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A model for mode-locking in quantum dot lasers

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Abstract

We propose a model for passive mode-locking in quantum dot laser and report on specific dynamical properties of the regime which is characterized by a fast gain recovery. No Q-switching instability has been found accompanying the mode-locking. Bistability can occur between the mode-locking regime and zero intensity steady state.

Lasers and amplifiers based on self-assembled quantum dots attract significant attention due to reduced threshold current, low chirp, weak temperature dependence [1] and a reduced sensitivity to optical feedback [2] at telecom wavelengths. Among the regimes displayed by multimode lasers, passive mode-locking (ML) is a powerful method to generate short pulses for time domain multiplexing and for optical comb generator. The first experiments have been reported on ML in quantum dot (QD) lasers at 1.3 μm up to 50 GHz [3, 4] and demonstrated their superiority to quantum well lasers for network applications [5]. The physics of the passive ML in a QD laser remains the same as in other lasers: the absorbing medium saturates faster than the amplifying one, and, therefore, a short window of net gain emerges for the pulse amplification [6]. However, the nonlinear dynamics of ML depends on the specific characteristics of QD material and deserves detailed theoretical analysis in order to optimize QD lasers for practical applications.

In this Letter we construct a delay differential model to describe ML in quantum dot lasers using the approach proposed in [7, 8]. In our model the dynamics of each of the two quantum dot laser sections, gain and absorber, is governed by two rate equations for the time evolution of the quantum dot occupation probabilities and the carrier densities in the wetting layer. We find that ML in quantum dot lasers exhibits specific dynamical properties which we associate with the escape and the capture processes in the quantum dot material. In particular, we explain the absence of the low frequency Q-switching instability and the bistability with the nonlasing state.

Let us consider a ring laser consisting of three sections. The first section with the length L_q, and the second section with the length L_g, contain the saturable absorber and the gain medium, respectively. The third section acts as a spectral filter that limits the bandwidth of the laser radiation. The equations describing the evolution of the electric field envelope $A(t)$ at the entrance of the absorber section can be written in the form [7]:

$$\gamma^{-1}\partial_t A(t) + A(t) = \sqrt{\kappa}e^{(1-i\alpha_g)G(t-T)/2-(1-i\alpha_q)Q(t-T)/2}A(t-T) \tag{1}$$

where $A(t)$ is the normalized complex amplitude of the electric field, γ is the dimensionless bandwidth of the spectral filtering section, α_g (α_q) the linewidth enhancement factors in the gain (absorber) section. The delay parameter T is equal to the cold cavity round trip time. The attenuation factor $\kappa < 1$ describes total non-resonant linear intensity losses per cavity round trip.

The variables $G(t)$ and $Q(t)$ are the dimensionless saturable gain and absorption:

$$G(t) = 2g_g L_g [2\rho_g(t) - 1], \tag{2}$$
$$Q(t) = -2g_q L_q [2\rho_q(t) - 1], \tag{3}$$

where the variables $\rho_g(t)$ and $\rho_q(t)$ describe the occupation probabilities in a dot located either in the amplifier or absorber section, respectively. A detailed derivation of Eq. (1) and the relations among the input and output fields in the amplifier and absorber sections are given in [8]. Using these relations we write the following four equations for the amplifying and absorbing sections:

$$\partial_t \rho_g = -\gamma_g \rho_g + F_g(\rho_g, N_g) - e^{-Q}\left(e^G - 1\right)|A|^2, \tag{4}$$

$$\partial_t \rho_q = -\gamma_q \rho_q + F_q(\rho_q, N_q) - s\left(1 - e^{-Q}\right)|A|^2, \tag{5}$$

$$\partial_t N_g = N_{g0} - \Gamma_g N_g - 2F_g(\rho_g, N_q), \tag{6}$$

$$\partial_t N_q = N_{q0} - \Gamma_q N_q - 2F_q(\rho_q, N_q). \tag{7}$$

Here the variables $N_{g,q}(t)$ describe the carrier densities in the wetting layers, scaled to the QD carrier density. The parameters $g_{g,q}$, $\Gamma_{g,q}$, and $\gamma_{g,q}$ are, respectively, the differential gains, the carrier relaxation rate in the wetting layers and the carrier relaxation rates in the dots. The dimensionless parameters N_{g0} and N_{g0} describe pumping processes in the amplifier and the absorber sections. The parameter $s = g_q/g_g$ is the ratio of the saturation intensities in the gain and absorber sections. The factor 2 in Eqs. (2), (3) and (6),(7) accounts for the twofold spin degeneracy in the quantum dots energy levels. The functions $F_{g,q}(\rho_{g,q}, N_{g,q})$ describe the carrier exchange rate between the wetting layers and the dots. In the most general form, the carrier exchange can be written as [9]:

$$F_{g,q}(\rho_{g,q}, N_{g,q}) = R^{cap}_{g,q}(1 - \rho_{g,q}) - R^{esc}_{g,q}\rho_{g,q}, \tag{8}$$

where $1 - \rho_{g,q}$ is the Pauli blocking factor, $R^{cap}_{g,q} = B_{g,q}N_{g,q}$ describes the carrier capture from the wetting layer to the dots with the rate $B_{g,q}$. $R^{esc}_{g,q}$ is a temperature-dependent coefficient defining carrier escape from the dots to the wetting layer. Together with Eq. (1), the equations (4)-(7) constitute a closed set of equations.

