Control and removal of modulational instabilities in low-dispersion photonic crystal fiber cavities

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Taking up to fourth-order dispersion effects into account, we show that fiber resonators become stable for a large intensity regime. The range of pump intensities leading to modulational instability becomes finite and controllable. Moreover, by computing analytically the thresholds and frequencies of these instabilities, we demonstrate the existence of a new unstable frequency at the primary threshold. This frequency exists for an arbitrary small but nonzero fourth-order dispersion coefficient. Numerical simulations for a low and flattened dispersion photonic crystal fiber resonator confirm analytical predictions and open the way to experimental implementation. © 2007 Optical Society of America

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existence domain from their rise up to their disappearance. Finally, in view of an experimental implementation, we perform numerical simulations for a realistic experimental configuration with a flattened dispersion photonic crystal fiber and find excellent agreement with the analytical predictions.

The fiber resonator is schematically depicted in Fig. 1. A continuous wave of power $E_i^2$ is launched into the cavity by means of a beam splitter, propagates inside the fiber, and experiences dispersion and the Kerr effect. At each round trip the light inside the fiber is coherently superimposed with the input beam. This can be described by the boundary conditions $E(z=0, \tau+t_R)=T \times E_{in}(\tau)+R \times E(L, \tau)\exp(-i\Phi_0)$ and by the extended NLSE $\partial_z E(z, \tau)=(-i\beta_2/2\partial_{\tau^2} + \beta_4/6\partial_{\tau^2}+i\beta_2/24\partial_{\tau^4}+i\gamma|E|^2)E$, with round-trip time $t_R$, linear phase shift $\Phi_0$, intensity mirror transmissivity (reflectivity) $T^2$ ($R^2$), and cavity length $L$. The electric field inside the cavity is denoted $E$. $\beta_{2,3,4}$ are the second-, third-, and fourth-order dispersion terms, respectively. $\gamma$ is the nonlinear coefficient, $z$ is the longitudinal coordinate, and $\tau$ is the time in a reference frame moving at the group velocity of the light. This infinite-dimensional map can be simplified to the following single normalized equation by applying the mean field approximation:

$$\frac{\partial \psi}{\partial \tau'} = S - (1+i\Delta)\psi + i|\psi|^2\psi - i\beta_2 \frac{\partial^2 \psi}{\partial \tau'^2} + B_3 \frac{\partial^3 \psi}{\partial \tau'^3} + iB_4 \frac{\partial^4 \psi}{\partial \tau'^4},$$

(1)

where $\tau'=tT^2/2t_R$, $\tau' = \pi(T^2/L)^{1/2}$, $\psi=E\sqrt{2}\gamma L/T^2$, $S=2/(2\gamma L/T^2)^{1/2}E_i$ is the normalized input field, $B_3 = \beta_2T^2/\sqrt{5L}$, $B_4 = \beta_2T^4/12L$, and $\Delta = 2\Phi_0/T^2$ is the cavity detuning. We carry out the analytical study in a low-dispersion fiber with a small dispersion slope. Thus, $B_3$ can be neglected. The steady state (SS) response $\psi_{SS}$ of Eq. (1) satisfies $S_{SS} = [1+i(\Delta - |\psi_{SS}|^2)]\psi_{SS}$. This solution is identical to that of the LL model leading to a monostable (bistable) regime for $\Delta < \sqrt{3}$ ($> \sqrt{3}$). Its stability with respect to finite frequency perturbations, i.e., of the form $\exp(i\Omega'\tau + \lambda \tau')$, shows that the MI frequencies that can be destabilized at the primary threshold $I_{1m} = |\psi_{1m}|^2 = 1$ are

$$\Omega^2_{UL} = -\frac{\beta_2 \pm \sqrt{\beta_2^2 + 4(\Delta - 2)B_4}}{2B_4},$$

