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Uniqueness in determining polyhedral sound-hard obstacles with a single incoming wave

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Abstract

We consider the inverse acoustic scattering problem of determining a sound-hard obstacle by far field measurements. It is proved that a polyhedral scatterer in $\mathbb{R}^n$, $n \geq 2$, consisting of finitely many solid polyhedra, is uniquely determined by a single incoming plane wave.

1 Introduction

Let D be a compact subset of $\mathbb{R}^n$, $n \geq 2$, and assume that a time harmonic plane wave $u^{in}(x) = \exp(ikd \cdot x)$, $x \in \mathbb{R}^n$, is incident on the scatterer D. Here $k > 0$ is the wave number, which is kept fixed throughout the paper, and $d \in \mathbb{S}^{n-1}$ is the incident direction. In the case of a sound-hard scatterer D, the total field u which is the sum of u^{in} and the scattered field u^{sc} satisfies the following exterior boundary value problem in $D^c := \mathbb{R}^n \backslash D$:

$$\left.\begin{aligned} \Delta u + k^2 u = 0 \quad \text{in } D^c\,, \quad \partial_\nu u = 0 \quad \text{on } \partial D\,, \\ \lim_{r\to\infty} r^{(n-1)/2}(\partial_r u^{sc} - iku^{sc}) = 0\,, \quad r = |x|\,. \end{aligned}\right\} \tag{1.1}$$

Here ν is the unit normal to ∂D pointing to D^c, and the relation in the second line of (1.1) is the Sommerfeld radiation condition which holds uniformly in all directions $\hat{x} = x/|x| \in \mathbb{S}^{n-1}$ as $|x| \to \infty$. This condition implies that the asymptotic behaviour at infinity of the scattered field u^{sc} is governed by the relation

$$u^{sc}(x) = r^{(1-n)/2} \exp(ikr)\{u_\infty(\hat{x}) + O(r^{-1})\}\,,\ r \to \infty\,, \tag{1.2}$$

holding uniformly in all directions $\hat{x} \in \mathbb{S}^{n-1}$; see, e.g., [3], [11]. The function u_∞ defined by (1.2) on $\mathbb{S}^{n-1}$ is called the far field pattern of u^{sc}. In the case of a sound-soft scatterer D, the boundary condition on ∂D in (1.1) is replaced by $u = 0$.

The inverse acoustic scattering problem consists in determining a (sound-soft or sound-hard) scatterer by its far field pattern u_∞ for one or several incident directions, and its uniqueness presents important and challenging open problems since many years; see, e.g., [3], [4], [7]. Uniqueness results with a minimal number of incident waves have recently been obtained within the class of polygonal and polyhedral scatterers.

Definition 1 *A compact set $D \subset \mathbb{R}^n$ is called a polyhedral obstacle if D is the union of finitely many convex polyhedra and its exterior D^c is connected. We shall say that a compact set $D \subset \mathbb{R}^n$ with connected exterior D^c is a polyhedral scatterer if D is the union of a polyhedral obstacle and finitely many cells, where a cell is defined as the closure of an open connected subset of an $(n-1)$-dimensional hyperplane.*

Note that a polyhedral obstacle coincides with the closure of its interior, whereas a polyhedral scatterer may also contain $(n-1)$-dimensional components (e.g., screens).

It was proved in [1] that any sound-soft polyhedral scatterer is uniquely determined by the far field pattern of a single incident wave. The method in [1] is based on a careful study of the nodal set $\mathcal{N} = \{x : u(x) = 0\}$ of the direct solution in D^c, the reflection principle for the Helmholtz equation, and the construction of a path in D^c connecting a point on ∂D to infinity and intersecting $\mathcal{N}$ suitably.

The approach of [1] was considerably simplified in [8] to obtain a shorter proof in the sound-soft case, together with the result that any sound-hard polyhedral scatterer is uniquely determined by the far field patterns for n linearly independent incident directions, generalizing previous work in the 2D case [2]. Moreover, these uniqueness results can be extended to scatterers with impedance and mixed type (Dirichlet/Neumann) boundary conditions [9], [10].

