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Martingale approach in pricing European options under regime-switching

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submitted: September 22, 2011

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Preprint No. 1645
Berlin 2011

2010 *Mathematics Subject Classification.* 60H35, 91G10.

Key words and phrases. Incomplete markets, martingale measure, generalized, self-financing strategy, attainability, self-financing in mean.

This research was supported by the Deutsche Forschungsgemeinschaft through the CRC 649 “Economic Risk”.

Edited by
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Abstract

The paper focuses on the problem of pricing and hedging a European contingent claim for an incomplete market model, in which evolution of price processes for a saving account and stocks depends on an observable Markov chain. The pricing function is evaluated using the martingale approach. The equivalent martingale measure is introduced in a way that the Markov chain remains the historical one, and the pricing function satisfies the Cauchy problem for a system of linear parabolic equations. It is shown that any European contingent claim is attainable using a generalized self-financing replicating strategy. For such a strategy, apart from the initial endowment, some additional funds are required both step-wise at the jump moments of the Markov chain and continuously between the jump moments. It is proved that the additional funds (the additional investments and consumptions) are present in the proposed strategy in the risk-neutral manner, hence the generalized self-financing strategy is self-financing in mean. A payment for the considered option should consist of two parts: the initial endowment and a fair insurance premium in order to compensate for contributions and consumptions arising in future.

1 Introduction

Let a financial market consist of a saving account $B(t)$ (riskless asset) and d stocks (risky assets) with price processes $X^i(t)$, $i = 1, \dots, d$. The system (B, X) is assumed to satisfy the stochastic differential equations

$$dB = r(X,\mu)B ds, \tag{1.1}$$

$$dX^i = X^i\Big[b^i(X,\mu)ds + \sum_{j=1}^{d}\sigma^{ij}(X,\mu)dW^j(s)\Big], \quad 0 \le s \le T, \quad i = 1, \dots, d, \tag{1.2}$$

where $W = (W^1, \dots, W^d)^\intercal$ is a d-dimensional standard Wiener process and μ is a Markov chain with finite state space $\{\mu_1, \dots, \mu_m\}$.

Let $X^{t,x,\mu_k}(s), \mu^{t,\mu_k}(s)$, $0 \le t \le s \le T$, be a trajectory of the Markov process (X, μ) (we consider the coefficients of (1.2) to be deterministic), where $\mu^{t,\mu_k}(s)$ is the Markov chain starting from μ_k at the moment t and $X^{t,x,\mu_k}(s)$ is the solution of (1.2) starting from x at the moment t with $\mu = \mu^{t,\mu_k}(s)$. We consider the problem of pricing and hedging a European claim at a maturity time T, specified by a payoff function f which depends on $X(T)$ and $\mu(T)$ by constructing a generalized self-financing strategy.

The model has a Markovian structure, hence the price of the contingent claim should be associated with a function $u(t,x,\mu_k)$. For evaluating $u(t,x,\mu_k)$, we use the martingale approach. To this end we choose an equivalent probability measure under which the discounted underlying assets are martingales. This martingale measure does not change the probabilistic characteristics of the Markov chain μ which remains a historical one. It turns out that the function $u(t,x,\mu_k)$ (or the collection $u_k(t,x) := u(t,x,\mu_k)$, $k = 1,\dots,m$) satisfies the Cauchy problem for a system of linear parabolic equations. The system can be solved by the Monte Carlo approach.

Analogously to the classical Black-Scholes case, one can expect that the price of the contingent claim $f(X(T),\mu(T))$ along the trajectory $X(s) = X^{t,x,\mu_k}(s)$, $\mu(s) = \mu^{t,\mu_k}(s)$ is connected with a wealth process

$$U(s) = u(s, X(s), \mu(s)).$$

Both X and μ are observable and, consequently, we can construct a trading strategy depending not only on X but also on μ. The system (1.1)–(1.2) has two sources of randomness: the d-dimensional Wiener process W and the Markov chain μ. However only the d-dimensional risky asset X and the saving account B are tradable. Hence the model is incomplete. Besides, in contrast to the classical case, the wealth process $U(s)$ here is discontinuous because of discontinuity of the Markov chain μ (we note that the stock prices $X^i(s)$ remain continuous). The process $U(s)$ is right-continuous with left limits (RCLL or càdlàg process). The discontinuities of $U(s)$ coincide with the jump times of the Markov chain μ.

Let τ be a jump time of μ from the state μ_l to the state μ_r. Then

$$\begin{aligned} U(\tau-) &= u(\tau, X(\tau), \mu_l) = u_l(\tau, X(\tau)), \\ U(\tau) &= u(\tau, X(\tau), \mu_r) = u_r(\tau, X(\tau)) = U(\tau+). \end{aligned}$$

Let $t < \tau_1 < \ldots < \tau_\nu < T$, $\nu = 1, 2, \ldots$, be all times between t and T where $\mu(s)$ has a jump. Between the jumps, i.e. on the intervals $[t,\tau_1)$, $(\tau_1,\tau_2), \dots,$ $(\tau_\nu, T]$, the wealth process $U(s)$ is constructed according to a trading strategy $(\Phi(s),\Psi(s)) = (\Phi(s),\Psi^1(s),\dots,\Psi^d(s))$ depending on $(s, X(s), \mu(s))$:

$$\Phi(s) = \varphi(s, X(s), \mu(s)), \quad \Psi^i(s) = \psi^i(s, X(s), \mu(s)), \quad i = 1, \dots, d,$$

and it has the value

$$U(s) = \Phi(s)B(s) + \sum_{i=1}^{d} \Psi^i(s)X^i(s). \tag{1.3}$$

If the strategy is self-financing then

$$U(s) = \Phi(s)dB(s) + \sum_{i=1}^{d} \Psi^i(s)dX^i(s). \tag{1.4}$$

However, the self-financing strategy which is able to replicate the price $u(s, X(s), \mu(s))$, is impossible (see Subsection 4.1). That is why we construct the wealth process in the form of generalized self-financing strategy, i.e. we admit (1.3) with

$$U(s) = \Phi(s)dB(s) + \sum_{i=1}^{d} \Psi^i(s)dX^i(s) + dD_s - dC_s \tag{1.5}$$

instead of (1.4), allowing some contributions dD_s and consumptions dC_s to the wealth process between the jump times.

