Weierstraß–Institut
für Angewandte Analysis und Stochastik

im Forschungsverbund Berlin e.V.

On the stabilization of trigonometric collocation methods for a class of ill–posed first kind equations

Gottfried Bruckner

submitted: 19th July 1994

Weierstrass Institute
for Applied Analysis
and Stochastics
Mohrenstraße 39
D – 10117 Berlin
Germany

Preprint No. 109
Berlin 1994

1991 Mathematics Subject Classification. 65R30.
Key words and phrases. Regularization–discretization procedures, moderately ill–posed, boundary integral equations, convergence rates.

Edited by
Weierstraß–Institut für Angewandte Analysis und Stochastik (WIAS)
Mohrenstraße 39
D — 10117 Berlin
Germany

Fax: + 49 30 2004975
e–mail (X.400): c=de;a=d400;p=iaas-berlin;s=preprint
e–mail (Internet): preprint@iaas-berlin.d400.de

ABSTRACT. In this paper regularization–discretization procedures are developed for the numerical solution of moderately ill–posed linear first kind equations appearing as boundary integral equations for Dirichlet boundary value problems, e.g. the Dirichlet–Laplace problem. The method consists in firstly regularizing the noisy right–hand side by trigonometric interpolation and then applying a trigonometric collocation procedure to the regularized data. Convergence rates are obtained in Sobolev spaces, Hölder–Zygmund spaces or Hölder spaces according to the error analysis of the used procedures for exact data. The method can be generalized to other kinds of equations and approximation procedures.

1. Introduction

In this paper we will consider linear integral equations of the first kind

$$Au = g \tag{1.1}$$

with a smoothing operator A. More precisely, in a Banach scale B^s let the operator A map a space B^μ injectively onto a space B^ν, where

$$\varrho = \nu - \mu > 0$$

is a measure of the smoothing effect of A. Considering A as a mapping of B^μ into B^μ with a dense range the number ϱ also characterizes the degree of ill–posedness of the problem (1.1) as weakly $(0 < \varrho < 1)$ or moderately $(1 \leq \varrho < \infty)$ ill–posed. Besides, in the language of pseudodifferential operators the negative number $-\varrho$ is the order of the pseudodifferential operator A.

Equations (1.1) of that type appear as boundary integral equations for Dirichlet boundary value problems. They enjoy a growing interest of engineers and mathematicians who are providing approximation procedures for their numerical solution in an extending rate. If one starts at exactly given data such not severely ill–posed problems can be investigated without regularization.

On one hand the solutions of the considered integral equations are auxiliary quantities for the solution of boundary value problems given by a single–layer potential where integration removes the ill–posedness if the boundary value problem is well–posed.

On the other hand in many cases the solution of the integral equation has its own independent physical meaning. For instance, in the case of Symm's equation the solution can be a distribution of forces, mass or charge density or potential flow, while the right–hand side has the meaning of displacements, Newton or Coulomb potential or temperature (cf. [5], [4], [9]).

Therefore, problems (1.1) with noisy data g can be understood as indirect measurement problems where regularization techniques are in order.

In this paper we will develop regularization procedures for the considered problem (1.1) by stabilizing trigonometric collocation procedures taken from [13], [10], [3] in the one–dimensional, periodical, elliptic case. Our method is outlined in section 2. It can be called a direct discretization method where the pointwise measurements are used directly in the computation and the number of equidistant measurement points is taken as the regularization parameter. Concerning other regularization methods for that kind of problems, especially Symm's equation, we refer to [2], [6], [7], [8]. Section 3 gives some preliminaries about trigonometric polynomials and interpolation. The sections 4, 5, 6 concern the regularization procedure and its error analysis in Sobolev spaces, Hölder–Zygmund spaces and Hölder spaces according to [13], [10] and [3], respectively.

I am indepted to S. Prößdorf and G. Vainikko for discussions and useful remarks.

Throughout the paper the letter c will describe a generic constant.

2. Description of the method

Let us consider the problem

$$Au = g \tag{2.1}$$

where A is an injective bounded linear mapping of the Banach space X onto the Banach space Y, i.e. $D(A) = X$, $R(A) = Y$. Then A^{-1} as a mapping from Y onto X has the same properties. The problem (2.1) is well–posed.