What about uniqueness in the inverse Neumann problem with only one incident direction? The counter-examples of [9] show that, in general, a polyhedral scatterer cannot be uniquely reconstructed using less than n incident waves. On the other hand, it was shown in [6] and [10] that one incident wave is enough to recover a polygonal obstacle (in the sense of Definition 1 for $n = 2$). The aim of this paper is to prove the higher dimensional analogue of that result.

Theorem. *For fixed $k > 0$ and $d \in \mathbb{S}^{n-1}$, a polyhedral obstacle $D \subset \mathbb{R}^n$ $(n \geq 2)$ is uniquely determined by the far field pattern u_∞.*

For the proof of our uniqueness result, the notion of a Neumann plane is of importance (cf. also [8]).

Definition 2 *Let $\Pi \subset \mathbb{R}^n$ be an $(n-1)$-dimensional hyperplane. A non-void open connected component π of $\Pi \cap D^c$ such that $\partial_\nu u = 0$ in π (where u is the solution of* (1.1)*) is called a Neumann plane of u.*

Note that in contrast to a Dirichlet plane in the sound-soft case, which is always a bounded set (see [1], [8]), a Neumann plane may be unbounded. As a key preliminary result, we will show in Section 2 that there are at most finitely many unbounded Neumann planes of u (Lemma 2). To carry out the proof of the theorem in Section 3, we modify the path and reflection arguments of [1] and [8] in that we construct a path to infinity avoiding the unbounded Neumann planes. Note that our approach here differs essentially from that in the 2D case [6], [10], which is mainly based on the finiteness of the set of bounded Neumann planes (lines) and is difficult to extend to higher dimensions.

2 Preliminaries

Let $D \subset \mathbb{R}^n$ be a polyhedral obstacle, and let $u \in H^1_{loc}(D^c)$ be the unique solution of the direct problem (1.1); see, e.g., [5, Chap. 3.4]. Note that the solution u to the homogeneous Helmholtz equation is real-analytic in D^c. We first collect some properties of the Neumann planes of u, which will be needed in the sequel.

Let $\pi \subset \Pi$ be a Neumann plane of u, where $\Pi \subset \mathbb{R}^n$ is an $(n-1)$-dimensional hyperplane. Then its boundary $\partial\pi$ is always a subset of ∂D (but may be void) since otherwise the zero set of $\partial_\nu u$ on $\Pi \cap D^c$ could be extended.

A Neumann plane $\pi \subset \Pi$ may be bounded or unbounded, but Π can contain at most one unbounded Neumann plane for $n \geq 3$ and at most two Neumann planes (lines) if $n = 2$. This is clear for $n = 2$ since a connected open set in $\mathbb{R}$ is a (bounded or unbounded) interval, and there cannot lie more than two unbounded intervals on an infinite straight line. Here and in the following, we refer to [12, Chaps. 2.9, 2.10] for the properties of connected sets. For $n \geq 3$, assume there are two different unbounded Neumann planes π_1, π_2 on Π. However, outside a sufficiently large ball $B \subset \Pi$, there is no continuous curve on Π connecting points from π_1 and π_2 since it would intersect ∂D. Note that ∂D is bounded and B^c is connected.

Moreover, a hyperplane Π contains at most finitely many bounded Neumann planes. Note that $\overline{D^c}$ is the union of Q^c and finitely many bounded convex polyhedra, where Q is a sufficiently large closed cube. Hence $\overline{D^c}$ is the union of finitely many (possibly unbounded) convex polyhedra. Moreover, the intersection of the interior of such a polyhedron with Π is either void or an $(n-1)$-dimensional open convex polyhedron. Thus the intersection of Π with D^c can only have finitely many connected components.

Lemma 1 *The normal to an unbounded Neumann plane is always orthogonal to the wave vector d of the incident wave u^{in}.*

The proof is analogous to that of Lemma 9 in [2], using the fact that $\lim_{r\to\infty} |\nabla u^{sc}| = 0$; see also Lemma 2 in [8].

