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Stochastic stability of structures under active control with distributed time delays

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STOCHASTIC STABILITY OF STRUCTURES UNDER ACTIVE CONTROL WITH DISTRIBUTED TIME DELAYS

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Abstract
The pathwise behaviour of a single degree of freedom (SDOF) system with symmetric nonlinearity and distributed delays is investigated under the presence of seismic excitation and multiplicative noise. Besides distributed time delays and finite build-up time of control force are taken into consideration. The system is modelled as stochastic integro–differential equation with exponential type kernels. Interpreting stochastic equations in Stratonovich sense, stochastic stability is analyzed in terms of Lyapunov exponents. Estimates of frequencies with which sample paths of displacement of SDOF system cross certain critical values are also obtained. Studies of stochastic linear and nonlinear systems are carried out by resorting to numerical techniques for the solution of (ordinary) stochastic differential equations.

Keywords: Stochastic stability; Lyapunov exponents; Exit frequencies; Weak and strong time delay; Seismic excitation; Active control; Stochastic differential equations; Implicit numerical methods; Numerical mean square and almost sure stability

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1. Introduction

The idea of applying active control for vibration suppression of structures in civil engineering has attracted a lot of attention in recent years, see e.g. Abdel–Rohmann (1987), Pu and Kelly (1990), Agrawal et al. (1993) or Zhang et al. (1993). This idea is emerging as a powerful technique to reduce the damage caused by earthquakes, wind and other dynamic excitations. One of the major concerns associated with the application of control is the existence of unavoidable time delay which may lead to instability in system's response. Time delay arises due to a number of factors such as time required for data acquisition, online calculation or unsynchronized application of large forces.
Most of the studies concerning stability of structures deal with deterministic models. In a more realistic model seismic excitations and external noise (environmental) should be incorporated. Karmeshu and Schurz (1994) have investigated effects of seismic excitations and state–dependent environmental noise on SDOF structures with distributed delays. They have also discussed stochastic stability in the moment sense, obtained explicit conditions in the linear case and numerical results in the nonlinear situation. Zhang et al. (1993) examined the stability of deterministic SDOF system with a modified control algorithm which takes into account the finite time required for a mechanical system to build up a control force.

In this paper we investigate effects of seismic excitations and stochastic environmental noise on the SDOF system with distributed delays in the presence of the modified control force. In the limit when the build up time delay of the control force tends to zero, this model reduces to the one considered by Karmeshu and Schurz (1994). The behaviour of the system under stochastic excitations is investigated for different values of build-up time delay of the control force and for different intensities of environmental noise. The issue of almost sure stability of the system is explored in terms of Lyapunov exponents. Furthermore, we carry out some phase plane analysis and obtain estimates of the frequency with which sample paths cross 'critical' values. Calculations for almost sure stability, exit frequencies and phase plane analysis of the system are numerically carried out in the case of weak delay.

The paper is organized as follows. Section 2 contains the formulation of the model as a system of stochastic differential equations (SDEs) with multiplicative noise interpreted in Stratonovich sense. Sections 3 and 4 are devoted to the study of various aspects of stochastic analysis of seismic systems with weak and strong delay. Some analysis concerning almost sure stability for corresponding linear systems is presented in section 3. Section 4 exclusively deals with the investigation for the case of symmetric nonlinearity. Here phase diagrams and probabilistic exit frequencies are briefly analyzed for the given stochastic SDOF system. The paper is finished with a summary, remarks and a brief discussion on open problems.

2. Stochastic model with distributed delays and active control

The displacement $x = x(t)$ of an SDOF structure follows the differential system

$$m\ddot{x} + kx + c\dot{x} + \gamma x^3 \;=\; F(x, \dot{x}, t) - z(t) \tag{1}$$

where m, k, c and γ are nonnegative real parameters. These parameters can be interpreted respectively as mass, stiffness, damping and nonlinearity coefficients of the system, F as excitation force and z as control force. Without loss of generality, we take $m = 1$. The control force $z(t)$ is based on distributed delays instead of constant time delay. It seems that nonconstant delay is more realistic as the delay magnitude varies from one sensor to another, from one actuator to another, etc. Now, let the force $z = z(t)$ be given by

$$z(t) \;=\; \alpha g_1 \omega_0 k \int_0^t K(t-s)x(s)\,ds + \alpha g_2 \omega_0 k \int_0^t \hat{K}(t-s)\dot{x}(s)\,ds\,. \tag{2}$$

