Dynamical regimes in a monolithic passively mode-locked quantum dot laser

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Received May 20, 2010; revised August 10, 2010; accepted August 11, 2010;
posted August 12, 2010 (Doc. ID 128778); published September 22, 2010

Operation regimes of a two section monolithic quantum dot (QD) mode-locked laser are studied experimentally with InGaAs lasers and theoretically, using a model that takes into account carrier exchange between QD ground state and two-dimensional reservoir of a QD-in-a-well structure. It is shown analytically and numerically that, when the absorber section is long enough, the laser exhibits bistability between laser off state and different mode-locking regimes. The Q-switching instability leading to slow modulation of the mode-locked pulse peak intensity is completely eliminated in this case. When, on the contrary, the absorber length is rather short, in addition to usual Q-switched mode-locking, pure Q-switching regimes are predicted theoretically and observed experimentally. © 2010 Optical Society of America

OCIS codes: 140.5960, 140.4050.

1. INTRODUCTION

Mode-locking (ML) in lasers is a powerful tool for generating short optical pulses for different practical applications ranging from high speed communication to medical diagnostics. In particular, monolithic passively mode-locked semiconductor lasers are compact sources of picosecond pulses with high repetition rates suitable for applications in telecommunication technology [1,2]. Recently a new generation of mode-locked semiconductor lasers based on quantum dot (QD) material was developed [3]. These lasers demonstrate many advantages over conventional quantum well lasers, such as low threshold current, small alpha factor [4], low pulse chirp, high stability to noise, and external feedback, among others [5,6]. It was recently shown that QD mode-locked lasers can generate very short subpicosecond optical pulses [7,8].

Due to the discrete nature of electronic states in QD lasers, they demonstrate a number of characteristic features distinguishing them from conventional semiconductor devices [9]. Therefore, theoretical modeling of these lasers requires development of more sophisticated tools, which take into account these features. In particular, carrier dynamics in QD lasers must include carrier exchange processes between two-dimensional (2D) reservoir of a QD-in-a-well structure [10] and the discrete levels in QDs. These processes are characterized by a large number of quite different characteristic time scales, which have an important impact on the quality of mode-locked pulses and the dynamical behavior of the laser in general.

Rate equations describing the carrier exchange dynamics in QD lasers were proposed in [10–12]. These equations govern the time evolution of three quantities: carrier density in the 2D reservoir and occupation probabilities of two discrete levels in QDs, corresponding to the first excited state and the ground state. Assuming that the carrier relaxation rate from the excited states to the ground state is sufficiently fast, one gets only two rate equations for the occupation probability ρ of the QD ground state and the carrier density n in the 2D reservoir [13,14]. Similar pairs of carrier rate equations for gain and absorber sections were recently incorporated into the delay differential model to describe passive ML in semiconductor lasers [15,16]. It was shown that fast carrier capturing from the 2D reservoir to QDs can lead to the suppression of the undesirable Q-switching instability of the fundamental ML [14]. Such instability leading to a degradation of the quality of the ML regime is quite difficult to eliminate in quantum well mode-locked devices [2,17]. In this paper, using the delay differential equation (DDE) model [16] together with traveling wave equations [18], we perform a more systematic study of the effect of carrier exchange processes on the qualitative aspects of the dynamics of a QD mode-locked laser. We show theoretically that taking into consideration Pauli blocking in carrier exchange terms can lead to a qualitative change in the laser dynamical behavior. When the absorber section is long enough, the Q-switching instability of fundamental ML regime disappears, and bistability appears between the laser off state and different ML regimes. We also describe the period-doubling bifurcation of a harmonic ML regime leading to different amplitudes and separations of two pulses circulating in the cavity. In a laser with relatively short absorber section the Q-switching instability in the ML regime takes place at small injection currents. In this case we show theoretically and confirm experimentally that at sufficiently small injection currents and reverse biases applied to the absorber section, apart from usual Q-switched ML the laser can operate in a pure Q-switching regime.

2. MODEL EQUATIONS

We consider a model of passively mode-locked QD laser consisting of two sections: a forward biased amplifying section and a reversely biased saturable absorber section. In each section the spatial-temporal evolution of the amplitudes of two counter-propagating waves $E^{\pm}$ can be described by the so-called traveling wave equations [18,19]:

$$\frac{\partial E^{\pm}}{dt} \pm \frac{\partial E^{\pm}}{\partial z} = -\frac{\beta_{g,q}}{2}E^{\pm} + \frac{g_{g,q}}{2}(2\rho - 1)E^{\pm}, \qquad (1)$$

where the index g corresponds to the amplifying section, and the index q refers to the absorber section. The parameters $\beta_{g,q}$ describe linear internal losses in the semiconductor medium, and g_g (g_q) is the differential gain (loss) parameter in the amplifying (absorbing) section. In Eq. (1) for the sake of simplicity we have neglected the dependence of the refractive index of the semiconductor medium on the occupation probabilities of QD levels and carrier density in the 2D reservoir. It is known that the linewidth enhancement factors can be neglected in the modeling of Q-switching behavior; see, e.g., [17]. Investigation of the effect of the linewidth enhancement factors on the ML regimes requires separate consideration. In particular, it was shown in [16] that different α-factors in gain and absorber sections can lead to a transition from regular ML regime to irregular pulsations. Furthermore, the inhomogeneous spectral broadening effect due to QD nonuniformity is also neglected in our model. This allows one not only to reduce significantly the amount of computing time required for calculations, but also to derive some analytical formulas. Another reason for neglecting of the inhomogeneous broadening is the fact that the width of the experimentally measured generation spectrum in the ML regime was comparable to the homogeneous linewidth. Assuming that the relaxation of populations from upper QD levels to the ground state is fast enough, we use the following equations to describe carrier exchange processes between QDs and 2D reservoir [13,14]:

$$\frac{\partial \rho}{\partial t} = -\gamma_{g,q}\rho - r_{g,q}\rho + b_{g,q}n(1-\rho) - g_{g,q}(2\rho - 1)(|E^{+}|^2 + |E^{-}|^2), \qquad (2)$$

$$\frac{\partial n}{\partial t} = \eta_{g,q} - \delta_{g,q}n + 2r_{g,q}\rho - 2b_{g,q}n(1-\rho). \qquad (3)$$

