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Bragg localized structures in a passive cavity with transverse refractive index modulation

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Abstract

We consider a passive nonlinear optical cavity containing a photonic crystal inside it. The cavity is driven by a superposition of the two coherent beams forming a periodically modulated pump. Using a coupled mode reduction and direct numerical modeling of the full system we demonstrate existence of resting and moving transversely localized structures of light in this system.

Dissipative structures in optical systems have been the subject of intense research during the last years [1]. They result from the modulational (often called Turing [2]) instability that triggers a spontaneous transition from homogeneous steady states to self-organized or ordered structures. These can be either periodic or localized in space. The latter case corresponds to stationary localized pulses that are formed in the plane transverse to the beam propagation direction. They are often called cavity solitons, and have been observed experimentally in a wide class of optical systems: lasers with saturable absorber [3], liquid crystal light valve with optical feedback [4], and single-mirror feedback systems using sodium vapor [5]. The experimental realization of a write/erase system based on cavity solitons in semiconductor microresonator gives hope to achieve an integrable all-optical information processor [6].

Recently, the inclusion of the transverse refractive index modulation into models of intracavity nonlinear optics has revealed the existence of a new type of localized structures associated with Bragg reflection in lasers with a saturable absorber [7] and in discrete sets of coupled lasers [8] and resonantors [9]. It has also been shown that the modulation of the refractive index can inhibit a modulational instability [10]. Solitons in periodically patterned semiconductor amplifiers, i.e. without feedback, have been theoretically predicted in [11]. More recently, slowly moving dissipative localized structures of light have been found in a thin photonic crystal film with Kerr nonlinearity excited under the conditions of the so-called Fano resonance [12].

In this Letter, we consider a nonlinear passive cavity with a photonic crystal pumped by two plane waves beams. We show that the photonic crystal induces a modulational instability and creates conditions for existence of stable Bragg-like localized structures in the transverse direction. These structures have zero transverse velocity if the two coherent pumping beams are symmetric and the phase shift δ between the pump intensity profile and the refractive index modulation is an integer multiple of π. If these two conditions are not satisfied simultaneously then the localized structure move with constant velocity.

We consider a planar passive cavity with two adjacent media inside it, see Fig. 1. The first is a nonlinear two-level medium. The second is the photonic crystal introducing a refractive index modulation along the transverse directions. The cavity is driven by two coherent pump beams with amplitudes $P_{1,2}$. The interference between the two pumping waves produces a spatial modulation of the driving fields,

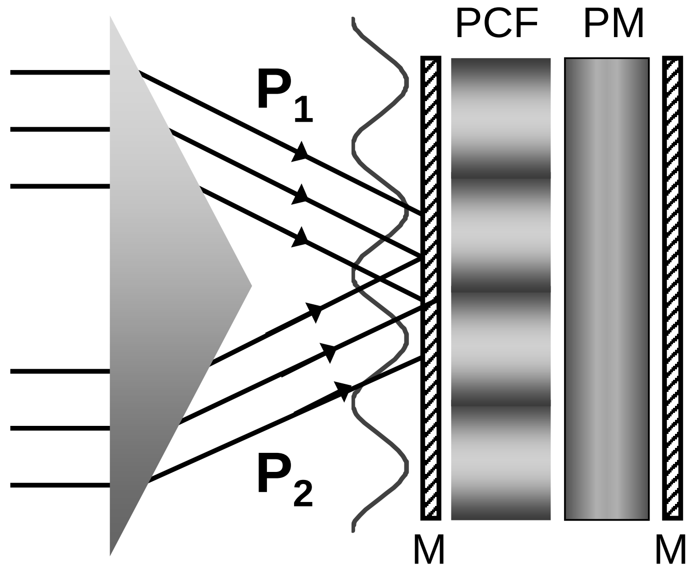

Figure 1: Schematic setup of the nonlinear cavity filled with a passive medium (PM) and a photonic crystal film (PCF). The Fabry-Perot cavity with flat Mirrors (M) is driven by two pumping beam $P_{1,2}$.

