore, it is possible to derive from this theorem the well-known integral equations of potential theory.

It is our aim to use this theorem to develop a discrete potential theory. The first step in this direction is the description of a discrete fundamental solution. In case of the Laplace equation we use the discrete Fourier transform. We apply a sum formula given in a paper by van der Pol to compute this fundamental solution on the diagonal points of an equidistant mesh. We use symmetry and the Laplace equation to find the solution in all the other mesh points.

By the help of our discrete fundamental solution it is possible to solve some simple canonical problems. Interface problems are the next point of view. We are able to solve the problem of two coupled half spaces and, under an additional condition, the problem of four coupled quadrants. We have some ideas to treat the last problem if the additional condition is not fulfilled, but up to now the expressions are very complicated.

The next problem is the description of the discrete boundary. We illustrated our results on a L–shaped domain.

Our ideas and results have not been published yet.

On Boundary Value Problems with Interface Corners

L. Jentsch (Chemnitz)

In the framework of elastostatics formulae for singular exponents in the asymptotic expansion of boundary value problem solutions at interface corners are presented. An interface corner occurs, if the interface between two different materials meets with the boundary of the whole body.

In the first part we look for the solution of the traction boundary value problem in the form of a single layer potential which satisfies a priori the transmission conditions on the interface. For the asymptotics of the potential density at a corner point as well as for the Fredholm property of the boundary integral operator the symbol of a Mellin convolution operator is responsible. The Mellin symbol is calculated explicitly. It can be expressed by elementary transcendental functions. The case of homogeneous material is discussed in detail. The asymptotics of the solution of the boundary value problem can be obtained by evaluation of the potential.

The disadvantage of the boundary integral method consists in the fact that a singular exponent of the potential density must not lead to a singular function in the asymptotics of the boundary value problem solution.

In the second part formulae for the singular exponents are obtained using the direct method of Mellin transformation. In connection with that some new formulae for corner problems with unusual boundary conditions are included. In the case of the general boundary transmission problem the singular exponents are zeros of a determinant with eight rows. These determinants are calculated for several boundary conditions.

References

1. Jentsch, J.: On the boundary transmission problem of thermoelastostatics in a plane domain with interface corners (submitted to Mathematical Methods in Applied Sciences).

Algebras of Convolution Type Operators with Shifts

Yu. I. Karlovich (Odessa)

A review of some results on Fredholm theory of algebras of convolution type operators with discrete groups of shifts and coefficients having discontinuities of the first and second kind. This class includes algebras of singular integral operators with shifts and coefficients with discontinuities of the semi–almost–periodic type, algebras of integrodifference Wiener–Hopf operators with piecewise–continuous or oscillating coefficients and others. In the general case these algebras consist of non-local convolution type operators with two discrete groups of shifts with respect to direct and dual Fourier variables in the presence of discontinuities of the coefficients and presymbols. The investigation of these algebras is based on two methods of local–trajectory studying the Fredholm property of considered operators in Banach and Hilbert spaces, respectively (see [Yu. I. Karlovich, Dokl. Akad. Nauk SSSR, 299 (1988), 546–550; 304 (1989), 274–280]). The theory essentially depends on structure of the set of fixed points in shifts. In general case the representations are infinite and inhomogeneous.

Parabolic Boundary–Value Problems in Domains with Conical Points

V. Kozlov (Linköping)

The talk is devoted to general parabolic boundary–value problems in a domain G with a conical point $O \in \partial G$.

If the right–hand sides satisfy the consistency conditions then the asymptotics of the solution has the form

$$u(x,t) \sim \Sigma U_j(x, \partial_t) h_j(t) \,,$$

where $U_j(x,p)$ is a polynomial in p with coefficients determined by solving a model elliptic boundary–value problem in the tangent cone and U_j depends only on the asymptotics of the operators and the boundary near conical point. The coefficient h_j is expressed by some Volterra operator applied to the right–hand sides. The kernel of this operator satisfies the homogeneous adjoint problem and has a singularity at $x = 0$, $t = 0$.

Consider the case when the consistency condition is not valid and let us confine ourselves to the heat equation. Then the following asymptotics holds

$$u(x,t) \sim \sum_{k \geq 0} t^{k/2} \left(\zeta(r) U_k \left(w, \frac{r}{2\sqrt{t}} \right) + r\chi(\nu/r) V_k \left(x', \frac{\nu}{2\sqrt{t}} \right) \right) .$$

Here ν is the distance to the boundary, x' is the projection of the point x onto ∂G, ζ and χ are cutoff functions. The functions $U_k(\cdot,\cdot)$ and $V_k(x',\cdot)$ are found successively by solving model problems in the cone and on the half line.

The next topic relates to the Green function and the Poisson kernels of the general parabolic problem in a cone. Pointwise estimates as well as asymptotics near the vertex of the cone and as $t \to \infty$ are obtained.

On a Quadrature Method for a Logarithmic Integral Equation of the First Kind

R. Kress (Göttingen)

We present a quadrature method for the numerical solution of the logarithmic integral equation of the first kind arising from a single–layer approach to solving the Dirichlet boundary value problem for the two–dimensional Helmholtz equation. Our method is based on weighted trigonometric interpolation quadratures on an equidistant mesh with appropriately chosen weight functions taking care of the logarithmic singularity. The convergence analysis is based on the theory of collectively compact operators and yields error estimates with respect to uniform convergence. For analytic boundaries and boundary data the convergence is exponential. We also briefly indicate how the method can be extended to the corresponding integral equation on open arcs by using a graded mesh where the grading is based upon the idea of substituting a new variable. Numerical examples illustrate rapid convergence of the method both for closed and open contours.

This is joint work with R. Chapko (Lwow) and will be published in due source as R. Chapko and R. Kress: On a quadrature method for a logarithmic integral equation of the first kind.

Extensions of Fredholm Symbols

I. Gohberg (Tel Aviv) and N. Krupnik (Ramat–Gan)

Let $\mathbb{A}$ be an algebra of linear bounded operators acting in some Banach space $\mathbb{B}$ and $\{\gamma_\tau\}$ $(\tau \in \mathcal{T})$ a set of homomorphisms $\gamma_\tau : \mathbb{A} \to \mathbb{C}^{l\times l}$ $(l = l(\tau) \leq n)$. One says that the set $\{\gamma_\tau\}$ $(\tau \in \mathcal{T})$ generates a Fredholm symbol of order n for algebra $\mathbb{A}$ in the space $\mathbb{B}$ if any operator $A \in \mathbb{A}$ is Fredholm in $\mathbb{B}$ iff $\det \gamma_\tau(A) \neq 0$ for all $\tau \in \mathcal{T}$. In this case we write $\mathbb{A} \in FS(n, \mathbb{B})$.

Theorem. Let $\mathbb{K}$ be a dense subalgebra of $\mathbb{A}$ and let a set $\{\gamma_\tau\}$ $(\tau \in \mathcal{T})$ generates a Fredholm symbol for algebra $\mathbb{K}$. If $\mathbb{A} \in FS(m, \mathbb{B})$ then for every $\tau \in \mathcal{T}$ the function $\det(A)$ is continuous on $\mathbb{K}$ and any operator $A \in \mathbb{A}$ is Fredholm in $\mathbb{B}$ iff

$$\inf_{\tau \in \mathcal{T}} |\det \gamma_\tau(A)| > 0 . \tag{1}$$

Here is one of the applications of this theorem. Let Γ be a non–simple contour on the complex plane $\mathbb{C}$, ϱ – some weight function such that the operator of singular integration S_τ is bounded in $L_p(\Gamma, \varrho)$. Denote by $\mathbb{K}$ the algebra (nonclosed) generated by singular integral operators with piecewise continuous coefficients. If a set $\{\gamma_\tau\}$ $(\tau \in \mathcal{T})$ generates a Fredholm symbol of order n for algebra $\mathbb{K}$ then any operator $A \in \mathbb{A}(= \overline{\mathbb{K}})$ is Fredholm in $\mathbb{B}$ iff (1) holds.

Some New Results on Lagrange Interpolation

G. Mastroianni (Naples)

Lagrange polynomial interpolation is useful in polynomial approximation of functions and in searching numerical solutions of functional equations by collocation methods.

Unfortunately, classical literature gives few examples of "good" interpolatory processes. One of the aim of this talk is to give two procedures (extended interpolation and the method of additional nodes), by which it is possible to construct infinite optimal interpolation processes. The previous methods allow to obtain simultaneous approximation theorems.

The error estimates are expressed by the best uniform approximation error. The previous estimates are natural, if we consider the interpolation operator as an operator mapping bounded functions into functions belonging to L^p. However, the previous estimates are unsuitable and often it is needed to estimate the L^p–norm of Lagrange error by the same norm of the derivatives of the function.

Finally I want to remark that such estimates are possible for more general cases.

Asymptotic Behaviour of Solutions of Ordinary Differential Equations with Variable Operator Coefficients

V. Maz'ya (Linköping)

This is a survey of results obtained together with V. Kozlov.

