Weierstraß–Institut
für Angewandte Analysis und Stochastik

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Error bounds of discretization methods
for boundary integral equations
with noisy data

Gottfried Bruckner[1] , Siegfried Prößdorf[1], Gennadi Vainikko[2]

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[1] Weierstraß-Institut
für Angewandte Analysis
und Stochastik
Mohrenstraße 39
D – 10117 Berlin
Germany

[2] Institute of Mathematics
Helsinki University of Technology
Otakaari 1
SF – 02150 Espoo
Finland

Dedicated to Professor George C. Hsiao on the occasion of his 60th birthday.

ABSTRACT. The influence of small perturbations in the kernel and the right–hand side of boundary integral equations, e.g. of Symm's integral equation, discretized by collocation or quadrature formula methods, is analyzed in Sobolev and Hölder–Zygmund norms.

Introduction

In this paper we analyze the influence of small perturbations in the right–hand terms and kernels of some boundary integral equations (BIE) and pseudodifferential equations (PE), discretized by Galerkin, collocation, quadrature formulae or related methods. The rounding errors cause perturbations of order, say $10^{-10}\%$; the measurement and modelling errors may cause much larger perturbations, say of order $0, 1 - 1\%$. Both types of perturbations are hard to be controlled, therefore we assume to be given only their possible magnitude. Of course, also controllable perturbations, caused e.g. by a data compression in the stiffness matrix or by numerical integrations to complete a discretization, may be taken into account.

An important feature of (elliptic) BIE and PE is that the underlying operators build isomorphisms between appropriate pairs of spaces in Sobolev and Hölder scales. Effective discretizations present similar isomorphisms uniformly with respect to the discretization parameter – this is the essence of the stability property of a discretization method. An establishment of the stability is not a purpose of this paper, we mainly consider methods with already known stability properties. In different norms, the influence of perturbations of data is of different magnitude. Considered as equations in L^2, BIE of the first kind and PE of a negative order are ill–posed. Our estimates give an insight how the discretization parameter should be chosen to obtain a regularization effect; no special regularization of the problem is needed. This phenomenon is sometimes called the self–regularization of an ill–posed problem through its discretization.

In some abstract settings, the self–regularization of ill–posed problems through projection methods has been analyzed by Natterer [8], and Vainikko and Hämarik [13]. In [8] only the right–hand term of the equation was perturbed; in [13] only a pair of spaces (and not scales) was used. It is reasonable to present newly and independently an error analysis in an abstract setting with consequences to the self–regularization. This is done in Section 1. The corresponding results and arguments are elementary. Our main results concern applications to concrete discretization methods (Sections 2-4). A key to these results is a sufficiently sharp analysis of operator perturbations in different norms corresponding to realistic models of data errors. Here we restrict ourselves to the case of Symm's integral equation postponing more general BIE and PE to other papers. We refer to papers of G.C. Hsiao and other authors (see e.g. [2, 4, 5, 1, 3]) where the influence of perturbations in the right–hand term is estimated. We analyze also the influence of perturbations in the parametrization $x = \gamma(t)$ of the boundary curve.

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1. Error bounds: an abstract consideration

Let E^λ and F^λ, $\lambda_0 \le \lambda < \infty$, be Banach spaces with the properties

$$E^{\lambda'} \subset E^\lambda, \; ||v||_{E^\lambda} \le c||v||_{E^{\lambda'}} \quad \text{for} \quad \lambda_0 \le \lambda \le \lambda' < \infty, \; v \in E^{\lambda'},$$

$$F^{\lambda'} \subset F^\lambda, \; ||g||_{F^\lambda} \le c||g||_{F^{\lambda'}} \quad \text{for} \quad \lambda_0 \le \lambda \le \lambda' < \infty, \; g \in F^{\lambda'}.$$

We consider the problem

$$Au = f \tag{1.1}$$

where $A \in \mathcal{L}(E^\lambda, F^\lambda)$ is an isomorphism for all $\lambda \ge \lambda_0$, and $f \in F^\mu$ with some $\mu > \lambda_0$. Let

$$E_n \subset \bigcap_{\lambda \ge \lambda_0} E^\lambda, \, F_n \subset \bigcap_{\lambda \ge \lambda_0} F^\lambda, \, n \in \mathbb{N},$$

be some finite dimensional subspaces where $\dim E_n = \dim F_n$. We approximate the problem (1.1) by the finite dimensional problems

$$A_n u_n = f_n \tag{1.2}$$

where $f_n \in F_n$ and $A_n \in \mathcal{L}(E_n, F_n)$ are approximations to f and A corresponding to a discretization of problem (1.1). We assume that the stability condition

$$||v_n||_{E^{\lambda_0}} \le c_0 ||A_n v_n||_{F^{\lambda_0}}, v_n \in E_n, \, n \ge n_0 \tag{1.3}$$

holds. Under suitable approximation conditions to f_n and A_n, this allows to establish an error estimate

$$||u_n - u||_{E^{\lambda_0}} \le cn^{\lambda_0 - \mu}||u||_{E^\mu}, n \ge n_0 \tag{1.4}$$

or something else of this type, e.g.

$$||u_n - u||_{E^{\lambda_0}} \le cn^{\lambda_0 - \mu} \log n ||u||_{E^\mu}, \; n \ge n_0$$

in some cases. For instance, usually there are operators $P_n : E \to E_n$ and $Q_n : F \to F_n$ (not necessarily linear) such that

$$||v - P_n v||_{E^\lambda} \le cn^{\lambda - \mu}||v||_{E^\mu}, \; \lambda_0 \le \lambda \le \mu, \; n \in \mathbb{N}, \; v \in E^\mu, \tag{1.5}$$

$$||A_n P_n v - Q_n A v||_{F^\lambda} \le cn^{\lambda - \mu}||v||_{E^\mu}, \; \lambda_0 \le \lambda \le \mu, \; n \in \mathbb{N}, \; v \in E^\mu,$$

$$||f_n - Q_n f||_{F^\lambda} \le cn^{\lambda - \mu}||f||_{F^\mu}, \; \lambda_0 \le \lambda \le \mu, \; n \in \mathbb{N},$$

and under those conditions (1.4) easily follows from (1.3):

$$||u_n - P_n u||_{E^{\lambda_0}} \le c_0 ||A_n(u_n - P_n u)||_{F^{\lambda_0}} = c_0 ||(f_n - Q_n f) + (Q_n A u - A_n P_n u)||_{F^{\lambda_0}}$$

$$\le cn^{\lambda_0 - \mu}(||f||_{F^\mu} + ||u||_{E^\mu}) \le c'n^{\lambda_0 - \mu}||u||_{E^\mu},$$

$$||u_n - u||_{E^{\lambda_0}} \le ||u_n - P_n u||_{E^{\lambda_0}} + ||u - P_n u||_{E^{\lambda_0}} \le cn^{\lambda_0 - \mu}||u||_{E^\mu}.$$

But we are indifferent to this argument assuming in the sequel both (1.3) and (1.4).