The purpose of this Letter is not to model a specific laser configuration, but to describe some generic properties of the ML regime in quantum dot lasers. Therefore, some of the parameters are taken identical for the amplifier and the absorber for simplicity. We consider low linewidth enhancement factors $\alpha_{g,q} = 2$ and relatively high differential gains. Carrier capture times in the dots were reported to be from

1ps to 100ps. We consider capture rates $B_{g,q} = 2$ which correspond to 5ps and take escape rates as $R^{esc}_{g,q} = 0.1 \ll B_{g,q}$. The devices for ML have usually a long amplifier section and a short absorber section with a typical ratio 6 to 1 [10]. The other parameters are conventional for QD materials and geometry. They are specified in the figure caption. The time t is normalized by the time interval $\Delta t = 10$ ps.

A typical dynamics of the ML regime is shown in Fig.1. The intensity time trace shown in Fig. 1(a) displays the usual sequence of equidistant mode-locked pulses. The time dependence of the carrier densities $N_{g,q}(t)$ in the wetting layer of the gain and absorber sections is shown in Fig. 1(d-e). They are similar to those of the saturable gain and loss in a quantum well laser, for which the gain section is characterized by a very slow, almost linear recovery [8]. On the contrary, the recovery of the saturable gain $G(t)$ and loss $Q(t)$ of the QD sections appears to be much faster [see Fig. 1(b-c)]. As can be seen in Fig. 1(b), after a fast recovery the saturable gain remains almost constant between two consecutive pulses, $G(t) \approx 2g_g L_g$. The existence of fast and slow recovery stages is a specific feature of quantum dots and is related to the occurrence of two competing mechanisms for the carrier transfer between the wetting layers and the dots. This transfer is described in our model by the function $F_g(\rho_g, N_g)$ which includes two parts: nonlinear capture $BN_g(1-\rho_g)$ and linear escape $R^{esc}_g \rho_g$. The nonlinear part of $F_g(\rho_g, N_g)$ describes intrinsic saturation introduced in Eq. (8) by the Pauli blocking factor. The strength of this saturation depends on the carrier capture rate B_g. The linear part of $F_g(\rho_g, N_g)$ describes the carrier escape from the dots to the wetting layer. Together with the carrier relaxation rates γ_g and Γ_g, it induces the slow monotonic recovery as in the quantum well semiconductor lasers (Fig. 1(d)). The slow recovery is controlled by the relaxation rates $\gamma_g + R^{esc}_g$ and Γ_g. Since $B_g \gg \gamma_g + R^{esc}_g$, it is much longer than the fast recovery stage. The fast gain recovery is known in optical amplifiers based on quantum dots, promising 1Tb/s speed for booster amplifier [5]. The fast recovery of the saturable gain prevents the appearance of low frequency self-pulsations which degrade the ML regime. It is directly related to the strong damping of the relaxation oscillation frequency in QD laser [2]. Still, we have observed the low frequency fluctuations in transients leading to stable ML.

The dynamics of the variable $Q(t)$ describing the quantum dot saturable absorber also shows two distinct stages in the recovery process (see Fig.1(c)). A similar feature in the decay of QD absorber was observed experimentally in Ref. [11]. In that paper fast recovery was attributed to the absorption saturation in the QD. Our modelling confirms this hypothesis as we find a strong dependence of the fast recovery time on the rate B_q which defines the saturation of the QD in our model.

The nonlinear dynamics associated with the ML regime is rich and can include chaos. An important dynamical feature we have observed is a bistability between the ML regime and the non-lasing steady state regime. This bistability and the corresponding hysteresis loops in the characteristics were already experimentally observed and related to the nonlinear saturation of the QD absorption [4]. This saturation occurs due to the state filling and, therefore, is directly linked to the fast recovery stage.

In Fig. 1, the round-trip net gain parameter $\mathcal{G}(t) = G(t) - Q(t) + \ln\kappa$ is negative between pulses and becomes positive only during short time intervals when the pulse amplitude is large. Therefore according to New's criterion [6], the regime shown in this Figure is stable with respect to small perturbations of the zero-intensity background. During the slow stage of the gain recovery, the net gain parameter $\mathcal{G}(t)$ is always negative.

The complete parameter and bifurcation analysis of Eqs. (1)-(7) will be reported elsewhere. We only briefly mention the main tendencies of the ML regime. Increasing the capture rates $B_{g,q}$ leads to shorter pulses which is different from the results obtained for actively modelocked lasers [12]. The escape rates $R^{esc}_{g,q}$ do not significantly influence the pulse duration if $R^{esc}_{g,q} \ll B_{g,q}$. The trends for the other parameters are similar to the quantum well lasers.