Here ρ is the QD ground state occupation probability, and n is the carrier density in the 2D reservoir. The factor of 2 in Eq. (3) accounts for the double degeneracy of the ground state in QDs. The parameters $\gamma_{g,q}$ ($\delta_{g,q}$) are the inverse radiative lifetimes in the QDs (inverse effective residence times in the 2D reservoir). The parameters $r_{g,q}$ and $b_{g,q}$ describe, respectively, the rate of carrier escape from QDs to the 2D reservoir and the carrier capture rate from the reservoir to the QDs. The term η_g in Eq. (3) describes linear gain due to injection current. Since there is no injection current in the absorber section, we have $\eta_q = 0$. However, we assume that the relaxation time δ_q increases with the increase in the reverse voltage applied to the absorber section. Boundary conditions for Eq. (1) are given by $E^{+}(0,t) = \sqrt{\kappa_1}E^{-}(0,t)$ and $E^{-}(L,t) = \Gamma\sqrt{\kappa_2}\int_0^{\infty} e^{-\Gamma\tau}E^{+}(L,t-\tau)d\tau$, where $z=0$ ($z=L$) corresponds to the left (right) laser facet. These conditions account for the reflectivities of the facets $\kappa_{1,2}$ and the gain dispersion, which is described by Lorentzian filter of width Γ [20].

Equations (1)–(3) have been used for numerical modeling of dynamics in monolithic QD lasers. A numerical scheme for solving these equations is described in [20]. Along with these equations a simplified model assuming ring cavity geometry and unidirectional lasing approximation [15,16,21] was used. This model can be written as a set of five DDEs [14]:

$$\Gamma^{-1}\frac{dA}{dt} + A = \sqrt{\kappa}e^{G(t-T)/2 + Q(t-T)/2}A(t-T), \qquad (4)$$

$$\frac{dP_g}{dt} = -(\gamma_g + r_g)P_g + b_gN_g(1-P_g) - e^{Q}(e^{G} - 1)|A|^2, \qquad (5)$$

$$\frac{dP_q}{dt} = -(\gamma_q + r_q)P_q + b_qN_q(1-P_q) - s(e^{Q} - 1)|A|^2, \qquad (6)$$

$$\frac{dN_g}{dt} = \delta_g(g_0 - N_g) + 2r_gP_g - 2b_gN_g(1-P_g), \qquad (7)$$

$$\frac{dN_q}{dt} = -\delta_qN_q + 2r_qP_q - 2b_qN_q(1-P_q). \qquad (8)$$

Here A is the normalized electric field envelope at the entrance of the absorber section. The variables $P_{g,q} = \int_{g,q}\rho dl/l_{g,q}$ and $N_{g,q} = \int_{g,a} n dl/l_{g,q}$ are normalized integrals of the occupation probability ρ and carrier density n along the corresponding section, where l_g (l_q) denotes the length of the gain (absorber) section. The cumulative saturable gain G and loss Q introduced by the gain and absorber sections are given by $G = g_gl_g(2P_g - 1)$ and $Q = g_ql_q(2P_q - 1)$. The parameter T is the cavity round trip time, Γ is the spectral filtering width, and κ is the attenuation factor that accounts for the linear nonresonant losses $\beta_{g,q}$ and reflectivities of the laser facets $\kappa_{1,2}$. $g_0 = \int_g \eta_g dl/l_g$ is the dimensionless injection parameter defined in [16]. Finally, the parameter $s = g_g/g_q$ in Eq. (6) is the ratio of the differential gain in the gain section and the differential absorption in the absorber section.

3. CONTINUOUS WAVE REGIMES

In this section we study the solutions of Eqs. (4)–(8) corresponding to the continuous wave (cw) laser output. The simplest cw solution is that corresponding to zero laser intensity. It is given by

$$A=0, \quad P_g=P_{g0}, \quad N_g=N_{g0}, \quad P_q=N_q=0, \tag{9}$$

with $P_{g0}=(1+\xi)/2-\sqrt{(1+\xi)^2/4-(g_0\zeta)/2}$, $N_{g0}=g_0-2P_{g0}/\zeta$, and $\xi=(\zeta/2)[g_0+(r_g+\gamma_g)/b_g]$, $\zeta=\delta_g/\gamma_g$. The dependence of the quantities $P_g=P_{g0}$ and $N_g=N_{g0}$ on the injection parameter g_0 in the amplifying section is shown in Fig. 1(a). We see that unlike the integral carrier density N_g in the 2D reservoir, which grows linearly at large injections g_0, the growth of the occupation probability P_g of QDs saturates at $P_g=1$ when the injection current becomes sufficiently large. The smaller is the ratio $(r_g+\gamma_g)/b_g$ and the larger is ζ, the faster P_g saturates. This saturation is related to the presence of the Pauli blocking factor $1-P_g$ in the capture rate term in Eq. (6); hence it is a consequence of the fact that the number of free places for electrons in QDs is limited. Saturation of the occupation probability results in saturation of the gain in the amplifying section. This gain cannot exceed the maximal value $G=g_g l_g$ corresponding to a state with fully occupied QDs, $P_g=\rho_g=1$. Noteworthy is that the assumption of fast relaxation from the first exited state to the ground state in QDs is probably not valid in the regime of strong gain saturation.