Let us discuss the influence of noises in the data. The noises may be caused e.g. by rounding errors preparing the problem to a discretization, measurement errors, and modelling errors. As a result, instead of f_n and A_n we have at our disposal some $f_{n,\delta} \in F_n$ and $A_{n,\varepsilon} \in \mathcal{L}(E_n, F_n)$ where the parameters $\delta > 0$ and $\varepsilon \ge 0$ characterize the level of

the noises in the data. A discretization procedure may magnificate these quantities. We accept the following model:

$$\left.\begin{array}{rcl} ||f_{n,\delta} - f_n||_{F^{\lambda_0}} & \leq & \delta_n ||f||_{F^{\mu}} \,, \; n \in \mathbb{N} \\ ||(A_{n,\varepsilon} - A_n)v_n||_{F^{\lambda_0}} & \leq & \varepsilon_n^0 ||v_n||_{E^{\lambda_0}} \,, v_n \in E_n \,, \; n \in \mathbb{N} \,, \\ ||(A_{n,\varepsilon} - A_n)v_n||_{F^{\lambda_0}} & \leq & \varepsilon_n ||v_n||_{E^{\mu}} \,, \; v_n \in E_n \,, \; n \in \mathbb{N} \,. \end{array}\right\} \tag{1.6}$$

Typically $\delta_n \leq cn^d \delta$, $\varepsilon_n^0 \leq cn^{d_0}\varepsilon$, $\varepsilon_n \leq cn^{d_1}\varepsilon$ with $d \geq 0$, $d_0 \geq d_1 \geq 0$ but also more complicated magnifications may occur, e.g., $\varepsilon_n \leq cn^d(\log n)\varepsilon$ or something else. Actually, the establishment of inequalities of type (1.6) will be the main task analyzing the stability of concrete discretization methods with respect to the noises.

Lemma 1.1. *Let (1.3) and (1.6) hold. Then for $n \geq n_0$ satisfying*

$$\varepsilon_n^0 \leq qc_0^{-1} \,, \; q \in (0,1) \,, \tag{1.7}$$

the operator $A_{n,\varepsilon}$ is invertible, and for $u_{n,\varepsilon,\delta} = A_{n,\varepsilon}^{-1}f_{n,\delta}$ and $u_n = A_n^{-1}f_n$ we have

$$||u_{n,\varepsilon,\delta} - u_n||_{E^{\lambda_0}} \leq \frac{c_0}{1-q}\left(\varepsilon_n ||u_n||_{E^{\mu}} + \delta_n ||f||_{F^{\mu}}\right) \,.$$

Proof. It follows from (1.3) and (1.6) that, for n satisfying (1.7), the stability inequality for $A_{n,\varepsilon}$ holds true:

$$||v_n||_{E^{\lambda_0}} \leq \frac{c_0}{1-q}||A_{n,\varepsilon}v_n||_{F^{\lambda_0}} \,, \; v_n \in E_n \,.$$

Therefore

$$\begin{aligned} ||u_{n,\varepsilon,\delta} - u_n||_{E^{\lambda_0}} & \leq & \frac{c_0}{1-q}||A_{n,\varepsilon}(u_{n,\varepsilon,\delta} - u_n)||_{F^{\lambda_0}} \\ & = & \frac{c_0}{1-q}||(f_{n,\delta} - f_n) + (A_n - A_{n,\varepsilon})u_n||_{F^{\lambda_0}} \\ & \leq & \frac{c_0}{1-q}(\delta_n ||f||_{F^{\mu}} + \varepsilon_n ||u_n||_{E^{\mu}}) \,. \end{aligned}$$

$\square$

Theorem 1.1. *Let $A \in \mathcal{L}(E^{\lambda}, F^{\lambda})$ be an isomorphism for $\lambda_0 \leq \lambda \leq \mu$ and let $f \in F^{\mu}$ with some $\mu > \lambda_0$. Let (1.3)–(1.6) hold. Finally, let the following inverse inequality hold:*

$$||v_n||_{E^{\lambda}} \leq c_1 n^{\lambda-\lambda_0}||v_n||_{E^{\lambda_0}} \,, \; \lambda_0 \leq \lambda \leq \mu \,, \; v_n \in E_n \,, \; n \in \mathbb{N} \,. \tag{1.8}$$

Then for $n \geq n_0$ satisfying (1.7) we have

$$||u_{n,\varepsilon,\delta} - u||_{E^{\lambda}} \leq c\left[n^{\lambda-\mu} + n^{\lambda-\lambda_0}(\varepsilon_n + \delta_n)\right]||u||_{E^{\mu}} \,, \; \lambda_0 \leq \lambda \leq \mu \,, \tag{1.9}$$

where $u = A^{-1}f \in E^{\mu}$ is the solution of (1.1) and $u_{n,\varepsilon,\delta} = A_{n,\varepsilon}^{-1}f_{\delta,n}$ is the solution of (1.2) corresponding to the noisy data.