Let further the problem (2.1) have been solved numerically by the procedure

$$T_h g\,, \; h > 0\,,$$

with a bounded linear operator T_h from Y into X such that

$$||A^{-1}g - T_h g||_X \to 0 \quad (h \to 0) \quad \text{for all} \quad g \in Y \tag{2.2}$$

with a known rate of convergence depending on the discretization parameter h. Then by the Banach–Steinhaus theorem the stability result

$$||T_h|| \leq c \tag{2.3}$$

holds uniformly in h.

Now, let X and Y be function spaces over a certain domain and let us assume that g is observed by the values

$$g_j \in \mathbb{C}\,, \; j = 1, ..., M \tag{2.4}$$

at the points

$$t_j\,, \; j = 1, ..., M \tag{2.5}$$

of a grid G_d characterized by a parameter d. (In the one–dimensional case of equidistant meshpoints G_d can be characterized by the real parameter $d = 1/M$.) Let the observed quantities g_j have the property

$$|g_j - g(t_j)| \leq \delta\,, \quad j = 1, ..., M \tag{2.6}$$

where $\delta > 0$ is a given noise level. Clearly, to approximate g for $\delta \to 0$ we must have also $M \to \infty$.

In the case when g is given by an observation (2.4), (2.5), (2.6), a crucial point for the numerical solution of (2.1) is to find a number

$$d = d(\delta)$$

and elements

$$P(d,\delta) \in Y$$

with the property

$$||g - P(d,\delta)||_Y \to 0 \quad (\delta \to 0).$$

Then, as an approximation of the solution $A^{-1}g$ of (2.1)

$$T_h P(d,\delta)$$

can be taken where h and d are chosen appropriately. (In our approach we will take $h = d = d(\delta)$.)

This can be seen from the triangle inequalities

$$||A^{-1}g - T_h P(d,\delta)|| \leq ||T_h g - T_h P(d,\delta)|| + ||A^{-1}g - T_h g|| \tag{2.7}$$

or

$$||A^{-1}g - T_h P(d,\delta)|| \leq ||A^{-1}g - A^{-1}P(d,\delta)|| + ||A^{-1}P(d,\delta) - T_h P(d,\delta)|| \tag{2.8}$$

by estimating the first terms on the right–hand side of (2.7) resp. (2.8) using the stability (2.3) of T_h resp. the continuity of A^{-1}, and the second terms using the convergence (2.2) of the procedure T_h.

More generally, $P(d,\delta)$ can be considered as a regularized approximation for the exact data g where d is the regularization parameter. The construction of $P(d,\delta)$ depends on Y (the range space of A) and the data and may be tuned to an a–priori information about the solution u (translated to $g = Au$). It must not depend on the operator A and the given procedure T_h. The ill–posedness of the problem (2.1) in the case of disturbed data then appears as the ill–posedness of the approximation of g in the state space Y.

In [2] the auxiliary regularization process was the regularization of the (compact) imbedding $H^s \subset L_2$ that can be interpreted as data smoothing of observations in L_2.

In this paper the construction of $P(d,\delta)$ is closely connected to the regularization of an ill–posed approximation problem: to reconstruct g from pointwise measurements using trigonometric interpolation. Here the number of equidistant measurements serves as the regularization parameter. The parameter choice answers the question of how many measurements are needed for given δ to get convergence in Y of trigonometric interpolation polynomials to the exact data g.

It should be remarked that in the here considered special case where a trigonometric collocation procedure T_h is combined with trigonometric approximation for the right–hand side g in the just described sense the resulting estimates could be gained more directly by a mere application of the inverse property of trigonometric polynomials (G. Vainikko).

3. Notations and trigonometric interpolation

In this paper we are interested in periodic problems over smooth closed curves in $\mathbb{R}_2$. (Open arcs can be treated by a similar approach.)