For $n = 2$, it was proved in [10, Cor. 2.16] that all unbounded Neumann lines must lie on one infinite straight line, so that there exist at most two unbounded Neumann lines of u. Using Lemma 1, we can prove the following weaker version of this result which is valid in any dimensions and is sufficient for our purposes.

Lemma 2 *There is at most a finite number of unbounded Neumann planes of u, say $\pi_j \subset \Pi_j$, $j = 1, \cdots, N$, where Π_j are $(n-1)$-dimensional hyperplanes. Of course, the set of unbounded Neumann planes may be void, $N = 0$.*

Note that the hyperplanes Π_j must be mutually different for $n \geq 3$, whereas this need not be the case for $n = 2$.

Proof of Lemma 2. Assume there exists a sequence of (different) hyperplanes $\{\Pi_j : j \in \mathbb{N}\}$ such that there is (at least) one unbounded Neumann plane $\pi_j \subset \Pi_j$ for each j. We first show that the convex hull $\mathcal{D}$ of the polyhedral obstacle D must be symmetric with respect to each $\Pi \in \{\Pi_j\}$.

Let R denote the reflection with respect to Π. If $\mathcal{D}$ were not symmetric, i.e. $\mathcal{D} \neq R(\mathcal{D})$, there would exist a vertex $P \in R(\mathcal{D})$ such that $P \in \mathcal{D}^c$. Applying even reflection to the solution u of (1.1) and using the fact that at least n faces (cells) of D meet at the vertex $R(P)$ of $\mathcal{D}$ (and D), we obtain n unbounded Neumann planes passing P and having linearly independent normal vectors. Note that $P \in \mathcal{D}^c \subset D^c$ is a vertex of the convex set $R(\mathcal{D})$, and u is even symmetric with respect to Π and analytic in D^c. This contradicts Lemma 1 since there do not exist more than $n-1$ linearly independent vectors orthogonal to x_n-direction.

Finally, we observe that $\mathcal{D}$ cannot be symmetric with respect to infinitely many hyperplanes since the number of vertices of $\mathcal{D}$ is finite. This contradiction finishes the proof of the lemma. □

3 Proof of Theorem

3.1 The case $N = 0$

To prove the theorem in this case (where no unbounded Neumann plane of u exists), we employ path and reflection arguments due to [1] and later modified in [8]. Here we follow [8] in spirit, but present a shorter version.

Step 1: existence of a Neumann plane
Assume contrarily that there is another polyhedral obstacle $D_1 \neq D$ such that the far fields of u and u_1 (the solution of problem (1.1) for D_1) coincide on $\mathbb{S}^{n-1}$. The following arguments are standard, and we refer to [6], [8] for the details. We have

$$u_1 = u \quad \text{in the unbounded connected component } \ \Omega \ \text{ of } \ \mathbb{R}^n \backslash (D \cup D_1)\,. \tag{3.1}$$

Furthermore, since D^c and D_1^c are connected, we obtain $\partial\Omega \not\subset D \cup D_1$ and can assume without loss of generality that

$$S := (\partial D_1 \backslash D) \cap \partial\Omega \neq \emptyset\,. \tag{3.2}$$

By (3.1) and (3.2), there is a cell $F \subset S$ such that $\partial_\nu u = 0$ on F, and denoting by Π the hyperplane containing F and by $\mathrm{int}(F)$ the interior of the set F, we find a Neumann plane π of u such that $\mathrm{int}(F) \subset \pi \subset \Pi$ which must be bounded by our assumption.

Step 2: path argument
Choose a point $P \in \mathrm{int}(F)$ and a continuous and injective path $\gamma(t)$, $t \geq 0$, starting at $P = \gamma(0)$ and leading to infinity in the connected set Ω. In fact, if B is a sufficiently large ball (centered at the origin), we can first connect P with some point $Q \in B^c \cap \Omega$ by finitely many segments parallel to the coordinate axes, and Q may be connected to infinity e.g. by a ray parallel to x_n-direction. Note that the set $\gamma := \{\gamma(t) = t \geq 0\}$ is homeomorphic to $[0, \infty)$.