Conditions of uniqueness, existence and asymptotic properties of solutions to the differential equation $\mathcal{A}(t, D_t)u = f$ on R^1 with unbounded operator coefficients are found. One of the results is a theorem on the preservation of a "powerexponential" asymptotics under the Dini type condition

$$\int_0^\infty \omega(t) t^{2(m-1)} dt < \infty , \qquad (*)$$

where $\omega(t)$ is the continuity modulus of the coefficients at infinity (understood in the sense of corresponding operator norms) and m is the maximal length of Jordan chains corresponding to some eigenvalues of the operator pencil $\mathcal{A}(\infty, \lambda)$. The condition $(*)$ is best possible in a sense. The proof is based on estimates for the inverse operator of the equation in question. An auxiliary result of independent interest is a comparison theorem for solutions of the operator differential equation $\mathcal{A}(t, D_t)u = f$ and those of a certain higher-order ordinary differential equation. Results of this type have immediate applications to the theory of partial differential equations in domains with boundary singularities.

Some Interior and Exterior Boundary–Value Problems for the Helmholtz Equation in a Quadrant

E. Meister, F. Penzel (Darmstadt), F.–O. Speck, F.S. Teixeira (Lisbon)

The variety of problems in classical mathematical physics which allow a solution in closed analytical form is relatively small. However, those canonical problems are of particular interest, since they give a clear understanding of correct settings of functional spaces which includes unique solvability and continuous dependence of the given data. Moreover they can be used for more detailed studies of the regularity of the solutions, asymptotics, convergence and stability of numerical procedures. This analysis is straightforward, if the resolvent is known as a bounded operator in a suitable form.

In this talk the first author presents a method to solve the Dirichlet, Neumann and mixed D/N boundary–value problem for the interior and exterior of a right–angled wedge in the energy space $H^1(Q;\Delta)$ or $H^1(\mathbb{R}^3\backslash Q;\Delta)$ using a Fourier transform representation based on $H^{\pm 1/2}(\mathbb{R}_+)$–data and a reduction method where data on one face of the wedge are equal to zero. With the help of the corresponding explicit solutions of the interior problems, the exterior one for $\mathbb{R}^3\backslash Q_3$ is reduced to a 2×2–system of singular integral equations of Wiener–Hopf–Hankel type in $\tilde{H}^{-1/2}(\mathbb{R}_+)^2$ for the pair of unknown Dirichlet–Cauchy–data (f_1, f_2) on the 1st and 2nd semi–axes, the boundary parts of the cross–section of the 1st wedge. After lifting to $(L^2(\mathbb{R}_+))^2$–space this system can be resolved by a method introduced by the fourth author treating wedge boundary–value problems by formulating them as 2×2 Wiener–Hopf problems with special symbol matrices which allow for a canonical factorization.

The details of this subject presented in the talk will appear in one of the next issues of the Proceedings of the Royal Society, Edinburgh.

Integral Operators with Fixed Singularities on the Half Plane with Applications

D. Mirschinka (Chemnitz)

The background of our investigations is a local treatise of boundary integral operators appearing in the solution of so–called bimetal problems.

As an example we consider the Dirichlet problem for stationary heat conduction in a nonhomogeneous medium. More precisely, we look for a function u which is harmonic in a domain D consisting of two subdomains D_+ and D_- with a common plane interface S_o. On this interface natural transmission conditions are given. Furthermore, we assume that the interface intersects the outer boundary S in a smooth curve Γ which is an edge of S. The ansatz of the double layer potential with the Green's function for two coupled half spaces leads to an integral equation on the outer boundary only. One can show that this operator on S is a zero index Fredholm operator iff a family of 2×2–matrix operators on the half plane related to the points of Γ is invertible. These operators are studied via a Harmonic Analysis approach on the "$ax+b$" group. We show that the aforementioned operators admit a representation as convolution operators on this group. For nonperfect heat contact between the two medias we can prove the invertibility of the matrix operators in L_p–spaces with $p \geq 2$ for arbitrary wedge angles. Unfortunately, this result can not be improved in the case of perfect heat contact. We give some special results in terms of the wedge angles.

Finally, we want to note that the presented approach is applicable to a wide class of operators including classical singular integral operators, Green type operators and operators with piecewise continuous symbols.

Nyström Interpolants Based on Zeros of Legendre Polynomials for Non Compact One–Dimensional Integral Equations

Giovanni Monegato (Turin)

Several problems of Mathematical Physics lead to integral equations of the type

$$u(y) + \lambda \int_{-1}^{1} k(x,y)u(x)dx = h(y) \qquad (1)$$

where the kernel $k(x,y)$ is either weakly or strongly singular. When the input functions $h(y)$ are smooth, and the kernels satisfy certain conditions, these equations have solutions which are smooth everywhere in $(-1,1)$ except at the endpoints ± 1. For instance, in the case $k(x,y) = k(|x-y|)$, with $k(1+t) \in C^{q-1}(-1,1]$, $h \in C^q[-1,1]$, $q \geq 1$, if we also assume that

$$\begin{aligned} |k^{(i)}(1+t)| &\leq \gamma_i(1+t)^{-\alpha-i}, \quad -1 < t \leq 1, \; i = 0,1,...,q-1, \\ |k^{(i)}(1+t)| &\geq \delta_i(1+t)^{-\alpha_o-i}, \quad -1 < t \leq t_o, \; i = 0,1,...,q-1, \end{aligned}$$

where γ_i, δ_i, $0 < \alpha < 1$, $-1 < \alpha_o \leq \alpha$, t_o are constants, then it has been shown (see [5]) that the solution of (1) belongs to $C[-1,1] \cap C^q(-1,1)$. Furthermore, we can also claim that $u \in \overline{C}^q_o[-1,1]$, where

$$\overline{C}^q_p[-1,1] = \{g : (1-x^2)^{i-p} g^{(i)}(x) \in C[-1,1], \; i = 0,1,...,q\}.$$

In the case of certain classes of Mellin convolution equations of form (1), when $h \in C^q_p[-1,1] + \Pi_d$ for some $q \geq 1$, $p \geq 0$, where

$$C^q_p[-1,1] = \{g : (1-x)^{i-p} g^{(i)}(x) \in C[-1,1], \; i = 0,...,q\}$$

and Π_d denotes the space of polynomials of degree d, we have (see [1]) $u \in C^q_p[-1,1] + \Pi_d$.

Among the numerical methods proposed to solve such equations we recall the Nyström methods, which are based on quadrature formulas for the discretization of the integral present in (1). Here we consider the use of product rules of interpolatory type, based on zeros of Jacobi polynomials. They are obtained by replacing the function $u(x)$ by its Lagrange interpolation polynomial. For this type of rules we derive accurate uniform error estimates in the cases

(i) $|k(x,y)| \leq c|x-y|^{\nu}$, $-1 < \nu < 0$, $u \in \overline{C}^q_o[-1,1]$
(ii) $k(x,y) = k^*\left(\frac{1-y}{1-x}\right)\frac{1}{1-x}$, $k^*(\cdot)$ bounded, $u \in C^q_p[-1,1]$.

Once we prove stability of our numerical methods, these estimates describe the behaviour of the errors associated with the Nyström interpolants.

Since in the case of weakly singular kernels the stability issue has already been characterized by several authors, we have confined our attention to the case of Mellin convolution equations. In particular we have considered the following equation

$$u(y)+\frac{4}{\pi}\int_0^1 \frac{y^2x}{(y^2+x^2)^2}u(x)dx = y\,,\ 0<y\le 1\,,$$

for which it is known (see [2]) that $u \in C_1^\infty[0,1]+\Pi_1$.

At the moment we are not able to prove stability for the method associated with a quadrature rule of interpolatory type which integrates exactly the kernel. Numerical testing shows however that the method is stable, convergent, and more accurate than what was expected.

We have proved stability, hence convergence, for a proper modified discrete operator associated with the well–known Gauss–Legendre formula. In spite of the presence of the singularity at $x=y=0$, its convergence rate and numerical performances are very similar to those we obtain by using the previous product type rule.

References

1. Elschner J.: Spline approximation methods for a class of non compact integral equations. Math. Nachr. 1990.
2. Monegato, G., S. Prößdorf: On the numerical treatment of an integral equation arising from a cruciform crack problem. Math. Meth. Appl. Sci., 1990.
3. Mastroianni, G., G. Monegato: Nyström interpolants based on the zeros of Legendre polynomials for a non compact integral operator equation. To appear in: IMA J. Numer. Anal.
4. Mastroianni, G., G. Monegato: Polynomial approximations of functions with endpoint singularities and product integration formulas, submitted.
5. Vainikko, G., A. Pedas: The properties of solutions of weakly singular integral equations. J. Austral. Math. Soc. (B), 1981.

Numerical Solution of a Nonlinear Integro–Differential Equation of Prandtl's Type

D. Oestreich (Freiberg)

The analysis of crack initiation and growth in brittle solids such as ceramics composites leads to the following nonlinear integro–differential equation

$$-\frac{1}{\pi}\int_{-1}^{1}\frac{u'(s)}{s-t}\,ds + F(t,u(t)) = G(t),\ (|t|<1) \tag{1}$$

with homogeneous boundary conditions

$$u(-1) = u(1) = 0 \tag{2}$$

An existence theorem for the problem (1), (2) was proved by v. WOLFERSDORF.