Consequently, let us consider the one–dimensional torus

$$\mathbb{T} = \mathbb{R}/\mathbb{Z}$$

and the spaces of complex–valued functions

$$H^s(\mathbb{T}),\ s > 0,\ L_2(\mathbb{T}), C(\mathbb{T})$$

with the norms

$$||\cdot||_s,\ ||\cdot||_0, ||\cdot||_C$$

respectively, where $H^s(\mathbb{T}) = H^s$ is the scale of Sobolev spaces over $\mathbb{T}$. Let be further

$$\begin{aligned} \varphi_l(x) &= e^{2\pi i l x} \quad x \in \mathbb{T}\,,\ l \in \mathbb{Z}\,, \\ \mathbb{Z}_M &= \{l \in \mathbb{Z}\,,\ -M/2 < l \leq M/2\}\,,\ M \in \mathbb{N}\,, \\ \mathfrak{T}_M &= \text{span}\{\varphi_l\,,\ l \in \mathbb{Z}_M\}\,. \end{aligned} \tag{3.1}$$

Additionally, let us consider the equidistant mesh on $\mathbb{T}$

$$G_d = \{jd\,,\ 1 \leq j \leq M\}\,,\ d = 1/M\,, \tag{3.2}$$

$\mathbb{C}_M$ the M–dimensional vector space over $\mathbb{C}$. For a given $g_d \in \mathbb{C}_M$,

$$g_d = (g_1, ..., g_M)\,,$$

there is a unique trigonometric interpolation polynomial

$$S_d g_d(x) = \sum_{l \in \mathbb{Z}_M} \left(d \sum_{1 \leq j \leq M} g_j e^{-2\pi i l j d} \right) \varphi_l(x)$$

with the properties

$$\begin{aligned} &S_d g_d \in \mathfrak{T}_M\,, \\ &S_d g_d(jd) = g_j\,, \quad j = 1, ..., M\,. \end{aligned} \tag{3.3}$$

Let for $g \in C(\mathbb{T})$

$$S_d g$$

be the unique d–th interpolation polynomial of g, i.e. $S_d g \in \mathfrak{T}_M$ and

$$S_d g(jd) = g(jd)\,, \quad j = 1, ..., M \tag{3.4}$$

hold.

Lemma 3.1. *Let $\varphi, \psi \in \mathfrak{T}_M$. Then*

$$\int_{\mathbb{T}} \varphi(x)\overline{\psi(x)}dx = d \sum_{1 \leq j \leq M} \varphi(jd)\overline{\psi(jd)}\,.$$

Especially, for $\psi \in \mathfrak{T}_M$

$$||\psi||_0^2 = d \sum_{1 \leq j \leq M} |\psi(jd)|^2\,. \tag{3.5}$$

Proof. Using the orthonormality of the basis (3.1) in L_2 the lemma follows straightforwardly from orthogonality relations for trigonometric sums: If $1 \le j, k \le M$,

$$\sum_{l \in \mathbb{Z}_M} e^{-2\pi i l d(j-k)} = M\delta_{jk}\,.$$

Lemma 3.2. *(Inverse property.) Let $\psi \in \mathfrak{T}_M$. Then for $r \ge s$*

$$||\psi||_r \le C_{r,s} d^{s-r} ||\psi||_s\,. \tag{3.6}$$

Now, let us consider the trigonometric interpolation as a regularization method for the reconstruction of a function f from measured values on an equidistant mesh.

Let $f \in H^\sigma$, $\sigma > 1/2$, $0 \le s < \sigma$. Then the approximation property

$$||f - S_d f||_s \le c d^{\sigma - s} ||f||_\sigma \tag{3.7}$$

holds. (See e.g. [1] or [12].)

Let $f_d^\delta \in \mathbb{C}_M$, $f_d^\delta = (f_1^\delta, ..., f_M^\delta)$, $d = 1/M$, with

$$|f_j^\delta - f(jd)| \le \delta\,. \tag{3.8}$$

Lemma 3.3. *For $f \in H^\sigma$, $\sigma > 1/2$, $0 \le s < \sigma$, we obtain under the assumptions (3.7) and (3.8)*

$$||f - S_d f_d^\delta||_s \le c(d^{\sigma - s}||f||_\sigma + d^{-s}\delta)\,. \tag{3.9}$$

If

$$d \sim \delta^{1/\sigma} \tag{3.10}$$

then

$$||f - S_d f_d^\delta||_s = O(\delta^{\frac{\sigma - s}{\sigma}})\,. \tag{3.11}$$