Let $\mathcal{M}$ be the set of intersection points of γ with all (bounded) Neumann planes of u. By Step 1, $\mathcal{M} \neq \emptyset$. Moreover, $\mathcal{M}$ is bounded since there is no bounded Neumann plane outside a sufficiently large ball B. (A Neumann plane π, with $\pi \cap B^c \neq \emptyset$ and B large, must be unbounded since ∂D is bounded.)

By Lemma 2 in [8], $\mathcal{M}$ is also closed, hence compact. Thus there exists $t_0 \geq 0$ such that no Neumann plane of u can intersect $\gamma(t)$ for $t > t_0$. Let $\pi_0 \subset \Pi_0$ be a Neumann plane passing $\gamma(t_0)$, where Π_0 is an $(n-1)$-dimensional hyperplane.

Step 3: reflection argument and final contradiction
We now apply the reflection argument of [1, Lemma 3.7] to prove the existence of a Neumann plane π' intersecting $\gamma(t)$ at some $t' > t_0$ which is a contradiction.

Let R denote the reflection with respect to the plane Π_0, and choose $x^+ = \gamma(t_0 + \varepsilon)$ for $\varepsilon > 0$ sufficiently small and $x^- = R(x^+)$. Let $G^\pm$ be the connected component of $D^c \backslash \pi_0$ containing $x^\pm$, and denote by $E^\pm$ the connected component of $G^\pm \cap R(G^\mp)$ containing $x^\pm$. We set $E = E^+ \cup \pi_0 \cup E^-$. Note that E is a connected open set whose boundary consists of cells of ∂D and $R(\partial D)$.

Then, by the (even) reflection principle for the Helmholtz equation in D^c, we obtain that u is even symmetric in E (with respect to Π_0), so that $\partial_\nu u = 0$ on ∂E and $E \cap \Pi_0$. Moreover, E is bounded since otherwise Π_0 would contain an unbounded Neumann plane.

Hence, $\gamma(t)$ must intersect ∂E at some $t' > t_0$, so that there exists a Neumann plane π' passing $\gamma(t')$. □

Remark 1 *The decisive step in the above argument is the boundedness of the set E. Here this is ensured by the fact that Π_0 does not contain an unbounded Neumann plane.*

Another possibility to prove boundedness of E is to use a bounded connected component G^- of $D^c \backslash \pi_0$ with $\partial_\nu u = 0$ on ∂G^-, in which case Π_0 may contain an unbounded Neumann plane. We will employ a version of this argument to prove existence of a bounded Neumann plane in the case $N \geq 1$.

3.2 The case $N \geq 1$

We now assume that there is at least one unbounded Neumann plane of u.

Definition 3 *Let π be a Neumann plane of u. We write $\pi \in \mathcal{P}_0$ if π is either an unbounded Neumann plane ($\pi_j \subset \Pi_j$, $j = 1, \cdots, N$) or a bounded Neumann plane lying on one of the hyperplanes Π_j ($j = 1, \cdots, N$). Otherwise, if π is bounded and not contained in one of the hyperplanes Π_j, we write $\pi \in \mathcal{P}_1$.*

To prove the theorem for $N \geq 1$ by contradiction along the lines of Section 3.1, we will modify the path and reflection arguments appropriately. More precisely, we construct a path $\gamma(t)$, $t \geq 0$, which starts at a Neumann plane of u and leads to infinity avoiding the finitely many (cf. Section 2) Neumann planes of $\mathcal{P}_0$ for all $t > 0$. Then we prove existence of a bounded Neumann plane intersecting $\gamma(t)$ at some $t^* > 0$ by using the reflection argument. Finally, the path and reflection arguments of Section 3.2 are applied again to obtain a contradiction to the existence of a "last" intersection point of $\{\gamma(t) : t \geq t^*\}$ with the Neumann planes of $\mathcal{P}_1$.