Proof. First we show that the estimate (1.4) can be extended to E^{λ} norms as follows:

$$||u_n - u||_{E^{\lambda}} \leq cn^{\lambda-\mu}||u||_{E^{\mu}} \,, \; \lambda_0 \leq \lambda \leq \mu \,. \tag{1.10}$$

Indeed, due to (1.8), (1.5) and (1.4),

$$\begin{aligned} ||u_n - u||_{E^{\lambda}} & \leq & ||u_n - P_n u||_{E^{\lambda}} + ||u - P_n u||_{E^{\lambda}} \\ & \leq & cn^{\lambda-\lambda_0}||u_n - P_n u||_{E^{\lambda_0}} + cn^{\lambda-\mu}||u||_{E^{\mu}} \\ & \leq & cn^{\lambda-\lambda_0}\left(||u_n - u||_{E^{\lambda_0}} + ||u - P_n u||_{E^{\lambda_0}}\right) + cn^{\lambda-\mu}||u||_{E^{\mu}} \\ & \leq & c'n^{\lambda-\mu}||u||_{E^{\mu}} \,. \end{aligned}$$

Further, using Lemma 2.1 we find

$$
\begin{aligned}
||u_{n,\varepsilon,\delta} - u||_{E^\lambda} &\leq ||u_{n,\varepsilon,\delta} - u_n||_{E^\lambda} + ||u_n - u||_{E^\lambda} \\
&\leq c_1 n^{\lambda-\lambda_0} ||u_{n,\varepsilon,\delta} - u_n||_{E^{\lambda_0}} + c' n^{\lambda-\mu} ||u||_{E^\mu} \\
&\leq \frac{c_0 c_1}{1-q} n^{\lambda-\lambda_0} \left(\varepsilon_n ||u_n||_{E^\mu} + \delta_n ||f||_{F^\mu} \right) + c' n^{\lambda-\mu} ||u||_{E^\mu} .
\end{aligned}
$$

Noticing that due to (1.10) $||u_n||_{E^\mu} \leq c ||u||_{E^\mu}$, we obtain (1.9). $\qquad\square$

Remark 1.1. If in Theorem 1.1, instead of (1.4), we have

$$
||u_n - u||_{E^{\lambda_0}} \leq c n^{\lambda_0-\mu} \log n ||u||_{E^\mu} , \; n \geq n_0 ,
$$

then, instead of (1.9), we obtain

$$
||u_{n,\varepsilon,\delta} - u||_{E^\lambda} \leq c \left[n^{\lambda-\mu} \log n + n^{\lambda-\lambda_0}(\varepsilon_n + \delta_n) \right] ||u||_{E^\mu} , \; \lambda_0 \leq \lambda \leq \mu .
$$

Remark 1.2. If

$$
\varepsilon_n \leq c n^{\lambda_0-\mu} , \; \delta_n \leq c n^{\lambda_0-\mu} , \tag{1.11}
$$

then (1.9) yields

$$
||u_{n,\varepsilon,\delta} - u||_{E^\lambda} \leq c n^{\lambda-\mu} ||u||_{E^\mu} . \tag{1.12}
$$

The error estimate (1.12) is of the same order as in the case of exact data (cf. (1.10)). Note that conditions (1.11) are realistic only in the case of controllable perturbations of the data, e.g., in the cases of data compression and/or numerical integration completing a discretization.

Let us shortly discuss the case of (in general, non–controllable) perturbations of the data with a simplest magnification model in (1.6):

$$
\delta_n \leq c n^d \delta , \; \varepsilon_n \leq c n^d \varepsilon , \; d \geq 0 . \tag{1.13}
$$

Then (1.9) takes the form

$$
||u_{n,\varepsilon,\delta} - u||_{E^\lambda} \leq c \left(n^{\lambda-\mu} + n^{d+\lambda-\lambda_0}(\varepsilon + \delta) \right) ||u||_{E^\mu} , \; \lambda_0 \leq \lambda \leq \mu .
$$

The best results will be obtained for n such that $n^{\lambda-\mu}$ and $n^{d+\lambda-\lambda_0}(\varepsilon + \delta)$ are of the same order, i.e. $n \sim (\varepsilon + \delta)^{-1/(d+\mu-\lambda_0)}$, resulting to

$$
||u_{n,\varepsilon,\delta} - u||_{E^\lambda} \leq c(\varepsilon + \delta)^{(\mu-\lambda)/(d+\mu-\lambda_0)} ||u||_{E^\mu} , \; \lambda_0 \leq \lambda \leq \mu . \tag{1.14}
$$

This estimate is of highest order for $\lambda = \lambda_0$:

$$
||u_{n,\varepsilon,\delta} - u||_{E^{\lambda_0}} \leq c(\varepsilon + \delta)^{(\mu-\lambda_0)/(d+\mu-\lambda_0)} ||u||_{E^\mu} . \tag{1.15}
$$

Estimates (1.14) and (1.15) characterize the self–regularization of problem (1.1), if considered in an ill–posed setting, through its discretizations (1.2). Similar results can be easily obtained for more complicated magnification models rather than (1.6), (1.13).

2. TRIGONOMETRIC COLLOCATION FOR SYMM'S INTEGRAL EQUATION. ERROR BOUNDS IN SOBOLEV NORMS

Symm's integral equation

$$\int_\Gamma \log|x - y| v(y) ds_y = g(x), \quad x \in \Gamma,$$

arises from solving the Dirichlet boundary value problem for the Laplace equation in a region $\Omega \subset \mathbb{R}^2$ with a Jordan curve $\Gamma = \partial\Omega$ as the boundary. We assume that Γ is C^∞-smooth and we have a C^∞-smooth 1-periodic parametrization $t \to \gamma(t) : \mathbb{R} \to \Gamma$ of Γ such that $|\gamma'(t)| \neq 0$ for all $t \in \mathbb{R}$. The equation reduces to

$$\int_0^1 \log|\gamma(t) - \gamma(s)| u(s) ds = f(t), \quad t \in [0,1], \tag{2.1}$$

where $u(t) = v(\gamma(t))|\gamma'(t)|$, $f(t) = g(\gamma(t))$. It is known that (2.1) is uniquely solvable if and only if the capacity of Γ is different from 1. Introduce the standard representation

$$Au := A_0 u + Bu = f \tag{2.2}$$

of (2.1) where

$$(A_0 u)(t) = \int_0^1 \log|\sin \pi(t - s)| u(s) ds, \tag{2.3}$$

$$(Bu)(t) = \int_0^1 b(t,s) u(s) ds, \quad b(t,s) = \begin{cases} \log \dfrac{|\gamma(t) - \gamma(s)|}{|\sin \pi(t - s)|}, & t \neq s, \\ \log(|\gamma'(t)|/\pi), & t = s. \end{cases} \tag{2.4}$$

The operator A_0 has the property

$$A_0 e^{im2\pi t} = \begin{cases} -\dfrac{1}{2|m|} e^{im2\pi t}, & 0 \neq m \in \mathbb{Z}, \\ -\log 2, & m = 0. \end{cases} \tag{2.5}$$

The kernel $b(t,s)$ of the operator B is C^∞-smooth and 1-biperiodic.