Since at $A=0$ the absorber section is completely unsaturated, we have $P_q=0$; and, hence, the cumulative loss introduced by this section is $Q=-g_q l_q$. Therefore, for $g_0\to\infty$ when all the states in QDs are fully occupied, i.e., $G=g_g l_g$, the stability of the zero intensity cw solution (9) is determined by the inequality

$$g_g l_g - g_q l_q + \ln\kappa < 0, \tag{10}$$

where $\ln\kappa$ describes the linear nonresonant losses. This inequality ensures that the absolute value of the coefficient in front of the delayed electric field term in Eq. (4) is less than 1. From this condition we see that when the absorber section length is large enough, namely, $g_q l_q > g_g l_g + \ln\kappa$ the zero intensity steady state (9) remains stable at arbitrary large injection currents (arbitrary large g_0). In this case the maximal achievable linear cumulative gain $G=g_g l_g$ is smaller than total unsaturated losses $|Q|=g_q l_q$ introduced by the absorber section plus linear nonsaturated losses $-\ln\kappa$. However, even in this case laser generation is possible in a regime with strongly saturated absorber. Indeed, in the limit $g_0\to\infty$ one obtains that apart from zero intensity steady state (9) there exist two cw solutions with $|A|^2\neq 0$. The first of these cw solutions corresponds to a fully saturated absorber $P_q=1/2$ and is always stable in this limit [see the upper branch of curve 2 in Fig. 1(b)]. The second (unstable) solution [see the lower branch of curve 2 in Fig. 1(b)] corresponds to positive occupation probability $P_q=(g_q l_q - g_g l_g - \ln\kappa)/(2g_q l_q)>0$ only when inequality (10) is satisfied. Otherwise, this solution corresponds to negative occupation probability, $\rho_q<0$, and laser intensity, $|A|^2<0$, and, therefore, is nonphysical. Thus, if inequality (10) holds, the branch of nonzero-intensity cw solutions is isolated from the zero intensity state [curve 2 in Fig. 1(b)], and the bistability exists between the two cw states: one with zero and the other with positive laser intensity. If the absorber section length is sufficiently short, so that the inequality $g_q l_q > g_g l_g + \ln\kappa$ is not satisfied, the solution with nonzero laser intensity bifurcates from the zero intensity state at gain parameter value $g_0=(g_g l_g+g_q l_q-\ln\kappa)[1/g_g l_g\zeta+r_g+\gamma_g/b_g(g_g l_g-g_q l_q+\ln\kappa)]$ which corresponds to the linear lasing threshold [see curve 1 in Fig. 1(b)]. In this case, either there is no bistability or the bistability domain is rather small.

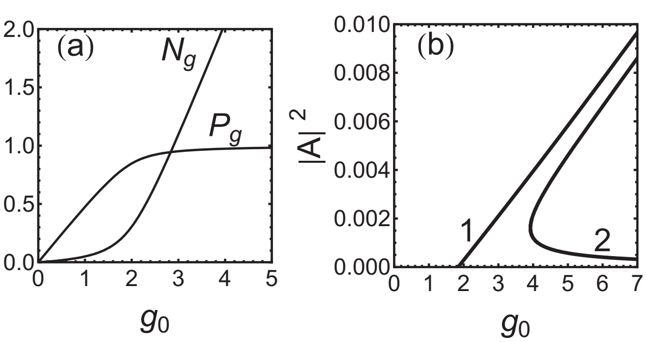

Fig. 1. Left: Dependence of the integral carrier density in the 2D reservoir N_g and occupation probability in QDs P_g in the gain section on the injection parameter g_0 for the solution with zero laser intensity. Right: Dependence of the laser intensity on the injection parameter g_0. 1—monostable behavior, short absorber section $l_g=0.9$ mm, $l_q=0.1$ mm. 2—bistable behavior, longer absorber section, $l_g=0.8$ mm, $l_q=0.2$ mm. Other parameters are $T=25$ ps, $s=15$, $\Gamma^{-1}=0.4$ ps, $\kappa=0.3$, $\gamma_g^{-1}=1$ ns, $\gamma_q^{-1}=1$ ns, $\delta_g^{-1}=1$ ns, $\delta_q^{-1}=10$ ps, $g_g=4$ mm^{-1}, $g_q=10$ mm^{-1}, $b_g^{-1}=5$ ps, $b_q^{-1}=5$ ps, $r_g^{-1}=100$ ps, $r_q^{-1}=10$ ps.

4. MODE-LOCKING REGIMES

The results of numerical bifurcation analysis of Eqs. (4)–(8) performed using the path following software package DDEBIFTOOL [22] and direct numerical simulations [23] are summarized in Fig. 2. This figure shows the dependence of the laser field peak intensity on the injection parameter g_0 in lasers with two different lengths of the absorber section. The parameter values for the calculations have been chosen using the estimations of the capture and escape rates in the gain and absorber sections performed in [24,25].

