Bragg localized structures in a passive cavity with transverse modulation of the refractive index and the pump

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Abstract: We consider a passive optical cavity containing a photonic crystal and a purely absorptive two-level medium. The cavity is driven by a superposition of two coherent beams forming a periodically modulated pump. Using a coupled mode reduction and direct numerical modeling of the full system we demonstrate the existence of bistability between uniformly periodic states, modulational instabilities and localized structures of light. All are found to exist within the conduction band of the photonic material. Moreover, contrary to similar previously found intra-band structures, we show that these localized structures can be truly stationary states.

OCIS codes: (050.2770) Gratings; (190.1450) Bistability; (190.4420) Nonlinear Optics, transverse effects in

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References and links

1. P. Mandel and M. Tlidi, "Transverse dynamics in cavity nonlinear optics (2000-2003)," J. Opt. B: Quantum and semiclass. Opt. **6**, R60-R75 (2004).
2. N. Akhmediev and A. Ankiewicz, Dissipative Solitons, Series: Lecture Notes in Physics, Vol. **661** (Springer, 2005).
3. N. N. Rosanov, Spatial hysteresis and optical patterns (Springer, Berlin 2002).
4. L. A. Lugiato and R. Lefever, "Spatial dissipative structures in passive optical systems," Phys. Rev. Lett. **58**, 2209-2211 (1987).
5. V. B. Taranenko, K. Staliunas, and C. O. Weiss, "Spatial soliton laser: Localized structures in a laser with a saturable absorber in a self-imaging resonator," Phys. Rev. A **56**, 1582-1591 (1997).
6. Y. F. Chen and Y. P. Lan, "Formation of repetitively nanosecond spatial solitons in a saturable absorber Q-switched laser," Phys. Rev. Lett. **93**, 013901 (2004).
7. J. Tredicce, M. Guidici, and P. Glorieux, "Comment on "Formation of repetitively nanosecond spatial solitons in a saturable absorber Q-switched laser," Phys. Rev. Lett. **94**, 249401 (2005).
8. A. Schreiber, B. Thuring, M. Kreuzer, and T. Tschudi, "Experimental investigation of solitary structures in a nonlinear optical feedback system," Opt. Commun. **136**, 415-418 (1997).
9. P. L. Ramazza, S. Ducci, S. Boccaletti, and F. T. Arecchi, "Localized versus delocalized patterns in a nonlinear optical interferometer," J. Opt. B: Quantum and semiclass. Opt. **2**, 399-405 (2000).
10. U. Bortolozzo, L. Pastur, P. L. Ramazza, M. Tlidi, and G. Kozyreff "Bistability between different localized structures in nonlinear optics," Phys. Rev. Lett. **93**, 253901 (2004).
11. M. Pesch, E. Große Westhoff, T. Ackemann, and W. Lange, "Observation of a discrete family of dissipative solitons in a nonlinear optical system," Phys. Rev. Lett. **95**, 143906 (2005).

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12. S. Barland, J. R. Tredicce, M. Brambilla, L. A. Lugiato, S. Balle, M. Giudici, T. Maggipinto, L. Spinelli, G. Tissoni, T. Knödl, M. Miller, and R. Jäger "Cavity solitons as pixels in semiconductor microcavities," Nature (London) **419**, 699-702 (2002).
13. K. Staliunas, "Midband dissipative spatial solitons," Phys. Rev. Lett. **91**, 053901 (2003).
14. K. Staliunas, "Midband solitons in nonlinear photonic crystal resonators," Phys. Rev. E **70**, 016602 (2004).
15. N.K. Efremidis and D.N. Christodoulides, "Discrete Ginzburg-Landau solitons," Phys. Rev. E **67**, 026606 (2003).
16. K. Maruno, A. Ankiewicz, and N. Akhmediev, "Exact localized and periodic solutions of the discrete complex Ginzburg-Landau equation," Opt. Commun. **221**, 199-209 (2003).
17. U. Peschel, O. Egorov, and F. Lederer, "Discrete cavity solitons," Opt. Lett. **29**, 1909-1911 (2004).
18. A. V. Yulin, D. V. Skryabin, and P. St. J. Russell, "Dissipative localized structures of light in photonic crystal films," Opt. Express **13**, 3529-3534 (2005). http://www.opticsexpress.org/abstract.cfm?URI=OPEX-13-9-3529
19. E.A. Ultanir, G.I. Stegeman, and D.N. Christodoulides, "Dissipative photonic lattice solitons," Opt. Lett. **29**, 845-847 (2004).
20. D. Gomila, R. Zambrini, and G.L. Oppo, "Photonic band-gap inhibition of modulational instabilities," Phys. Rev. Lett. **92**, 253904 (2004).
21. D. Gomila and G.L. Oppo, "Coupled-mode theory for photonic band-gap inhibition of spatial instabilities," Phys. Rev. E **72**, 016614 (2005).
22. L. Spinelli, G. Tissoni, M. Brambilla, F. Prati, and L.A. Lugiato, "Spatial solitons in semiconductor microcavities," Phys. Rev. A **58**, 2542-2559 (1998).
23. Y.S. Kivshar and G.P. Agrawal, Optical solitons: from a fiber to photonic crystals (Academic Press, 2003).