Starting from the state (x, μ_k) at the moment t with the initial endowment $u(t, x, \mu_k)$, we construct the process $U(s)$ on the interval $[t, \tau_1)$ according to (1.3) and (1.5). At the moment τ_1 the Markov chain μ switches from μ_k to μ_r and the value of the portfolio changes from $u(\tau_1, X(\tau_1), \mu_k)$ to $u(\tau_1, X(\tau_1), \mu_r)$. Such change in the wealth of the portfolio requires an additional capital (a contribution to the wealth process) if

$$U(\tau_1) = u(\tau_1, X(\tau_1), \mu_r) > u(\tau_1, X(\tau_1), \mu_k) = U(\tau_1-)$$

or withdrawal (for instance, for consumption) in the case

$$U(\tau_1) = u(\tau_1, X(\tau_1), \mu_r) < u(\tau_1, X(\tau_1), \mu_k) = U(\tau_1-).$$

We proceed in the same way on the intervals $(\tau_1, \tau_2), \dots,\ (\tau_\nu, T]$. Thus, we should take into account the future necessary additional investments (contributions)

$$D_{\tau_i} := \max\left\{u(\tau_i, X(\tau_i), \mu_{r_i}) - u(\tau_i, X(\tau_i), \mu_{l_i}), 0\right\}$$

and consumptions

$$C_{\tau_i} := \max\left\{u(\tau_i, X(\tau_i), \mu_{l_i}) - u(\tau_i, X(\tau_i), \mu_{r_i}), 0\right\},$$

$i = 1, \dots, \nu.$

It is shown that any European contingent claim is attainable due to a generalized self-financing strategy. This property brings the considered model closer to the classical Black-Scholes model. The generalized replicating strategy on the interval $[t,T]$ is determined not only by the initial endowment (which is equal to $u(t,X(t),\mu(t))$) and by means of evolution of underlying assets but also due to contributions dD_s with consumptions dC_s and due to additional funds at the jump moments $\tau_1,\dots,\tau_\nu$. So, the proposed strategy requires some additional investments and consumptions. However, and this is very remarkable, they appear in the risk-neutral manner. To be more precise, it is proved that the mean of all the discounted additional funds on any interval $[t,\bar{t}\,]$ is equal to zero. One may say that the constructed here generalized self-financing strategy is self-financing in mean. Let us emphasize that the strategy $(\Phi(s),\Psi(s))$, the additional contributions dD_s and consumptions dC_s, and the additional funds at the jump moments are uniquely defined by the function $u(t,x,\mu)$. A payment for the considered option should consist of two parts: the initial endowment (the initial value of the wealth process) and a fair insurance premium in order to compensate for contributions and consumptions arising in future. We see that both financial and insurance aspects appear together in the considered model. However, here we restrict ourselves to constructing the wealth process and the generalized replicating strategy and to determining the necessary additional funds.

In Section 2, we briefly recall the well-known results concerning the classical Black-Scholes model (see, e.g. [5], [14]) in the required form. In Section 3, we choose a martingale measure fixing the chain μ as a historical one and construct the price $u(t,x,\mu_k)$. Section 4 is devoted to the generalized self-financing strategy and it contains derivation of formulas for $(\Phi(s),\Psi(s))$, dD_s, dC_s, and $U(\tau_i)-U(\tau_i-)$. In Section 5, it is proved that the mean of all the discounted additional funds on any interval $[t,\bar{t}\,]$ is equal to zero, hence the constructed generalized self-financing strategy is self-financing in mean. Using a Markov chain with single jump on the interval $[t,T]$, the proposed approach can be exploited for modelling defaults which is illustrated in Section 6.

2 Preliminary

Let us remind the problem of pricing and hedging a European claim at a maturity time T, specified by a payoff function f which depends on $X(T)$ in the classical Black-Scholes model:

$$dB = r(X)B\,ds, \tag{2.1}$$

$$dX^i = X^i\Big[b^i(X)ds + \sum_{j=1}^{d}\sigma^{ij}(X)dW^j(s)\Big], \quad 0\le s\le T, \quad i=1,\dots,d. \tag{2.2}$$

In (2.1)–(2.2), $W = (W^1, \ldots, W^d)^\intercal$ is a d-dimensional standard Wiener process on a probability space $(\Omega, \mathcal{F}, P)$. As usual, the P-augmentation of the filtration generated by W is denoted by $\mathcal{F}_t^W$. We write $\mathcal{F}_t$ for the σ-field $\mathcal{F}_t^W$. It is assumed that the functions $r(x)$, $x^i b^i(x)$, $x^i \sigma^{ij}(x)$, $i, j = 1, \ldots, d$, $k = 1, \ldots, m$, $x \in \mathbf{R}_+^d := \{x : x^1 > 0, \ldots, x^d > 0\}$, have bounded derivatives with respect to x up to some order. In addition, we assume that the volatility matrix $\sigma(x) = \{\sigma^{ij}(x)\}$ has full rank for any $x \in \mathbf{R}_+^d$.