(2)

and one can see immediately that two frequencies can be destabilized at the primary threshold for a suitable choice of $\beta_2$ and $\Delta$. Thus, taking into account $\beta$ expansion up to the fourth order in Eq. (1) evidences the existence of a second frequency of instability that has not yet been reported experimentally or theoretically when working in quite strong dispersion regions. This result is illustrated by the closed marginal stability curve in Fig. 2(a), where two destabilization frequencies ($\Omega_L$ and $\Omega_U$) exist at the primary threshold $I_{1m}$ in the monostable regime [Fig. 2(b)]. The finite extent of the MI domain is also evidenced by the lower and upper values of cavity power, $|\psi_{2m}|^2 = I_{2m} = 1$ and $|\psi_{2m}|^2 = I_{2m} = (2\Delta_0 + \sqrt{\Delta_0^2 - 3})/3$. The lower value fixes the minimum input power required for the MI process to occur, while the upper one can be tuned as a function of the physical parameter $\Delta_{eff} = \beta_2^2/(4B_4) + \Delta$. The critical value of the frequency at the upper bifurcation point $I_{2m}$ is given by $\Omega^2_{2m} = -\beta_2/2B_4$, and we note that it satisfies the averaging relation $\Omega^2_{2m} = \Omega^2_L + \Omega^2_U$. This result strongly contrasts with the usual cavity MI where the instability domain is not bounded as shown in Fig. 2(a) by the gray curves. So the two main results of this stability analysis are that (i) two instabilities at frequencies $\Omega_L$ and $\Omega_U$ occur simultaneously at the primary threshold ($I_{1m}$) and (ii) it is possible to restabilize or recover the stationary state by driving the system to the large intensity regime ($I > I_{2m}$).

In view of the above analysis, an important question arises: how do the first two frequencies $\Omega_L$ and $\Omega_U$ evolve and connect to $\Omega_L$ upon increasing the input intensity $P = |\psi|^2$? The linear stability analysis can give us some insight on this point through the evolution of the most unstable frequencies of the SS,

\[\frac{\partial \psi}{\partial \tau'} = -S - (1+i\Delta)\psi + i|\psi|^2\psi - i\beta_2 \frac{\partial^2 \psi}{\partial \tau'^2} + B_3 \frac{\partial^3 \psi}{\partial \tau'^3} + iB_4 \frac{\partial^4 \psi}{\partial \tau'^4},\]

\[\frac{\partial \psi}{\partial \tau'} = S - (1+i\Delta)\psi + i|\psi|^2\psi - i\beta_2 \frac{\partial^2 \psi}{\partial \tau'^2} + B_3 \frac{\partial^3 \psi}{\partial \tau'^3} + iB_4 \frac{\partial^4 \psi}{\partial \tau'^4},\]

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\[\frac{\partial \psi}{\partial \tau'} = S - (1+i\Delta)\psi + i|\psi|^2\psi - i\beta_2 \frac{\partial^2 \psi}{\partial \tau'^2} + B_3 \frac{\partial^3 \psi}{\partial \tau'^3} + iB_4 \frac{\partial^4 \psi}{\partial \tau'^4},\]
the two main predictions of our analytical study are numerically verified. This linear stability analysis provides an excellent insight into the frequency evolution scenario within the instability domain, except for 50 mW < I < 300 mW [Fig. 4(a)]. In this last region only a nonlinear analysis as in Refs. 14 and 15 will figure out the dynamic evolution of the system. This work is in progress.

To summarize, we presented an analytical and numerical study of a coherently driven photonic crystal fiber resonator. We showed that it is necessary to take into account dispersion up to the fourth order to capture the full temporal dynamics of the system. Namely, there exist two frequencies at the primary MI threshold, and their domain of existence is finite or bounded such that the stationary state is recovered for pumping of high enough intensity. In addition, numerical simulations carried out for realistic experimental parameters provide the evolution of these instabilities with the input field. They confirm our analytical results and constitute a step towards a future experimental demonstration.

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