Let H^λ, $\lambda \in \mathbb{R}$, denote the Sobolev space of 1-periodic functions (distributions) on the real line with the norm

$$\|u\|_\lambda = \left(|\hat{u}(0)|^2 + \sum_{0 \neq m \in \mathbb{Z}} |m|^{2\lambda} |\hat{u}(m)|^2 \right)^{1/2}$$

where $\hat{u}(m) = \int_0^1 u(s) e^{-im2\pi s} ds$, $m \in \mathbb{Z}$, are the Fourier coefficients of $u(t) = \sum_{m \in \mathbb{Z}} \hat{u}(m) e^{im2\pi t}$. Due to (2.5), $A_0 \in \mathcal{L}(H^\lambda, H^{\lambda+1})$ is an isomorphism for all $\lambda \in \mathbb{R}$. Since $B \in \mathcal{L}(H^\lambda, H^{\lambda+1})$ is compact, the operator $A = A_0 + B \in \mathcal{L}(H^\lambda, H^{\lambda+1})$ is also an isomorphism for all $\lambda \in \mathbb{R}$ (we assume that cap $\Gamma \neq 1$).

Introduce the n-dimensional space of trigonometric functions

$$\mathcal{T}_n = \left\{ v_n = \sum_{m \in Z_n} c_m e^{im2\pi t}, c_m \in \mathbb{C} \right\}, \quad Z_n = \left\{ m \in \mathbb{Z} : -\frac{n}{2} < m \leq \frac{n}{2} \right\}.$$

Let P_n and Q_n denote the corresponding orthogonal and interpolation projections, respectively:

$$P_n u \;=\; \sum_{m \in Z_n} \hat{u}(m) e^{im2\pi t} \in \mathcal{T}_n \,,$$

$$Q_n u \in \mathcal{T}_n \,, \quad (Q_n u)(jn^{-1}) \;=\; u(jn^{-1}) \,, \quad j = 0, ..., n-1 \,.$$

It is known that (see e.g. [9], [12])

$$\|u - P_n u\|_\lambda \;\leq\; \left(\frac{n}{2}\right)^{\lambda-\mu} \|u\|_\mu \,, \quad \lambda \leq \mu \,, \; u \in H^\mu \,, \tag{2.6}$$

$$\|u - Q_n u\|_\lambda \;\leq\; c_{\lambda,\mu} n^{\lambda-\mu} \|u\|_\mu \,, \quad 0 \leq \lambda \leq \mu \,, \; u \in H^\mu \,, \; \mu > \frac{1}{2} \,. \tag{2.7}$$

Introduce the operator (cf. (2.4))

$$(B_n u)(t) = n^{-1} \sum_{j=0}^{n-1} b(t, jn^{-1}) u(jn^{-1}) \,.$$

Approximate the equation (2.2) by the equation

$$A_n u_n := A_0 u_n + Q_n B_n u_n = Q_n f \,, \quad u_n \in \mathcal{T}_n \tag{2.8}$$

(fully discretized trigonometric collocation method). A possible matrix form of (2.8) is

$$(\mathcal{A}_n + \mathcal{B}_n)\underline{u}_n = \underline{f}_n$$

where $\underline{u}_n = (u_n(jn^{-1}))_{j=0}^{n-1}$, $\underline{f}_n = (f(jn^{-1}))_{j=0}^{n-1}$ are n-vectors, and $\mathcal{A}_n = (a_{kj})$, $\mathcal{B}_n = (b_{kj})$, $0 \leq k,j \leq n-1$, are $n \times n$-matrices with entries

$$a_{kj} = \alpha_{k-j} \,, \; \alpha_k = n^{-1}\left(-\log 2 - \frac{1}{2} \sum_{0 \neq l \in Z_n} |l|^{-1} e^{il2\pi k n^{-1}} \right) \,, \quad |k| \leq n-1 \,, \tag{2.9}$$

$$b_{kj} = n^{-1} b(kn^{-1}, jn^{-1}) \,.$$

We recall the convergence result (see [10]).

Theorem 2.1. *Assume cap* $\Gamma \neq 1$ *and* $f \in H^{\mu+1}$, $\mu > -\frac{1}{2}$. *Then there is some* n_0 *such that the stability inequality*

$$\|v_n\|_\lambda \leq c_\lambda \|(A_0 + Q_n B_n) v_n\|_{\lambda+1} \,, \quad v_n \in \mathcal{T}_n \,, \; n \geq n_0$$

holds for all $\lambda \in \mathbb{R}$, *and*

$$\|u_n - u\|_\lambda \leq c_{\lambda,\mu} n^{\lambda-\mu} \|u\|_\mu \,, \quad -1 \leq \lambda \leq \mu$$

for the solutions $u = A^{-1} f \in H^\mu$ *and* $u_n = A_n^{-1} Q_n f \in \mathcal{T}_n$ *of (2.2) and (2.8), respectively.*

Thus conditions (1.3)–(1.5) with $E^\lambda = H^\lambda$, $F^\lambda = H^{\lambda+1}$, $\lambda \geq \lambda_0 = -1$, are fulfilled. Clearly, also (1.8) holds true, and to apply Theorem 1.1, we only have to establish the inequalities of type (1.6) corresponding to disturbations of $f(t)$ and $\gamma(t)$. Assume that

$$\left(n^{-1} \sum_{j=0}^{n-1} |f_\delta(jn^{-1}) - f(jn^{-1})|^2 \right)^{1/2} \leq \delta \|f\|_{\mu+1} \tag{2.10}$$

$$|\gamma_\varepsilon(jn^{-1}) - \gamma(jn^{-1})| \leq \varepsilon \,, \quad |\gamma_\varepsilon'(jn^{-1}) - \gamma'(jn^{-1})| \leq n\varepsilon \,, \quad j = 0, ..., n-1 \,. \tag{2.11}$$

Only the grid values of f or f_δ are used in the method (2.8), therefore we may assume that $f_\delta = f_{n,\delta} \in \mathcal{T}_n$.