Discretization by a collocation method using special trigonometrical polynoms yields a nonlinear algebraic equation system $A\overline{x} + \Phi(\overline{x}) = 0$ with a well–defined matrix A and vector–function Φ. If the function $F(t,u)$ is continuous and non–decreasing in $u \in I\!R$ for almost all $t \in [-1,1]$ and bounded in $t \in [-1,1]$ for all $u \in I\!R$ and $G(t)$ is bounded in $t \in [-1,1]$, this nonlinear algebraic equation system can be solved by the Jacobi method. The convergence of the approximated solution to the exact solution of the problem (1), (2) is proved.

The other numerical approach, namely at first linearization of (1) and after that discretization of the linearized equation, will be considered too. For linearization a Newton–method is used. If $F(t,u)$ has the above mentioned properties and possesses a derivative $F_u(t,u)$ which is continuous in $u \in I\!R$ for all $t \in [-1,1]$ and bounded in $t \in [-1,1]$ for all $u \in I\!R$ the convergence of the approximated solution in $C(-1,1)$ can be easily shown.

The practical computing for relevant examples shows that the most effective way may be the combination of the global, but slow Jacobi–method with the local Newton–method which converges Q–superlinear.

References

1. Oestreich, D.: Numerical solution of a nonlinear integro–differential equation of Prandtl's type. Math. Nachr. (submitted).

Operational Formulae for the Construction of Fundamental Solutions

N. Ortner (Innsbruck)

The main methods for constructing fundamental solutions of linear differential operators are the *Fourier– and the Laplace transform* for tempered distributions (see L. Schwartz 1966, L. Hörmander 1983, V.S. Vladimirov 1971, 1979). In contrast to these methods, operational procedures are procedures allowing the construction of fundamental solutions from other fundamental solutions. Often, the result of an operational procedure is an operational formula.

5 Examples of operational formulae were presented. The first operational formula relates to the so–called *"difference trick"* and to convolution (see N. Ortner, Methods of Construction of Fundamental Solutions of Decomposable Linear Differential Operators. In: BEM IX, Vol. 1, ed. by C.A. Brebbia et al., Springer, 1987; Prop. 2, p. 86). It allows the construction of a fundamental solution of a product of operators as a convolution of a fundamental solution with a sum of fundamental solutions of the factors of the product. This procedure was applied for the first time by G. Herglotz in 1926 to construct the fundamental solution of a product of wave operators with different speeds. Other examples are: product of Helmholtz operators, Reissner's system for the static, elastic plate, $\Delta(\Delta - 2a\partial_1)^{(2)}$.

A second type of operational formulae are the *parameter–integration formulae.* In the talk, the one–dimensional version was presented (loc. cit., Prop. 5, p. 92). To apply the parameter–integration method it is necessary to know fundamental solutions of "simpler" operators "with parameters" and of iterates of these "simpler" operators. These fundamental solutions can be found as well by some operational formulae.

A last type of operational formula is the Sommerfeld formula permitting the construction of the fundamental solution $\partial_t^2 + P(\partial_x)$ from the fundamental solution of $\partial_t^2 + P(\partial_x) + c^2$, $c \in \mathbb{C}$.

On the Uniform Invertibility of Regular Approximations of Singular Integral Operators

V. Pilidi (Rostov a.D.)

Let a and b be piecewise continuous functions defined on the real axis $\mathbb{R}$, and let T be a compact operator in $L_p(\mathbb{R})$. We introduce the operators $A = aI + bS + T$ and $A_\varepsilon = aI + bS_\varepsilon + T$ $(\varepsilon > 0)$ acting in $L_p(\mathbb{R})$. Here S is the singular integration operator in the latter space, S_ε acts in this space as follows:

$$(S_\varepsilon f)(x) = \frac{1}{\pi i}\left(\int_{-\infty}^{x-\varepsilon} + \int_{x+\varepsilon}^{\infty}\right)\frac{f(y)}{y-x}\,dy\,,\ x \in \mathbb{R}\,.$$

Assume in addition that A is invertible. We get necessary and sufficient conditions for the operators A_ε to be invertible for all sufficiently small ε, and for their inverse operators to converge strongly to A^{-1} as $\varepsilon \to +0$. The property just formulated is equivalent to the condition that the operators A_ε are invertible for all sufficiently small ε and the norms of the operators inverse to them are uniformly bounded. We say that such family is *uniformly invertible.*

We also investigate other classes of operators and other types of approximations.

The proofs use approaches from [1], [2] and local principle of Gohberg–Krupnik.

References

1. Kozak, A.V.: A local principle in the theory of projection methods (in russian). Dokl. Akad. Nauk SSSR. **212** (1973), 1287–1289.
2. Silbermann, B.: Lokale Theorie des Reduktionsverfahrens für Toeplitzoperatoren. Math. Nachr. **104** (1981), 137–146.
3. Pilidi, V.S.: A criterion for the uniform invertibility of regular approximations for one-dimensional singular integral operators with piecewise continuous coefficients (in russian). Izv. Akad. Nauk SSSR, Ser. Mat. **54** (1990), 6.
4. Pilidi, V.S: On the method of cutting off the singularities for bisingular integral operators with continuous coefficients (in russian). Funkts. Anal. Prilozh. **23** (1989), N.1, 82–83.
5. Pilidi, V.S: On the uniform invertibility of the regular approximations of one-dimensional singular integral operators with non-compact perturbations (in russian). Diff. Uravn. **26** (1990), N.12, 2127–2136.

Marcinkiewicz–Zygmund–Type Inequalities and Interpolation

J. Prestin (Rostock)

The underlying question is the convergence order of interpolatory processes on $[-1, 1]$ for non-smooth functions in L^p–norms. In particular, we are interested in the interpolation of functions of bounded variation and Riemann–integrable functions where the interpolation nodes coincide with the zeros x_k of certain generalized Jacobi polynomials $P_n^{(\alpha,\beta)}$. Furthermore we use the concept of additional nodes y_j, z_j near the boundary ± 1 to control large values of Jacobi parameters α and β. For this extended node–system we investigate the corresponding Lagrange interpolant and compare it with a Lagrange–Hermite interpolant defined by additional conditions on the derivatives in ± 1.

The main result are Marcinkiewicz–Zygmund–type inequalities for algebraic polynomials which give an equivalence of the weighted L^p–norm to some weighted l^p–norm of polynomial values on the zeros x_k, y_j, z_j. By the help of these inequalities we carry over the interpolation error problem to the theory of best one–sided approximation. I.e., results which connect the degree of weighted L^p–approximation of a bounded measureable function f by polynomials p_n with $p_n(x) \leq f(x)$ to smoothness properties of the function f. In other words, we are able to estimate the interpolation error by the best one–sided approximation. This demonstrates that here the best one–sided approximation plays the same role than the ordinary best approximation for the L^p–error of the Fourier–Jacobi series. Moreover we obtain estimates for the simultaneous approximation of the derivatives of the interpolants.

Convergence Theory of Wavelet Approximation Methods for Pseudodifferential Equations

S. Prößdorf (Berlin)

This is a survey of some recent results obtained together with W. Dahmen and R. Schneider [1].

A significant number of recent papers treats Galerkin or collocation methods separately for special cases of operators and for various special choices of trial and test functions. Here we attempt to propose a general framework that allows us to develop a unified approach to all these cases and also to extend previous results. It seems that ascending sequences of nested spaces which are generated by the translates and dilates of a single *refinable* function provide a suitable setting for that purpose. Of course, spline spaces form a typical example which fits into this context. More precisely, we are concerned with generalized Petrov–Galerkin schemes for elliptic periodic pseudodifferential equations in $\mathbb{R}^n$ covering e.g. classical Galerkin methods, collocation and quasiinterpolation. It turns out that the essential conditions that entail optimal convergence rates and stability of the methods can be conveniently formulated in terms of the Fourier transform of the refinable function (ellipticity of the *symbol* of the Petrov–Galerkin scheme under consideration). A crucial point of our method is the observation that the matrices corresponding to the discretized equations for convolution operators with positively homogeneous symbol are circulants. The key to the convergence analysis for pseudodifferential operators with variable symbols is a local principle due to the author. This principle enables us to deduce the stability of the corresponding approximation method for the general pseudodifferential operator A from the stability for a family of convolution operators derived from A by freezing its principal symbol. Its applicability relies here on a sufficiently general version of a so–called discrete commutator property (i.e. a certain *super–approximation* result) in combination with the equivalence of Sobolev norms and certain discrete norms. Moreover, optimal error estimates in the scale of Sobolev norms are established.

Notice that here we focus on the model case of periodic pseudodifferential equations to exploit the full advantages of Fourier transform techniques in connection with appropriate representations for the class of operators under consideration. However, the analysis in the papers [1], [2] is mainly of local nature and therefore remains valid in a nonperiodic setting.