We denote $X^{t,x}(s)$, $0 \le t \le s \le T$, the solution of (2.2) starting from x at the moment t and we denote $X(s) := X^{0,x}(s)$, $0 \le s \le T$. The price $U(t)$ of the contingent claim $f(X(T))$ is defined as the expectation of the discounted value of claim under the martingale measure $\tilde{P}$:

$$U(t) = E^{\tilde{P}} \left[B(t) B^{-1}(T) f(X(T)) \mid \mathcal{F}_t \right], \tag{2.3}$$

with $B(s) = B(0) e^{\int_0^s r(X(s'))ds'}$. Here $\tilde{P}$ is an equivalent probability measure under which the discounted stock price processes $\tilde{X}^i(s) := B^{-1}(s) X^i(s)$ are all martingales. It is known that

$$\begin{aligned} \frac{d\tilde{P}}{dP} &:= Z(T), \\ Z(s) &:= \exp\left\{ -\sum_{i=1}^d \left(\int_0^s \theta^i dW^i(s') + \frac{1}{2} \int_0^s [\theta^i]^2 ds' \right) \right\}, \end{aligned} \tag{2.4}$$

with the vector $\theta = (\theta^1, \ldots, \theta^d)^\intercal$ to be equal to

$$\theta = \sigma^{-1}(b - r\mathbf{1}), \quad \mathbf{1} = (1, \ldots, 1)^\intercal, \ b = (b^1, \ldots, b^d)^\intercal, \quad \sigma = \{\sigma^{ij}\},$$

where r, b, and σ are calculated at $(s', X(s'))$. Here it is assumed that $Z(s)$ is a martingale. A well known sufficient condition for the martingale property is

$$E \exp\left\{ \frac{1}{2} \int_0^T \sum_{i=1}^d [\theta^i]^2 ds \right\} < \infty. \tag{2.5}$$

We have

$$dX^i = X^i \Big[r ds + \sum_{j=1}^d \sigma^{ij} d\tilde{W}^j(s) \Big], \quad i = 1, \ldots, d, \quad X(0) = x, \tag{2.6}$$

where

$$\tilde{W}^j(s) = W^j(s) + \int_0^s \theta^j(s') ds'$$

is a $\tilde{P}$-Brownian motion due to the Girsanov theorem. Now the formula (2.3) can be written in the form

$$U(t) = E^{(2.6)} \left[e^{-\int_t^T r(X(s))ds} f(X(T)) \mid \mathcal{F}_t \right], \tag{2.7}$$

where the sign $E^{(2.6)}$ means that the averaging is carried by virtue of the system (2.6) with $\tilde{W}$ being the Brownian motion on the probability space $(\Omega, \mathcal{F}, \tilde{P})$ with filtration $\mathcal{F}_t$.

Due to the Markov property, the price $U(t)$ can be represented in the form

$$U(t) = u(t, X(t)), \tag{2.8}$$

where the function $u(t,x)$ is found according to

$$u(t,x) = E^{(2.6)}\left[e^{-\int_t^T r(X^{t,x}(s))ds} f(X^{t,x}(T))\right]. \tag{2.9}$$

This yields that the function $u(t,x)$ satisfies the following Cauchy problem for the parabolic partial differential equation

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{1}{2}\sum_{i,j=1}^{d} a^{ij}(x)\frac{\partial^2 u}{\partial x^i \partial x^j} + r(x)\sum_{i=1}^{d} x^i \frac{\partial u}{\partial x^i} - r(x)u &= 0, & (2.10)\\ u(T,x) &= f(x), & (2.11) \end{aligned}$$

where

$$a^{ij}(x) = x^i x^j \sum_{l=1}^{d} \sigma^{il}(x)\sigma^{jl}(x), \quad i,j = 1,\ldots,d.$$

The price $U(t) = u(t, X(t))$ coincides with the portfolio value of a trading strategy $(\Phi(t), \Psi(t)) = (\Phi(t), \Psi^1(t), \ldots, \Psi^d(t))$, where Φ and Ψ^i denote the portfolio positions in bond $B(t)$ and stocks $X^i(s)$, respectively. Hence $U(t)$ is given by

$$U(t) = \Phi(t)B(t) + \sum_{i=1}^{d} \Psi^i(t)X^i(t).$$

Self-financing property of the trading strategy implies

$$dU(t) = \Phi(t)dB(t) + \sum_{i=1}^{d} \Psi^i(t)dX^i(t). \tag{2.12}$$

On the other hand,

$$dU(t) = \frac{\partial u}{\partial t}dt + \sum_{i=1}^{d} \frac{\partial u}{\partial x^i}dX^i + \frac{1}{2}\sum_{i,j=1}^{d} \frac{\partial^2 u}{\partial x^i \partial x^j}dX^i dX^j. \tag{2.13}$$

The equations (2.12) and (2.13) imply in view of (2.6) and (2.10) that

$$\begin{aligned} \Phi(t) &= \varphi(t, X(t)) = \frac{1}{B(t)}\Big[u(t, X(t)) - \sum_{i=1}^{d} \frac{\partial u}{\partial x^i}(t, X(t)) \cdot X^i(t)\Big], & (2.14)\\ \Psi^i(t) &= \psi^i(t, X(t))) = \frac{\partial u}{\partial x^i}(t, X(t)), \end{aligned}$$

with

$$\varphi(t,x) = \frac{1}{B(t)}\Big[u(t,x) - \sum_{i=1}^{d}\frac{\partial u}{\partial x^i}(t,x)\cdot x^i\Big], \quad \psi^i(t,x)) = \frac{\partial u}{\partial x^i}(t,x), \quad i = 1,\dots,d.$$

3 The pricing function $u(t,x,\mu)$

Let us return to the system (1.1)–(1.2)

$$dB = r(X,\mu)Bds, \tag{3.1}$$

$$dX^i = X^i\Big[b^i(X,\mu)ds + \sum_{j=1}^{d}\sigma^{ij}(X,\mu)dW^j(s)\Big], \quad 0 \le s \le T, \quad i = 1,\dots,d. \tag{3.2}$$

In (3.1)–(3.2), $W = (W^1,\dots,W^d)^\intercal$ is a d-dimensional standard Wiener process and μ is a Markov chain with finite state space $\{\mu_1,\dots,\mu_m\}$ on a probability space $(\Omega,\mathcal{F},P)$. As usual, the P-augmentation of the filtration generated by W (by μ) is denoted by $\mathcal{F}_t^W$ (by $\mathcal{F}_t^\mu$). We write $\mathcal{F}_t$ for the σ-field $\mathcal{F}_t^W \cup \mathcal{F}_t^\mu$.