1. Introduction

Dissipative structures in optical systems have been the subject of intense research during the last years [1, 2, 3]. They result from the modulational instability [4] that triggers a spontaneous transition from homogeneous steady states (HSS) to self-organized or ordered structures. These can be either periodic or localized in space. The latter case corresponds to stationary localized pulses that are formed in the plane transverse to the beam propagation direction. They are often called cavity solitons, and have been observed experimentally in a wide class of optical systems: lasers with saturable absorber [5, 6, 7], liquid crystal light valve with optical feedback [8, 9, 10], single-mirror feedback systems using sodium vapor [11] and in semiconductor microresonators [12].

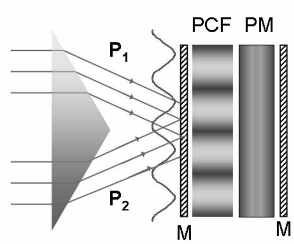

Fig. 1. Schematic setup of the nonlinear cavity filled with a passive two-level medium (PM) and a photonic crystal film (PCF). The Fabry-Perot cavity with flat Mirrors (M) is driven by two pumping beam $P_{1,2}$.

Recent research has demonstrated the existence of a new type of cavity localized structure associated with Bragg reflection in lasers with saturable absorbers [13, 14], in discrete sets of coupled lasers [15, 16] and resonators [17] and in photonic crystal films with Kerr nonlinearity under Fano resonance conditions [18]. In the same vein, solitons in periodically patterned semiconductor amplifiers, i.e. without feedback, have been theoretically predicted in [19]. On the other hand, it has also been shown that the modulation of the refractive index can inhibit modulational instability [20, 21].

In this article, we consider a nonlinear passive cavity with a photonic crystal pumped by two plane waves beams, see Fig. 1. Contrary to the purely refractive (Kerr) case in [18], we focus on the purely dissipative response of a two-level medium driven at atomic resonance. We show that the photonic crystal induces a modulational instability and creates conditions for existence of stable Bragg-like localized structures in the transverse direction. These structures have zero transverse velocity if the two coherent pumping beams are symmetric and the phase shift δ between the pump intensity profile and the refractive index modulation is an integer multiple of π. If these two conditions are not satisfied simultaneously then the localized structure drifts with constant velocity.

Let the cavity decay rate and resonant frequency be denoted by $\kappa = cT/(2L)$ and ω_c, respectively. Here L is the cavity length, T is the mirror transmissivity, and c is the velocity of light. The driving field, on the other hand, is characterized by a frequency ω_0 and $k_0 = \omega_0/c$. The photonic crystal introduces a refractive index modulation $\varepsilon = \varepsilon_0(1+\Delta\varepsilon\cos k_m x)$ along the transverse direction. With a two-level medium having an unsaturated absorption coefficient α, the electric field envelope F and the population difference N can be described, in the mean-field approximation, by the partial differential equations

$$\frac{\partial F}{\partial t} = P(x) - \kappa F - i(\omega_c-\omega_0)F - \alpha c N F + \frac{ic}{2k_0}\frac{\partial^2 F}{\partial x^2} + i\omega_0\Delta\varepsilon\cos(k_m x)F, \tag{1}$$

$$\frac{\partial N}{\partial t} = \Gamma\left[1 - N\left(1+|F|^2\right)\right]. \tag{2}$$