On the other hand, the above mentioned sequences of refinable spaces, often called *multiresolution analysis*, offer convenient ways of constructing wavelets bases. Thus one expects that the present setting should be able to take advantage to the recent interesting developments in this direction. In this regard, there are two issues which are of central importance for the present purposes, namely the preconditioning effect of wavelet bases, as well as the possibility of compressing stiffness matrices relative to wavelet bases in order to obtain sparse matrices (see the paper [2] as well as the lecture given by R. Schneider at this symposium).

References

1. Dahmen, W., S. Prößdorf and R. Schneider: Wavelet approximation methods for pseudodifferential equations I: stability and convergence. Institut für Angewandte Analysis und Stochastik, Berlin, Preprint No. 7, Berlin 1992.
2. Dahmen, W., S. Prößdorf and R. Schneider: Wavelet approximation methods for pseudodifferential equations II: matrix compression and fast solution. Institut für Angewandte Analysis und Stochastik, Berlin, Preprint (to appear).

Smooth Solutions to Linear Equations with Right Invertible Operators

D. Przeworska–Rolewicz (Warsaw)

Let X be a linear space. Consider a linear equation

$$(*) \qquad P(D)x = y\,, \quad \text{where } y \in E \subset X$$

with a right invertible operator $D \in L(X)$ and, in general, operator coefficients. The main purpose of this paper is to characterize these subspaces $E \subset X$ for which all solutions of (*) belong to E (provided that they exist). The largest space with this property is the space

$$D_\infty = \bigcap_{k \geq 1} \operatorname{dom} D^k$$

of *smooth* elements. This leads, even in the classical case of ordinary differential equations with scalar coefficients to a new class of C^∞–functions which properly contains the classes of analytic functions of a real variable and of functions vanishing together with all derivatives at a given point. Some results of [1] proved in the case when X is a Banach space and a right inverse R of D is quasinilpotent are here generalized for complete linear metric space and almost quasinilpotent right inverses. These last results are not yet published.

REFERENCES are included in the author's paper

[1] Przeworska–Rolewicz, D.: Spaces of D-R paraanalytic functions. Dissertationes Math. 302, Warszawa, 1990, pp. 99.

Numerical Solution of the Double Layer Potential Equation over Polyhedral Boundaries

A. Rathsfeld (Berlin)

One popular method for the solution of boundary value problems for elliptic differential equations consists in the reduction to boundary integral equations. For instance, the Dirichlet problem for Laplace's equation in a bounded and simply connected polyhedron $\Omega \subseteq \mathbb{R}^3$ can be reduced to the second kind integral equation $Ax = \{I + 2W_S\}x = y$ over the boundary $S := \partial\Omega$, where W_S denotes the double layer integral operator defined over S. Note that, since S is not smooth, W_S is not compact. For the numerical solution of $Ax = y$, various methods have been introduced. For instance, Wendland has considered the so–called panel method, i.e., piecewise constant collocation, and Angell, Kleinman, Kral, and Wendland have shown that this method is stable for the case of certain rectangular domains Ω. Arbitrary polyhedral domains have been considered by the author. Elschner has analysed the Galerkin method with piecewise polynomial trial functions over arbitrary polyhedrons, and the Galerkin method together with an approximation of the Lipschitz boundary by smooth surfaces has been investigated by Dahlberg and Verchota. For all these procedures, the question arises how to compute the entries of the discretized system of equations. In order to avoid this problem, the author has analysed simple quadrature methods which are similar to those of Graham and Chandler, Kress, and Elschner for the corresponding equation over polygonal boundaries. However, for two dimensional boundaries, these quadrature methods improve the complexity only up to a certain order.

In order to get a fully discretized numerical method which reduces the complexity similarly to the one–dimensional case one needs quadrature methods, where the quadratures and the grids depend on the collocation points, in other words one needs certain discretized collocation methods. The first step in this direction is the stability analysis of piecewise polynomial collocation due to Angell, Kleinman, Kral, Wendland, Atkinson, and Chien. However, all these authors consider uniform grids only. In the present paper we shall consider a method, where the trial functions are taken from a certain space of higher degree tensor product splines defined over a geometrically graded mesh. For this method, we can prove stability and nearly optimal error estimates in the L^2–space. The stability proof is a discretized version of the invertibility proof for the continuous operator A. Analogously to the well-known reduction via Mellin transform to the corresponding invertibility problem for one–dimensional double layer operators, we can reduce the stability problem to the investigation of the collocation method applied to one–dimensional double layer operators (to the "discretized Mellin symbol"). The method of proof requires a certain stability condition. Namely, we have to suppose that certain finite section operators are invertible.

Spline Approximation Methods Cutting Off Singularities

S. Roch (Chemnitz)

Let ϕ be a bounded, measurable, and compactly supported function, set $\varphi_{kn}(t) = \varphi(nt-k)$, and let S_n denote the smallest closed subspace of $L^2(\mathbb{R})$ which contains all functions φ_{kn}, $k \in \mathbb{Z}$. For solving the equation $Au = f$ where A stands, e.g., for a singular integral operator or a Mellin operator, consider approximation equations $L_n A u_n = L_n f$ $(*)$ with L_n referring to a projection from $L^2(\mathbb{R})$ onto S_n (one can take Galerkin, collocation or qualocation projections) and with u_n being seeked in S_n. Standard theory of spline projection methods for SIO (Prößdorf/Rathsfeld, Hagen/Silbermann) entails that applicability of method $(*)$ is equivalent to invertibility of two families of operators, W^t with $t \in \mathbb{T}$ and W_s with $s \in \dot{\mathbb{R}}$, which depend on the sequence $(L_n A|S_n)$ in a natural way. These operators are often of the form $B + K$ where B is a well behaving operator but K is a compact perturbation which essentially complicates the exploitation of this criterion.

In a special situation, Chandler and Graham proposed to overcome these difficulties (and, by the way, some others) by replacing the spline space S_n by the space $S_{n,i}$ which is spanned by functions φ_{kn} with $|k| \geq i$ only. The author shows that the modified spline approximation methods $L_{n,i} A u_{n,i} = L_{n,i} f$ with $L_{n,i} : L^2(\mathbb{R}) \to S_{n,i}$ and $u_{n,i} \in S_{n,i}$ (actually depending on two parameters) are stable iff the family (W^t) of the operators is invertible and if another family, (W_s), which is now formed by operator sequences, is stable. The latter sequences can be thought of as infinite section method sequences for operators in a Toeplitz algebra whose stability can be completely (and verifiably) studied by the author's earlier results concerning finite sections of operators in Toeplitz algebras.

A publication is in preparation.

On Φ–Subdifferentiability and Φ–Differentiability

S. Rolewicz (Warsaw)

Let (X, d) be a metric space. Let Φ be a family of real–valued functions on X. Let f be a real–valued function. We say that $\varphi \in \Phi$ is a (global) Φ–subgradient of the function f at a point $x_o \in X$ if

$$f(x) - f(x_o) \geq \varphi(x) - \varphi(x_o) \tag{1}$$

for all $x \in X$. If there is a neighbourhood U of x_o such that for $x \in U$ (1) holds we say that φ is a local Φ–subgradient of the function f at x_o.

We say that the family Φ has globalization property if from the fact that for all $x_o \in X$ there is a local Φ–subgradient φ_{x_o} of a function f at x_o follows that φ_{x_o} are global Φ–subgradients.

It is easy to see that if (Y, d) is a linear metric space, X is a convex set in Y, and Φ is a restriction of linear functions to X, then Φ has globalization property. There is a natural question, which subsets X of linear metric spaces have the property that the restrictions of linear functions to X have globalization property. In the talk it was shown that for one–connected domain it holds if and only if X is convex. It was also shown that the restrictions of linear functions to the surface of a convex body in R^n have globalization property.

As a consequence we obtain that the family of functions $\Phi = \{\varphi : \varphi(t) = a \sin t + b \cos t\}$ has globalization property. There was presented an example of simple sets $X \in R^n$ about which we do not know: do restrictions of linear functions to X, have globalization property?

Let X, Y be two normed spaces. Let $\mathcal{M}$ be a family of subsets of X. We say that a linear operator φ is an $(\mathcal{M})$–differential of $f : X \to Y$ at x if for each $M \in \mathcal{M}$, each $\varepsilon > 0$ there is a $\delta > 0$ such that

$$||f(x + h) - f(x) - \varphi(h)|| \leq \varepsilon ||h|| \tag{2}$$

for such h that $h \in M$ and $||h|| < \delta$ (in other words: for $h \in M \cap \delta B$, where B is the unit ball in X). We say that φ is an $\mathcal{M}$–differential of f at x if (2) holds for all $h \in \delta M$. For important classes $\mathcal{M}$ both the definitions coincide. There is an example of a normed non–complete space, where they differ.

The following problem is open. Suppose that X is a Banach space. Suppose that $X \subset \cup_{M \in \mathcal{M}} M$. Are both the definitions equivalent?