Let

$$Q = \begin{bmatrix} -q_1 & q_{12} & \dots & q_{1m} \\ q_{21} & -q_2 & \dots & q_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ q_{m1} & q_{m2} & \dots & -q_m \end{bmatrix}$$

be the infinitesimal generator matrix of the chain μ with $q_{kl} \ge 0$, $k \ne l$, and

$$\sum_{l\ne k} q_{kl} = q_k. \tag{3.3}$$

It is assumed that the functions $r(x,\mu_k)$, $x^i b^i(x,\mu_k)$, $x^i\sigma^{ij}(x,\mu_k)$, $i,j = 1,\dots,d$, $k = 1,\dots,m$, $x \in \mathbf{R}_+^d := \{x : x^1 > 0,\dots,x^d > 0\}$, have bounded derivatives with respect to x up to some order. In addition, we assume that the volatility matrix $\sigma(x,\mu_k) = \{\sigma^{ij}(x,\mu_k)\}$ has full rank for any $0 \le s \le T$, $x \in \mathbf{R}_+^d$, and μ_k, $k = 1,\dots,m$.

Let $X^{t,x,\mu_k}(s), \mu^{t,\mu_k}(s), 0 \le t \le s \le T$, be a trajectory of the Markov process (X,μ), where $\mu^{t,\mu_k}(s)$ is the Markov chain starting from μ_k at the moment t, $X^{t,x,\mu_k}(s)$ is the solution of (3.2) starting from x at the moment t with $\mu = \mu^{t,\mu_k}(s)$. We denote $X(s) := X^{0,x,\mu_k}(s)$, $\mu(s) := \mu^{0,\mu_k}(s)$, $0 \le s \le T$. Consider the problem of pricing and hedging a European claim at a maturity time T, specified by a payoff function f which depends on $X(T)$ and $\mu(T)$. Applying the martingale approach, we introduce the option's price using a natural analogue of the formula (2.3). So, the price of the option appears here in a postulated way. This price as a function of t, x, μ plays the key role in construction of the replicating self-financing in mean strategy.

3.1 The equivalent martingale measure

Analogously to the classical Black-Scholes case, one can expect that the price of the contingent claim $f(X(T), \mu(T))$ along the trajectory $X(s)$, $\mu(s)$, $0 \le s \le T$, is given by

$$\begin{aligned} U(t) &= E^{\tilde{P}}\left[B(t)B^{-1}(T)f(X(T), \mu(T)) \mid \mathcal{F}_t\right], \\ B(s) &= B(0)e^{\int_0^s r(X(s'), \mu(s'))ds'}, \quad 0 \le s \le T. \end{aligned} \tag{3.4}$$

In (3.4), $\mathcal{F}_t = \mathcal{F}_t^W \cup \mathcal{F}_t^\mu$, $\tilde{P}$ is an equivalent probability measure under which the discounted stock price processes $\tilde{X}^i(s) := B^{-1}(s)X^i(s)$ are all martingales.

Let us verify directly that such a measure is defined by the density (in comparison with (2.4), we have ϑ instead of θ)

$$\frac{d\tilde{P}}{dP} := Z(T), \quad Z(s) := \exp\left\{-\sum_{i=1}^d \left(\int_0^s \vartheta^i(s')dW^i(s') + \frac{1}{2}\int_0^s [\vartheta^i(s')]^2 ds'\right)\right\},$$

where the vector $\vartheta = (\vartheta^1, \dots, \vartheta^d)^\intercal$ is equal to

$$\vartheta = \sigma^{-1}(b - r\mathbf{1}), \quad \mathbf{1} = (1, \dots, 1)^\intercal, \quad b = (b^1, \dots, b^d)^\intercal, \quad \sigma = \{\sigma^{ij}\},$$

and r, b, and σ are evaluated at $(X(s'), \mu(s'))$.

Due to the Girsanov theorem (of course, we assume that the condition (2.5) with ϑ instead of θ is fulfilled), the process $\tilde{W}(s) = (\tilde{W}^1(s), \dots, \tilde{W}^d(s))$ with

$$\tilde{W}^j(s) = W^j(s) + \int_0^s \vartheta^j(s')ds', \quad j = 1, \dots, d,$$

is an $\mathcal{F}_t$-standard Wiener process. This yields

$$dX^i = X^i[b^i ds + \sum_{j=1}^d \sigma^{ij} dW^j(s)] = X^i\Big[b^i ds - \sum_{j=1}^d \sigma^{ij}\vartheta^j(s)ds + \sum_{j=1}^d \sigma^{ij} d\tilde{W}^j(s)\Big],$$

i.e.,

$$dX^i = X^i\Big[r ds + \sum_{j=1}^d \sigma^{ij} d\tilde{W}^j(s)\Big], \quad i = 1, \dots, d. \tag{3.5}$$

It follows easily from (3.5) that the processes $\tilde{X}^i(s) := B^{-1}(s)X^i(s)$ are $\tilde{P}$-martingales.

Further, (3.4) can be written in the form

$$U(t) = E^{(3.5)} \left[e^{-\int_t^T r(X(s),\mu(s))ds} f(X(T),\mu(T)) \mid \mathcal{F}_t \right],$$

where the sign $E^{(3.5)}$ means that the averaging is carried out with respect to $\mu^{t,\mu_k}(s)$ and $X^{t,x,\mu_k}(s)$, $t \le s \le T$, which is the solution of (3.5).