The nonnegative parameters g_1 and g_2 represent feedback gains of the displacement and velocity of the oscillations, whereas ω_0 is the natural frequency. $K(t)$ and $\hat{K}(t)$ are absolutely integrable weight functions specifying the distributed delays. For simplicity, it is assumed that these functions are normalized, i.e.

$$\int_0^\infty |K(u)|\,du \;=\; 1 \qquad \text{and} \qquad \int_0^\infty |\hat{K}(u)|\,du \;=\; 1\,. \tag{3}$$

Zhang et al. (1993) noted that a modified control algorithm for mechanical system takes a finite time building up a control force. They have captured this aspect by incorporating a control speed parameter α, related to the build-up time delay t_b as follows

$$\alpha \;=\; \frac{1}{\omega_0 t_b} \;>\; 0\,. \tag{4}$$

Thus, following Zhang et al. (1993), the dynamics of the control force is modified to

$$\dot{z}(t) + \alpha\omega_0\, z(t) \;=\; \alpha g_1 \omega_0 k \int_0^t K(t-s)x(s)\,ds + \alpha g_2 \omega_0 k \int_0^t \hat{K}(t-s)\dot{x}(s)\,ds\,. \tag{5}$$

When the system is subjected to environmental fluctuations and seismic excitations, the stochastic force $F = F(x, \dot{x}, t)$ can be specified as

$$F(x,\dot{x},t) = \sigma_1 x(t)\xi_1(t) + \sigma_2\dot{x}(t)\xi_2(t) + \sigma_3\eta(t) . \tag{6}$$

The first two terms on the right side of (6) correspond to stochastic environmental perturbations, and the last term represents the ground level acceleration corresponding to seismic excitations. The random environmental perturbations per unit displacement and per unit velocity are modelled by independent white noise processes $\xi_1(t)$ and $\xi_2(t)$. σ_i $(i = 1, 2)$ in (6) give the magnitude of fluctuations. The first two expressions are called multiplicative noise because of their state–dependence.
Several attempts to model seismic excitations by an appropriate stochastic process $\eta(t)$ have been made, e.g. Bolotin (1960), Shinozuka (1967, 1972) and Kozin (1977). Seismic excitation can be modelled by the process

$$\eta(t) = I(t)\,\xi_3(t) \tag{7}$$

where $\xi_3(t)$ is a white noise process being independent of $\xi_1(t)$ and $\xi_2(t)$. $I(t)$ with parameters β_1 and β_2 is assumed to be of the form

$$I(t) = \exp(-\beta_1 t) - \exp(-\beta_2 t), \quad 0 < \beta_1 < \beta_2 . \tag{8}$$

The resulting integro–differential equation (1) can be interpreted in many different ways according the type of stochastic integration calculus. Two major interpretations have crystallized out, namely Itô and Stratonovich calculus. These two calculi are related to each other in the sense that the results of one of them can be transformed to the other via a transformation formula, cf. Arnold (1974). Here we have adopted the Stratonovich interpretation as it is preferable for modelling physical phenomena, cf. Wong and Zakai (1965).
To analyze the model we need specific forms of the weight functions $K(t)$ and $\hat{K}(t)$. For the sake of simplicity we examine the case $K(t) = \hat{K}(t) = K_i(t)(i = 1, 2)$ in this paper. Main attention is drawn to the two forms

$$K_1(t) = \nu\exp(-\nu t) \qquad \text{and} \qquad K_2(t) = \nu^2 t\exp(-\nu t) . \tag{9}$$

In consonance with similar approaches in population ecology, e.g. Mac Donald (1978), the first form in (9) is termed as '**weak delay**' and the second one as '**strong delay**'. The corresponding stochastic systems turn out to be very complex for analytic analysis. Thus, for system analysis, one has to resort to numerical techniques, as described in the next section. For completeness and numerical treatment, we state both systems in their equivalent Itô prescriptions.