Step 1. By Lemma 2 we can assume that $\mathcal{P}_0 = \{\pi_1, \cdots, \pi_{N+M}\}$, where $M \geq 0$ and π_j, $j > N$, are bounded Neumann planes lying on the hyperplanes Π_k, $k = 1, \cdots, N$. We introduce the open set (cf. (3.1))

$$\Sigma := \Omega \backslash \bigcup_{j=1}^{N+M} \pi_j \,, \tag{3.3}$$

which has only finitely many bounded and unbounded connected components, but at least one unbounded connected component. In fact, $\bar{\Omega}$ is the union of finitely many (possibly unbounded) convex polyhedra, and by the hyperplanes $\Pi_1, \cdots, \Pi_N$ each of these convex polyhedra is cut into a finite number of polyhedral connected components. Recall that each π_j $(j = 1, \cdots, N + M)$, i.e. any Neumann plane on $\Pi_1, \cdots, \Pi_N$, extends to the boundary of Ω and/or to infinity.

Let $\Omega_1, \cdots, \Omega_l$ be the bounded components of Σ if there is any, and the case $l = 0$ is not excluded. We remove the bounded components (which may block the exit to infinity) from Σ by setting

$$\Sigma_1 := \Sigma \setminus \bigcup_{j=1}^{l} \bar{\Omega}_j \tag{3.4}$$

and observe that $\partial\Sigma_1$ consists of cells lying on $\partial\Omega$ and the Neumann planes from $\mathcal{P}_0$. The following lemma is crucial for the path and reflection arguments of the next step.

Lemma 3 *There is a cell $F^* \subset \partial\Sigma_1$ with the following properties.*

(i) int(F^*) is contained in a Neumann plane π_0 (which may be unbounded),

(ii) F^* lies on the boundary of a bounded connected component, say G^-, of $\mathbb{R}^n \setminus (\bar{\Sigma}_1 \cup D)$ such that $\partial_\nu u = 0$ on ∂G^-,

(iii) F^* belongs to the boundary of some (unbounded) connected component Ω^* of Σ_1.

Proof of Lemma 3. Let first $l \geq 1$ and set $\mathcal{K} = \bigcup_{j=1}^{l} \bar{\Omega}_j$. Note that $\bar{\Omega}$, $\bar{\Sigma}_1$ and $\mathcal{K}$ are unions of finitely many convex polyhedra (bounded ones in case of $\mathcal{K}$), and by (3.3), (3.4) we have

$$\bar{\Omega} = \bar{\Sigma} = \mathcal{K} \cup \bar{\Sigma}_1 , \quad \text{int}(\mathcal{K}) \cap \text{int}(\bar{\Sigma}_1) = \emptyset . \tag{3.5}$$

The connectedness of Ω and (3.5) imply

$$S^* := \partial\bar{\Sigma}_1 \cap \partial\mathcal{K} \cap \Omega \neq \emptyset , \tag{3.6}$$

and since $\partial\bar{\Sigma}_1$ and $\partial\mathcal{K}$ consist of cells, there exists a cell F with

$$F \subset S^* \subset \partial\bar{\Sigma}_1 \subset \partial\Sigma_1 . \tag{3.7}$$

Moreover, there is a subcell $F' \subset F$ such that

$$F' \subset \partial\Omega_{j^*} \quad \text{for some } \; j^* \leq l . \tag{3.8}$$

To verify (3.8), we note that each $\bar{\Omega}_j$ is the union of finitely many convex polyhedra, and the intersection of the boundary of such a polyhedron with F is either a cell or a convex set of dimension $\leq n - 2$ (possibly void). However, the latter case cannot occur for each of those convex polyhedra since its intersection with F is nowhere dense in F.

From (3.6)–(3.8) we now observe that the cell F' satisfies (i) and (ii). In particular, it is contained in a Neumann plane of $\mathcal{P}_0$, and we can choose as G^- the bounded component Ω_{j^*}.

