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A reduced-order modeling for efficient design study of artificial valve in enlarged ventricular outflow tracts

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Abstract

A computational approach is proposed for efficient design study of a reducer stent to be percutaneously implanted in enlarged right ventricular outflow tracts (RVOT). The need for such a device is driven by the absence of bovine or artificial valves which could be implanted in these RVOT to replace the absent or incompetent native valve, as is often the case over time after Tetralogy of Fallot repair. Hemodynamics are simulated in the stented RVOT via a reduce order model based on proper orthogonal decomposition (POD), while the artificial valve is modeled as a thin resistive surface. The reduced order model is obtained from the numerical solution on a reference device configuration, then varying the geometrical parameters (diameter) for design purposes. To validate the approach, forces exerted on the valve and on the reducer are monitored, varying with geometrical parameters, and compared with the results of full CFD simulations. Such an approach could also be useful for uncertainty quantification. Device design, percutaneous pulmonary valve replacement, proper orthogonal decomposition (POD), finite element method, blood flow CFD, repaired Tetralogy of Fallot

1 Introduction

Tetralogy of Fallot (ToF) is a congenital heart disease characterized after the initial repair, by the absence of a functioning pulmonary valve, which causes blood flow regurgitation in the right ventricle during diastole. Although valve replacement is thus often warranted at some course of the disease progression, valved stents (Melody valve, Medtronic Inc.) are available in diameter up to 22 mm making percutaneous pulmonary valve implantation impossible in post-operative Fallot patients [4], often characterized by enlarged right ventricle outflow tracts (RVOT) [1]. To avoid redo-surgery in these patients, some authors have advocated to create a landing zone by reducing the RVOT externally similar to a pulmonary artery (PA) implanting a *reducer stent* [1, 4, 11]. This self-expandable device, which has shown promising results on animals, has not been implanted yet in patients.

Device design typically requires many three-dimensional time-dependent simulations (see, e.g., [12, 16]). The goal of this study is to test a reduced-order model (i) to efficiently predict hemodynamics changes due to percutaneous pulmonary valve reducer, and (ii) to quantify the forces acting on the device that could lead to its migration or rupture.

The valve is modeled with a simplified model [2], which focuses only on the main dynamics. Moreover, proper orthogonal decomposition (POD) is used to further reduce the size of the discrete problem. In the context of blood flows, the POD was already used to interpolate flow field measured on medical images [9, 10], while a POD model for ToF patients – without any device – was proposed in [7]. See also [8] for alternative model-order reduction techniques in hemodynamics. In this work, we employ the POD to precompute reduced order models of the device when varying the diameter, hence allowing a faster device design. In order to validate the method, we perform both reduced and full finite element simulations, monitoring the difference in the main quantities of interest.

2 Computational models

The surface representation of ToF patient pulmonary arteries (pulmonary outflow tract and first pulmonary artery bifurcation) was extracted from medical imaging (Figure 1, left). A geometrical model of the toroïdal reducer stent described in [11] was manually created in intersecting the original surface with a device of length 27 mm and internal diameter of 19 mm. Next, from

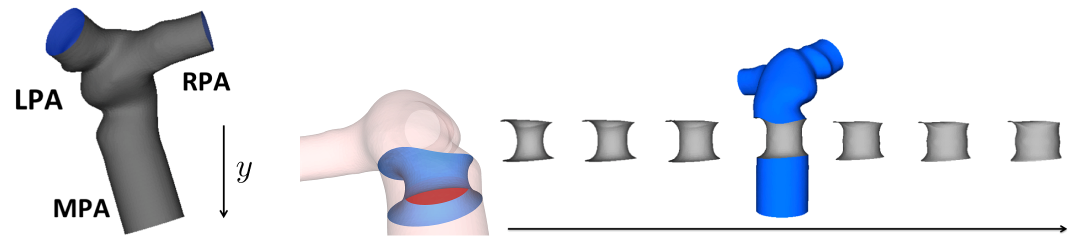

Figure 1: Left: The surface used for the computational study, consisting of a segment of main pulmonary artery (MPA) with an inlet diameter of 13.6 mm, and two branches of length 26 mm (left PA) and 34 mm (right PA), respectively, with outlet boundaries mean diameters of 19 mm (LPA) and 16 mm (RPA). The orientation of the y-axis is also shown. Center: Computational model of the reducer (blue) and of the valve (thin surface, in red). Right: Model of reducers of different diameters obtained with reducer length 27 mm and diameter 19 mm (middle figure). The resulting reducer have inner diameters varying from 14 mm to 24 mm.

the obtained mesh a set of geometries with reducers of different diameters (from 14 mm up to 24 mm) was generated, smoothly deforming the original configuration. This step consists in solving numerically a hyperelasticity problem on the original three-dimensional geometry (imagined as filled by an elastic material) and applying a surface normal force on the reducer surface. With this approach, the meshes with different diameters maintain exactly the same discrete topology as the reference one, in terms of nodes connectivity and elements.