Weak delay. Writing $x_1(t)$ for the displacement $x(t)$, $x_2(t)$ for the velocity $\dot{x}(t)$, $x_5(t)$ for the control force $z(t)$, system (1) with kernel $K_1(t)$ is described by the set

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -(kx_1 + (c - \frac{1}{2}\sigma_2^2)x_2 + \gamma x_1^3 + x_5) + \sigma_1 x_1\xi_1(t) + \sigma_2 x_2\xi_2(t) + \sigma_3 I(t)\xi_3(t) \qquad (10)\\
\dot{x}_3 &= -\nu x_3 + \nu x_1 , \quad \dot{x}_4 = -\nu x_4 + \nu x_2 , \quad \dot{x}_5 = \alpha\omega_0 k(g_1 x_3 + g_2 x_4) - \alpha\omega_0 x_5
\end{aligned}$$

where $x_3(t)$ and $x_4(t)$ represent the time integrals of the right hand side of equation (5).

Strong delay. The system with kernel $K_2(t)$ can be similarly rewritten to

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -(kx_1 + (c - \frac{1}{2}\sigma_2^2)x_2 + \gamma x_1^3 + x_7) + \sigma_1 x_1\xi_1(t) + \sigma_2 x_2\xi_2(t) + \sigma_3 I(t)\xi_3(t) \\
\dot{x}_3 &= -\nu x_3 + \nu x_4 , \quad \dot{x}_4 = -\nu x_4 + \nu x_1 , \quad \dot{x}_5 = -\nu x_5 + \nu x_6 , \qquad (11)\\
\dot{x}_6 &= -\nu x_6 + \nu x_2 , \quad \dot{x}_7 = \alpha\omega_0 k(g_1 x_3 + g_2 x_5) - \alpha\omega_0 x_7 ,
\end{aligned}$$

where $x_7(t)$ corresponds to the control force $z(t)$ and

$$\begin{aligned} x_3(t) &= \int_0^t K_2(t-s)x(s)\,ds\ , \quad x_4(t) = \int_0^t K_1(t-s)x(s)\,ds \\ x_5(t) &= \int_0^t K_2(t-s)\dot{x}(s)\,ds\ , \quad x_6(t) = \int_0^t K_1(t-s)\dot{x}(s)\,ds\ . \end{aligned}$$

Consequently, the original integro–differential systems with both weak and strong delays have been transformed to sets of coupled SDEs without time delay. This is due to the special kernel structure of the distributed lag.

3. Stability analysis of stochastic model: The suggested methology

The commonly adopted approach for dealing with stochastic stability starts with linearization of the nonlinear system around an equilibrium point. Then by analyzing the resulting linearized system one infers about the stability behaviour of the original nonlinear one. For a more mathematical description and justification of this approach see Has'minskiĭ (1980) or Kloeden et al. (1994). However, we are conscious that we have not clarified when the stability of the linearized system exactly implies stability of the nonlinear system (1), cf. Has'minskiĭ (1980) for some sufficient conditions. We will simply consider a corresponding linear system as a reasonable description of the nonlinear one in view of its stability behaviour, at least in a small neighbourhood.
Now we take $W_t^j = \int_0^t \xi_t^j dt$ as independent Wiener processes ($j = 1, 2, 3$). Assume that the previously introduced model is linear or has been linearized around zero in the form

$$dX_t \ = \ A\,X_t\,dt + B^1\,X_t\,dW_t^1 + B^2\,X_t\,dW_t^2 + b_t\,dW_t^3 \tag{12}$$

with corresponding drift matrix A, diffusion matrices B^1 and B^2 and additive noise vector b_t. The (a.s.) stability behaviour of stochastic dynamical systems governed by (12) can be characterized by their largest Lyapunov exponent. By the theorem of Oseledets (1968) there are nonrandom real numbers $\lambda_i (i = 1, .., d)$ (Lyapunov exponents) with

$$\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_d \quad \text{and} \quad \lambda_i \ = \ \lambda(x_0^i) \ = \ \limsup_{t\to\infty} \ \frac{1}{t} \ \log \|X_t(x_0^i)\| \tag{13}$$