A. Long Absorber Section

Bifurcation diagram shown in Fig. 2(a) corresponds to a laser with rather long absorber section when the branch of nonzero intensity cw solutions is isolated from laser off state [see Fig. 1(b)]. In Fig. 2(a) the stable parts of the branches of fundamental (ml) and harmonic (ml2, ml2′) ML regimes coexist with the stable laser off state. The lower unstable part of the fundamental ML branch extends to infinite values of g_0. Furthermore, the Q-switching instability of the ML regime does not appear. Thus, by increasing the absorber section length the Q-switching instability can be completely eliminated.

Figure 2(a) demonstrates another peculiar feature of Eqs. (4)–(8) describing a mode-locked QD laser. For certain parameter values these equations can exhibit a period-doubling bifurcation of the harmonic mode-locked solution with the pulse repetition rate twice larger than that of the fundamental regime. After this bifurcation of the ML regime with two pulses circulating in the cavity, the pulses acquire different amplitudes and separations. This regime is indicated ml2′ in Fig. 2. Similar period-doubling bifurcation was found in numerical simulations of the traveling wave equations (1)–(3), see the bifurca-

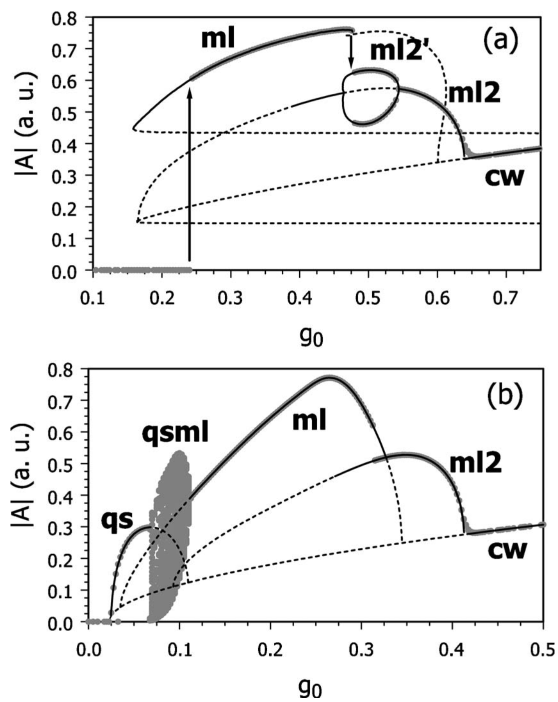

Fig. 2. Bifurcation diagram illustrating the sequence of dynamical regimes that takes place with the increase in the injection parameter g_0 Stable (unstable) solutions are shown by solid (dotted) lines. cw, ml, and ml2 correspond to continuous wave, fundamental ML, and harmonic ML regimes, respectively. ml2′ indicates the harmonic ML regime with two different pulses. The gray dots indicate the extrema of the absolute value of the electric field amplitude $|A|$ obtained by means of direct numerical simulation of Eqs. (4)–(8). Arrows indicate jumps between different regimes; $\Gamma^{-1}=0.5$ ps. (a) $l_g=0.8$ mm, $l_q=0.2$ mm, $g_g=2.22$ mm^{-1}, $g_q=20$ mm^{-1}, $\delta_q^{-1}=10$ ps, $b_g^{-1}=1$ ps, $b_q^{-1}=20$ ps, $r_g^{-1}=1$ ns, $r_q^{-1}=10$ ps. (b) $l_g=0.9$ mm, $l_q=0.1$ mm, $g_g=4$ mm^{-1}, $g_q=20$ mm^{-1}, $\delta_q^{-1}=5$ ps, $r_g^{-1}=250$ ps, $r_q^{-1}=6.67$ ps. Other parameters are the same as for Fig. 1(b).

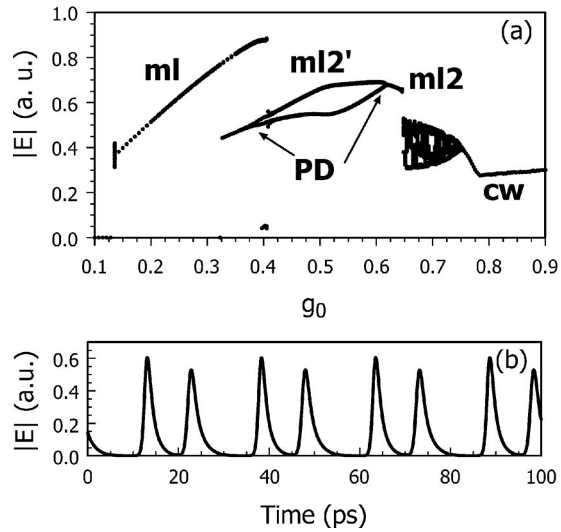

Fig. 3. (a) Bifurcation tree obtained by numerical simulation of Eqs. (1)–(3). Black dots correspond to maxima of intensity time traces calculated at different values of the injection parameter $g_0=\eta_g l_g$. Period-doubling bifurcations of the harmonic ML regime are labeled PD. (b) Harmonic ML regime with two pulses having different peak intensities and separations calculated for $g_0=\eta_g l_g=0.5$. $l_g=1.125$ mm, $l_q=0.125$ mm, $\Gamma^{-1}=0.25$ ps, $\kappa_{1,2}=0.55$, $\beta_g=\beta_q=0$, $g_g=2$ mm^{-1}, $g_q=9$ mm^{-1}, $r_g^{-1}=20$ ps, $r_q^{-1}=1$ ns, $\gamma_g^{-1}=\gamma_q^{-1}=1$ ns, $\delta_g^{-1}=1$ ns, $\delta_q^{-1}=10$ ps.