We write the driving field P in the form

$$P(x) = \frac{\omega_0\Delta\varepsilon}{2}\left[P_1 e^{-\frac{i}{2}(k_m(1+\phi)x-\delta)} + P_2 e^{\frac{i}{2}(k_m(1-\phi)x-\delta)}\right], \tag{3}$$

where the scale $\omega_0\Delta\varepsilon/2$ is chosen for maximum interaction with the refractive index grating. Angle ϕ characterizes the detuning of the transverse component of the incident wave vector from k_m and δ is the phase shift. For $\phi = 0$ the driving field is in the Bragg resonance with the photonic crystal. P_1, P_2 and F are expressed in units of the saturation field of the two-level resonance. Had we chosen $P(x)$ to consist of a single normally incident plane wave, the resulting localized structures would essentially be the same as those existing with $\Delta\varepsilon = 0$ but with a small $\cos(k_m x)$-like modulation superimposed on them. Therefore, their size can be estimated as [22]

$$\Delta x \sim \frac{1}{k_0}\sqrt{\frac{\omega_0}{2\kappa}} = \sqrt{\frac{2L}{k_0 T}}. \tag{4}$$

By contrast with this type of localized structures, the structures studied below have no counterpart in the homogeneous case.

2. Coupled-mode reduction and scalings

The pump profile $P(x)$ naturally leads us to seek for an electric field envelope of the form

$$F(x,t) = A_1(x,t)\, e^{-\frac{i}{2}(k_m x-\delta)} + A_2(x,t)\, e^{\frac{i}{2}(k_m x-\delta)} \tag{5}$$

Substituting the expression (5) into Eq. (2) and neglecting higher harmonics we get $N = N_0 + N_1 e^{-i(k_m x-\delta)} + N_2 e^{i(k_m x-\delta)}$, where

$$N_0 = \frac{1+|A_1|^2+|A_2|^2}{S}, \quad N_1 = \frac{-A_1 A_2^*}{S}, \quad N_2 = \frac{-A_1^* A_2}{S}, \tag{6}$$

$$S = \left(1+|A_1|^2+|A_2|^2\right)^2 - 2|A_1|^2|A_2|^2. \tag{7}$$

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Substituting back into (1), we thus obtain, with the new time and space scales $\tau = \omega_0\, \Delta\varepsilon\, t/2$, $\xi = \left(k_0^2\, \Delta\varepsilon/k_m\right) x$,

$$\frac{\partial A_1}{\partial \tau} = P_1 \exp^{iq\xi} - (\gamma + i\Omega) A_1 + iA_2 e^{-i\delta} + \frac{\partial A_1}{\partial \xi} - 2C \frac{1+|A_1|^2}{S} A_1, \tag{8}$$

$$\frac{\partial A_2}{\partial \tau} = P_2 \exp^{iq\xi} - (\gamma + i\Omega) A_2 + iA_1 e^{i\delta} - \frac{\partial A_2}{\partial \xi} - 2C \frac{1+|A_2|^2}{S} A_2, \tag{9}$$

where the normalized cavity decay rate γ, the effective detuning Ω, the cooperativity parameter C, and the normalized wave number shift q are defined by

$$\gamma = \frac{2\kappa}{\omega_0 \Delta\varepsilon}, \;\; \Omega = \frac{1}{\omega_0 \Delta\varepsilon} \left(\omega_c - \omega_0 + \frac{c\, k_m^2}{4 k_0} \right), \;\; C = \frac{\alpha\, c}{\omega_0 \Delta\varepsilon}, \;\; q = \frac{k_m^2 \phi}{2 k_0^2 \Delta\varepsilon}. \tag{10}$$

In the coupled mode equations (8) and (9), we have neglected the second order derivatives $\partial^2 A_j/\partial \xi^2$. This is valid in the case when the size Δx_B of the Bragg localized structures studied below is much larger than the characteristic scale k_m^{-1} of the refractive index modulation. From Eqs. (8) and (9) the dimensionless size of these structures can be estimated as $\Delta\xi_B \sim \gamma^{-1/2}$. Therefore, using the relations (4) and (10) we get the following estimate for Δx_B

$$\Delta x_B = \frac{k_m}{k_0^2 \Delta\varepsilon} \Delta\xi_B \sim \sqrt{\frac{\omega_0}{2\kappa \Delta\varepsilon}} \frac{k_m}{k_0^2} = \frac{k_m}{k_0 \sqrt{\Delta\varepsilon}} \Delta x. \tag{11}$$

Using these relations the condition $k_m^{-1} \ll \Delta x_B$ can be rewritten in the form

$$\Delta\varepsilon \gamma \ll k_m^2/k_0^2 \ll 1, \tag{12}$$

where the second inequality corresponds to the paraxial approximation under which Eqs. (1) and (2) are valid. It follows from (11) and (12) that $\Delta x_B \gg \sqrt{\gamma} \Delta x$ which means that when γ is not very small the size of the Bragg localized structures is usually larger than Δx. Note, that $k_m^2/k_0^2 \ll 1$ simply corresponds to the paraxial approximation. It will be shown below, that in this case the numerical simulations of Eqs. (8) and (9) agree very well with those performed on the full model (1) and (2).