under ergodicity of stochastic process X_t ($X_0 = x_0^i$ is an element of random eigenspace E_i of the underlying probability space Ω). $\|.\|$ denotes the Euclidean vector norm. There is a standard algorithm due to Has'minskiĭ (1980) to calculate the top exponent of these characteristics. This algorithm relies on analytic calculation of invariant measures which can be a very difficult task for general systems. Thus one is mostly forced to use numerical computations for obtaining an appropriate estimate of the top Lyapunov exponent. Besides, in contrast to deterministic linear models, one generally has to resort to numerical methods for the pathwise analysis of SDEs (11). This is due to the fact that the matrices A and B^j ($j = 1, 2$) are not commuting as in our case, provided that $\sigma^1 \neq 0$. Hence a closed expression for the analytic solution is not available. Only a few multi–dimensional examples are known where one can exactly compute their 'pathwise stability behaviour', see e.g. Mil'shtein and Auslaender (1982), Baxendale (1986) or Ariaratnam and Xie (1991).
For pathwise analysis we make use of numerical techniques described in monographs Mil'shtein (1988), Kloeden et al. (1994), Talay (1990) or Artemiev (1993). Methods with lower order of mean square convergence are prefered by the authors. This can be justified by a lack of extensive stability investigations, by very unhandy generation of multiple integrals with higher multiplicity and by stronger requirements on smoothness and boundedness of drift and diffusion – conditions which are imposed on the usage of

numerical methods with higher order of convergence. Thus we will only use the class of Balanced implicit methods (BIMs) introduced in Mil'shtein et al. (1994). These methods can be interpreted as stochastic, linear–implicit corrections of well–known Euler methods which converge with same order 0.5 as the Euler method does and possess the iterative scheme

$$Y_{n+1} = Y_n + \sum_{j=0}^{r} b^j(t_n, Y_n)\Delta W_n^j + \sum_{j=0}^{r} C^j(t_n, Y_n)|\Delta W_n^j|(Y_{n+1} - Y_n) \tag{14}$$

where $Y_n = Y(t_n)$ is the value of approximation Y for process X_t at time t_n and

$$b^0(t,x) = a(t,x), \Delta W_n^j = W_{t_{n+1}}^j - W_{t_n}^j, \Delta W_n^0 = \Delta_n = t_{n+1} - t_n, n = 0, 1, ...,$$

along a given discretization of finite time interval $[0, T]$. For existence and convergence, the weight matrices C^j must satisfy some boundedness conditions, cf. Mil'shtein et al. (1994). Recent investigations have shown appropriate usage and superiority of BIMs with respect to convergence and stability under a minimum of smoothness requirements, cf. Schurz (1994). For practical use weight matrices C^j need to be specified in relation to given SDE. We suggest that these matrices for our model are chosen as follows:

$$C^0 = 0.5A, C^1 = \eta_1 B^1, C^2 = \eta_2 B^2 \quad \text{and} \quad C^3 = 0 \quad (\eta_i \in \mathbb{R}^1, \eta_i \geq 0, i = 1, 2)\,. \tag{15}$$

Thus, we can guarantee both numerical mean square and almost sure stability under some conditions. It is worth mentioning that by method (14) we present an alternative to the numerical procedure suggested by Talay (1991). Another numerical alternative would be to apply stochastic Runge–Kutta methods. However, in general, there is no very efficient and A–stable stochastic Runge–Kutta method with higher order of convergence known so far. Thus we prefer the application of method (14). For more details on stochastic numerical analysis, see e.g. Kloeden et al. (1994). In passing, we also note that it is advisable to take

$$\lambda_i^n = \frac{1}{n\Delta}\left[\log\|Y_0\| + \sum_{k=1}^{n}\log\left(\frac{\|Y_k\|}{\|Y_{k-1}\|}\right)\right], \quad n = 1, 2, ... \tag{16}$$

for more efficient and robust estimation of the top Lyapunov exponent using numerical methods with equidistant step size Δ instead of direct discretization of (13). Besides, Talay (1991) has shown that discretizations of expression (13) using numerical values $(Y_n)_{n=0,1,...}$ converge to the largest (top) Lyapunov exponent λ_1 as the maximum step size Δ tends to zero. The same result can be carried over to methods (14) under some mild requirements.

Numerical results for top Lyapunov exponents

The numerical results presented have been confined to the system with weak delay. The same analysis can be easily extended to the system with strong delay. Investigations to be followed will be understood as a suggestion for further intensive studies concerning interesting parameter–dependences and other practical aspects.