Calculation of Singularities for Inclusions with Conical Points

A.–M. Sändig (Rostock)

Let Ω_2 be a two– or threedimensional domain with an inclusion Ω_1 in the interior. The domain Ω_1 is polygonal in the twodimensional case or has a rotationally symmetric conical boundary point in the threedimensional case. It follows from the general theory that the solutions of elliptic differential equations in Ω_i, $i = 1, 2$, which satisfy certain transmission conditions on the common boundary $\partial\Omega_1$, have an asymptotic expansion in singular and regular terms. The method, how to calculate these singularities is demonstrated for different examples, namely for the 2D and 3D–Poisson equations, the 2D plate equations and for the 2D and 3D–Lamé equation systems. The singular terms can have the following form: $r^\alpha s_i(\alpha, \varphi, \vartheta)$, $i = 1, 2$, where (r, φ, ϑ) are the spherical coordinates and r is the distance to a conical point. The real parts of the exponents α determine the regularity of the solutions belonging to weighted or usual Sobolev spaces.

They are calculated numerically for some materials and for all openings of the conical points. The corresponding graphs show, when oscillating singularities (α is complex) and when instabilities in the asymptotics (branching points or crossing points with integer–lines) appear.

Convolution Equations on a Union of Two Intervals

A. dos Santos (Lisbon)

Solvability conditions for convolution equations on a union of two disjoint intervals (I) are studied. The solution is sought in a Sobolev space $\tilde{H}_\alpha(I)$ and the right–hand side is given in another Sobolev space $H_\beta(I)$. The method of analysis is based on the reduction of the problem to a vector Wiener–Hopf equation with a symbol that is a 4×4 matrix valued function with oscillating elements. It is shown that the Wiener–Hopf operator associated with this equation has the same Fredholm properties as the original convolution operator on the union of the two intervals.

By a standard procedure the Wiener–Hopf operator is shown to be equivalent to a Wiener–Hopf operator acting on the space $[L_2^+(\mathbb{R})]^4$.

A constructive procedure is given which permits the reduction of the Wiener–Hopf operator with the oscillating symbol to one that is piecewise continuous on $\mathbb{R}$. From this the Fredholm properties of the operator are easily derived.

Finally a sufficient condition for the invertibility of the Wiener–Hopf operator is given.

The work mentioned above was done jointly with M. Amélia Bastos and is incorporated (except for the invertibility section) in the paper "Wiener–Hopf operators with oscillating symbols and convolution operators on a union of intervals" to appear in "Integral Equations and Operator Theory".

Quadrature Methods for Boundary Integral Equations on Curves

J. Saranen (Oulu)

Consider the boundary integral equation which after using a parametric representation takes the form

$$(Au)(t) = \int_0^1 k(t,\tau)u(\tau)d\tau = f(t)\,. \tag{1}$$

We apply the simple $\epsilon-$ and the modified quadrature methods for the numerical solution of (1) assuming that the operator A has the order $\beta < 0$. As particular examples we have Symm's operator with the logarithmic kernel and $\beta = -1$, and the biharmonic single layer operator with the kernel $k(t,\tau) = |x(t)-x(\tau)|^2 ln|x(t)-x(\tau)|$ (where $x : \mathbb{R} \to \Gamma$ is a regular 1–periodic parametrization of the curve Γ) and $\beta = -3$. In both methods we use the composite trapezoidal rule to replace the integral in (1), in the latter method after subtraction of the singularity of the kernel. We set up the quadrature equations by requiring collocation at the evenly spaced meshpoints $t_i^\epsilon = t_i + \epsilon h$, $t_i = ih$, which gives

$$h\sum_{j=1}^{N} k(t_i^\epsilon, t_j)(u_j) = f(t_i^\epsilon)\,,\ 1 \le i \le N \tag{2}$$

for the simple ϵ–quadrature method and, with $\epsilon = 0$,

$$\alpha(t_i)u_i + h\sum_{j=1}^{N} k(t_i,t_j)(u_j - u_i) = f(t_i)\,,\ 1 \le i \le N \tag{3}$$

for the modified quadrature method. Here $\alpha(t) = \int_0^1 k(t,\tau)d\tau$. For the first method we have the stability and the convergence with the maximal rate $O(h^{-\beta})$, $h = \frac{1}{N}$, for classical strongly elliptic pseudodifferential operators of negative order. Moreover, for operators of a special form covering the typical applications the maximal rate $O(h^{-\beta+1})$ is achieved with a special choice of ϵ. Correspondingly, the maximal rates $O(h^{-\beta+1})$, $O(h^{-\beta+2})$ hold in the case of the modified quadrature method. Since for the general curves the function $\alpha(t)$ is not explicitly known, we introduce a numerical approximation $\alpha_h(t)$ for $\alpha(t)$ such that the previous convergence results are retained when $\alpha(t)$ is replaced by $\alpha_h(t)$. We point out that the method (3) yields a symmetric equation for solution of the unknown quantities $u_i \simeq u(t_i)$. Some numerical experiments confirming our theoretical results are also presented.

References

1. Saranen J. and L. Schroderus: Quadrature methods for strongly elliptic equations of negative order on smooth closed curves. SIAM J. Numer. Anal. (to appear).
2. Saranen J. and L. Schroderus: The modified quadrature method for classical pseudodifferential equations of negative order on smooth closed curves (to appear).

A Collocation Method for Singular Integral Equations on Hölder Spaces

U. Schmid (Munich)

For the singular integral equation $(A+K)u = f$, where

$$Au(t) := c(t)\cdot u(t) + \frac{1}{\pi i}\int_{-1}^{1} \frac{d(\tau)}{\tau - t} u(\tau) t\tau$$

and K is a compact perturbation, the classical polynomial collocation method is investigated in a weighted Hölder space

$H_o(\mu,\alpha) := \{f :]-1,1[\rightarrow \mathbb{K} \mid f\cdot\varrho_\alpha \quad \text{is Hölder continuous of order } \mu\}$,

where $\varrho_\alpha(t) := (1+t)^{\alpha_1}(1-t)^{\alpha_2}$.

For this method you construct a function s, depending on the functions c and d, such that $A(s\cdot\mathbb{P}) \subset \mathbb{P}$ (c.f. Junghanns/Silbermann 1981) and look for an approximate solution $u_n \in s\cdot\mathbb{P}_{n-1}$.

In the case considered here the nodes of the Chebyshev polynomials $t_{j,m} = \cos\left(\frac{2j-1}{2m}\pi\right)$, $j = 1,\dots m$ are used as collocation points.

Under certain conditions on the functions c and d A is a Fredholm operator in $H_o(\mu,\alpha)$ (c.f. Duduchava 1970). If its index $k \geq 0$, you can show that under less restrictive assumptions on the k side–conditions $Nu = v$ and under the additional assumption $K \in \mathcal{L}(H_o(\mu,\alpha), C^{r,\gamma}[-1,1])$, where $r \in \mathbb{N}_o$, $\gamma \in]0,1]$ and $2\mu < r+\gamma$, there exists an unique solution of

$$\begin{pmatrix} A+K \\ N \end{pmatrix} u = \begin{pmatrix} f \\ v \end{pmatrix}, \; f \in H_o(\mu,\alpha),\; v \in \mathbb{K}^k$$

and for $n \geq n_o$ you can solve

$$\begin{aligned}(A+K)u_n(t_{j,n-k}) &= f(t_{j,n-k}),\; j = 1,\dots,n-k \\ Nu_n &= v\end{aligned}$$

uniquely in $s\cdot\mathbb{P}_{n-1}$.

The proof is based on a theoretical result of Junghanns and Silbermann (1981). Using a lemma due to Kalandiya (1957) and assuming $f \in C^{r,\gamma}[-1,1]$ one can derive the following error estimate for $n \geq n_o$:

$$||(u-u_n)\varrho_\alpha||_\infty \leq ||u-u_n||_{\mu,\alpha} \leq c\cdot\frac{ln(n-k)}{(n-k-1)^{r+\gamma-2\mu}},$$

where

$$||f||_{\mu,\alpha} = ||f\cdot\varrho_\alpha||_\infty + \sup_{s\neq t}, \frac{|(f\cdot\varrho_\alpha)(s) - (f\cdot\varrho_\alpha)(t)|}{|s-t|^\mu}.$$

Numerical examples confirm these results, but the given order of convergence seems not to be optimal.

Approximation of Poincaré–Steklov Operators with Boundary Elements

Gunther Schmidt (Berlin)

Poincaré–Steklov operators are natural mathematical tools for the investigation of boundary value problems and their numerical solution with boundary decomposition methods. If, for example, the domain is decomposed into nonoverlapping subdomains, then the continuity conditions on common boundaries for the solutions in adjacent subdomains can be interpreted as equations with Poincaré–Steklov operators of these subdomains (cf. [2]). The iterative solution of the equations is known as iterative substructuring, which convergence depends strongly on the mapping properties of Poincaré–Steklov operators and their discretizations determined by the approximation method for the corresponding subproblem.