Blood flow in the PA is assumed to be described by the incompressible Navier-Stokes equations, which were solved numerically with a time-discretization based on a Chorin-Temam projection scheme [5, 14] and piecewise linear finite elements. In particular, blood density and viscosity were set as $\rho = 1$ g/cm^3 and $\mu = 0.04$ Poise, respectively, assuming a Newtonian fluid. The pulmonary valve is modeled as a resistive immersed surface [2], i.e. considering the valve as a static thin porous interface which resistance to flow depends on the current flow and pressure conditions across the valve surface, varying dynamically between zero, when the valve is open, to a very high value, when the valve is closed. In particular, the valve closes whenever the flow becomes negative (or below a given threshold), while it reopens when the pressure gradient is favorable (decreasing in the direction of the pulmonary artery). This approach does not requires neither a spatial resolution of the three-dimensional valve geometry, nor a coupled fluid-structure solver, while being very robust [2]. This choice is further justified by the fact that this study is not focused on the detailed blood flow around the valve itself, but rather on the general hemodynamics features of the entire device.

At the inlet boundary, a time-varying right ventricular pressure is prescribed, while the outlets are treated using three elements lumped parameter models in order to take into account the effect of the downstream vasculature [6, 15, 7]. Finally, no slip boundary conditions are imposed for the velocity on the vessel wall. Note that since the different reducers have a covered stent [1, 11], the reducer was considered as part of the wall.

3 Proper orthogonal decomposition for device design study

The goal of proper orthogonal decomposition (POD) is to determine a set of functions which, even if containing a small number of elements, can represent sufficiently well a given set of input data [3, 13]. Within a finite element method, a POD basis allows to restrict the space of the numerical solution from a standard finite element space (e.g. $O(10^5)$ degrees of freedom for the cases considered here) to a low order subspace (typically consisting of few tenths of basis functions), hence considerably reducing the complexity of the model. In order to compute the bases for velocity and pressure, we used the method of snapshots: the POD bases are obtained via singular value decomposition of the two *snapshot matrices*, i.e. matrices whose columns are defined by the numerical solutions at different time steps. In particular, 400 snapshots per heart beat were considered, from which 30 basis elements were extracted for velocity and pressure. Hence, the discrete problems for velocity and pressure can be formulated in terms of *reduced solutions* requiring, at each time step, only the solution of small linear systems (two systems 30×30 in this case).

The aim of this work is to employ the POD to perform faster fluid simulations when varying the geometrical parameters using the approach similar to the one described in [7]. First, POD bases were extracted from a full finite element simulation on a *reference* geometry (the one in the middle of Figure 1, right); next, for each different shape obtained by deforming the reference mesh, a suitable POD basis for the reduced model was precomputed by mapping the reference POD basis functions onto the new meshes (without the need of performing additional finite element simulations); finally, the fluid solution on the new domain was obtained by solving a reduced model, i.e. only a reduced simulation (two problems of size 30×30) needs to be carried out online.

4 Validation

In terms of efficiency, the reduced simulations resulted up to three times faster than the full simulations. Although this allows already a good speed-up, note that the simulations were carried out using an in-house finite element code, which is not optimized for reduced order modeling. Thus, the reduced model efficiency is expected to be further improvable.

Next, in order to validate the accuracy of reduced-order approach, we performed both *full* (finite element) and *reduced* (POD) simulations of various configurations (reducer stents with different diameters). Two snapshots of the solutions are shown in Figure 2 for the most defavorable case, i.e. the smallest device. In both systole and diastole the comparison demonstrates that, although the velocity magnitude is slightly lower, the POD can reproduce the main flow characteristics of the full simulations. Figure 3 presents the pressure forces on the device (reducer and valve) and the mean flow rates, further demonstrating that the reduced model is able to reproduce very accurately the full simulation. Moreover, for each quantity the average and maximum errors were monitored:

$$E_{\text{mean}} = \frac{1}{N}\sum_{n=1}^{N} \frac{\|X^{\text{POD}}(t_n) - X^{\text{full}}(t_n)\|}{\max \|X^{\text{full}}\|}, \; E_{\max} = \frac{\max \|X^{\text{POD}} - X^{\text{full}}\|}{\max \|X^{\text{full}}\|}, \tag{1}$$