Because the process $(X(t),\mu(t))$ is Markovian, the price $U(t)$ can be represented as a function u of the position (t,x,μ) :

$$U(t) = u(t,X(t),\mu(t)), \quad 0 \le t \le T. \tag{3.6}$$

Due to the Markov property,

$$u_k(t,x) := u(t,x,\mu_k) = E^{(3.5)} \left[e^{-\int_t^T r(X^{t,x,\mu_k}(s),\mu^{t,\mu_k}(s))ds} f(X^{t,x,\mu_k}(T),\mu^{t,\mu_k}(T)) \right]. \tag{3.7}$$

3.2 The system of parabolic equations for $u(t,x,\mu_k)$

The infinitesimal generator A of the Markov process (X,μ) governed by the system (3.5) is given by (see [1, 3, 7, 8, 15])

$$\begin{aligned} Af(x,\mu_k) = \frac{1}{2}\sum_{i,j=1}^d a^{ij}(x,\mu_k)\frac{\partial^2 f}{\partial x^i \partial x^j}(x,\mu_k) + \sum_{i=1}^d x^i r(x,\mu_k)\frac{\partial f}{\partial x^i}(x,\mu_k) \\ -q_k f(x,\mu_k) + \sum_{l\neq k} q_{kl} f(x,\mu_l), \quad k=1,\dots,m, \end{aligned} \tag{3.8}$$

We note also that the discounted price

$$\begin{aligned} &e^{-\int_0^t r(X^{0,x,\mu_k}(s),\mu^{0,\mu_k}(s))ds} u(t,X^{0,x,\mu_k}(t),\mu^{0,\mu_k}(t)) \\ &\quad = E^{(3.5)} \left[e^{-\int_0^T r(X^{0,x,\mu_k}(s),\mu^{0,\mu_k}(s))ds} f(X^{0,x,\mu_k}(T),\mu^{0,\mu_k}(T)) \mid \mathcal{F}_t \right] \end{aligned}$$

is a $\tilde{P}$-martingale.

It follows from (3.8) that for sufficiently good functions $f_k(x) := f(x,\mu_k)$, $k=1,\dots,m$, (for instance, for functions with bounded derivatives up to some order), the functions (3.7) satisfy the following Cauchy problem for linear system of parabolic partial differential equations:

$$\begin{aligned} \frac{\partial u_k}{\partial s}(s,x) + \frac{1}{2}\sum_{i,j=1}^d a^{ij}(x,\mu_k)\frac{\partial^2 u_k}{\partial x^i \partial x^j}(s,x) + \sum_{i=1}^d x^i r(x,\mu_k)\frac{\partial u_k}{\partial x^i}(s,x) \\ -ru_k(s,x) - q_k u_k(s,x) + \sum_{l\neq k} q_{kl} u_l(s,x) = 0, \end{aligned} \tag{3.9}$$

$$u_k(T,x) = f(x,\mu_k), \quad k=1,\dots,m. \tag{3.10}$$

Because $\mu(s)$ is constant on any of the intervals $[t, \tau_1)$, $(\tau_1, \tau_2), \ldots,\ (\tau_\nu, T]$, the equation (3.9) implies:

$$\frac{\partial u}{\partial s}(s,x,\mu(s)) + \frac{1}{2}\sum_{i,j=1}^{d} a^{ij}(x,\mu(s))\frac{\partial^2 u}{\partial x^i \partial x^j}(s,x,\mu(s)) + \sum_{i=1}^{d} r(x,\mu(s))x^i \frac{\partial u}{\partial x^i}(s,x,\mu(s))$$
$$-ru(s,x,\mu(s)) - q_{\mu(s)}u(s,x,\mu(s)) + \sum_{l\neq\mu(s)} q_{\mu(s)l}u(s,x,l) = 0, \tag{3.11}$$
$$s \in (t,\tau_1)\cup(\tau_1,\tau_2)\cup\ldots\cup\ (\tau_\nu,T).$$

We show that using the price function $u(s, x, \mu)$, one is able to construct the replicating wealth process.

Remark 3.1. Suppose that an infinitesimal generator matrix $Q^{(1)}$ is such that $q_{kl}^{(1)} > 0$ if and only if $q_{kl} > 0$. It is known ([13], P. 39) that the new law of the Markov chain is equivalent to the old one. Basing on the new law we can introduce the equivalent martingale measure $\tilde{P}^{(1)}$ instead of the measure $\tilde{P}$ and repeat the previous construction. Clearly, we obtain the new pricing function $u^{(1)}(t, x, \mu_k)$ which satisfies the system (3.9)–(3.10) with $q_{kl}^{(1)}$ instead of q_{kl}. The questions concerning a choice of an equivalent martingale measure are not considered here. However, bearing in mind the problem of evaluating a fair insurance premium, the real world probability measure for μ given by Q seems to be suitable.

4 The wealth process, the generalized self-financing strategy, attainability

Let $t < \tau_1 < \ldots < \tau_\nu < \bar{t}$, $0 \le t < \bar{t} \le T$, be all the times between t and $\bar{t}$ where $\mu(s)$ has a jump. Using the Ito formula for $u(s, X(s), \mu(s))$ on the intervals $[\tau_0, \tau_1),\ (\tau_1, \tau_2), \ldots,\ (\tau_\nu, \tau_{\nu+1}]$, $\tau_0 := t$, $\tau_{\nu+1} := \bar{t}$, we obtain

$$du = \frac{\partial u}{\partial s}ds + \sum_{i=1}^{d} r\frac{\partial u}{\partial x^i}X^i ds + \frac{1}{2}\sum_{i,j=1}^{d} a^{ij}\frac{\partial^2 u}{\partial x^i\partial x^j}ds \tag{4.1}$$
$$+\sum_{i=1}^{d}\frac{\partial u}{\partial x^i}X^i\sum_{j=1}^{d}\sigma^{ij}d\tilde{W}^j(s), \quad \tau_i < s < \tau_{i+1}, \quad i = 0,\ldots,\nu, \quad \nu = 0,1,\ldots$$

On an interval $[t, \bar{t}\,]$, $0 \le t \le \bar{t} \le T$, we get (see [15])

$$\begin{aligned} &u(\bar{t}, X(\bar{t}), \mu(\bar{t})) - u(t, X(t), \mu(t)) \qquad (4.2)\\ &= \int_t^{\bar{t}} \left(\frac{\partial u}{\partial s} + \sum_{i=1}^d r \frac{\partial u}{\partial x^i} X^i + \frac{1}{2} \sum_{i,j=1}^d a^{ij} \frac{\partial^2 u}{\partial x^i \partial x^j} \right) ds + \int_t^{\bar{t}} \sum_{i=1}^d \frac{\partial u}{\partial x^i} X^i \sum_{j=1}^d \sigma^{ij} d\tilde{W}^j(s) \\ &\quad + \sum_{i=1}^{\nu} u(\tau_i, X(\tau_i), \mu(\tau_i)) - u(\tau_i, X(\tau_i), \mu(\tau_i -)), \end{aligned}$$

with the preceding notation the sum $\sum_{i=1}^0 := 0$.