Let now $l = 0$, i.e. $\Sigma_1 = \Sigma$. As in Step 1 in Section 3.1 we can choose a cell $F \subset S \subset \partial\Omega$, which additionally belongs to the boundary of a bounded connected component of $D^c \backslash \partial D_1$, say G^-, for which $\partial_\nu u = 0$ on ∂G^-. This follows immediately from (3.1) and (3.2), provided there exists a bounded connected component of $D^c \backslash (\partial D_1 \cap \partial\Omega)$. If the latter set is unbounded and connected, we obtain $\partial_\nu u = 0$ on $\partial(D^c \backslash \partial D_1)$. Note that we have $D^c \backslash \partial D_1 \subset D^c \backslash (\partial D_1 \cap \partial\Omega)$ and thus $u = u_1$ in $D^c \backslash \partial D_1$. Therefore, since $\partial(D^c \backslash \partial D_1) \subset \partial D \cup \partial D_1$, $\partial_\nu u = 0$ on ∂D and $\partial_\nu u_1 = 0$ on ∂D_1, we obtain $\partial_\nu u = 0$ on $\partial(D^c \backslash \partial D_1)$. Furthermore, by (3.2), and since D and D_1 are assumed to be polyhedral obstacles, there is a bounded connected component of $D^c \backslash \partial D_1$, so that we have $\partial_\nu u = 0$ on its boundary.

Therefore, in any case, there exists a cell F satisfying (i) and (ii). Finally, since Σ_1 (where $\Sigma_1 = \Sigma$ for $l = 0$) only consists of finitely many (unbounded) polyhedral components, we can choose a subcell $F^* \subset F$ which also satisfies (iii); see the proof of (3.8). □

Step 2. Now we select a (continuous and injective) path $\gamma(t)$, $t \geq 0$, starting at some point $P^* = \gamma(0) \in \mathrm{int}(F^*)$ and leading to infinity in the connected set Ω^* (from Lemma 3).

Indeed, for a sufficiently large ball B centred at the origin, we first connect P^* to some point $Q^* \in B^c \cap \Omega^*$ by finitely many segments parallel to the coordinate axes. Then, by Lemma 1 and because there is no bounded Neumann plane outside B, we may connect Q^* to infinity by a ray parallel to x_n-direction.

Next we show that γ must intersect another Neumann plane π at some point $P = \gamma(t^*)$, $t^* > 0$. For this we apply the reflection argument of Section 3.1 to u with respect to the hyperplane $\Pi_0 \supset F^*$, where F^* is the cell from Lemma 3.

In particular, we use the bounded component G^- from Lemma 3 and choose G^+ as the connected component of $D^c \backslash \pi_0$ containing $\gamma(\varepsilon)$ for sufficiently small $\varepsilon > 0$ (cf. Section 3.1, Step 3). Thus the existence of such a Neumann plane π is guaranteed (cf. also Remark 1), and by our construction of the path γ, π must belong to $\mathcal{P}_1$.

Denoting by $\mathcal{M}^*$ the set of intersection points of $\gamma^* = \{\gamma(t) : t \geq t^*\}$ with all Neumann planes of $\mathcal{P}_1$, we can repeat the arguments of Section 3.1 to show that $\mathcal{M}^*$ is compact. Hence, there exists a “last” intersection point of γ^* with the (bounded) Neumann planes from $\mathcal{P}_1$.

Step 3. Performing another reflection step, we obtain a final contradiction as in Section 3.1 since γ^* can only intersect Neumann planes from $\mathcal{P}_1$. This finishes the proof of the theorem. □

Remark 2 *The theorem is not true for polyhedral scatterers in the sense of Definition 1 as counter-examples in [9] show. In fact, the results of Liu and Zou [8] on the uniqueness of polyhedral scatterers in the inverse Neumann problem with n linearly independent incident directions cannot be improved in general. We also note that the arguments of Section 3.1 are sufficient to prove these uniqueness results with n incident waves since the corresponding (simultaneous) Neumann planes are always bounded.*

Remark 3 *Combining our approach with some of the arguments in [9], [10], the theorem can be extended to polyhedral obstacles with mixed type and impedance boundary conditions.*

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