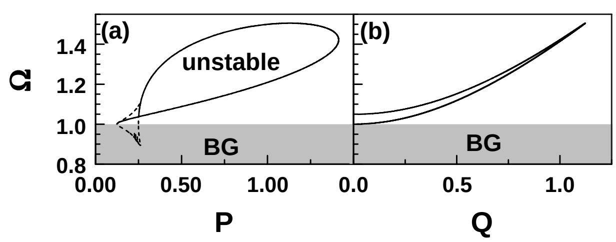

Fig. 2. Instability boundaries as a function of the effective detuning parameter Ω. (a) Pump $P_1 = P_2 = P$ versus Ω. The solid curve is the modulational instability boundary. The bistability region is delimited by the three dashed curves. Grey region indicates photonic band gap (BG). (b) Critical wavenumber at the modulational instability versus Ω. Parameters are $\gamma = 0.01$, $C = 0.4$, $\delta = 0$, and $q = 0$.

#9444 - $15.00 USD Received 10 November 2005; revised 19 December 2005; accepted 19 December 2005

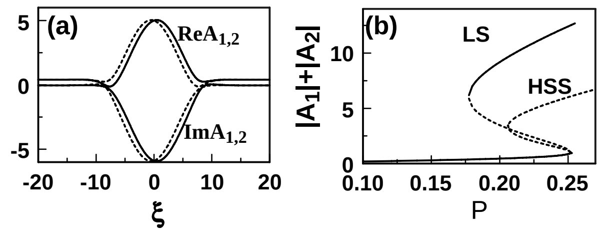

Fig. 3. Stationary localized structures. $\Omega = 1.05$. Other parameters are the same as in Fig. 2. (a) Real and imaginary parts of the field amplitudes $A_{1,2}$ for $P_1 = P_2 = 0.225$. Solid (broken) lines correspond to A_1 (A_2). (b) Bifurcation diagram. LS: localized structures, HSS: homogeneous steady state. Broken lines correspond to unstable solutions.

3. Numerical results

Let us first examine the symmetric pumping situation, i.e., $P_{1,2} \equiv P$, with a zero phase shift between the pump intensity and the refractive index, i.e., $\delta = 0$ and at the exact Bragg resonance $q = \phi = 0$. The HSS's can be either monostable or bistable. In the $(P - \Omega)$ parameter space, the region of bistability is delimited by the approximately triangular domain in dotted line [see Fig. 2]. In addition, the linear stability analysis of the HSS solutions of Eqs. (8) and (9) with respect to a finite wavenumber perturbations shows that the system exhibits a modulational instability. The results of this analysis is summarized in Fig. 2 where we plot the critical pump amplitude as a function of the effective detuning. The critical wavenumbers Q corresponding to the maximum gain are plotted in Fig. 2(b). From Fig. 2, we see that the modulational instability takes place outside the photonic band gap. The band gap, indicated in this figure by the grey area, is calculated from Eqs. (8) and (9) without dissipative terms as the region of non-existence of solutions of the form $\exp(\pm iQ\xi)$ for real Q and Ω.

When a modulational instability appears subcritically, localized structures are formed in the hysteresis loop involving the HSS and periodic patterns [1, 2, 3]. In what follows, we focus on the localized structures whose existence is ensured by the Bragg scattering at the periodic refractive index modulation. These structures can not be generally traced back to the limit with transversely homogeneous refractive index, where photonic band gap disappears. We find the transverse profiles of the localized structures by solving numerically the nonlinear coupled mode equations. Figure 3(a) represents typical profiles of the amplitudes $A_{1,2}$ corresponding to bright stationary localized solutions which have been calculated for the case of symmetric pumping, $\delta = 0$, $\phi = 0$, and $P_1 = P_2$, when the coupled mode equations are invariant under the reflection transformation $\xi \to -\xi$, $A_1 \leftrightarrow A_2$. The branch of the localized structures obtained by varying the pump strength parameter $P = P_1 = P_2$ is shown in Fig. 3(b) together with the branch of spatially homogeneous solutions of the coupled mode equations.