In contrast to the classical case, the process

$$U(s) = u(s, X(s), \mu(s)) \qquad (4.3)$$

here is discontinuous on $[t, T]$ because of discontinuity of the Markov chain μ (we note that the asset prices $X^i(s)$ remain continuous). The process $U(s)$ is right-continuous with left limits (RCLL process), i.e. it is càdlàg. The discontinuity moments of $U(s)$ coincide with the jump times τ_i of the Markov chain μ which are observable.

4.1 Impossibility of replicating self-financing strategy

Let us suppose for a while that $U(s)$ is the value of a self-financing portfolio defined by $(\Phi_s, \Psi_s) = (\Phi_s, \Psi_s^1, \dots, \Psi_s^d)$ on the interval (τ_i, τ_{i+1}), where Φ_s and Ψ_s^i denote the portfolio positions in the bond $B(s)$ and stocks $X^i(s)$, respectively:

$$U(s) = \Phi_s B(s) + \sum_{i=1}^d \Psi_s^i X^i(s), \quad \tau_i < s < \tau_{i+1}. \qquad (4.4)$$

The trading strategy (Φ_s, Ψ_s) is self-financing on (τ_i, τ_{i+1}) if

$$\begin{aligned} dU &= \Phi_s dB + \sum_{i=1}^d \Psi_s^i dX^i \\ &= \Phi_s r B ds + \sum_{i=1}^d \Psi_s^i X^i \Big[r ds + \sum_{j=1}^d \sigma^{ij} d\tilde{W}^j(s) \Big], \quad \tau_i < s < \tau_{i+1}. \qquad (4.5) \end{aligned}$$

Assuming that $U(s)$ of the form (4.4) replicates the price u, we get from (4.3)

$$du = dU, \quad \tau_i < s < \tau_{i+1}. \qquad (4.6)$$

Let $\mu(s) = \mu_k$ under $\tau_i < s < \tau_{i+1}$. Comparing (4.1) and (4.5), we obtain

$$\Psi_s^i = \psi^i(s, X(s), \mu(s)) = \frac{\partial u}{\partial x^i}(s, X(s), \mu(s)) = \frac{\partial u_k}{\partial x^i}(s, X(s)), \qquad (4.7)$$

and

$$\frac{\partial u_k}{\partial s} + \frac{1}{2}\sum_{i,j=1}^{d} a^{ij}\frac{\partial^2 u_k}{\partial x^i \partial x^j} = \Phi_s r B, \quad \tau_i < s < \tau_{i+1}. \tag{4.8}$$

From (4.6), (4.4), and (4.7) we get

$$\Phi_s B(s) = u_k - \sum_{i=1}^{d} \Psi_s^i X^i(s) = u_k - \sum_{i=1}^{d} \frac{\partial u_k}{\partial x^i} X^i(s). \tag{4.9}$$

Substituting this in (4.8), we obtain

$$\frac{\partial u_k}{\partial s} + \frac{1}{2}\sum_{i,j=1}^{d} a^{ij}\frac{\partial^2 u_k}{\partial x^i \partial x^j} - r u_k + \sum_{i=1}^{d} r \frac{\partial u_k}{\partial x^i} X^i = 0. \tag{4.10}$$

However, (4.10) contradicts to (3.9). Hence the trading strategy of the form (4.4)–(4.5) is impossible and we should allow some contributions dD_s and consumption dC_s to the wealth process.

4.2 The generalized self-financing strategy

We propose to construct the wealth process in the form (4.4) but with the generalized self-financing strategy (see, e.g. [2]):

$$dU = \Phi_s r B ds + \sum_{i=1}^{d} \Psi_s^i X^i \Big[r ds + \sum_{j=1}^{d} \sigma^{ij} d\tilde{W}^j(s) \Big] + dD_s - dC_s, \tag{4.11}$$

$$\mu(s) = \mu_k, \quad \tau_i < s < \tau_{i+1},$$

with contributions dD_s and consumptions dC_s.

Let us choose the following contributions and consumptions:

$$dD_s = q_k u_k(s, X(s)) ds, \quad dC_s = \sum_{l \neq k} q_{kl} u_l(s, X(s)) ds. \tag{4.12}$$

The equality (4.11) is equivalent to

$$B d\Phi_s + \sum_{i=1}^{d} X^i d\Psi_s^i + \sum_{i=1}^{d} dX^i d\Psi_s^i = q_k u_k ds - \sum_{l \neq k} q_{kl} u_l ds, \quad \tau_i < s < \tau_{i+1}. \tag{4.13}$$

Comparing (4.1) and (4.11), we obtain

$$\Psi_s^i = \frac{\partial u_k}{\partial x^i}(s, X(s)), \quad \tau_i < s < \tau_{i+1}, \tag{4.14}$$

and from (4.3) and (4.14)

$$\Phi_s = \frac{1}{B(s)}\Big[u_k(s,X(s)) - \sum_{i=1}^{d} X^i(s)\frac{\partial u_k}{\partial x^i}(s,X(s))\Big], \quad \tau_i < s < \tau_{i+1}. \tag{4.15}$$

We pay attention that the formulas (4.14)–(4.15) coincide with (2.14). However, the strategy (4.14)–(4.15) is not self-financing in contrast to the classical case.