tion diagram in Fig. 3(a), which is qualitatively very similar to that in Fig. 2(a), and time trace in Fig. 3(b). A regime with two different pulses existing in the cavity simultaneously was recently observed experimentally in a 20 GHz two-section mode-locked QD laser [26]. The period-doubling bifurcation shown in Figs. 2(a) and 3(a) is different from that described in [20] for a mode-locked quantum well laser. The latter bifurcation is possible only in the case of the linear cavity geometry when an additional passive section is present in the laser cavity. This contrasts to the period-doubling bifurcation shown in Fig. 2(a), which appears in the DDE model (4)–(8) of a two-section laser assuming a ring laser cavity. This bifurcation may be attributed to a more complicated carrier dynamics in the QD laser model (4)–(8) as compared to the models [15,16,20,21] of quantum well lasers. We note that the results obtained with the help of the traveling wave model (1)–(3) are in a very good qualitative agreement with those of the DDE model (4)–(8) based on the ring cavity and unidirectional lasing approximations.

B. Short Absorber Section

Figure 2(b) presents a diagram similar to that of Fig. 2(a), but corresponding to a shorter absorber section. The bifurcation sequence depicted in this figure is qualitatively similar to that described earlier for a model of a monolithic quantum well mode-locked device [15,16]. ML solutions bifurcate from the cw solution corresponding to nonzero laser intensity. At sufficiently small injection currents g_0 the fundamental ML regime exhibits an instability toward a regime with undamped oscillations of the pulse peak intensity at the Q-switching frequency. This regime of "Q-switched" ML is indicated qsml in Fig. 2(b) [see also the time trace in Fig. 4(b)]. However, unlike the model of quantum well laser described in [15,16], the QD ML model (4)–(8) apart from Q-switched ML can exhibit a pure Q-switching regime. In the latter regime, indicated qs in Fig. 2(b), the laser emits a sequence of pulses with the repetition frequency of the order of 1 GHz. The corresponding time trace and power spectrum are shown in Fig. 4(c). With the increase in the injection current the Q-switching regime looses stability and a transition to Q-switched ML with quasiperiodic laser intensity takes place, as it is illustrated in Fig. 4(b). At even higher injection currents, the fundamental ML regime labeled ml becomes stable. This regime is characterized by a sequence of short pulses with the repetition period close to the cavity round trip time and a single peak at the frequency close to 40 GHz in the power spectrum [see Fig. 4(c)]. With further increase in the injection parameter g_0, transitions to harmonic ML regime with approximately twice higher repetition rate and finally to a stable cw regime take place. These regimes are labeled ml2 and cw in Fig. 2(b).

Two-parameter plots illustrating the dependence of the ML range on the capture and escape rates in the gain and absorber sections are presented in Figs. 5 and 6. They have been obtained by direct numerical solution of Eqs. (4)–(8). In these figures different dynamical regimes are indicated by different gray levels. As we see from the left

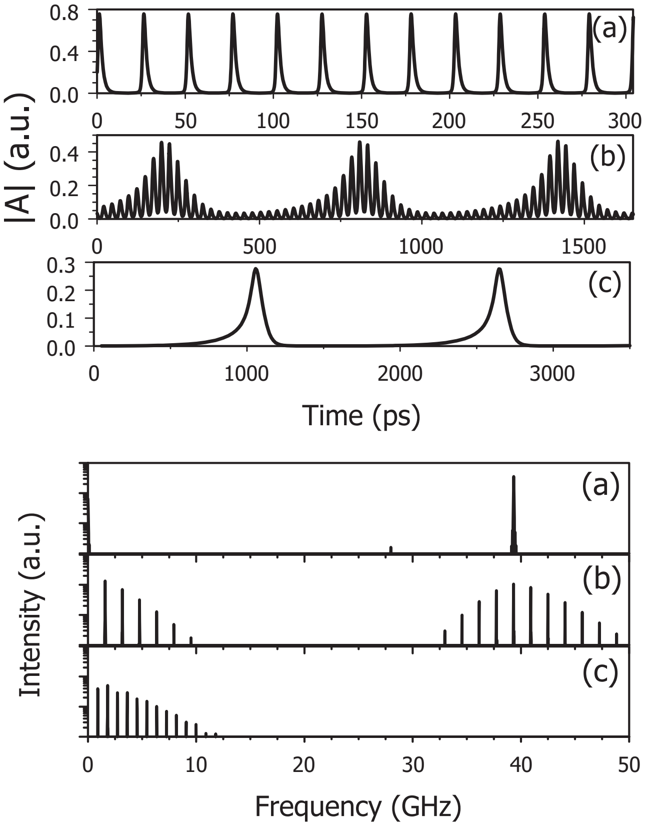

Fig. 4. Field amplitude time traces (top) and power spectra (bottom) illustrating dynamical regimes shown in Fig. 2(b). (a) Fundamental ML; (b) Q-switched ML; (c) pure Q-switching (qs).

