Weierstrass Institute for
Applied Analysis and Stochastics

Inverse problems for the scatterometric measuring of grating structures

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Outline

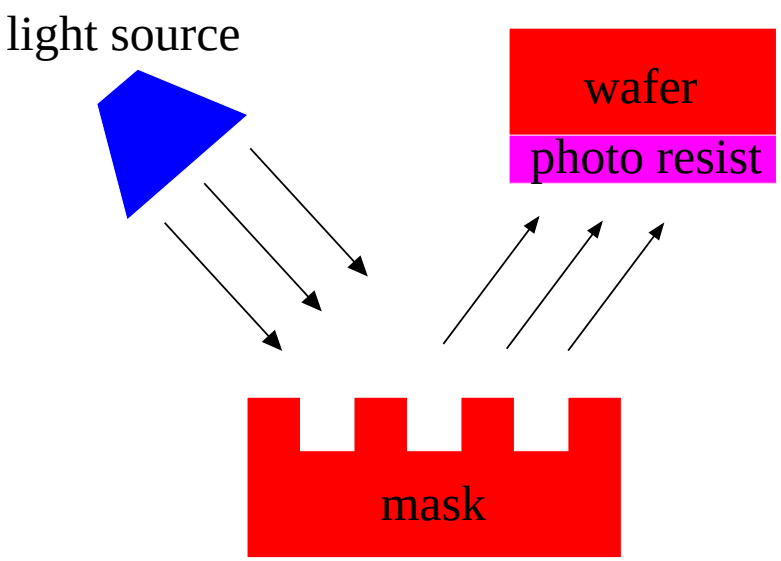
Outline

Lithography

chip production like old-fashioned photography:

photoresist layer illuminated by light scattered from mask,

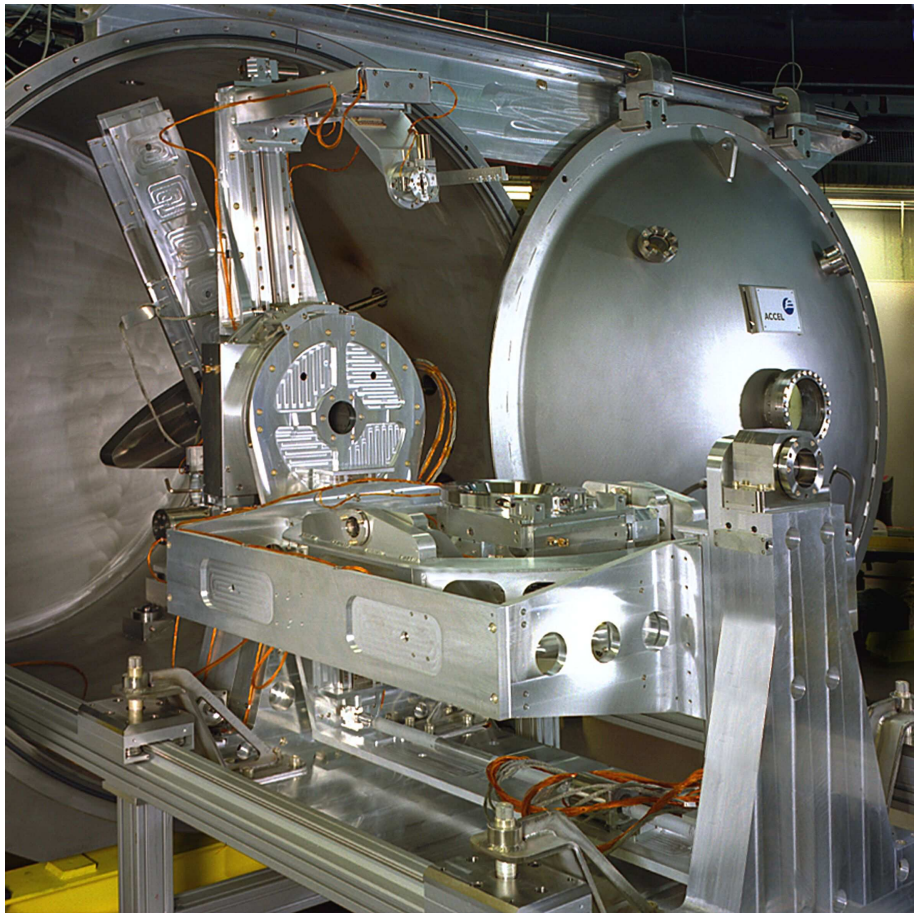
development: baking and etching procedures $\longrightarrow$ chip