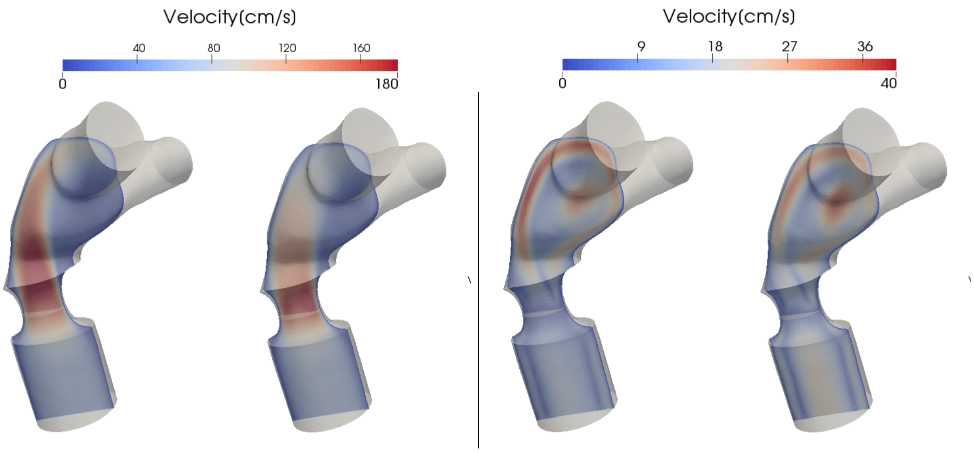

Figure 2: Velocity magnitude on a longitudinal cut at the time instants of (left) systole and (right) diastole, comparing the full simulation results (1rst & 3rd columns) and the POD solution (2nd & 4th columns).

defined as the difference in a quantity of interest (e.g. with X corresponding to the pressure forces or to the flow rates) between the full and POD simulation (renormalized by their maximum values over a heart beat). The results are summarized in Figure 4 for different diameters. The errors remain in all cases very low, on the order of 5%, reaching 11% only in the largest deformation case. In general reducing the diameter yields to slightly larger errors than increasing it. This could be due to the fact that reducing the diameter significantly increases the peak velocity (by 15% from 19mm to 14mm).

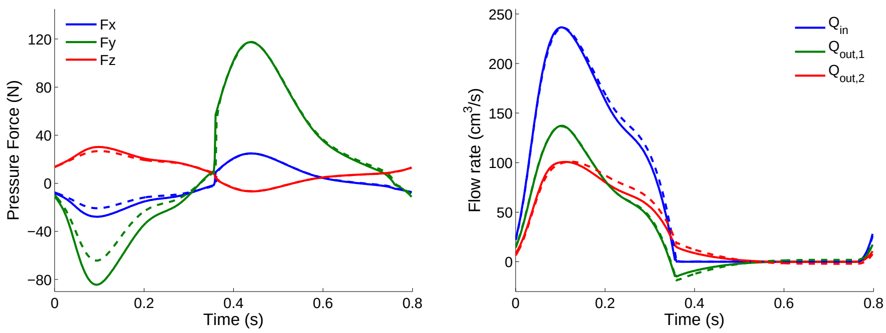

Figure 3: Continuous line: full finite element simulation; dashed line: reduced order simulation with 30 basis functions computed from the reference geometry (diameter 19 mm). Left: pressure force over time for the simulation with the smallest diameter (14 mm); Right: inlet and outlet flow rates over time for the simulation with the smallest diameter (14 mm).

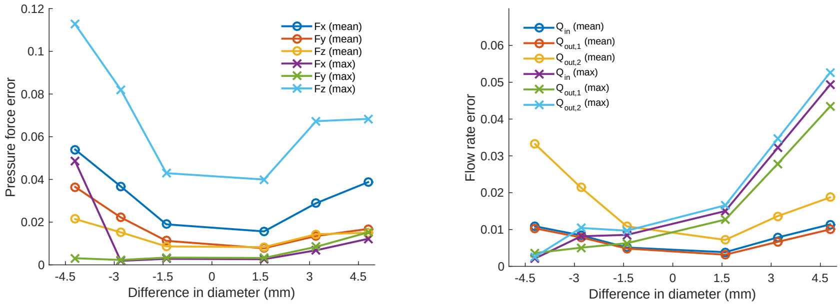

Figure 4: Average and maximum renormalized errors (1) of the reduced order model against a full finite element simulation, as a function of the diameter of the reducer. Left: Pressure forces. Right: Flow rates.

5 Conclusion

The POD approach offers a promising way to reduce computational complexity for optimal device design. The main flow characteristics are preserved. The mean errors in forces on the device and flow rates were all below 6%, with larger errors for decreasing diameters. This may be caused by larger geometrical deformation and peak velocity. Moreover, the computational time for the reduced simulation was, without optimization of the code, in all cases between 25% and 30% of the time needed for the full solution. Such strategy could also be effective in uncertainty quantification of geometrical parameters due to low medical image resolution issues.

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