In this talk we consider Poincaré–Steklov operators for elliptic selfadjoint partial differential equations of second order, which map given Dirichlet data on some part Γ of the piecewise smooth boundary of the domain Ω to the conormal derivative on Γ of the solution of the homogeneous equation with homogeneous boundary conditions on $\partial\Omega\backslash\Gamma$. This operator maps boundedly $\tilde{H}^{1/2}(\Gamma)$ onto its dual, is selfadjoint and positive definite. The finite element solution of the given problem yields the FE–discretization of the Poincaré–Steklov operator, which maps the space of traces on Γ of finite element functions endowed with the $\tilde{H}^{1/2}(\Gamma)$–norm onto its dual space. This mapping is selfadjoint, bounded and positive definite independent of the discretization parameter.

We show that the symmetric Galerkin BEM (cf. [1]) can be used to construct a discretization of the Poincaré–Steklov operator which possesses the same mapping properties as the FE–discretization, if the boundary elements on Γ satisfy some natural conditions.

Therefore in iterative substructuring procedures for elliptic problems the finite element solution of subproblems can be replaced by the solution with the symmetric Galerkin BEM. For the special case of Laplace equation the mentioned results are obtained in [3].

References

1. M. Costabel, E.P. Stephan: The method of Mellin transformation for boundary integral equations on curves with corners. In "Numerical Solutions of Singular Integral Equations", A. Gerasoulis et al., eds., IMACS, 1985, 95–102.
2. Quarteroni, A, A. Valli: Theory and application of Steklov–Poincaré operators for boundary value problems. In "Applied and Industrial Mathematics", R. Spigler ed., Kluwer, Dordrecht 1991, 179–203.
3. G. Schmidt: Boundary element discretization of Poincaré–Steklov operators. IAAS-Preprint No. 9, Berlin 1992, submitted to Num. Math.

Wavelet Approximation Methods for Periodic Pseudodifferential Operators: Matrix Compression and Fast Solution

R. Schneider (Darmstadt)

The advantages of the use of wavelets for the numerical solution of periodic pseudodifferential equations will be discussed in principle.

Based on the framework of multiresolution analysis, biorthogonal wavelets are constructed. They provide a hierarchical basis in L^2 which forms also a Riesz basis. A recursive cascade (pyramide) algorithm performs the transformation from the scaling function basis into a wavelet basis. If the wavelets are compactly supported, such an algorithm requires only $O(N)$ operations, where N denotes the number of unknowns. This algorithm is sometimes called Fast (discrete) Wavelet Transform. An important property of a wavelet basis is that it provides a discrete Littlewood Paley decomposition, i.e., Sobolev (and Besov–) norms of functions can be characterized equivalently by certain weighted l_2–norms of the corresponding coefficients. This result explains the possibility of immediate preconditioning and adaptive approximation. The latter fact is used e.g. for image data compression.

A recent paper of Beylkin, Coifman and Rohklin (C.P.A.M. 1991) demonstrates the adaptive approximation of the Schwartz kernel of Calderon Zygmund operators using a wavelet Galerkin method. Therein they derived a drastical reduction of complexity.

The corresponding algorithm for generalized Petrov Galerkin schemes for the numerical solution of pseudodifferential equations of arbitrary order is examined, taking stability for granted. Two possibilities of representation, namely, the wavelet representation given by the stiffness matrix arising from wavelet bases, and secondly, the atomic representation based on the atomic decomposition of the operator are discussed in parallel. Applying analytical tools developed by Y. Meyer for a recent framework of Calderon Zygmund theory, we show that only $O(N)$ coefficients are required for numerical computation if a fixed, but arbitrary, error bound should not be exceeded. Modifying the above compression, certain order of convergence can also be achieved with an expense of at most $O(N)$, and in the extreme case, $O(N \log^a N)$ operations.

The connection to former fast algorithms as multipole expansion or panel clustering is mentioned. All these algorithms are more or less multilevel algorithms.

Boundary Value Problems for Elliptic Pseudodifferential Operators and some of their Applications

E. Shargorodsky (Tiflis)

The boundary value problems for elliptic pseudodifferential operators (PDO) without transmission property are considered in the Besov and the Bessel potential spaces. Applications to the mathematical theory of cracks, mixed boundary value problems of the elasticity theory for isotropic and anisotropic, homogeneous and nonhomogeneous bodies are discussed as well. The existence and uniqueness theorems for boundary value problems of statics, steady–state oscillation and dynamics are established. Regularity of solutions is treated. Analogous results are valid for boundary value problems of electrodynamics and linearized hydrodynamics.

Mixed initial–boundary value problems (and screen type problems) are studied for nonclassical integro–differential equations, generalizing Sobolev's equation, which describes rotation of fluid, and equation of gravitational gyroscopic waves in exponentially stratified fluid.

Boundary value problems in the half space are investigated for anisotropic elliptic PDO–s with "constant coefficients". The elliptic and 2b–parabolic operators are examples of such PDO–s. From the obtained results it follows, for example, that for the heat conduction equation $\partial u/\partial t - (\Delta - 1)u = f$, $t > 0$, the Cauchy problem $u|_{t=0} = \varphi$ is uniquely solvable in proper functional spaces, while for the heat conduction equation "with inverted time" $\partial u/\partial t + (\Delta - 1)u = f$, $t > 0$, the initial conditions are superfluous. The last equation is uniquely solvable in the corresponding anisotropic Besov and Bessel potential spaces.

The case of boundary value problems on two–dimensional manifolds is studied in detail. The problems with boundary conditions containing either the operator of complex conjugation or analytic projections are considered. They generalize the Hilbert problem and the problem of linear conjugation for analytic functions. Applications to the differential boundary value problems for the generalized analytic vectors and for the equation $\partial^{m+n}u/\partial\bar{z}^m\partial z^n = f(z)$, $z \in \Omega = \mathbb{C}$ are given. In particular, it is proved that for the Bitsadze equation $\partial^2 u/\partial\bar{z}^2 = f(z)$ not only the Dirichlet and the Neumann problems are non–Noetherian (= non–Fredholm), but also the problem $Bu|_{\partial\Omega} = \varphi$, where B is an arbitrary $\mathbb{C}$–linear differential operator (more generally, for the Bitsadze equation any boundary value problem from the Boutet de Monvel's algebra would be non–Noetherian).

Non–Strongly Converging Approximating Methods

B. Silbermann (Chemnitz)

The aim of my talk is to present some concept of non–strongly converging approximating methods.

Let $\mathcal{M}$ stand for the set of all finite unions of left–sided closed and right–sided open intervals of $\mathbb{R}_+$. Assume that to each $U \in \mathcal{M}$ a projection operator R_U (acting on some Banach space X) is associated such that

- $R_U + R_{\mathbb{R}_+ \setminus U} = I$,
  $R_U R_V = R_V R_U = R_{U \cap V}$,
  $R_U + R_V = R_{U \cup V} \quad \text{if } U \cap V = \emptyset$
- $\sup\limits_{U \in \mathcal{M}} \|R_U\| < \infty$
- $\bigcap\limits_{w \in \mathbb{R}_+} \ker R_{[0,w)} = \{0\}$.

The collection of all these projection operators is denoted by R. With R there are connected three important notions

- R–convergence of elements
- R–convergence of operators
- operators of R–local type

Note that R–convergence is in general weaker than norm or strong convergence, respectively. Then a special case of a general converging theorem reads as follows:

Suppose that the bounded operator $A : X \to X$ is invertible and of R–local type. Then the projection method $\left\{P_{[0,\tau)} A P_{[0,\tau)}\right\}$ R–converges for the operator A if and only if the sequence $\left\{P_{[0,\tau)} A P_{[0,\tau)}\right\}$ is stable.

References

1. Roch, S., B. Silbermann: Non–strongly converging approximation methods. Demonstratio Mathematica, vol. XXII, Nr. 3, 1989.
2. Prößdorf, S., B. Silbermann: Numerical Analysis for Integral and Related Operator Equations. Akademie–Verlag und Birkhäuser–Verlag, 1991 (Chapter 4).

Error Estimates for the Approximation of Semi–Coercive Variational Inequalities

W. Spann (Munich)

Semi–coercive variational inequalities arise in a wide area of physical phenomena, for example Signorini problems in elasticity where no Dirichlet data on parts of the boundary are prescribed. The abstract variational inequality

$$u \in K\,,\ a(u, v-u) \geq f(v-u) \quad \text{for all } v \in K$$

with a positive semidefinite, not necessarily symmetric, bilinear form satisfying Garding's inequality is considered. Here K denotes a closed convex set in a Hilbert space V. The discretization yields a variational inequality

$$u_h \in K_h\,,\ a_h(u_h, v_h-u_h) \geq f_h(v_h-u_h) \quad \text{for all } v_h \in K_h$$

where $V_h \subset V$ is a closed linear subspace, $K_h \subset V_h$ is closed and convex without assuming $K_h \subset K$, and a_h, f_h are approximations of a, f respectively. For the case of coercive bilinear form a (on $\overline{\text{span}K}$) and uniformly coercive a_h (on $\overline{\text{span}K_h}$) error estimates are well–known. However, this does not apply to semi–coercive variational inequalities. For the approximation we require that a_h is positive semidefinite uniformly satisfying Garding's inequality and that a_h, f_h approximate a, f in the sense of Strang's conditions. Furthermore, dist $(u, K_h) \to 0$ and dist $(v_h, K) \to 0$ uniformly in $\{v_h \in K_h : ||v_h|| \leq r\}$ for $r > r_o$ is supposed.