Lemma 2.1. *Under the conditions (2.10) we have*

$$||f_\delta - Q_n f||_0 \le \delta ||f||_{\mu+1} , \tag{2.12}$$

and under the condition (2.11) we obtain

$$||(A_{n,\varepsilon} - A_n)v_n||_0 \le c(\log n)\varepsilon ||v_n||_0 , \quad v_n \in \mathcal{T}_n , \quad n \in I\!N , \tag{2.13}$$

where $A_{n,\varepsilon}$ corresponds to the perturbed data (cf. (2.4), (2.8), (2.9)):

$$A_{n,\varepsilon} = A_0 + Q_n B_{n,\varepsilon} , \quad (B_{n,\varepsilon}v)(t) = n^{-1} \sum_{j=0}^{n-1} b_\varepsilon(t, jn^{-1})v(jn^{-1}) ,$$

$$b_\varepsilon(t, s) = \begin{cases} \log \dfrac{|\gamma_\varepsilon(t) - \gamma_\varepsilon(s)|}{|\sin \pi(t - s)|} , & t \ne s , \\ \log(|\gamma_\varepsilon'(t)|/\pi) , & t = s . \end{cases}$$

Proof. It is well known that

$$||v_n||_0 = \left(\int_0^1 |v_n(t)|^2 dt \right)^{1/2} = \left(n^{-1} \sum_{j=0}^{n-1} |v_n(jn^{-1})|^2 \right)^{1/2} , \quad v_n \in \mathcal{T}_n . \tag{2.14}$$

Since $f_\delta - Q_n f \in \mathcal{T}_n$, (2.12) is equivalent to (2.10). Let us prove (2.13). Due to (2.14) we have

$$||v_n||_0 = ||\underline{v}_n||_* := \left(n^{-1} \sum_{j=0}^{n-1} |v_n(jn^{-1})|^2 \right)^{1/2} , \quad ||A_n v_n||_0 = ||\mathcal{A}_n \underline{v}_n||_* ,$$

$$||A_{n,\varepsilon} - A_n)v_n||_0 = ||(\mathcal{A}_{n,\varepsilon} - \mathcal{A}_n)\underline{v}_n||_* = ||(\mathcal{B}_{n,\varepsilon} - \mathcal{B}_n)\underline{v}_n||_*$$
$$\le ||\mathcal{B}_{n,\varepsilon} - \mathcal{B}_n||_* ||\underline{v}_n||_* = ||\mathcal{B}_{n,\varepsilon} - \mathcal{B}_n||_* ||v_n||_0$$

where $||\mathcal{B}_n||_*$ is the usual spectral norm of the $n \times n$–matrix. Thus,

$$||\mathcal{B}_{n,\varepsilon} - \mathcal{B}_n||_* \le \max \left\{ \max_k n^{-1} \sum_j |b_\varepsilon(kn^{-1}, jn^{-1}) - b(kn^{-1}, jn^{-1})| , \right.$$

$$\left. \max_j n^{-1} \sum_k |b_\varepsilon(kn^{-1}, jn^{-1}) - b(kn^{-1}, jn^{-1})| \right\} .$$

It follows from (2.11) that

$$|b_\varepsilon(kn^{-1}, jn^{-1}) - b(kn^{-1}, jn^{-1})| \le c \frac{\varepsilon}{|\sin \pi(k - j)n^{-1}|} , \quad 0 \le k, j \le n - 1 , \quad k \ne j ,$$

$$|b_\varepsilon(jn^{-1}, jn^{-1}) - b(jn^{-1}, jn^{-1})| \le cn\varepsilon , \quad 0 \le j \le n - 1 ,$$

and this results to $||B_{n,\varepsilon} - B_n||_* \le c(\log n)\varepsilon$ proving (2.13). $\qquad\square$

As a consequence of (2.13) we obtain

$$||(A_{n,\varepsilon} - A_n)v_n||_0 \le c_2 n(\log n)\varepsilon ||v_n||_{-1} ,$$
$$||(A_{n,\varepsilon} - A_n)v_n||_0 \le cn^{\max(0,-\mu)}(\log n)\varepsilon ||v_n||_\mu .$$

These are two last inequalities (1.6) in the present case. Now Theorem 1.1 yields the following result.

Theorem 2.2. *Assume the conditions of Theorem 2.1 and (2.10), (2.11). Then for $n \geq n_0$ satisfying*

$$c_2 n(\text{lcg}\, n)\varepsilon < q/c_{-1}\,, \quad q \in (0,1)\,,$$

equation (2.8) with perturbed data is uniquely solvable, and

$$||u_{n,\varepsilon,\delta} - u||_\lambda \leq c\left[n^{\lambda-\mu} + n^{\lambda+1}\delta + n^{\lambda+1+\max(0,-\mu)}(\log n)\varepsilon\right]||u||_\mu\,, \quad -1 \leq \lambda \leq \mu\,. \quad (2.15)$$

In the case $\mu \geq 0$ (2.15) simplifies to the form

$$||u_{n,\varepsilon,\delta} - u||_\lambda \leq c\left[n^{\lambda-\mu} + n^{\lambda+1}\delta + n^{\lambda+1}(\log n)\varepsilon\right]||u||_\mu\,, \quad -1 \leq \lambda \leq \mu\,.$$

With $n \sim (\varepsilon + \delta)^{-1/(\mu+1)}$ this yields

$$||u_{n,\varepsilon,\delta} - u||_\lambda \leq c\left[\delta^{(\mu-\lambda)/(\mu+1)} + \varepsilon^{(\mu-\lambda)/(\mu+1)}|\log(\varepsilon + \delta)|\right]||u||_\mu\,, \quad -1 \leq \lambda \leq \mu\,. \quad (2.16)$$

The problem (2.1) is ill–posed if considered in $H^0 = L^2(0,1)$. The inequality (2.16) with $\lambda = 0$ characterizes the (self) regularization properties of the discrete collocation method (2.8).

