Howe Work 01

due 2018-12-02
Please send pdf of your homework to `si@wias-berlin.de`
please include your student ID in the title of your email.

Problem 1 (4 credit), Simplices and Simplicial Complex

Let σ be a k-simplex. The $\emptyset$ ((-1)-simplex) and σ are called *trivial faces* of σ. All other faces of σ are called *proper* faces of σ.

The *boundary* of σ, denoted as bdσ, is the union of all proper faces of σ. The *interior* of σ is $\sigma - \mathrm{bd}\sigma$. For an example, Figure 1 shows the boundary and interior of a triangle (a 2-simplex).

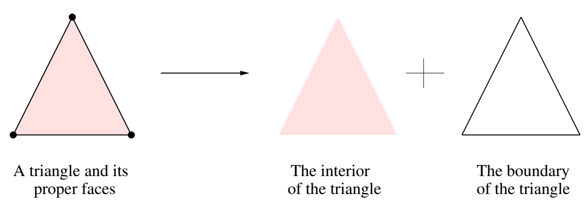

Figure 1: The interior and boundary of a triangle.

Let $\mathcal{K}$ be a non-empty two-dimensional simplicial complex. The underlying space of $\mathcal{K}$, denoted as $|\mathcal{K}|$, is the union of all simplices of $\mathcal{K}$, see Figure 2.

A *partition* of a $|\mathcal{K}|$ is a cell complex $\mathcal{C}$ which is a collection of cells (including the $\emptyset$) such that:

(i) the intersection of every two cells of $\mathcal{C}$ is also a cell of $\mathcal{C}$, and

(ii) the union of cells equal to $|\mathcal{K}|$.

Claim: The interior of simplices of $\mathcal{K}$ partition the underlying space of $\mathcal{K}$.

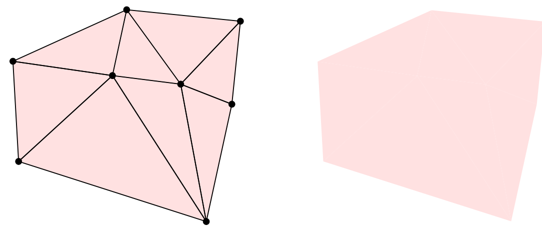

Figure 2: A 2d simplicial complex (left) and its underlying space (right).

Question: Is the above claim true or false? Please give your reasons to support your answer.

Problem 2 (4 credit), Euler's Formula

Given a triangulation $\mathcal{T}$ of a point set S in $\mathbb{R}^2$. Let the number of vertices, edges, and triangles in $\mathcal{T}$ are v, e, and f, respectively. Since we do not count the exterior of $|\mathcal{T}|$ as a face of $\mathcal{T}$, By Euler's formula, we have the following equality

$$v - e + f = 1.$$

Every triangle of $\mathcal{T}$ has three edges, every interior edge of $\mathcal{T}$ belongs two triangles, hence we have

$$3f = 2e - h,$$

where h is the number of edges of $\mathcal{T}$ which lies on the convex hull of S. Since $h > 0$, we have

$$3f < 2e.$$

Check whether the following inequalities hold for $\mathcal{T}$ or not. (Hint: If it is false, show a counter example is enough).

(1) $f < 2v - 2$;

(2) $e < 3v - 3$;

(3) $2e \geq 3v$;

(4) $v \leq 2f - 4$;

(5) $e \leq 3f - 6$;

Problem 3 (4 credit), Planar Graphs

A graph is *complete* if it contains maximum number of edges, which is $\binom{|V|}{2}$. For example, Figure 3 shows three complete graphs of 4 vertices. Show that the complete graph of 5 vertices is not planar. (**Hint**: it is an application of the Euler's formula.)

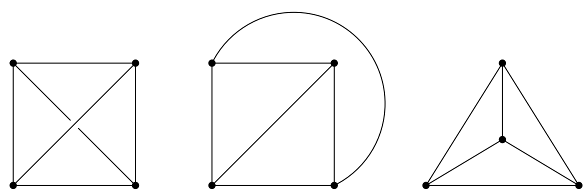

Figure 3: Three complete graph of 4 vertices.