The variational inequality is assumed to be uniquely solvable and to satisfy the intersection condition $\{v \in V : a(v,v) = 0\} \cap \{v \in V : a(u,v) = f(v)\} \cap \overline{\text{cone}(K-u)} = \{0\}$. For the discrete variational inequality we suppose that a solution exists. Then an error estimate generalizing Falk's estimate for the coercive case is obtained. It can be proven that the intersection condition is even necessary in order to imply that the error estimate holds.

These results are applied to the obstacle problem for the beam with free ends. The intersection condition turns out to be satisfied if the center of the external forces belongs to the interior of the coincidence set. For the case of sufficiently smooth data and assuming that the coincidence set is a finite union of closed nondegenerate intervals the approximation by cubic splines that satisfy the obstacle condition in the grid points gives the optimal convergence order $0(h^{3/2-\epsilon})$ in the H^2–norm. This is also observed in the numerical examples presented.

Meromorphic Factorization, Partial Index Estimates and Elastodynamic Diffraction Problems

M.C. Câmara, A.B. Lebre and F.-O. Speck (Lisbon)

This work is motivated by some elliptic boundary and transmission problems in mathematical physics, in particular by elastodynamic wave propagation. The analytical solution of the boundary pseudodifferential equations requires a generalized factorization of the lifted Fourier symbol which is a non-rational matrix-function. In the factorization procedure poles and increasing terms appear, and cause enormous practical and theoretical problems due to the possible occurance of partial indices different from zero. The paper presents an approach which avoids those difficulties by use of a factorization of the symbol matrix into meromorphic factors. An operator theoretic interpretation yields resolvents up to finite dimensional operators, whose ranks are closely related to partial indices, order of the algebraic increase at infinity, and the multiplicities of the poles in the factors.

For details, see [1].

1. Meromorphic Factorization, Partial Index Estimates and Elastodynamic Diffraction Problems. Math. Nachr. **157** (1992) 291–317.

h-p Version of the Boundary Element Method for Two– and Threedimensional Problems

E.P. Stephan (Hanover)

This lecture presents the h-p–version of the boundary element Galerkin method for first kind integral equations on polygons and on open surface pieces. The h-p version is a combination of both the h version (which achieves higher accuracy by refining the mesh) and the p version (which achieves higher accuracy by increasing the polynomial degree of the trial functions). The integral equations under consideration are those with the single layer potential and the normal derivative of the double layer potential, respectively. In case of quasiuniform meshes it is shown that the convergence of the p version is twice as fast as the h version, and the rate of convergence is restricted by the corner singularities and the edge singularities of the exact solutions of the integral equations. If a geometric mesh refinement towards the corners (in the 2D case, i.e. on the polygon) and towards the edges (in the 3D case, i.e. on the surface pieces) is used together with an appropriate p–refinement (i.e. increasing polynomial degrees away from corners or edges), we obtain exponentially fast convergence of the Galerkin error in the energy norm. Also adaptive refinement strategies based on the local residues are given both for the h and the h-p version. Furthermore, numerical experiments are given which underline the theoretical results.

Wiener–Hopf–Hankel Operators and Diffraction by Wedges

F.S. Teixeira (Lisbon)

The subject of this lecture lies in the topic "Applications of integral and pseudodifferential equations". It is divided into two parts: in the first one we present a survey on some problems in Mathematical Physics which give rise to convolution type operators, namely problems of Diffraction Theory and their operator–theoretic approach. In the second part, which follows a paper by E. Meister, A. Lebre and F.S. Teixeira, entitled "Some results on the invertibility of Wiener–Hopf–Hankel Operators" and published in Z.A.A., vol. 11 (1992) 1, 57–76, a study is presented on the invertibility properties of scalar operators defined as the sum of a Wiener–Hopf and a Hankel operator on $L_2(\mathbb{R}^+)$ with symbol in $L_\infty(\mathbb{R})$. This study is based on the properties of a vector Wiener–Hopf operator naturally associated with each of the operators mentioned above. In particular, it is shown that the invertibility of such an associated operator is equivalent to the simultaneous invertibility of the Wiener–Hopf–Hankel operators $W(a) + H(b)$ and $W(a) - H(b)$, and explicit analytic formulas are given for the inverse operators. At the end we consider an application of these results to the diffraction problem of a time–harmonic electromagnetic wave by a rectangular wedge, one of whose faces is perfectly conducting and the other having a prescribed impedance (finite or infinite). The problem, initially formulated as an exterior boundary value problem for the two–dimensional Helmholtz equation in the Sobolev space $H_1(\Omega)$, with a Dirichlet condition on one face of the wedge and a third kind boundary condition on the other, is reduced to an equivalent pseudodifferential equation of Wiener–Hopf–Hankel type in the trace spaces $H^+_{\pm 1/2}(\mathbb{R})$. By the standard lifting procedure, using Bessel Potential operators, that equation is seen to be equivalent to a Wiener–Hopf–Hankel equation on $L^+_2(\mathbb{R})$, with piecewise continuous symbols. The Fredholm property is obtained, and for the case of infinite impedance we prove the existence and uniqueness of the solution to the corresponding equation. In that case an explicit solution is given in terms of the generalized factorization of the presymbol of the associated vector Wiener–Hopf operator.

Higher Order Collocation Methods for Multidimensional Weakly Singular Integral Equations

G.M. Vainikko (Tartu)

Consider the integral equation

$$u(x) = \int_G K(x,y)u(y)dy + f(x)$$

where $G \subset \mathbb{R}^n$ is a parallelepiped, f is defined and smooth on $\overline{G}$ and the kernel is weakly singular, e.g. $K(x,y) = a(x,y)\log|x-y|$ or $K(x,y) = a(x,y)|x-y|^{-\nu}$, $0 < \nu < n$, with a smooth coefficient a which is bounded on $G \times G$ together with derivatives. Dividing G into boxes which correspond to graded grids in coordinate directions, a collocation method with piecewise polynomial functions of degree $m-1$ with respect to any of arguments $x_1, ..., x_n$ is introduced; in any box, m^n collocation points are obtained by a standard affine transformation of fixed nodes $\xi^1, ..., \xi^m$ in $[-1,1]$ into corresponding subdivision intervals. The global estimates $\max_{x\in\overline{G}} |u_N(x) - u(x)| \leq ch^m$ as well the superconvergence estimates at collocation points are derived, e.g. in the case of Gaussian nodes $\xi^1, ..., \xi^m$,

$$\varepsilon_N \leq ch^m \left\{ \begin{array}{lll} h^m & , & n-\nu > m \\ h^m|\log h| & , & n-\nu = m \\ h^{n-\nu} & , & n-\nu < m \end{array} \right\}$$

where ε_N is the maximal error at collocation points.

For more details, see [1].

1. Vainikko, G.: Multidimensional Weakly Singular Integral Equations: Analysis and Numerics (to appear).

Stability and Convergence of a Hyperbolic Tangent Method for Singular Integral Equations

E. Venturino (Iowa)

In this paper we reexamine a previously proposed quadrature scheme [Venturino], based on a formula proposed by [Stenger, 1976]. Our goal is to establish a stability result for the dominant singular integral equation of index one, and from it derive the error analysis for the proposed numerical method. We show that the norm of the inverse matrix of the discretized linear algebraic system, to which the original equation is reduced, grows like the square root of the number of unknowns.

The convergence of the method nevertheless is not affected by this growth, since the consistency error is exponentially convergent, and this feature is retained by the error of the solution.

A technical point must be made here. In discretizing the system, a new unknown function is defined, which has the nice property of vanishing at the endpoints. The discretization leads to a rectangular system, but in view of the previous property, the parts of the unknown vector corresponding to nodes close to the endpoints are removed, and the corresponding columns of the matrix as well. These terms must be accounted for in the error analysis.

The convergence of the method can then be extended to the first kind complete equation, although in this case a very strong assumption on the coefficients must be made. The results are then shown also for the equation of index zero.

Finally the necessary modifications for applying this analysis to a recently proposed scheme [Zhang] for Hadamard finite part integral equations are examined. Because of the good stability result, where the norm of the inverse is of order h^2, the method gains in convergence rate. Indeed it should be remarked that the quadrature scheme is modified in order to adapt it to finite part integrals [Zhang], but in so doing the consistency error loses the exponential convergence and becomes only quadratically convergent. The final convergence rate for the method is shown to be three.

In all these proofs the necessary assumption we need to make is to restrict our considerations to a compact subset of the interval in which the equation is formulated, containing all nodes where the unknown is evaluated.