Data analysis has shown that the feedback gains $g_1 = 11.85$ and $g_2 = 4.87$ are a reasonable choice for experiments. The natural frequency ω_0 chosen according to the system of Chung et al. (1988) is $\omega_0 = 21.8$ rad/s which gives us the relation between control speed α and build-up time t_b of the control force via (6). Furthermore, the stiffness and damping coefficients take the values $k = 5.0$ and $c = 0.872$, respectively, throughout numerical experiments.

We study the dependence of estimates of the top Lyapunov exponent on parameters of the seismic system, in particular the dependence on control speed α. A negative sign of

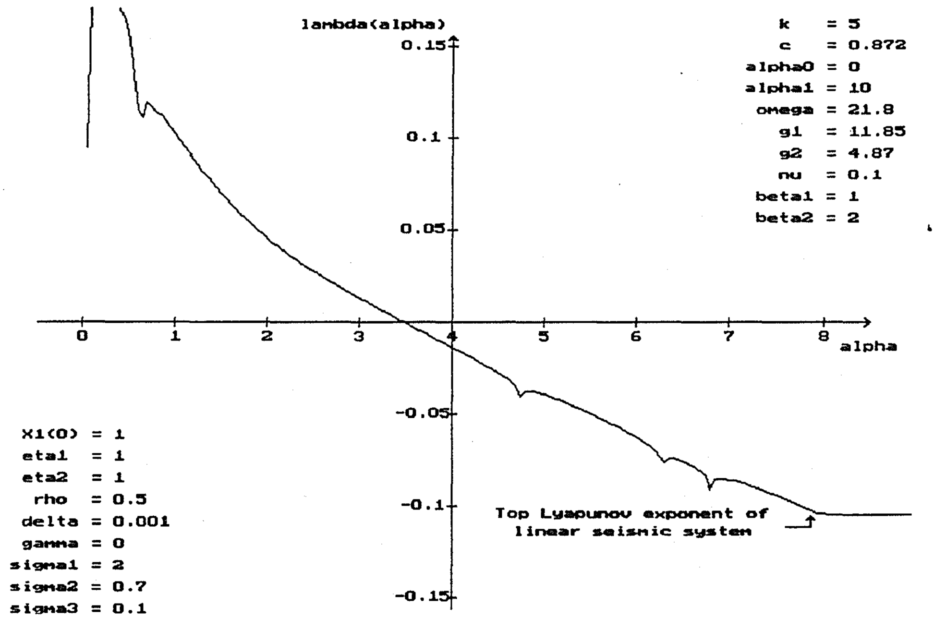

Figure 1: Dependence of the top Lyapunov exponent of the linear system on control speed α with $(\gamma, \sigma_1, \sigma_2, \sigma_3, \nu, \beta_1, \beta_2) = (0.0, 2.0, 0.7, 0.1, 0.1, 1.0, 2.0)$ estimated at time $T = 500$.