Proof. We have

$$||f - S_d f_d^\delta||_s \le ||f - S_d f||_s + ||S_d f - S_d f_d^\delta||_s\,.$$

Since $S_d f$, $S_d f_d^\delta \in \mathfrak{T}_M$ we get from (3.6), (3.5), (3.3), (3.4), (3.8)

$$\begin{aligned} ||S_d f - S_d f_d^\delta||_s &\le c d^{-s} ||S_d f - S_d f_d^\delta||_0 \\ &\le c d^{-s} \left(d \sum_{1 \le j \le M} |f(jd) - f_j^d|^2 \right)^{1/2} \\ &\le c d^{-s} \delta\,. \end{aligned}$$

This and (3.7) give (3.9), (3.11) is an easy consequence. □

Remarks.

(1) For $s = 0$ the reconstruction of f is well–posed. The number $s > 0$ is related to the degree of ill–posedness.
(2) The a–priori parameter choice (3.10) corresponds to the number of equidistant measurements being necessary at the noise level δ.

4. Stabilization in Sobolev spaces

In this and the following sections we consider elliptic first kind boundary integral equations of negative order β on a smooth closed boundary curve Γ in $\mathbb{R}_2$. By a suitable C^∞–parametrization such an equation can be transformed to a first kind integral equation

$$\int_{\mathbb{T}} K(t,\tau)u(\tau)d\tau = g(t)\,,\ t \in \mathbb{T} \tag{4.1}$$

over the one–dimensional torus $\mathbb{T}$. A special case is Symm's equation where

$$K(t,\tau) = -\log|\gamma(t) - \gamma(\tau)|$$

and γ is a parametrization of Γ.

Let us assume in this section that (4.1) is of the type (1.1) mentioned in the introduction, where the operator A maps the Sobolev space H^s injectively onto $H^{s-\beta}$ for some $s \geq 0$.

Following [13] we shall first give a numerical procedure T_h and its error analysis in Sobolev spaces. Then, we shall develop a regularization procedure that stabilizes T_h in the case of disturbed data.

Consider an equidistant mesh G_h on $\mathbb{T}$, $h = 1/N$, $N \in \mathbb{N}$ (cf. (3.2)). The procedure T_h is defined for $g \in H^{s-\beta}$ as

$$T_h g = S_h W_h$$

where W_h is the solution of the matrix equation

$$L_h W_h = \Gamma_h\,,\ \Gamma_h = (g(h), g(2h), ..., g(Nh))^T\,, \tag{4.2}$$

$$L_h = \left(l_{ij}^h\right)_{ij}\,,\quad l_{ij}^h = \begin{cases} hK(t_i,t_j) & i \neq j \\ \alpha(t_j) - h\sum\limits_{\substack{\nu=1\\ \nu\neq i}}^{N} K(t_i,t_\nu) & i = j\,, \end{cases}$$

$$\alpha(t) = \int_{\mathbb{T}} K(t,\tau)d\tau\,.$$

T_h is a continuous, linear operator from $H^{s-\beta}$ to H^s with the property

$$T_h g = T_h S_h g\,. \tag{4.3}$$

As to the error analysis of T_h we cite Theorem 3.2 from [13]:

"Let $u = A^{-1}g$ be the solution of (4.1) and let $u \in H^\sigma$, $\sigma > \beta + 1/2$. Then, for sufficiently small values of $h > 0$ the equation (4.2) is uniquely solvable and the error estimate

$$||u - T_h g||_s \leq ch^{\sigma-s}||u||_\sigma \tag{4.4}$$

holds for $\beta \leq s < \sigma \leq s - \beta + 1$".

Now, let us consider noisy data g. Let be $\delta > 0$, G_d, $d > 0$, an equidistant mesh, $g_d^\delta \in \mathbb{C}_M$, $d = 1/M$, and

$$|g(jd) - g_j^\delta| \le \delta\,, \quad 1 \le j \le M\,. \tag{4.5}$$

As an approximation for the solution u of (4.1) we regard

$$u_d^\delta := T_d S_d g_d^\delta\,, \tag{4.6}$$

i.e. we choose $h = d$ and

$$P(d,\delta) = S_d g_d^\delta$$

in the notation of section 2. This choice leads to an effective numerical procedure since the measured values are directly used.