3. Error bounds in Hölder–Zygmund norms

Let $\mathcal{H}^\lambda$, $\lambda \in \mathbb{R}$, be the scale of Hölder–Zygmund spaces of 1–periodic functions on $\mathbb{R}$ with the usual Hölder–Zygmund norm (cf. [6, 7, 9]). Let us again consider the procedure (2.8) for the numerical solution of Symm's equation (2.1). The following stability and error estimates are known (cf. [7]).

Theorem 3.1. *Assume cap $\Gamma \neq 1$ and $f \in \mathcal{H}^{\mu+1}$, $\mu > -1$. Then there is an n_0 such that for $n > n_0$ the stability inequality*

$$||v_n||_{\mathcal{H}^\lambda} \leq c||(A_0 + Q_n B_n)v_n||_{\mathcal{H}^{\lambda+1}}\,, \quad v_n \in \mathcal{T}_n \quad (3.1)$$

and the error estimate

$$||u_n - u||_{\mathcal{H}^\lambda} \leq c_{\lambda,\mu} n^{\lambda-\mu}(\log n)||u||_{\mathcal{H}^\mu}\,, \quad (3.2)$$

hold for $-1 < \lambda \leq \mu < \infty$ where $u = A^{-1}f \in \mathcal{H}^\mu$ and $u_n = A_n^{-1}Q_n f \in \mathcal{T}_n$ are the solutions of (2.2) and (2.8), resp. □

Thus, the conditions (1.3)–(1.5) with $E^\lambda = \mathcal{H}^\lambda$, $F^\lambda = \mathcal{H}^{\lambda+1}$, $\lambda \geq \lambda_0 = -1 + \sigma$, $\sigma > 0$, are fulfilled. Also the inverse inequality (1.8) holds in the form

$$||v_n||_{\mathcal{H}^\lambda} \leq cn^{\lambda-\mu}||v_n||_{\mathcal{H}^\mu}\,, \quad v_n \in \mathcal{T}_n\,, \quad \lambda,\mu \in \mathbb{R}\,, \quad \mu \leq \lambda\,. \quad (3.3)$$

Now, let us consider disturbed data with the property

$$|f_\delta(jn^{-1}) - f(jn^{-1})| \leq \delta||f||_{\mathcal{H}^{\mu+1}} \quad (3.4)$$

where $\delta > 0$, $j = 0, ..., n-1$ and $f_\delta(jn^{-1})$ is a disturbation of $f(jn^{-1})$. The goal now is to verify the estimates (1.6)–(1.8) in the case of Hölder–Zygmund norms. It is crucial for that to have estimates in the C–norm.

Lemma 3.1. *From the assumption (3.4) we obtain*

$$\left(n^{-1} \sum_{0 \le j \le n-1} |f_\delta(jn^{-1}) - f(jn^{-1})|^2 \right)^{1/2} \le \delta \|f\|_{\mathcal{H}^{\mu+1}} \tag{3.5}$$

and

$$|f_\delta - Q_n f|_C \le c(\log n)\delta \|f\|_{\mathcal{H}^{\mu+1}} \tag{3.6}$$

where $\mu > -1$ and f_δ is the trigonometric interpolation polynomial of the data $f_\delta(jn^{-1})$.

Proof. We have

$$|f_\delta - Q_n f|_C = |Q_n(f_\delta - f)|_C \le c(\log n) \max_{0 \le j \le n-1} |f_\delta(jn^{-1}) - f(jn^{-1})|$$

$$\le c(\log n)\delta \|f\|_{\mathcal{H}^{\mu+1}} .$$

$\square$

Lemma 3.2. *The assumptions (2.11) imply*

$$|(A_{n,\varepsilon} - A_n)v|_C \le c(\log n)^2 \varepsilon |v|_C , \quad v \in C . \tag{3.7}$$

Proof. We estimate

$$|(A_{n,\varepsilon} - A_n)v|_C = |Q_n(B_{n,\varepsilon} - B_n)v|_C$$

$$\le c(\log n) \max_{1 \le k \le n} |(B_{n,\varepsilon}v - B_n v)(kn^{-1})|$$

$$= c(\log n) \max_{1 \le k \le n} n^{-1} \left| \sum_{0 \le j \le n-1} \left[b_\varepsilon(kn^{-1}, jn^{-1}) - b(kn^{-1}, jn^{-1}) \right] v(jn^{-1}) \right|$$

$$\le c(\log n) \max_k n^{-1} \sum_j \left| b_\varepsilon(kn^{-1}, jn^{-1}) - b(kn^{-1}, jn^{-1}) \right| |v|_C$$

$$\le c'(\log n)^2 \varepsilon |v|_C ,$$

cf. the proof of Lemma 2.1. $\square$

Using the inverse inequality (3.3) for $s > 0$

$$|v_n|_C \le cn^s \|v_n\|_{\mathcal{H}^{-s}} , \quad v_n \in \mathcal{T}_n$$

and the imbedding $\mathcal{H}^s \subset C$, $s > 0$, we obtain from Lemma 3.2 the estimate

$$|(A_{n,\varepsilon} - A_n)v_n|_C \le c(\log n)^2 n^{\max(0,-s)} \varepsilon \|v_n\|_{\mathcal{H}^s} , \quad v_n \in \mathcal{T}_n$$

for $s \in \mathbb{R}$. By applying the inverse inequality (3.3) to $(A_{n,\varepsilon} - A_n)v_n \in \mathcal{T}_n$ this gives

$$\|(A_{n,\varepsilon} - A_n)v_n\|_{\mathcal{H}^\sigma} \le c(\log n)^2 n^{\sigma+\max(0,-s)} \varepsilon \|v_n\|_{\mathcal{H}^s} , \quad v_n \in \mathcal{T}_n , \ s \in \mathbb{R} \tag{3.8}$$

where $\sigma > 0$ may be taken arbitrarily small. This is the last inequality of (1.6) in the present case with $\lambda_0 = -1 + \sigma$.