Now instead of (4.8) we get from (4.1) and (4.11)

$$\frac{\partial u_k}{\partial s} + \frac{1}{2}\sum_{i,j=1}^{d} a^{ij}\frac{\partial^2 u_k}{\partial x^i \partial x^j} = \Phi_s r B + q_k u_k ds - \sum_{l\neq k} q_{kl} u_l ds$$

which is consistent with (3.9) if we take into account (4.4). The equality (4.13) for the strategy (4.14)–(4.15) can be checked directly.

4.3 Dynamics of the wealth replicating process

Let us describe the evolution of the wealth process and the additional funds on the interval $[t,T]$. The initial value $U(t)$ of the wealth process is equal to the initial endowment $u(t,x,\mu(t))$. Due to (4.3), we have

$$U(s) = u(s,X(s),\mu(s)), \quad \mu(s) = \mu(t), \quad t \le s < \tau_1.$$

Besides, $U(s)$ satisfies (4.3), (4.11) and the funds $D_s,\ C_s$ are found as (see (4.12)):

$$\begin{aligned} D_s &= \int_t^s q_{\mu(t)} u(s',X(s'),\mu(t))ds', \\ C_s &= \int_t^s \sum_{l\neq\mu(t)} q_{\mu(t)l} u(s',X(s'),l)ds', \quad t \le s < \tau_1. \end{aligned} \tag{4.16}$$

So, starting from the initial endowment, the portfolio value $U(s)$ changes on $[t,\tau_1)$ through trading in the saving account B and in the underlying assets $X^1,\dots,X^d$ according to (4.14)–(4.15) and due to the contributions dD_s and consumptions dC_s according to (4.16). Clearly, $U(\tau_1-) = u(\tau_1-,X(\tau_1),\mu(t))$. Let the chain μ jump at the moment τ_1 from the state $\mu(\tau_1-) = \mu(t)$ to the state $\mu(\tau_1)$. Then we set $U(\tau_1) = u(\tau_1,X(\tau_1),\mu(\tau_1))$. If the difference

$$U(\tau_1) - U(\tau_1-) = u(\tau_1,X(\tau_1),\mu(\tau_1)) - u(\tau_1-,X(\tau_1),\mu(t))$$

is positive, we need the additional contribution at the moment τ_1 in order to have $U(\tau_1)$ for the wealth process U. And we get the consumption $U(\tau_1-) - U(\tau_1)$ if the difference

$U(\tau_1) - U(\tau_1-)$ is negative. On the intervals $[\tau_1, \tau_2), \dots, [\tau_\nu, T]$ and at the jump moments $\tau_2, \dots, \tau_\nu$ the behavior of the wealth process is analogous.

In particular, we obtain

$$U(T) = u(T, X^{t,x,\mu(t)}(T), \mu^{t,\mu(t)}(T)) = f(X^{t,x,\mu(t)}(T), \mu^{t,\mu(t)}(T)).$$

This equality shows that any European contingent claim is attainable in the considered model. Such a property is usual for complete markets. However, it is not fulfilled for incomplete markets if one bears in mind self-financing strategies. Though the considered market is incomplete, we attain this property due to using the generalized self-financing strategy.

Remark 4.1. The computational aspects of the considered model can be developed on the base of numerical integration of stochastic differential equations (see [6, 9, 11]) using special methods from computational finance, in particular, Monte Carlo methods [4]. A lot of works are devoted to numerics in finance. Let us mention [10] and [12] among them.

5 The generalized self-financing strategy is self-financing in mean

Introduce the value at time t of the s-price u :

$$\begin{aligned} &v(s, X^{t,x,\mu(t)}(s), \mu^{t,\mu(t)}(s)) \\ &\quad := e^{-\int_t^s r(X^{t,x,\mu(t)}(s'), \mu^{t,\mu(t)}(s'))ds'} u(s, X^{t,x,\mu(t)}(s), \mu^{t,\mu(t)}(s)). \end{aligned} \tag{5.1}$$

The discounted additional funds F_1 during the time from t to T inside the intervals (τ_i, τ_{i+1}), $i = 0, \dots, \nu$ with $\tau_0 := t$ and $\tau_\nu + 1 := T$, are equal to

$$F_1 = \int_t^T e^{-\int_t^s r ds'} \Big[q_{\mu(s)} u(s, X(s), \mu(s)) ds - \sum_{l \neq \mu(s)} q_{\mu(s)l} u(s, X(s), l) \Big] ds, \tag{5.2}$$

and the discounted additional funds F_2 at the moments τ_i, $i = 1, \dots, \nu$, are equal to

$$F_2 = \sum_{i=1}^{\nu} e^{-\int_t^{\tau_i} r ds'} [u(\tau_i, X(\tau_i), \mu(\tau_i)) - u(\tau_i, X(\tau_i), \mu(\tau_i-))]. \tag{5.3}$$

Theorem 5.1. *The $\tilde{P}$-mean of all the discounted additional funds on the interval $[t, T]$ is equal to zero.*

Proof. Applying the Ito formula to the function v on the interval $[t,T]$ (see [3], [15]), we obtain

$$v(T,X(T),\mu(T))-v(t,X(t),\mu(t))=e^{-\int_t^T rds'}u(T,X(T),\mu(T))-u(t,x,\mu(t)) \tag{5.4}$$

$$=\int_t^T e^{-\int_t^s rds'}\Big[-ru+\frac{\partial u}{\partial s}+\frac{1}{2}\sum_{i,j=1}^d a^{ij}\frac{\partial^2 u}{\partial x^i\partial x^j}+\sum_{i=1}^d r\frac{\partial u}{\partial x^i}X^i\Big]ds$$
$$+\int_t^T e^{-\int_t^s rds'}\sum_{i=1}^d\frac{\partial u}{\partial x^i}X^i\sum_{j=1}^d\sigma^{ij}d\tilde{W}^j(s)+F_2.$$

Due to (3.11), we get from (5.5)

$$e^{-\int_t^T rds'}f(X(T),\mu(T))-u(t,x,\mu(t))$$
$$=F_1+F_2+\int_t^T e^{-\int_t^s rds'}\sum_{i=1}^d\frac{\partial u}{\partial x^i}X^i\sum_{j=1}^d\sigma^{ij}d\tilde{W}^j(s). \tag{5.5}$$

According to the definition of price u (see (3.7) the mean of the left-hand side of (5.5) is equal to zero, hence the $\tilde{P}$-mean of the right-hand side is equal to $E^{\tilde{P}}(F_1+F_2)=0$. Theorem 5.1 is proved.