panel of Fig. 5, the range of stable fundamental ML (shown by dark gray) increases with the capture rate b_g in the gain section. However, this dependence is rather weak. The dependence of ML range on the escape rate r_g in the gain section is, on the contrary, quite strong. As it is seen from the right panel of Fig. 5, the laser can exhibit stable ML only if the carrier escape rate from QDs to the 2D reservoir is sufficiently small. With the increase in this rate ML regime is replaced with either a cw or a Q-switching behavior. Figure 6 illustrates the dependence of the ML range on the capture and escape rates in the absorber section. According to this figure the range of fundamental and harmonic ML regimes increases with the decrease in the capture rate b_q and increases with the escape rate r_q. The increase in the ML range is usually accompanied by the appearance of harmonic ML with the pulse repetition rate close to twice the cavity round trip time. However, as it is seen from the right panel of Fig. 6, when the escape rate r_q to the 2D reservoir becomes significantly larger then the reservoir relaxation rate δ_q the ML range starts to decrease with r_q. Speaking more generally, stable ML is possible only if the relaxation rate δ_q and the escape rate r_q are large enough, i.e., the QD absorber is sufficiently fast. This is in agreement with the classical results of the ML theory [27]. At very small δ_q and/or r_q only Q-switching behavior is possible. We note that these two quantities increase with the reverse bias applied to the absorber section [28]. On the other hand an increase in the absorber capture rate b_q leads to a decrease in the ML range and the appearance of Q-switching behavior. In general, the situation when carriers spend sufficiently long average time in the absorber is favorable for the Q-switching and unfavorable for ML regimes.

Figure 7 illustrates the dependence of the ML range on the relaxation rate δ_q, which is the inverse of the effective

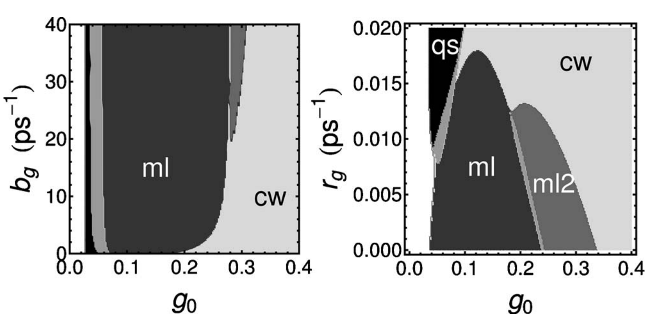

Fig. 5. Regions of different dynamical states in the plane of two parameters, the injection parameter g_0 and the capture rate b_g (escape rate r_g) in the gain section are shown in the left (right) panel. Different laser operation regimes are indicated by different levels of gray color. White, light gray, dark gray, gray, and black areas indicate, respectively, laser off, continuous wave (cw), fundamental ML (ml), harmonic ML (ml2), and Q-switching (qs) regimes. The gray area between fundamental ML and Q-switching domains corresponds to Q-switched ML regime (qsml); $s=15$, $\gamma_q^{-1}=10$ ps. Left (right) panel corresponds to $r_g^{-1}=250$ ps and $r_q^{-1}=5$ ps ($b_g^{-1}=5$ ps and $r_q^{-1}=2.5$ ps). Other parameters are the same as in Fig. 1(b).

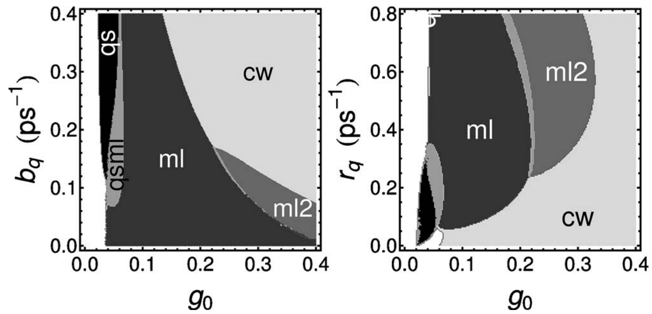

Fig. 6. Diagrams similar to those in Fig. 5 obtained by changing the capture rate b_q (left) and the escape rate r_g (right) in the absorber section; $b_g^{-1}=5$ ps, $r_g^{-1}=250$ ps. Left (right) panel corresponds to $r_q^{-1}=5$ ps ($b_q^{-1}=5$ ps). Other parameters are the same as in Fig. 5.

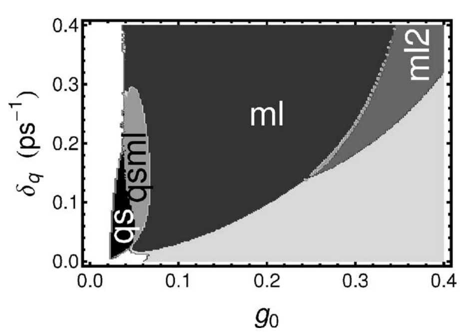

Fig. 7. Regions of different dynamical states in the plane of two parameters, injection parameter g_0 and absorber 2D reservoir relaxation rate δ_q. The notation is similar to that in Fig. 5: ml, fundamental ML regime; ml2, harmonic ML with approximately twice higher repetition rate; qs, Q-switching; qsml, Q-switched ML; cw, continuous wave regime. Parameters are the same as in Fig. 6.

residence time in the 2D reservoir. The result here is quite similar to that obtained earlier for a quantum well mode-locked laser [16,17,19], except for the existence of pure Q-switching regime at small values of δ_q, which corresponds to small absolute values of the reverse bias applied to the absorber section. The ML domain increases with the absorber relaxation rate and harmonic ML regimes appear. Finally, we note that the Q-switching domains in Figs. 5–7 are relatively small as compared to those calculated numerically [17] and measured experimentally [2,17] in a quantum well laser. This is in agreement with the results of [14], concerning a suppression of Q-switching instability in QD mode-locked lasers.