Theorem 4.1. *Let be $u \in H^\sigma$, $\sigma > \beta + 1/2$. Then for*

$$\beta \le s < \sigma \le s - \beta + 1$$

we have

$$||u_d^\delta - u||_s \le c(d^{\sigma-s}||u||_\sigma + d^{-s+\beta}\delta)\,. \tag{4.7}$$

Choosing

$$d \sim \delta^{\frac{1}{\sigma-\beta}} \tag{4.8}$$

we obtain

$$||u_d^\delta - u||_s = O\left(\delta^{\frac{\sigma-s}{\sigma-\beta}}\right) \tag{4.9}$$

Proof. From (4.6) and the triangle inequality we have

$$\begin{aligned} ||u_d^\delta - u||_s &= ||T_d S_d g_d^\delta - A^{-1}g||_s \\ &\le ||A^{-1}g - T_d g||_s + ||T_d g - T_d S_d g_d^\delta||_s\,. \end{aligned} \tag{4.10}$$

By (4.4)

$$||A^{-1}g - T_d g||_s \le c d^{\sigma-s}||u||_\sigma\,. \tag{4.11}$$

Then, using (4.3), (2.3), (3.6), (3.5) and (4.5) we get for the second term in (4.10)

$$\begin{aligned} ||T_d g - T_d S_d g_d^\delta||_s &= ||T_d S_d g - T_d S_d g_d^\delta||_s \\ &\le c||S_d g - S_d g_d^\delta||_{s-\beta} \\ &\le c d^{-s+\beta}||S_d g - S_d g_d^\delta||_0 \\ &\le c d^{-s+\beta} \max_{1\le j\le M} |g(jd) - g_j^\delta| \\ &\le c d^{-s+\beta}\delta\,. \end{aligned}$$

This and (4.11) are giving (4.7), then (4.9) follows by inserting (4.8). □

Remarks.

(1) The method (4.6), (4.8) has the same velocity (4.9) as it was proved in [2] for quite another regularization procedure. In the case $s = 0$, $\beta = -1$ the rate $O(\delta^{\frac{\sigma}{\sigma+1}})$ is optimal.

(2) The choice (4.8) is an a–priori parameter choice that needs an a–priori regularity information about the solution.
(3) Essential for the construction of our regularization procedure are the properties (2.3), (4.3), (4.4) of T_h and the properties (3.5) and (3.6) of trigonometric polynomials. The regularization could be based also on other procedures T_h with those properties and an convergence analysis in Sobolev spaces.
(4) Theorem 4.1 can also be proved using Lemma 3.3. Then instead of (4.3) the approximation property (3.7) is used.
(5) In the case $s > 1/2$ we have $H^s \subset C$ and

$$||\cdot||_C \leq c||\cdot||_s .$$

Then in the assumptions of Theorem 4.1 the estimates (4.7) and (4.9) also hold for the C–Norm instead of the H^s–norm.

5. Stabilization in Hölder–Zygmund spaces

In this section we again consider first kind integral equations (4.1) with a smoothing operator over the one–dimensional torus $\mathbb{T}$ that can be obtained by a transformation from boundary integral equations on a smooth closed curve in $\mathbb{R}_2$. Here, our investigations are based on the paper [10] where the studied numerical procedure is a fully discrete collocation method with an error analysis in Hölder–Zygmund spaces $\mathcal{H}^\nu$.

Let us consider the following Banach spaces of complex–valued functions on $\mathbb{T}$:

$$C^m = \{f \in C\,,\ D^j f \in C\,,\ 0 \leq j \leq m\}$$

with the norm

$$||f||_{C^m} = \sum_{0 \leq j \leq m} ||D^j f||_C\,,$$

and for $s = m + \alpha > 0$

$$\mathcal{H}^s = \{f \in C^m\,,\ [D^m f]^\alpha < \infty\}$$

with the norm

$$||f||_{\mathcal{H}^s} = ||f||_{C^m} + [D^m f]^\alpha$$

where $[D^m f]^\alpha$ is the Hölder–Zygmund seminorm. For $\psi \in \mathfrak{T}_N$ the Bernstein inequality

$$||D\psi||_C \leq cN||\psi||_C \tag{5.1}$$

and the estimate

$$[\psi]^\alpha \leq c \cdot N^\alpha ||\psi||_C \tag{5.2}$$

hold (cf. [11]). From (5.1) and (5.2) we have the following inverse properties for $\psi \in \mathfrak{T}_N$:

$$||\psi||_{\mathcal{H}^s} \leq cN^s||\psi||_C\,, \tag{5.3}$$

$$||\psi||_{C^m} \leq cN^m||\psi||_C\,, \tag{5.4}$$

where $s > 0$, $m \in \mathbb{N}$, and c can depend on s resp. m.