see Fig. 1. For simplicity, we consider the pure absorptive bistability, i.e. the atomic detuning is equal to zero. In the mean-field approximation [1], the electric field envelope F and the population difference N can be described by the dimensionless partial differential equations

$$\begin{aligned} \frac{\partial F}{\partial t} &= -\left[\gamma + i\theta + 2CN - 2i\cos(kx)\right] F + i\frac{\partial^2 F}{\partial x^2} \\ &+ e^{iqx}\left(P_1 e^{i(kx+\delta)/2} + P_2 e^{-i(kx+\delta)/2}\right), \qquad (1) \\ \frac{\partial N}{\partial t} &= \Gamma\left[N - 1 + N|F|^2\right]. \qquad (2) \end{aligned}$$

The modulation of the refractive index introduced by the photonic crystal is approximated by the $\cos kx$ function and its amplitude is rescaled to 2. Here k denotes the wavenumber of the refractive index modulation. θ is the cavity detuning and C is the cooperativity parameter [1]. The decay rates associated with the electric field and population difference are γ and Γ, respectively. q is the pump incidence angle. Diffraction is modeled by the second derivative with respect the transverse coordinate x.

To study Eqs. (1,2), we decompose the electric field and the population difference into a linear superposition of waves having opposite wavenumbers: $F = A_1 e^{ikx/2} + A_2 e^{-ikx/2}$ and $n = n_0 + n_1 e^{ikx} + n_2 e^{-ikx}$ where $A_{1,2}$ and $n_{0,1,2}$ are slowly varying envelopes with respect to the transverse coordinate. Substituting these decompositions into Eqs. (1,2) and performing adiabatic elimination of the variables $n_{0,1,2}$, we get the following nonlinear coupled mode equations:

$$\frac{\partial A_1}{\partial t} = -(\gamma + i\Omega)A_1 + iA_2 + P_1 e^{i(Q\xi+\delta/2)} + \frac{\partial A_1}{\partial \xi} - 2C(1 + |A_1|^2)GA_1, \tag{3}$$

$$\frac{\partial A_2}{\partial t} = -(\gamma + i\Omega)A_2 + iA_1 + P_2 e^{i(Q\xi-\delta/2)} - \frac{\partial A_2}{\partial \xi} - 2C(1 + |A_2|^2)GA_2, \tag{4}$$

where

$$G^{-1} = 1 + (1 + |A_1|^2)^2 + (1 + |A_2|^2)^2,$$

where $Q = kq$ is the rescaled sum of the incidence angle between the two pumping waves and $\xi = x/k$. The effective detuning is $\Omega = \theta + k^2/4$.