C. Experiment

Experimental studies have been performed with a ridge waveguide two-section QD monolithic mode-locked laser. The device dimensions were 4 μm ridge width and 1 mm overall length. The ratio of the absorber section length to the gain section length was 1:9 which corresponds to a rather short absorber section. The material incorporated is InGaAs forming 15 stacked layers of QDs [29]. Growth details can be found in [30]. The device was integrated in a module comprising a fiber pigtail, a microwave port, direct current contacts, and a thermoelectric cooler. Basic properties like *PIV*-curves, repetition frequency, pulse width/pulse shape dependencies on operating parameters, and temperature dependencies, as well as the characteristics of hybrid ML are already published in [28,31–33].

All measurements have been performed at room temperature in passive regime. The light output from the fiber was, after passing an optical isolator, given either to a high-speed photodetector or to an autocorrelator. The high-speed photodetector was connected to an electrical spectrum analyzer (ESA).

Previous measurements have not revealed any pure Q-switching in this QD mode-locked laser [26]. The reason is that the region of the instability is negligible compared to the ML region. High-resolution scans around the lasing threshold show the already seen Q-switched ML (qsml) and closer to the threshold pure Q-switching (qs); see the lower panel in Fig. 8.

In the upper panel of Fig. 8 three examples of ESA traces are given showing the evolution of the electrical spectrum with the increase in the injection current for a fixed reverse bias. This panel is an experimental counterpart of the lower panel in Fig. 4. The ESA traces are spanning a frequency range from 1 MHz up to 42 GHz. The resolution bandwidth was set to 100 kHz. At low injection currents, close to the laser threshold, only a spectral line and its harmonic at frequencies around 1 GHz are visible in the spectrum; see the lower trace in the upper panel of Fig. 8. This corresponds to a pure Q-switching regime. The only difference from Fig. 4(c) is that due to experimental limitations, in particular the noise floor of the ESA, the higher harmonics are not visible in Fig. 8. For the absorber bias of -8 V the Q-switching frequency starts with 0.54 GHz for an injection current of 24 mA and increases linearly up to 1.36 GHz at 30 mA of the current (not shown). This is in an agreement with the theoretical results shown in the lower panel of Fig. 4(c), where the Q-switching frequency is approximately equal to 0.9

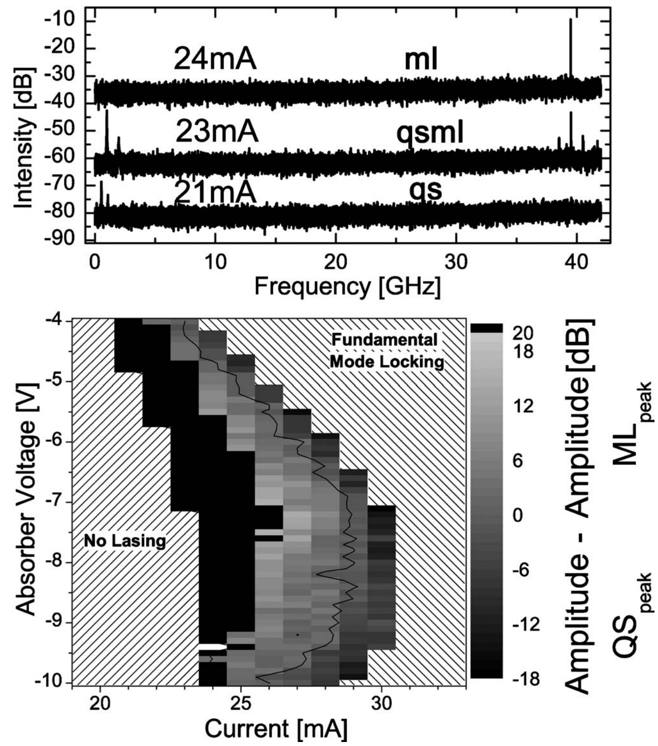

Fig. 8. Transition from Q-switching to fundamental ML regime. Top: ESA traces for three different currents at -4 V reverse bias. The offsets of the 23 and 24 mA traces have been shifted for better visibility. qs, Q-switching; qsml, Q-switched ML; ml, fundamental ML. Bottom: Different gray levels indicate the difference of the Q-switching spectral peak amplitude and the amplitude of the ML peak. Areas where no lasing or fundamental ML takes place are gray striped and labeled. The black region denotes pure Q-switching. The thin black line marks the value "0," where Q-switching and ML spectral peaks have equal amplitudes.

GHz. By fitting the measurement results we have obtained a slope of 0.128 GHz/mA for this linear increase. Going back to Fig. 8 we see that by increasing the current further, in this case 2 mA, the Q-switching spectral lines become more intense, and in addition a spectral line at the fundamental ML frequency appears. The ML spectral line is accompanied by two side peaks with a distance corresponding to the Q-switching frequency. At other operating parameters a pair of additional side peaks have been observed, which correspond to higher harmonics of the Q-switching frequency. A more intense pumping of the gain section results in a pure fundamental ML peak without any side peaks; see the upper trace in the upper panel of Fig. 8. The results of measurements of electrical spectrum traces for absorber voltages between -4 and -8 V and injection currents between 15 and 35 mA are summarized in the lower panel of Fig. 8. By calculating the difference between the amplitude of the Q-switching peak and the amplitude of the fundamental ML peak, the region where Q-switched ML takes place has been identified. Higher harmonics have not been taken into account. The region where only a Q-switching line appears in the electrical spectrum is shown in black. Negative (positive) numbers in the lower panel of Fig. 8 represent the measurements in which the ML line is more (less) intense than the Q-switching one. The thin black line within the gray-coded area in the lower panel of Fig. 8 is a guide to the eyes, showing the parameters where both spectral components, Q-switched and mode-locked, have equal amplitudes. No matter which driving current/absorber voltage combination is used, the pure Q-switching is only present in a range of 3 mA (in most cases the range is only 2 mA). Compared to the region where proper fundamental ML takes place, e.g., for an absorber voltage of -8 V from 30 mA to 100 mA, this range is less than 4%. The experimental results are in agreement with the theoretical ones shown in Fig. 7 where the domain of pure Q-switching is smaller than the domain of Q-switched ML and is much smaller than the domain of fundamental ML. This can be seen from the numbered grayscale bars at the bottom of the lower panel of Fig. 9. Besides the evaluation of the electrical spectra, one can gain information about the characteristics of mode-locked pulses by measuring autocorrelation traces, which represent a convolution of a pulse with itself. Details on the limitations of this method and real pulse shapes can be found in [33]. Despite these limitations, the autocorrelation traces can be used for monitoring the evolution of the pulse amplitude with the injection current. One has to keep in mind that the second-harmonic generation signal amplitude depends quadratically on the input amplitude.