Now, let again u_n be the solution of the discretized problem (2.8) and $u_{n,\varepsilon,\delta}$ be the solution of the perturbed problem. From the estimates (3.6), (3.7) and (3.8) we will derive error estimates in Hölder–Zygmund norms.

Theorem 3.2. *Let the assumptions of Lemmas 3.1 and 3.2 be fulfilled. We obtain for $f \in \mathcal{H}^{\mu+1}$ and $n \ge n_0$, $-1 < \lambda \le \mu < \infty$,*

$$\|u - u_{n,\varepsilon,\delta}\|_{\mathcal{H}^\lambda} \le c \left[n^{\lambda-\mu}(\log n) + n^{\lambda+1}(\log n)\delta + n^{\lambda+1+\max(0,-\mu)}(\log n)^2 \varepsilon \right] \|u\|_{\mathcal{H}^\mu} . \tag{3.9}$$

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Proof. In analogy to the proof of Theorem 1.1 for $\lambda_0 = -1 + \sigma$, $\sigma > 0$, using (3.3) and (3.8) and the Lemmas 3.1 and 3.2. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Finally, let us describe the error estimate (3.9) in dependence of the noise levels δ and ε where n is taken as a suitable function $n(\delta, \varepsilon)$.

Theorem 3.3. *Let the assumptions of Lemmas 3.1 and 3.2 be fulfilled and $\mu \geq 0$. If we take*

$$n \sim (\varepsilon + \delta)^{-1/(\mu+1)} \tag{3.10}$$

we obtain, for $n > n_0$ and $f \in \mathcal{H}^{\mu+1}$,

$$||u - u_{n,\varepsilon,\delta}||_{\mathcal{H}^\lambda} \leq c(\varepsilon + \delta)^{(\mu-\lambda)/(\mu+1)}(\log(\varepsilon + \delta))^2||u||_{\mathcal{H}^\mu}. \tag{3.11}$$

4. A QUADRATURE METHOD. ERROR BOUNDS IN SOBOLEV NORMS

Here we are concerned with the quadrature formula method considered by Saranen and Schroderus [11] for an exactly given operator and an exactly given right–hand side. In the special case of Symm's equation we impose errors to the data and investigate their influence on the approximated solution.

Let us approximate the solution of the equation (2.2) by the solution $u_n \in \mathcal{T}_n$ of

$$D_n u = Q_n f \tag{4.1}$$

where

$$(D_n u)(t) = Q_n \left(\frac{1}{n} \sum_{j=0}^{n-1} K(t, jn^{-1})(u(jn^{-1}) - u(t)) + \beta(t)u(t) \right),$$

$$K(t, s) = \log|\gamma(t) - \gamma(s)|$$

$$\beta(t) = \int_0^1 K(t, \tau)d\tau,$$

γ is the considered parametrization of the C^∞–curve Γ, and Q_n is the operator of trigonometric interpolation. The vector $\underline{u}_n \in \mathbf{C}_n$

$$\underline{u}_n = (u_n(jn^{-1}))_{j=0}^{n-1}$$

is the solution of the equation

$$\mathcal{D}_n \underline{u}_n = (f(kn^{-1}))_{k=0}^{n-1} \tag{4.2}$$

where $\mathcal{D}_n$ is the matrix with the entries $d_{k,j}$, $k, j = 1, ..., n$,

$$d_{k,j} = \begin{cases} \frac{1}{n}K(kn^{-1}, jn^{-1}), & k \neq j \\ \beta(kn^{-1}) - \frac{1}{n}\sum_{\nu \neq k} K(kn^{-1}, \nu n^{-1}), & k = j. \end{cases} \tag{4.3}$$

Let us recall the stability inequality and the convergence result from [11]:

Theorem 4.1. *Assume cap $\Gamma \neq 1$ and $f \in H^{\mu+1}$, $\mu > -1/2$. Then there is an n_0 such that for $n > n_0$ the stability inequality*

$$||v_n||_\lambda \leq c_\lambda ||D_n v_n||_{\lambda+1}, \quad v_n \in \mathcal{T}_n, \tag{4.4}$$

holds for $\lambda \in \mathbb{R}$ and the error estimate

$$||u_n - u||_\lambda \leq C_{\lambda,\mu} n^{\lambda-\mu}||u||_\mu \tag{4.5}$$

holds for $-1 \leq \lambda < \mu \leq \lambda + 2$, *where* u *and* u_n *are the solutions of* (2.2) *and* (4.1), *respectively.*

The method (4.2) is not fully discrete. To obtain a fully discrete method we approximate the integral (see (2.4))

$$\beta(t) = \int_0^1 \log |\gamma(t) - \gamma(s)| ds = \int_0^1 b(t, s) ds + \int_0^1 \log |\sin \pi(t - s)| ds$$
$$= \int_0^1 b(t, s) ds + \log 2$$

by

$$\beta_n(t) = \frac{1}{n} \sum_{0 \leq l \leq n-1} b(t, ln^{-1}) + \log 2 .$$

Then $d_{k,k}$ (see (4.3)) will be approximated by $\bar{d}_{k,k}$, where

$$\bar{d}_{k,k} := \beta_n(kn^{-1}) - \frac{1}{n} \sum_{\substack{0 \leq l \leq n-1 \\ l \neq k}} \log |\gamma(kn^{-1}) - \gamma(ln^{-1})|$$

$$= \log 2 - \frac{1}{n} \sum_{\substack{0 \leq l \leq n-1 \\ l \neq k}} \log |\sin \pi(k - l)n^{-1}| + \frac{1}{n} \log \frac{|\gamma'(kn^{-1})|}{\pi} .$$