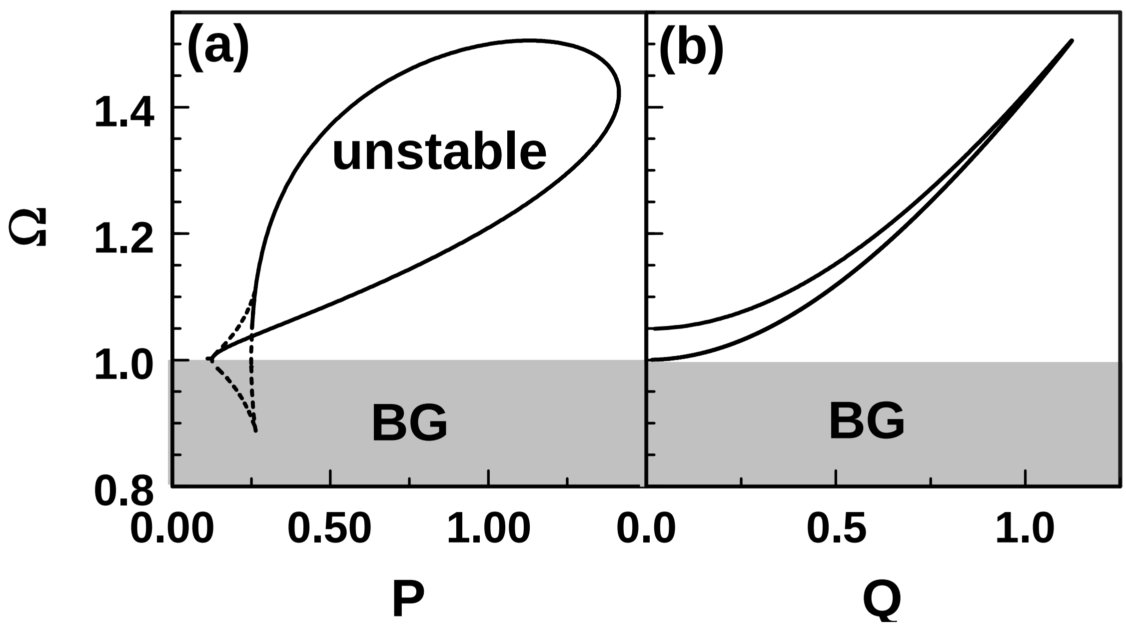

Figure 2: Instability boundaries as a function of the effective detuning parameter Ω. (a) Pump $P_1 = P_2 = P$ versus Ω. The solid curve is the modulational instability boundary. The bistability region is delimited by the three dashed curves. Grey region indicates photonic band gap (BG). (b) Critical wavenumber at the modulational instability versus Ω. Parameters are $\gamma = 0.01$, $C = 0.4$, $\delta = 0$, and $Q = 0$.

Let us first examine the symmetric pumping situation, i.e., $P_{1,2} \equiv P$, with a zero phase shift between the pump intensity and the refractive index, i.e., $\delta = 0$. In this case, the linear stability analysis of the uniform steady-state solutions of Eqs. (3,4) with respect to a finite wavenumber perturbations shows that the system exhibits a modulational instability in both the monostable and the bistable regimes. The results of this analysis is summarized in Fig. 2 where we plot the critical pump amplitude as a function of the effective detuning. The critical wavenumbers corresponding the maximum gain are plotted in Fig. 2 (b). From Fig. 2, we see that the modulational instability takes place outside the photonic band gap. The band gap indicated in this figure by the grey area is calculated from Eqs. (3,4) without dissipative terms as the region of non-existence of solutions of the form $\exp(\pm iQ\xi)$ for real Q and Ω.

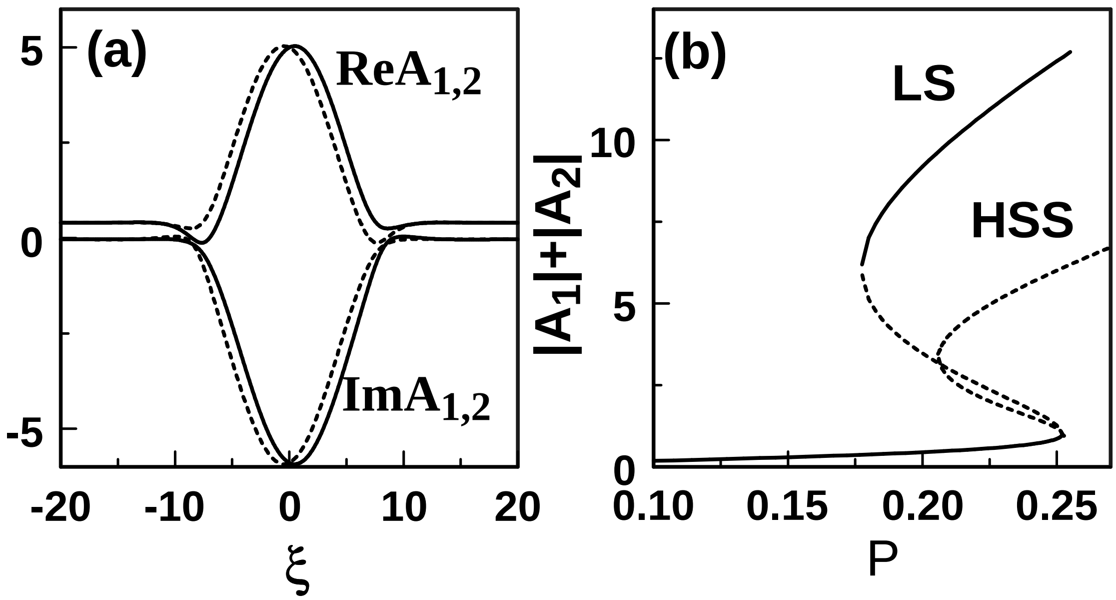

Figure 3: Stationary localized structures. $\Omega = 1.05$. Other parameters are the same as in Fig. 2. (a) Real and imaginary parts of the field amplitudes $A_{1,2}$ for $P_1 = P_2 = 0.225$. Solid (broken) lines correspond to A_1 (A_2). (b) Bifurcation diagram. LS: localized structures, HSS: homogeneous steady state. Broken lines correspond to unstable solutions.