As it is seen from Fig. 9, the peak amplitude of mode-locked pulses increases with the injection current in the range from 30 to 80 mA; above this range it starts to decrease up to 100 mA. At even higher injection currents the laser operates in a cw regime. This behavior is similar to that predicted theoretically for the case of short absorber section [see Fig. 2(b)]. Although the peak amplitude decreases in the range from 80 mA to 100 mA, the overall optical output gets steadily higher [31]. This is due to the fact that on one hand the pulses become broader, having therefore more energy per pulse, and on the other hand a

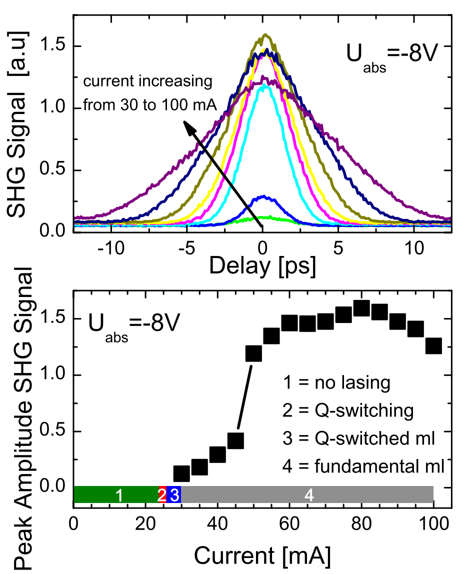

Fig. 9. (Color online) Experimentally measured evolution of the pulse peak amplitude and pulse width with the increase in the injection current. Top: Autocorrelator traces obtained for currents ranging from 30 to 100 mA at a fixed absorber voltage of -8 V. Bottom: Evolution of the peak amplitude with injection current extracted from the autocorrelator traces presented in the top figure. The operating regimes shown in Fig. 8 are marked using numbered grayscale bars.

cw-background shows up. Unlike the 20 GHz QD mode-locked laser studied in [26], no higher harmonics (pulse repetition frequency doubling) has been observed in the 40 GHz device using the autocorrelation technique. All other features which are Q-switching, Q-switched ML, fundamental ML, and the relation between the regimes as well the behavior of the amplitude with current are in a qualitative agreement with the results of numerical simulations.

5. CONCLUSION

We have studied the effect of carrier exchange processes between quantum dots (QDs) and 2D reservoir of a QD-in-a-well structure on the dynamical behavior of a monolithic mode-locked QD laser. We have presented a bifurcation analysis of the set of delay differential equations (DDEs) governing the time evolution of the electric field envelope, occupation probabilities of the ground state in QDs, and carrier densities in the 2D reservoir. In particular, these equations contain Pauli blocking terms that lead to a decrease in the capture rate when the occupation probability of the ground state in the QDs increases. We have shown that the dynamical behavior of the laser depends strongly on the relative length of the gain and absorber sections. When the absorber section is relatively long a bistability appears between the zero intensity state and ML regimes. In this case, the Q-switching behavior disappears completely. Another peculiar feature of the traveling wave equation (1)–(3) and DDE (4)–(8) QD laser models is the existence of period-doubling bifurcation of the harmonic ML regime with the repetition frequency approximately twice higher than that of the fundamental ML regime. As a result of this bifurcation a regime with two pulses having different amplitudes and separations in time develops. A similar regime was observed experimentally in a 20 GHz monolithic QD mode-locked laser [26]. The dynamical behavior of a mode-locked laser with a relatively short absorber is qualitatively quite similar to that of the quantum well ML laser model [15–17,19,21]. However, in contrast to the results reported in these papers, apart from Q-switched ML, numerical simulations of Eqs. (4)–(8) have revealed the existence of a pure Q-switching regime at very small injection currents. We have presented experimental evidence of the existence of such a regime in a 40 GHz monolithic passively mode-locked QD laser. The sequence of the dynamical regimes observed experimentally with the increase in injection current (no lasing → pure Q-switching → Q-switched ML → fundamental ML → cw) and the shapes and relative sizes of the domains corresponding to these regimes in a two-parameter plane, as well as the behavior of the amplitude with the current, are in qualitative agreement with the results obtained by simulations. The pulse widths obtained experimentally and by simulation for different parameter values are both in the picosecond range.

ACKNOWLEDGMENTS

A. G. Vladimirov is grateful to E. A. Viktorov and E. Avrutin for useful discussions. The authors from WIAS and TU Berlin acknowledge the funding of this work by the

SFB787 of the Deutsche Forschungsgemeinschaft (DFG). D. Rachinskii was partially supported by the Federal Programme "Scientists of Innovative Russia" (grant 2009-1.5-507-007).

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