Let now T_h be the discrete collocation procedure considered in [10]. Since the explicit form of that algorithm is not essential for our purpose we will not quote it here. Essential is that T_h is a continuous linear mapping from $\mathcal{H}^{s-\beta}$ into $\mathcal{H}^s$ with properties (2.3) and (4.3) and the following convergence property (cf. [10, Theorem 2.2]):

"Let the periodic elliptic pseudodifferential operator A map $\mathcal{H}^s$ injectively onto $\mathcal{H}^{s-\beta}$ and let the solution u of (1.1) be in $\mathcal{H}^\sigma$, $\beta < \sigma < \infty$.
Then for $\beta < s < \sigma < \infty$ we have the estimate

$$||T_h g - u||_{\mathcal{H}^s} \leq ch^{\sigma-s}|\log h|\,||u||_{\mathcal{H}^\sigma}\,.\text{"} \tag{5.5}$$

Now let us again consider disturbed data $g_d^\delta \in \mathbb{C}_M$, $\delta > 0$, on an equidistant mesh G_d on $\mathbb{T}$ with the property (4.5) and consider the procedure (4.6)

$$u_d^\delta := T_d S_d g_d^\delta\,,$$

where again d is the regularization parameter.

Theorem 5.1. *Let for $\beta < s < \sigma < \infty$ the operator A map $\mathcal{H}^s$ injectively onto $H^{s-\beta}$ and let $u = A^{-1}g \in \mathcal{H}^\sigma$. Then for an arbitrary $\lambda > 1/2$ we have the estimate*

$$||u_d^\delta - u||_{\mathcal{H}^s} \leq c(d^{\sigma-s}|\log d|\,||u||_{\mathcal{H}^\sigma} + d^{-s-\lambda+\beta}\delta)\,. \tag{5.6}$$

If

$$d \sim \delta^{\frac{1}{\sigma-\beta+\lambda}} \tag{5.7}$$

we obtain

$$||u_d^\delta - u||_{\mathcal{H}^s} = O\left(\delta^{\frac{\sigma-s}{\sigma-\beta+\lambda}}|\log\delta|\right)\,. \tag{5.8}$$

Proof. As in the proof of Theorem 4.1 we have

$$||u_d^\delta - u||_{\mathcal{H}^s} = ||T_d S_d g_d^\delta - A^{-1}g||_{\mathcal{H}^s} \leq ||A^{-1}g - T_d g||_{\mathcal{H}^s} + ||T_d g - T_d S_d g_d^\delta||_{\mathcal{H}^s}$$

and from (5.5)

$$||A^{-1}g - T_d g||_{\mathcal{H}^s} \leq c d^{\sigma-s}|\log d|\,||u||_{\mathcal{H}^\sigma}\,.$$

Besides, by using $||\cdot||_C \leq c||\cdot||_\lambda$ if $\lambda > 1/2$, and (4.3), (2.3), (5.3), (3.6), (3.5) and (4.5) we get

$$\begin{aligned}
||T_d g - T_d S_d g_d^\delta||_{\mathcal{H}^s} &= ||T_d S_d g - T_d S_d g_d^\delta||_{\mathcal{H}^s} \\
&\leq c||S_d g - S_d g_d^\delta||_{\mathcal{H}^{s-\beta}} \\
&\leq c d^{\beta-s}||S_d g - S_d g_d^\delta||_C \\
&\leq c d^{\beta-s}||S_d g - S_d g_d^\delta||_\lambda \\
&\leq c d^{\beta-s-\lambda}||S_d g - S_d g_d^\delta||_0 \\
&\leq c d^{\beta-s-\lambda}\delta\,.
\end{aligned}$$

The proof of (5.6) is complete, (5.8) follows by inserting (5.7). □

Remark. Again the remarks 2 and 3 of section 4 are true.