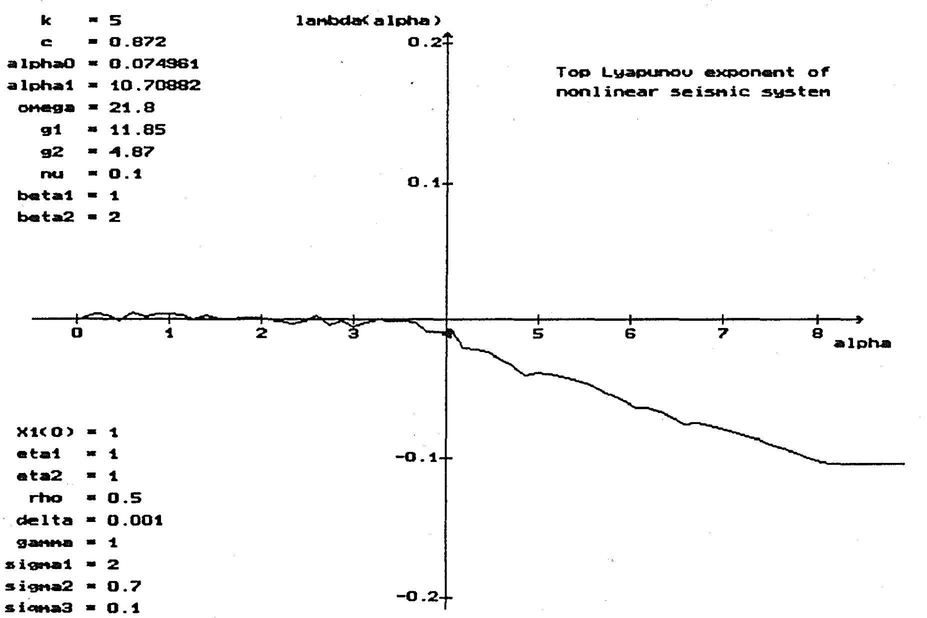

Figure 2: Dependence of the top Lyapunov exponent of the nonlinear seismic system on control speed α with $(\gamma, \sigma_1, \sigma_2, \sigma_3, \nu, \beta_1, \beta_2) = (1.0, 2.0, 0.7, 0.1, 0.1, 1.0, 2.0)$ estimated at time $T = 500$.

the top exponent indicates stochastic stability, whereas a positive sign indicates stochastic instability. Thus we interpret the graph depending on the control parameter $\alpha \in [\alpha_0, \alpha_1]$ which is plotted in figure 1. Obviously, negativity of the top exponent means stability (a.s.) of the system with the control speed ranging in the region $(\hat{\alpha}, \alpha_1]$. Thus we observe a critical control speed $\hat{\alpha}$ smaller than 3.5 (note that this corresponds to build-up time $t_b = (\omega_0 \alpha)^{-1} = 0.0131061$). From there on, for decreasing α (= increasing build-up time t_b), the seismic system becomes trapped in the region of stochastic instability.
As a supplement we state the results of estimation of top exponent of a nonlinear system. Although we have not found a complete, rigorous mathematical justification so far, we have used the same procedure of estimation as in the linear case with the same parameters and obtained plots as given in figure 2. Roughly speaking, one observes the same region of stability, comparing linear and nonlinear estimation. More precisely, using control speeds larger than 3.5, a remarkable coincidence of their graphs can be noticed. In contrast to figure 1, the region of instability has dramatically changed if one considers numerical results as being reliable in the nonlinear situation. We observe oscillations of the estimates around zero for control speeds smaller than $\hat{\alpha} = 3.5$ as in figure 2. However, a precise clarification of the latter remark requires a more detailed analysis, involving further experiments and rigorous mathematical investigation. This is omitted here.

4. Phase plane analysis and exit frequencies of nonlinear system (1)

Here we carry out some analysis for the nonlinear model in the presence of symmetric nonlinearity. First we look at phase diagrams plotted as stochastic flows, i.e. the displayed paths use one and the same underlying Wiener path and start with different initial values. Figures 3 and 5 show a typical spiralisation in the phase plane which indicates some stability (closeness of neighbouring paths). In contrast to this feature, one observes a small region where a limit cycle of the nonlinear dynamics might exist as an indicator of some 'structural stability', as seen in figure 4. Besides, figure 3 also shows how important an adequate estimation of Lyapunov exponents is for the evaluation of stochastic flows with respect to a.s. stability. Thus, without the knowledge on top exponent we would suspect some stochastic stability for large build-up times, what obviously is a very fuzzy or even wrong conjecture (cf. figures 1, 2 and 3). The numerical results give rise to clarify what is really happening in the region of very large build up times (i.e. small control speeds in the nonlinear system). This question is left open for future research work. An estimate for critical build-up time, i.e. when the system changes its qualitative behaviour (a.s.), is about $\hat{t}_b = 0.214176$ for the given constellation of parameters (cf. also figure 4).

An engineering definition of unstable control system is one in which the vibration of the controlled system leaves certain bounds, leading to catastrophic behaviour, otherwise the control is considered as stable. In this context we present estimates for probabilistic exit frequencies, i.e. estimates of the probability that paths of displacement leave a certain boundary level which can be interpreted as a critical value for structural stability. For simplicity, this level is taken as $\varepsilon = 1.0$. Consider

$$f_t^{\varepsilon}(t_b) \ := \ \mathbb{P}_{t_b}\{X_t^{(1)} > \varepsilon \,|\, X_0\} \, .$$

Estimates of these probabilities for different build-up times t_b are plotted in figure 6. We recognize that for the given choice of parameters sample paths exceed the level ε with high probability at times $t = 1...5$ sec. Naturally, for very small build-up times of the control force, sufficient control on these probabilities $f_t^{\varepsilon}(t_b)$ is achieved. For example, when build-up times are smaller than 0.01 the given level ε is not exceeded with high probability (larger than 90 %, as seen in figure 6). Once again one finds a critical region for a qualitative change in the stability behaviour of the seismic system in terms of build-up times t_b or control speeds α.