REFERENCES

F. Stenger (1976), Approximations via Whittaker's cardinal function. J. Approx. Theory 17, p. 222–240.

E. Venturino (1985), On solving singular integral equations via a hyperbolic tangent quadrature rule. Math. Comp. 47, p. 159–167.

F. Zhang (1991), A hyperbolic tangent quadrature rule for solving singular integral equations with Hadamard finite part integrals. Comp. Math. Applic. 22, p. 59–73.

(The paper is submitted to Mathematische Nachrichten.)

Discrete Nonliner Riemann–Hilbert Problems

E. Wegert (Freiberg)

Let $\{M_t\}_{t\in\mathbb{T}}$ be a given family of curves in the complex plane. A nonlinear Riemann–Hilbert problem consists in finding all functions w which are holomorphic in the complex unit disk $\mathbb{D}$, extend continuously up to its boundary $\mathbb{T}$, and satisfy the boundary relation

$$w(t) \in M_t \quad \forall t \in \mathbb{T}. \tag{1}$$

A solution to (1) that meets the additional condition

$$\alpha Re\, w(0) + \beta Im\, w(0) = \gamma \tag{2}$$

at the origin is called a regular solution of (1), (2) if

$$wind\, \nu = 0$$

and

$$\alpha \cos\delta + \beta \sin\delta = \gamma\,,$$

where $\nu(t)$ denotes the tangent to M_t at $w(t)$ and

$$\delta := \frac{1}{2\pi}\int_0^{2\pi} \arg \nu(e^{i\tau})d\tau\,.$$

We describe a (nondiscrete) quadratically convergent Newton type method for the iterative computation of a regular solution. A straightforward discretization, however, disturbs convergence. Alternatively, we propose a collocation method with trigonometric polynomials. The existence and local uniqueness of a solution and an optimal error estimate in the scale of Sobolev space $W_2^k(\mathbb{T})$ are shown. Applying Newton's method to the discrete problem yields a quadratically convergent projection–iteration method. If one uses fast solvers for Toeplitz systems the computational complexity is of order $N \log^2 N$ per iterative step.

In the special case of conformal mapping similar results go back to R. Wegmann [1], [2].

References

1. Wegmann, R.: Convergence proofs and error estimates for an iterative method for conformal mapping. Numer. Math. **44** (1984), 435–461.
2. Wegmann, R.: Discrete Riemann–Hilbert problems, interpolation of simply closed curves, and numerical conformal mapping. J. Comput. Appl. Math. **23** (1988), 323–352.
3. Wegert, E.: Iterative methods for discrete nonlinear Riemann–Hilbert problems. J. Comput. Appl. Math. (to appear).
4. Wegert, E.: Nonlinear Boundary Value Problems for Holomorphic Functions and Singular Integral Equations. Akademie–Verlag 1992, 240 p.

Substructuring for Boundary Integral Equations

W.L. Wendland (Stuttgart)

The lecture reports substructuring formulations involving boundary element methods and some corresponding analysis by G.C. Hsiao, B.N. Khoromskij and W.L. Wendland [4, 5, 6, 7].

1. Macro–elements and FEM–BEM coupling

For a combined FEM–BEM approximation of the Dirichlet problem in variational form, e.g. the Laplacian or the Lamé system, the given domain Ω is decomposed into subregions Ω_j corresponding for $j \in J_F$ to the finite element partition and for $j \in J_M$ to so–called macro–elements. The family of subdivisions is characterized by a mesh–width parameter H. $\mathcal{F}$ denotes a finite element space in $H^1(\Omega)$ associated with $\{\Omega_j\}_{j\in J_F}$. The space $\mathcal{F}_{|\tilde{\Gamma}}$ of traces on the sceleton $\tilde{\Gamma} = \dot{\cup}\partial\Omega_j$ is extended to the whole sceleton. The well known finite element variational formulation is to find $u_F \in \mathcal{F}$ satisfying the Dirichlet condition $u_F|_{\partial\Omega} = \phi_F$ such that for all $v_F \in \overset{o}{\mathcal{F}} \subset \overset{o}{H}{}^1(\Omega)$ there holds

$$\sum_{j\in J_F} \int_{\Omega_j} \sigma(u_F)\cdot e(v_F)dx + \sum_{j\in J_M} \int_{\Gamma_j} \lambda_j(u_F)v_F ds_j = 0$$

where $\lambda_j(u_F) = \tau(u_F)|_{\Gamma_j}$ denotes the Steklov–Poincaré mapping associated with the Laplacian or the Lamé system in Ω_j. These individual mappings can approximately be constructed by appropriate boundary element equations which require additional boundary element subdivisions and spline approximation on the macro–element boundaries Γ_j, $j \in J_M$. This coupling procedure is widely used in engineering structural analysis. Brezzi and Johnson provided asymptotic error estimates in [2] which were extended in [7], also to the opposite case of finer finite element refinements; a detailed analysis including boundary approximation is presented in [3].

2. Domain decomposition and a parallel algorithm

The above–mentioned coupling formulation can also be used for a domain decomposition method with boundary elements [4]. In [5], a parallel algorithm based on the so–called Glowinski–Wheeler preconditioner for two subdomains is presented whose convergence follows from [1]. In the lecture, this method is extended to a new two–level multigrid iteration where the coarse grid corresponds to the cross points of the sceleton. The method requires fast solution of individual Dirichlet problems, Neumann problems and mixed problems in each of the macro–elements and belongs to the class investigated in [6].

References

1. Agoshkov, V.I.: Poincaré–Steklov operators and domain decomposition methods in finite–dimensional spaces. Proc. First Intern. Symp. Domain Decomposition (eds. R. Glowinski et al.) SIAM, Philadelphia (1988) 73–112.
2. Brezzi, F., C. Johnson: On the coupling of boundary integral and finite element methods. Calcolo **16** (1979) 189–201.
3. Gatica, G.N., G.C. Hsiao: On the coupled BEM and FEM for a nonlinear exterior Dirichlet problem in $\mathbb{R}^2$. Numer. Math. **61** (1992) 171–214.
4. Hsiao, G.C., W.L. Wendland: Domain decomposition in boundary element methods. Domain Decomposition Methods for PDE. (eds. R. Glowinski et al.) SIAM, Philadelphia (1991) 41–49.
5. Hsiao, G.C., W.L. Wendland: Domain decomposition via boundary element methods. Proc. Intern. Congress Numer. Anal. Engng. and Appl. Sci., Concepcion 1992, to appear.
6. Khoromskij, B.N., W.L. Wendland: Spectrally equivalent preconditioners for boundary equations in substructuring techniques. East–West J. Numer. Analysis **1** (1992) 1–27.
7. Wendland, W.L.: On asymptotic error estimates for combined FEM and BEM. Finite Element and Boundary Element Techniques from Mathematical and Engineering Point of View (eds. E. Stein, W. Wendland). CISM No. 301, Springer–Verlag Wien (1988) 273–333.

On the Theory of Nonlinear Singular Integral Equations

L. v. Wolfersdorf (Freiberg)

In the lecture there was given an overview about some results and also open problems for nonlinear singular integral and integro–differential equations of Cauchy type.

(1) Application of Schauder's fixed point theorem and of methods of monotone operator theory to Hammerstein type and related integral equations and to a nonlinear version of Prandtl's integro–differential equation of airfoil theory. Further by monotone operator theory in combination with Banach's fixed point theorem and by Schauder's fixed point theorem, respectively, to mixed Volterra and singular integral equations of several kinds.

(2) As open problems there were mentioned: Nonlinear singular integro–differential equations in infinite intervals, for instance Prandtl's equation on the half axis and Peierls' equation in dislocation theory; nonlinear singular integral equations of Hammerstein and Uryson type with shift; nonlinear singular integral equations of polynomial type, for instance a bilinear singular integral equation from hydrodynamics for plane potential flow past and through a cylinder with porous surface.

(3) Briefly mentioned there were also three time–dependent equations, namely the Benjamin–Ono equation of wave theory and the Satsuma–Mimura equation from population dynamics both containing the Hilbert transform on the real axis and further the well–known Birkhoff–Rott equation describing the evolution of a vortex sheet in two–dimensional ideal flow.

On the Generalization of the Least Squares Method for the Operator Equations in Some Frechet Spaces

D. Zarnadze (Tiflis)

The generalization of the classical least squares method for the linear equation with an operator mapping a Frechet space into a Frechet space is given. The approximate solution is found by means of minimization the discrepancy with respect to the metric, which in the case of Hilbert spaces coincides with the metric, generated by the scalar product. The convergence of the sequence of approximate solutions to the exact solution is proved. Some estimates are also proved and the concretization of these estimates for the operators between the power series spaces of finite and infinite type are given.

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18. I. Schmelzer: Quantization and measurability in gauge theory and gravity.

19. A. Bovier V. Gayrard: Rigorous results on the thermodynamics of the dilute Hopfield model.

20. K. Gröger: Free energy estimates and asymptotic behaviour of reaction–diffusion processes.

21. E. Platen (ed.): Proceedings of the 1^{st} workshop on stochastic numerics.