6. Concerning the Helmholtz equation

Finally, in this section let us regard the equation

$$\int_\Gamma H_0^{(1)}(k|x-y|)u(y)dy = g(x) \tag{6.1}$$

where Γ is a smooth closed curve (or open arc) in $\mathbb{R}_2$ and $H_0^{(1)}$ is the Hankel function of order 0 of the first kind.

The problem (6.1) arises from an exterior Dirichlet problem for the Helmholtz equation. If the solution of this Dirichlet problem is denoted by w the solution of (6.1) can be interpreted as $\left[\frac{\partial w}{\partial n}\right]$, i.e. the jump of Neumann data at the boundary Γ (cf. [14]).

In [3] the problem (6.1) is transformed to an equation (1.1) on $\mathbb{T}$ where the operator A maps the Hölder space $C^{0,\alpha}$ bijectively onto $C^{1,\alpha}$. Besides, in [3] a trigonometric collocation–quadrature method T_h is developed with an error analysis in Hölder spaces.

It is not difficult to recognize that T_h is a continuous linear operator from $C^{1,\alpha}$ into $C^{0,\alpha}$ discretizing the problem on an equidistant mesh and having the properties (2.3) and (4.3). Then the application of our regularization method (4.6) is possible. Using (5.4) and being aware of $C^{1,\alpha} = \mathcal{H}^{1+\alpha}$ for $0 < \alpha < 1$ we find for $\lambda > 1/2$ similar as in Theorem 5.1

$$||u - T_d S_d g_d^\delta||_{0,\alpha} \leq ||u - T_d g||_{0,\alpha} + c d^{-1-\alpha-\lambda}\delta\,. \tag{6.2}$$

It is proved in [3] that $||u - T_d g||_{0,\alpha} \to 0$ for $d \to 0$ if $\alpha < 1/2$. Clearly from (6.2) our method converges if $d(\delta)$ is such that $d^{-1-\alpha-\lambda}\delta \to 0$ and $d \to 0$ for $\delta \to 0$. If $\alpha + \lambda < 1$ this is fulfilled for $d \sim \delta^{1/2}$.

References

1. Amosov, B.A., On the approximate solution of elliptic pseudo–differential equations on a smooth closed curve, Z. Anal. Anw. 9, 545–563 (1990).
2. Bruckner, G., On the regularization of the ill–posed logarithmic kernel integral equation of the first kind, Preprint No. 46, Institut für Angewandte Analysis und Stochastik, Berlin 1993.
3. Chapko, R., Kress, R., On a quadrature method for a logarithmic integral equation of the first kind, WSSIAA 2, 127–140 (1993).
4. Dautray, R., Lions, J.L., Mathematical Analysis and numerical methods for science and technology, Springer 1984, Vol. 4, Chapter XI.
5. Hartmann, F., Methode der Randelemente, Springer 1987.
6. Hsiao, G.C., The finite element method for a class of improperly posed integral equations, in: Hämmerlin/Hoffmann, Improperly posed problems and their numerical treatment, 117–131 (1983).
7. Hsiao, G.C., On the stability of integral equations of the first kind with logarithmic kernels, Arc. Rat. Mech. Anal. 94, 179–192 (1986).
8. Hsiao, G.C., Prößdorf, S., On the stability of the spline collocation method for a class of integral equations of the first kind, Appl. Anal. 30, 249–261 (1988).
9. Kress, R., Linear integral equations, Springer 1989.
10. McLean, W., Prößdorf, S., Wendland, W.L., A fully–discrete trigonometric collocation method, J. Integral Equations Appl. 5, 103–129 (1993).
11. McLean, W., Prößdorf, S., Wendland, W.L., Pointwise error estimates for the trigonometric collocation method applied to singular integral equations and periodic pseudodifferential equations, J. Integral Equations Appl. 2, 125–146 (1989).
12. Prößdorf, S., Silbermann, B., Numerical Analysis for Integral and Related Operator Equations, Akademie–Verlag, Berlin 1991, and Birkhäuser–Verlag, Basel, Boston, Stuttgart 1991, 1–542.
13. Saranen, J., Schroderus, L., The modified quadrature method for classical pseudodifferential equations of negative order on smooth closed curves, JCAM (to appear).
14. Stephan, E.P., Wendland, W.L., An angmented Galerkin procedure for the boundary integral method applied to two–dimensional screen and crack problems, Appl. Anal. 18, 183–219 (1984).