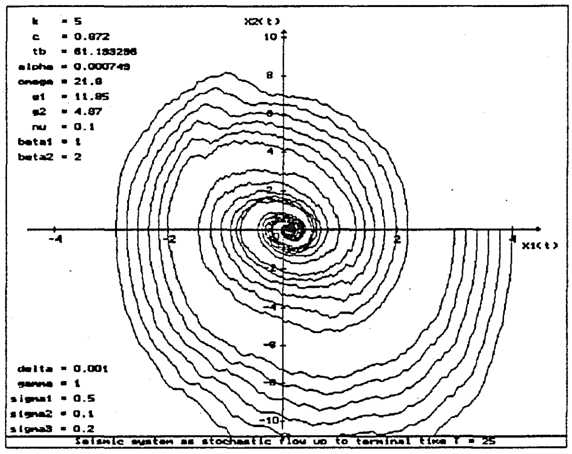

Figure 3: Phase diagram of the nonlinear seismic system with very large build-up time for control force on time interval $[0, 25]$.

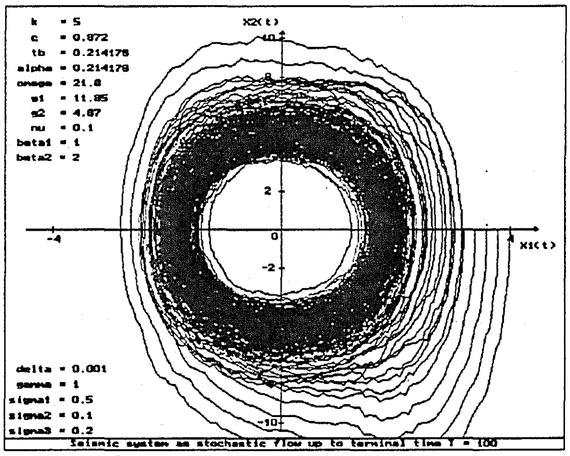

Figure 4: Phase diagram of the nonlinear seismic system with 'critical' build-up time for control force on time interval $[0, 100]$.

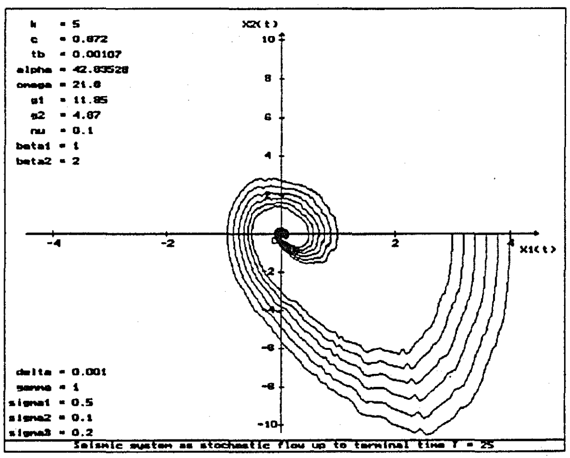

Figure 5: Phase diagram of the nonlinear seismic system with very small build-up time for control force on time interval $[0, 25]$.