Thus,

$$\bar{d}_{k,k} = \log 2 - \frac{1}{n} \sum_{1 \leq m \leq n-1} \log |\sin \pi m n^{-1}| + \frac{1}{n} \log \frac{|\gamma'(kn^{-1})|}{\pi} . \tag{4.6}$$

Lemma 4.1. *For any* $r > 0$ *there is a constant* c_r *such that*

$$\max_{1 \leq k \leq n-1} |\bar{d}_{k,k} - d_{k,k}| \leq c_r n^{-r} . \tag{4.7}$$

Proof. Since $b(t, s)$ is C^∞–smooth and 1–biperiodic,

$$\max_{0 \leq t \leq 1} |\beta_n(t) - \beta(t)| \leq \max_{0 \leq t \leq 1} \int_0^1 |b(t, s) - Q_{n,s} b(t, s)| ds \leq c_r n^{-r} ,$$

and (4.7) follows immediately. $\square$

Corollary 4.1. *Theorem* 4.1 *remains valid for the fully discrete method* (4.2) *with*

$$d_{k,j} = \begin{cases} \frac{1}{n} K(kn^{-1}, jn^{-1}) , & k \neq j , \\ \log 2 - \frac{1}{n} \sum_{1 \leq m \leq n-1} \log |\sin \pi m n^{-1}| + \frac{1}{n} \log \frac{|\gamma'(kn^{-1})|}{\pi} , & k = j . \end{cases} \tag{4.8}$$

In what follows we consider the fully discret method (4.2), (4.8). Let us turn to the perturbed data. Then (2.10) again yields (2.12), and from (2.11) we get the following L_2–estimate for $D_{n,\varepsilon} - D_n$.

Lemma 4.2. *For* D_n *corresponding to* (4.8), *the conditions* (2.11) *imply*

$$\|(D_{n,\varepsilon} - D_n)v\|_0 \leq c(\log n)\varepsilon \|v\|_0 , \quad v \in \mathcal{T}_n . \tag{4.9}$$

Proof. In the same way as in Section 2 we obtain

$$\|(D_{n,\varepsilon} - D_n)v_n\|_0 \leq \|\mathcal{D}_{n,\varepsilon} - \mathcal{D}_n\|_* \|v_n\|_0$$

and

$$\|\mathcal{D}_{n,\varepsilon} - \mathcal{D}_n\|_* \leq \max\left\{ \max_k \sum_j |d^\varepsilon_{k,j} - d_{k,j}|, \; \max_j \sum_k |d^\varepsilon_{k,j} - d_{k,j}| \right\}$$

$$\leq \max_k \sum_j |d^\varepsilon_{k,j} - d_{k,j}|$$

because of the symmetry of the kernel K. We estimate

$$\sum_j |d^\varepsilon_{k,j} - d_{k,j}| = \frac{1}{n} \sum_{j \neq k} |K_\varepsilon(kn^{-1}, jn^{-1}) - K(kn^{-1}, jn^{-1})|$$

$$+ \frac{1}{n} |\log|\gamma'_\varepsilon(kn^{-1})| - \log|\gamma'(kn^{-1})| \,|$$

$$\leq \frac{1}{n} \sum_{j \neq k} |K_\varepsilon(kn^{-1}, jn^{-1}) - K(kn^{-1}, jn^{-1})| + c\varepsilon \,.$$

We have

$$|K_\varepsilon(kn^{-1}, jn^{-1}) - K(kn^{-1}, jn^{-1})|$$
$$= \left| \log|\gamma(kn^{-1}) - \gamma(jn^{-1})| - \log|\gamma_\varepsilon(kn^{-1}) - \gamma_\varepsilon(jn^{-1})| \right|$$

$$= \left| \log \frac{|\gamma(kn^{-1}) - \gamma(jn^{-1})|}{|\sin \pi(kn^{-1} - jn^{-1})|} - \log \frac{|\gamma_\varepsilon(kn^{-1}) - \gamma_\varepsilon(jn^{-1})|}{|\sin \pi(kn^{-1} - jn^{-1})|} \right| \,.$$

Therefore, in the same way as in Section 2 we obtain

$$\frac{1}{n} \sum_{j \neq k} |K_\varepsilon(kn^{-1}, jn^{-1}) - K(kn^{-1}, jn^{-1})| \leq c \cdot \varepsilon \cdot \log n \,,$$

and the Lemma follows. $\qquad\square$

From (4.9) we obtain the inequality

$$\|(D_{n,\varepsilon} - D_n)v_n\|_0 \leq cn^{\max(0,-\mu)}(\log n)\varepsilon\|v_n\|_\mu \,. \tag{4.10}$$

Theorem 4.2. *Let the assumptions (2.10) and (2.11) be fulfilled and let $f \in H^{\mu+1}$, $\mu > -\frac{1}{2}$. Then for $n \geq n_0$ we have*

$$\|u_{n,\varepsilon,\delta} - u\|_\lambda \leq c \left[n^{\lambda-\mu} + n^{\lambda+1}\delta + n^{\lambda+1+\max(0,-\mu)}(\log n) \cdot \varepsilon \right] \|u\|_\mu \tag{4.11}$$

for $-1 \leq \lambda < \mu \leq \lambda + 2$. In the case $\mu \geq 0$ (4.11) has the form

$$\|u_{n,\varepsilon,\delta} - u\|_\lambda \leq c \left[n^{\lambda-\mu} + n^{\lambda+1}\delta + n^{\lambda+1}(\log n) \cdot \varepsilon \right] \|u\|_\mu \,. \tag{4.12}$$

Proof: Apply Theorem 1.1 using (2.12), (4.10).

Theorem 4.3. *Let the assumptions (2.10), (2.11) be fulfilled and let $f \in H^{\mu+1}$, $\mu \geq 0$. If we choose*

$$n \sim (\varepsilon + \delta)^{-\frac{1}{\mu+1}}$$

we obtain for $n > n_0$

$$\|u - u_{n,\varepsilon,\delta}\|_\lambda \le c(\varepsilon + \delta)^{\frac{\mu-\lambda}{\mu+1}} \cdot |\log(\varepsilon + \delta)| \, \|u\|_\mu$$

where $-1 \le \lambda < \mu \le \lambda + 2$.

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