The localized structures found within the framework of the coupled mode approach exist also in the full model, as we demonstrated by direct numerical modelling of Eqs. (1) and (2), see Fig. 4. The localized structure shown in this figure is formed by the two waves counter-propagating in the transverse direction and therefore it is characterized by oscillations of the electric field intensity with the spatial frequency k equal to that of the refractive index modulation and a phase shift π between the two neighboring intensity maxima, which fully complies with predictions of the coupled mode approach. In that respect our structures are similar to the so-called “staggered” solitary waves in discrete nonlinear systems [23] and different from the “unstaggered” solitons reported in [19]. The phase of the intensity oscillations of the localized structure shown in Fig. 4 coincides with that of the refractive index profile. From this figure we see that the HSS of the coupled mode equations which serves as a background for the localized

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solution in Fig. 3(a) corresponds to a spatially periodic solution of Eqs. (1) and (2). We now

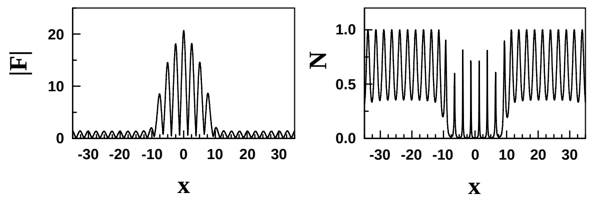

Fig. 4. Stationary localized structure obtained by direct numerical simulation of Eqs. (1,2). Parameters are $\gamma = 0.05$, $C = 2.0$, $\delta = 0$, $\phi = 0$, $P_1 = P_2 = 1.2$, $k_m = 2.5\sqrt{\Delta\varepsilon}\, k_0$, and $\omega_c - \omega_0 = -0.3125\, \omega_0\, \Delta\varepsilon$.

turn to the case of asymmetric pumping. The localized structures then move with a constant velocity $v = d\xi/dt$ and Fig. 5 illustrates the dependence of this velocity on the phase δ, the imbalance between the amplitudes of the pump beams $\delta P = (P_2 - P_1)/(P_1 + P_2)$, and the angle of incidence ϕ. From Fig. 5(a) we see that a phase shift δ between the pump intensity and the refractive index profiles results in a very small v, which is approximately four orders of magnitude smaller than δ itself. This suggests a predominant influence from the cavity decay rate $\gamma \ll 1$. However, v increases rapidly with δ and localized structures disappear as δ tends to $\pi/2$. On the other hand, v depends linearly on the pump imbalance δP and on ϕ, although much more sensitively so on the latter. This behavior is shown in Figs. 5(a) and (c).

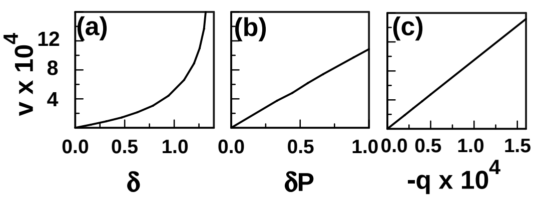

Fig. 5. Transverse velocity $v = d\xi/dt$ of a localized structure as a function of (a) the phase shift δ, (b) pump imbalance $\delta P = (P_2 - P_1)/(P_2 + P_1)$, where $P_2 + P_1 = 0.45$, (c) incidence angle ϕ. Other parameters are the same as in Fig. 3.

Finally, we note that Eqs. (8) and (9) are invariant with respect to the transformation $A_1 \to -A_1^*$, $A_2 \to A_2^*$, $\Omega \to -\Omega$, $\delta \to \delta + \pi$ applied together with complex conjugation. In particular, this means that the stationary localized structures found at $\delta = 0$ can be transformed into the structures with $\delta = \pm\pi$ for the same absolute value but opposite sign of the detuning parameter Ω. Hence the existence of a similar diagram as in Fig. 2 for negative Ω. Unlike the structures shown in Fig. 4, the structures with $\delta = \pm\pi$ are characterized by intensity oscillations in anti-phase with those of the refractive index profile.

In conclusion, using the nonlinear coupled mode approach and numerical modeling of the full system, we have demonstrated the existence of bistability, modulational instability and stable Bragg localized structures in the transverse section of an externally pumped passive cavity with photonic crystal. The localized structures move if the pumping is asymmetric or if the phase detuning δ is different from 0 or π. The coupled mode reduction similar to the one applied above can be used to study other driven nonlinear systems with a photonic band gap and one may expect that localized structures constitute a generic and general feature of such systems.

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