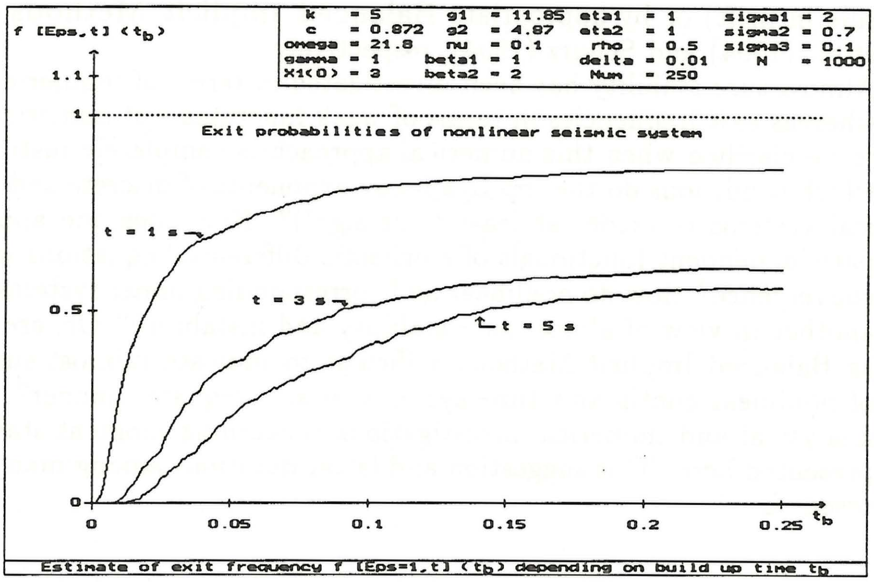

Figure 6: Estimate of the exit frequency $f_t^{\varepsilon}(t_b)$ with varying build-up time t_b and parameters $(\gamma, \sigma_1, \sigma_2, \sigma_3, \nu, \beta_1, \beta_2) = (1.0, 2.0, 0.7, 0.1, 0.1, 1.0, 2.0)$ started in $(3, 0, 0, 0, 0)$ and measured at times $t = 1, 3, 5$ sec.

5. Summary, remarks and open problems

The paper has examined the stability behaviour of stochastic–seismic systems under active control with distributed time delays. We have incorporated the modified control algorithm of Zhang et al. (1993) in order to take into account the finite time to build up the control force in mechanical systems. The role of interaction between structure and control can be specified in further studies, cf. Dyke et al. (1995). Our investigation is mainly concentrated on almost sure stability and mean exit frequencies of the arising systems under the presence of nonconstant time delay. When build-up time of mechanical control force vanishes, moment stability has already been investigated by Karmeshu and Schurz (1994).

The examination in this paper essentially relies on the removal of time delay by extension of dimension which is possible due to exponential–type kernels in modelling of time delay of control force. Other nontrivial modelling of time delay, e.g. random delay, would certainly cause serious troubles for rigorous analysis in practical applications, but this is a challenge for future research. We are either not sure what is better and simpler, to treat the original lower dimensional system with delay or the transformed higher dimensional one without delay.

Strong existence, uniqueness and regularity of exact solution of nonlinear stochastic systems presented here can be proved by using stochastic Lyapunov–type methods, whereas linear systems obviously satisfy classical requirements as linear–polynomial boundedness of coefficients of SDEs. In the presence of cubic nonlinearity, for 'nonclassical analysis', see Has'minskiĭ (1980). Thus, no spurious or exploding solutions come in after removal of exponential–type time delays by extension of dimension.

Critical values of build-up time for the change of qualitative behaviour could be numerically determined, whereas further analytical examination seems to be inaccessible in occuring high–dimensional problems. However, the difference between systems with weak and strong time delay was not really worked out, because of the computational burden to implement **adequate numerical methods** (preconditioning in high dimensions) as in

the case of strong delay. Adequate numerical solutions with respect to exact replication of mean square stability behaviour are provided by **Implicit Euler Methods** (trapezoidal rule in drift) or by appropriate **Balanced Implicit Methods (BIMs)**, see Mil'shtein et al. (1994) and Schurz (1994), respectively.
Almost sure stability has been investigated in terms of 'numerical Lyapunov exponents', whereas exit frequencies in terms of weak functionals of numerical solution. It still needs to be clarified when this numerical approach is completely justified. For example, under which conditions do the top Lyapunov exponents of discrete and continuous time dynamical systems coincide (at least their sign!)? How does one approximate nonsmooth or path–dependent functionals of stochastic differential equations with appropriate order of convergence? How do nonlinear and corresponding linear systems treated here relate each another in view of almost sure stability and instability? Or, are linear–implicit methods as Balanced Implicit Methods sufficient to indicate (almost surely) stochastic stability of nonlinear continuous time systems in an adequate manner? We also suggest further analytical and numerical investigations concerning moment stability of seismic systems presented here. This suggestion and latter questions among many others are left to future research.

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