When a modulational instability appears subcritically, localized structures are formed in the hysteresis loop involving the homogeneous steady state and periodic patterns [1]. In what follows, we focus on the localized structures whose existence is ensured by the Bragg scattering at the periodic refractive index modulation. These structures can not be generally traced back to the limit with transversely homogeneous refractive index, where photonic band gap disappears. We find the transverse profiles of the localized structures by solving numerically the nonlinear coupled mode Eqs. (3, 4). Fig. 3a represents typical profiles of the amplitudes $A_{1,2}$ corresponding to bright stationary localized solutions which have been calculated for the case of symmetric pumping, $\delta = 0$, $Q = 0$, and $P_1 = P_2$, when the coupled mode equations are invariant under the reflection transformation $\xi \to -\xi$, $A_1 \leftrightarrow A_2$. The branch of the localized structures obtained by varying the pump strength parameter $P = P_1 = P_2$ is shown in Fig. 3b together with the branch of spatially homogeneous solutions of the coupled mode equations.

The localized structures found within the framework of the coupled mode approach exist also in the full model, as we demonstrated by direct numerical modelling of Eqs. (1,2), see Fig. 4. The localized structure shown in this figure is formed by

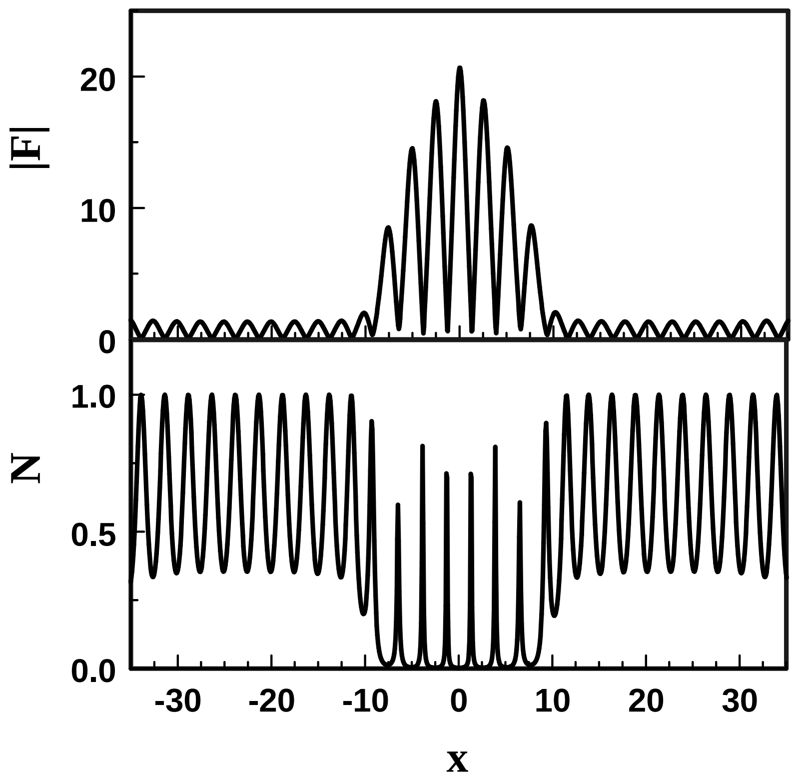

Figure 4: Stationary localized structure obtained by direct numerical simulation of Eqs. (1,2). Parameters are $\gamma = 0.05$, $C = 2.0$, $\delta = 0$, $Q = 0$, $P_1 = P_2 = 1.2$, $k = 2.5$, and $\theta = -0.3125$.

the two waves counterpropagating in the transverse direction and therefore it is characterized by oscillations of the electric field intensity with the spatial frequency k equal to that of the refractive index modulation and a phase shift π between the two neighboring intensity maxima, which fully complies with predictions of the coupled mode approach. In that respect our structures are similar to the so-called "staggered" solitary waves in discrete nonlinear systems [13] and different from the ünstaggeredßolitons reported in [11]. The phase of the intensity oscillations of the localized structure shown in Fig. 4 coincides with that of the refractive index profile. From this figure we see that the homogeneous steady state of the coupled mode equations which serves as a background for the localized solution in Fig. 3a corresponds to a spatially periodic solution of Eqs. (1,2).