G. Bruckner, Weierstrass Institute for Applied Analysis and Stochastics, Mohrenstrasse 39, D – 10117 Berlin, Germany

Recent publications of the Weierstraß–Institut für Angewandte Analysis und Stochastik

Preprints 1994

80. Roland Duduchava, Siegfried Prößdorf: On the approximation of singular integral equations by equations with smooth kernels.

81. Klaus Fleischmann, Jean–François Le Gall: A new approach to the single point catalytic super–Brownian motion.

82. Anton Bovier, Jean–Michel Ghez: Remarks on the spectral properties of tight binding and Kronig–Penney models with substitution sequences.

83. Klaus Matthes, Rainer Siegmund–Schultze, Anton Wakolbinger: Recurrence of ancestral lines and offspring trees in time stationary branching populations.

84. Karmeshu, Henri Schurz: Moment evolution of the outflow–rate from nonlinear conceptual reservoirs.

85. Wolfdietrich Müller, Klaus R. Schneider: Feedback stabilization of nonlinear discrete–time systems.

86. Gennadii A. Leonov: A method of constructing of dynamical systems with bounded nonperiodic trajectories.

87. Gennadii A. Leonov: Pendulum with positive and negative dry friction. Continuum of homoclinic orbits.

88. Reiner Lauterbach, Jan A. Sanders: Bifurcation analysis for spherically symmetric systems using invariant theory.

89. Milan Kučera: Stability of bifurcating periodic solutions of differential inequalities in $\mathbb{R}^3$.

90. Peter Knabner, Cornelius J. van Duijn, Sabine Hengst: An analysis of crystal dissolution fronts in flows through porous media Part I: Homogeneous charge distribution.

91. Werner Horn, Philippe Laurençot, Jürgen Sprekels: Global solutions to a Penrose–Fife phase–field model under flux boundary conditions for the inverse temperature.

92. Oleg V. Lepskii, Vladimir G. Spokoiny: Local adaptivity to inhomogeneous smoothness. 1. Resolution level.

93. Wolfgang Wagner: A functional law of large numbers for Boltzmann type stochastic particle systems.

94. Hermann Haaf: Existence of periodic travelling waves to reaction–diffusion equations with excitable–oscillatory kinetics.

95. Anton Bovier, Véronique Gayrard, Pierre Picco: Large deviation principles for the Hopfield model and the Kac–Hopfield model.

96. Wolfgang Wagner: Approximation of the Boltzmann equation by discrete velocity models.

97. Anton Bovier, Véronique Gayrard, Pierre Picco: Gibbs states of the Hopfield model with extensively many patterns.

98. Lev D. Pustyl'nikov, Jörg Schmeling: On some estimations of Weyl sums.

99. Michael H. Neumann: Spectral density estimation via nonlinear wavelet methods for stationary non–Gaussian time series.

100. Karmeshu, Henri Schurz: Effects of distributed delays on the stability of structures under seismic excitation and multiplicative noise.

101. Jörg Schmeling: Estimates of Weyl sums over subsequences of natural numbers.

102. Grigori N. Milstein, Michael V. Tret'yakov: Mean–square approximation for stochastic differential equations with small noises.

103. Valentin Konakov: On convergence rates of suprema in the presence of non–negligible trends.

104. Pierluigi Colli, Jürgen Sprekels: On a Penrose–Fife model with zero interfacial energy leading to a phase–field system of relaxed Stefan type.

105. Anton Bovier: Self–averaging in a class of generalized Hopfield models.

106. Andreas Rathsfeld: A wavelet algorithm for the solution of the double layer potential equation over polygonal boundaries.

107. Michael H. Neumann: Bootstrap confidence bands in nonparametric regression.

108. Henri Schurz: Asymptotical mean square stability of an equilibrium point of some linear numerical solutions with multiplicative noise.