In conclusion , we proposed a model based on delay differential equations for passive ML in quantum dot lasers. The model predicts the appearance of ML pulses with no accompanying self-pulsations. The gain recovery is specific to quantum dot lasers. It explains the absence of the low frequency self-pulsations and bistability between the ML regime and lasing-off state.

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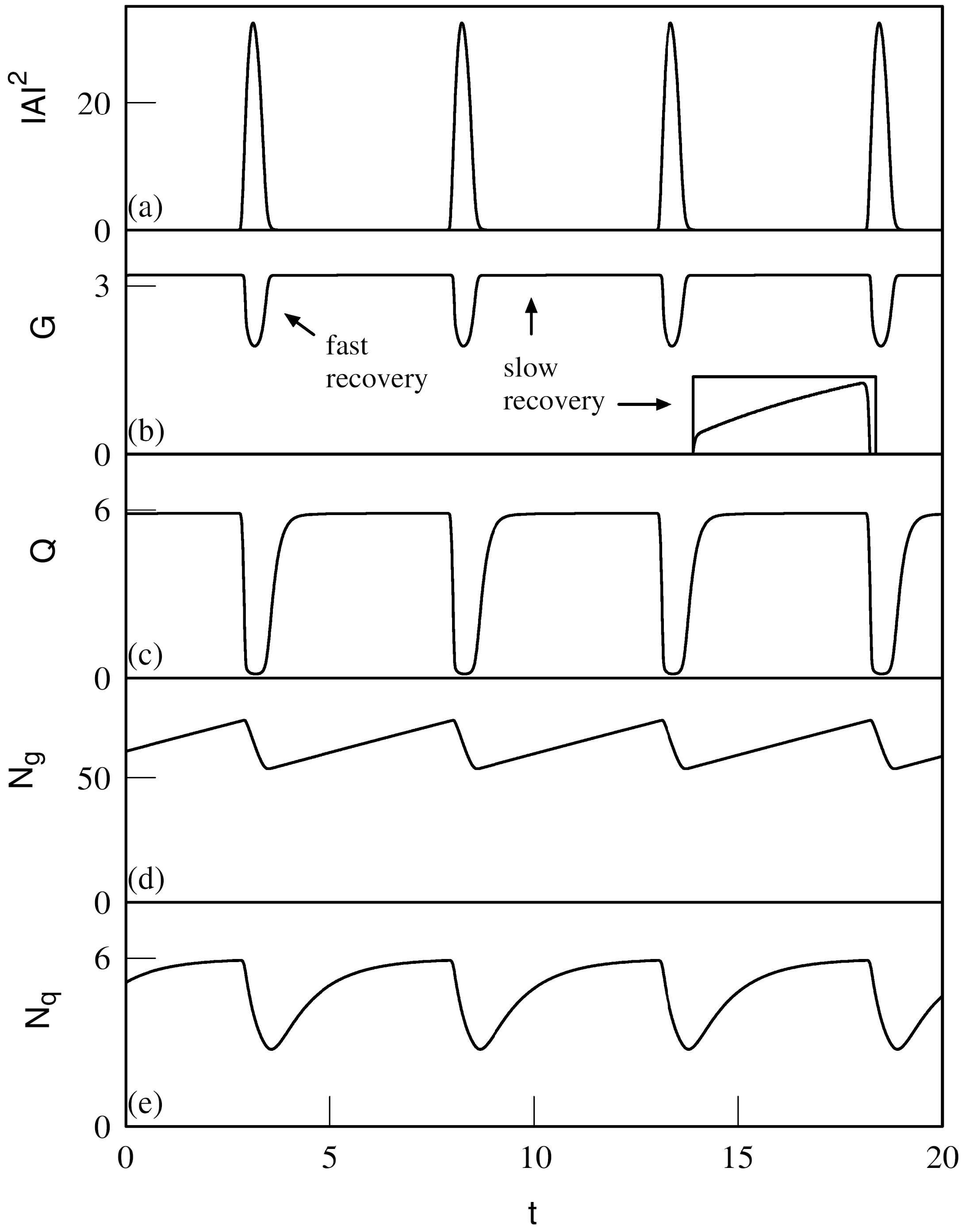

Figure 1: Time evolution of the electric field intensity $|A(t)|^2$, the saturable gain $G(t)$ and the saturable loss $Q(t)$ in the dots, the carrier densities $N_{g,q}(t)$ in the wetting layer in a mode-locked quantum dot laser with absorber. The recovery of $G(t)$ and $Q(t)$ consists of two stages: fast and slow. The parameters are: $\kappa = 0.2$; $T = 5$; $\gamma = 10$; $\gamma_g = \gamma_q = \Gamma_g = 0.01$; $\Gamma_q = 1$; $N_{g0} = 5$; $N_{q0} = 6$; $g_g L_g = 1.6$; $g_q L_q = 3$; $s = 10$.