In the case when the pumping is asymmetric, the localized structures move with a constant velocity. Fig. 5 illustrates the dependence of the localized structure velocity $v = d\xi/dt$ on the phase δ, imbalance between the amplitudes of the pump beams $\delta P = (P_2 - P_1)/(P_1 + P_2)$, and the angle of incidence Q. From Fig. 5 (a) we see that a phase shift δ between the pump intensity and the refractive index profiles results in a very small v, which is approximately four orders of magnitude smaller than δ itself. However, v increases rapidly with δ and localized structures disappear

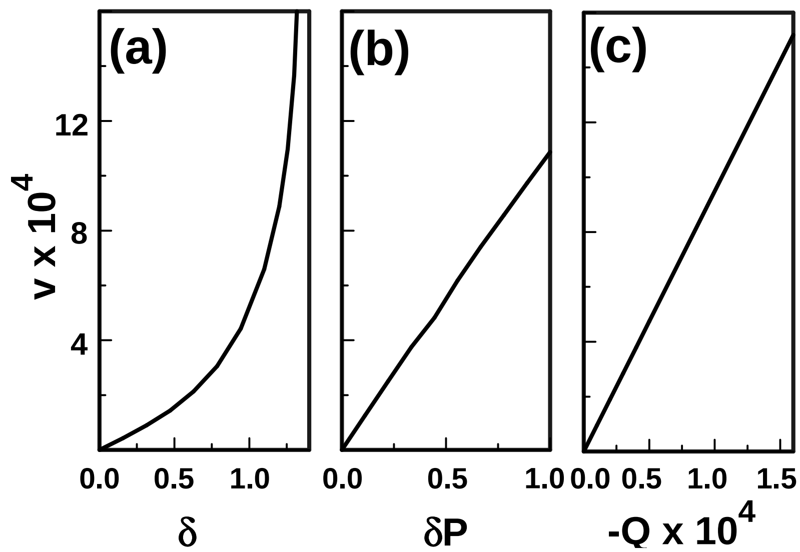

Figure 5: Transverse velocity $v = d\xi/dt$ of a localized structure as a function of (a) the phase shift δ, (b) pump imbalance $\delta P = (P_2 - P_1)/(P_2 + P_1)$, where $P_2 + P_1 = 0.45$, (c) the sum of incidence angles Q. Other parameters are the same as in Fig. 3.

as δ tends to $\pi/2$. On the other hand, v depends linearly on the pump imbalance δP and on Q, although much more sensitively so on the latter. This behavior is shown in Figs. 5 (a) and (c).

Finally, we note that Eqs. (3,4) are invariant with respect to the transformation $A_1 \to -A_1^*$, $A_2 \to A_2^*$, $\Omega \to -\Omega$, $\delta \to \delta + \pi$ applied together with complex conjugation. In particular, this means that the stationary localized structures found at $\delta = 0$ can be transformed into the structures with $\delta = \pm\pi$ for the same absolute value but opposite sign of the detuning parameter Ω. Unlike the structure shown in Fig. 4, the structures with $\delta = \pm\pi$ are characterized by intensity oscillations anti-phase with those of the refractive index profile.

In conclusion, using the nonlinear coupled mode approach and numerical modelling of the full system, we have demonstrated the existence of bistability, modulational instability and stable Bragg localized structures in the transverse section of an externally pumped passive cavity with photonic crystal. The localized structures move if the pumping is asymmetric or if the phase detuning δ, see Eq. (1), is different from 0 or π. The coupled mode reduction similar to the one applied above can be used to study other driven nonlinear systems with a photonic band gap and one may expect that localized structures constitute a generic and general feature of such systems.

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