Corollary 5.1.*By the same way it can be proved that the $\tilde{P}$-mean of all the discounted additional funds on the any interval $[t,\bar{t}\,]$, $t<\bar{t}\le T$, is equal to zero.*

6 A single jump case

As an example, let us consider the situation when there is only a single jump of the Markov chain on the interval $[t,T]$. Such a situation can be modelled by the system (3.1)–(3.2) where the chain μ has absorbing states. For definiteness, consider the chain with two states μ_1 and μ_2 where μ_2 is the absorbing state. The infinitesimal generator matrix of such chain is equal to

$$Q=\begin{bmatrix}-q & q\\ 0 & 0\end{bmatrix},\quad q>0.$$

Let $\tau>t$ be a (single) jump time of the chain $\mu^{t,\mu_1}(s)$. If $\tau>T$ then during the time $t\le s\le T$ the chain has no jumps, i.e., $\mu^{t,\mu_1}(s)\equiv\mu_1$, $t\le s\le T$. The price $u(t,x,\mu_k)$, $k=1,2$, is given by (3.7) where the functions $u_1(t,x),\ u_2(t,x)$ satisfy the system (see

(3.9)–(3.10)):

$$\frac{\partial u_1}{\partial s} + \frac{1}{2}\sum_{i,j=1}^{d} a_1^{ij}\frac{\partial^2 u_1}{\partial x^i \partial x^j} + \sum_{i=1}^{d} r_1 x^i \frac{\partial u_1}{\partial x^i} - r_1 u_1 - q u_1 + q u_2 = 0, \tag{6.1}$$

$$u_1(t,x) = f_1(x),$$

$$\frac{\partial u_2}{\partial s} + \frac{1}{2}\sum_{i,j=1}^{d} a_2^{ij}\frac{\partial^2 u_2}{\partial x^i \partial x^j} + \sum_{i=1}^{d} r_2 x^i \frac{\partial u_2}{\partial x^i} - r_2 u_2 = 0, \quad u_2(t,x) = f_2(x). \tag{6.2}$$

This system can be solved sequentially starting from the second equation. We also note that in this case, the generalized self-financing strategy on the interval $[\tau, T]$ reduces to self-financing one.

If $u_2(\tau, x) \gg u_1(\tau, x)$ then there is a possibility of default.

Consider the particular case when $r_1 = r_2 := r$, $a_1^{ij} = a_2^{ij} := a^{ij}$, but

$$f_2(x) \geq f_1(x). \tag{6.3}$$

Subtracting (6.1) from (6.2), we obtain

$$\frac{\partial(u_2 - u_1)}{\partial s} + \frac{1}{2}\sum_{i,j=1}^{d} a^{ij}\frac{\partial^2 (u_2 - u_1)}{\partial x^i \partial x^j} + \sum_{i=1}^{d} r x^i \frac{\partial(u_2 - u_1)}{\partial x^i} \tag{6.4}$$

$$-r(u_2 - u_1) - q(u_2 - u_1) = 0, \quad u_2 - u_1 = f_2 - f_1 \geq 0.$$

It follows from (6.3) and (6.4) that

$$u_2(s,x) - u_1(s,x) \geq 0, \quad t \leq s \leq T. \tag{6.5}$$

Therefore we have

$$F_1 = \int_t^{\tau\wedge T} e^{-\int_t^s r ds'} q[u_1(s, X(s)) - u_2(s, X(s))]ds \leq 0.$$

Because the integrand here is negative, the value $-e^{-\int_t^s r ds'} q[u_1(s, X(s)) - u_2(s, X(s))]ds$ is positive. This value is the discounted consumption on $[s, s + ds]$ and therefore $-F_1$ is the summarized discounted consumption of the wealth process on the interval $[t, \tau \wedge T]$. Further,

$$F_2 = \begin{cases} e^{-\int_t^\tau r ds'}[u_2(\tau, X(\tau)) - u_1(\tau, X(\tau))], & t < \tau \leq T, \\ 0, & \tau > T, \end{cases}$$

is positive, hence F_2 is the necessary additional contribution to the wealth process. According to Theorem 5.1 $E^{\tilde{P}} F_2 = -E^{\tilde{P}} F_1$. In the case of default, $E^{\tilde{P}} F_2 \gg 0$ and therefore the $\tilde{P}$-mean of the discounted consumption is large as well. This can be done only at cost of the sufficiently large endowment. This fact corresponds with our intuition: in the prevision of a serious default the price of the option should be large.

7 Summary and outlook

The paper considers the problem of pricing and hedging a European-type contingent claim in an incomplete market with regime switching. First we show that a replicating self-financing strategies in this situation does not exist. Instead, for any contingent claim, we construct a generalized self-financing strategy. Such a strategy requires not only an initial endowment but also some additional funds (additional investments and consumptions) which have to be involved both at the jump moments of the Markov chain and continuously between the jump moments. However, these additional funds are risk-neutral, so that the generalized self-financing strategy is self-financing in mean.

The construction is based on the general martingale approach and it is reduced to a system of linear parabolic differential equations which can be solved numerically by a Monte Carlo method.

A payment for the considered option should consist of two parts: the initial endowment and a fair insurance premium in order to compensate for contributions and consumptions arising in future. So, both financial and insurance aspects appear together in the considered model. This paper restricts itself to constructing the generalized replicating strategy and to determining the necessary additional funds. The insurance premium aspect will be studied elsewhere.

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