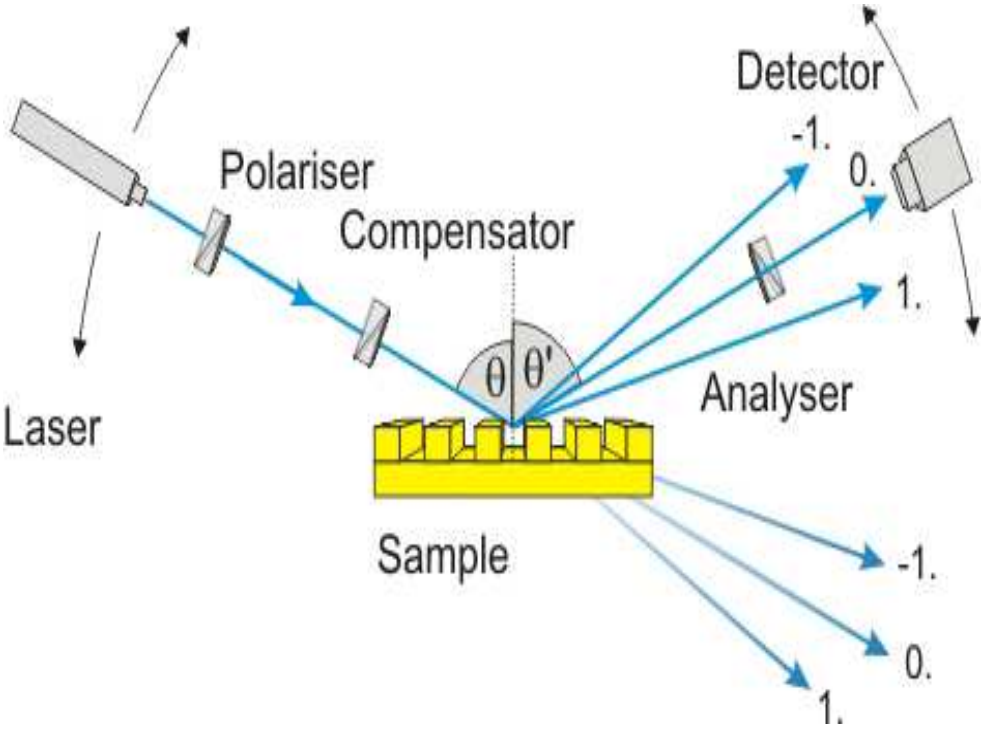


standard test configurations:

- periodic line-space structure (lines formed by bridges with trapezoidal cross section)
- biperiodic array of trapezoidal blocks resp. holes

Spectroscopic Reflectrometer, Operating Range ≈ 13 nm, BESSY II, PTB





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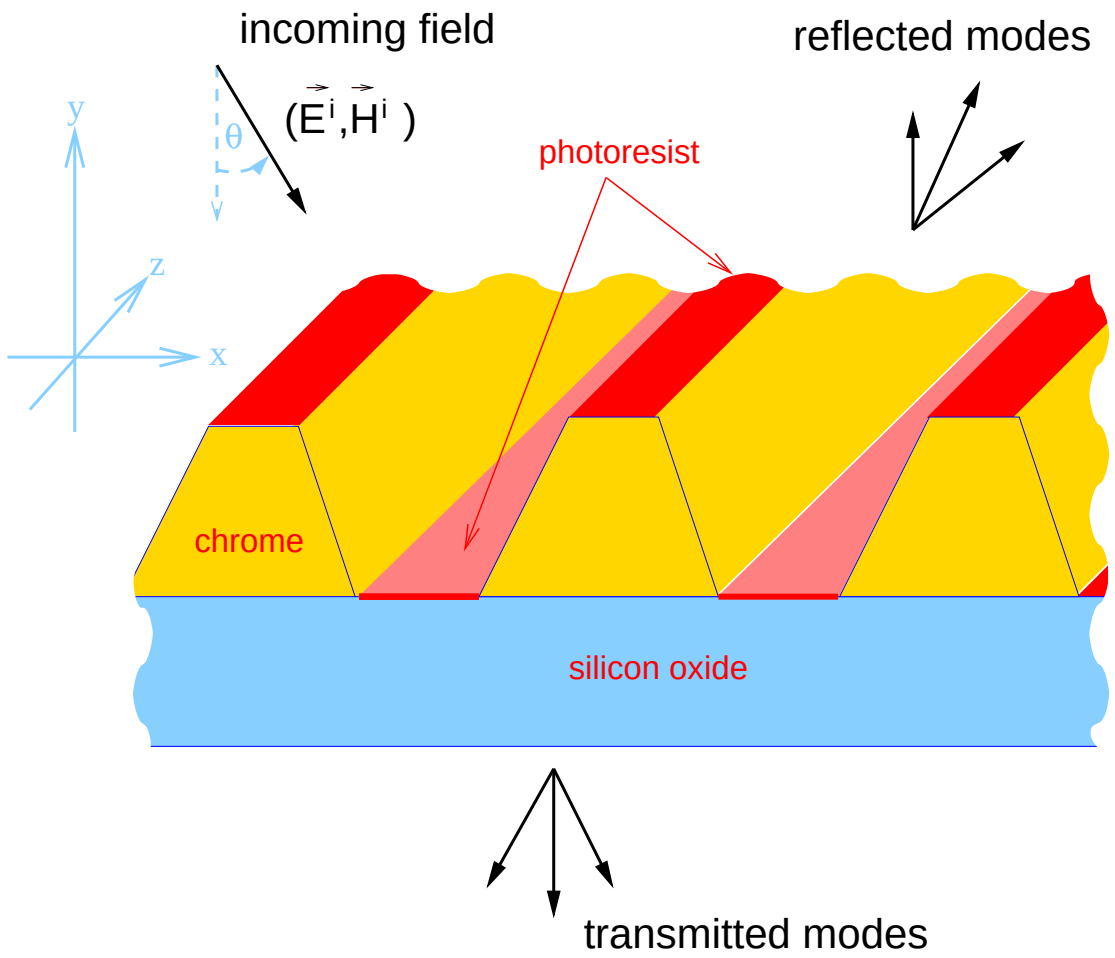
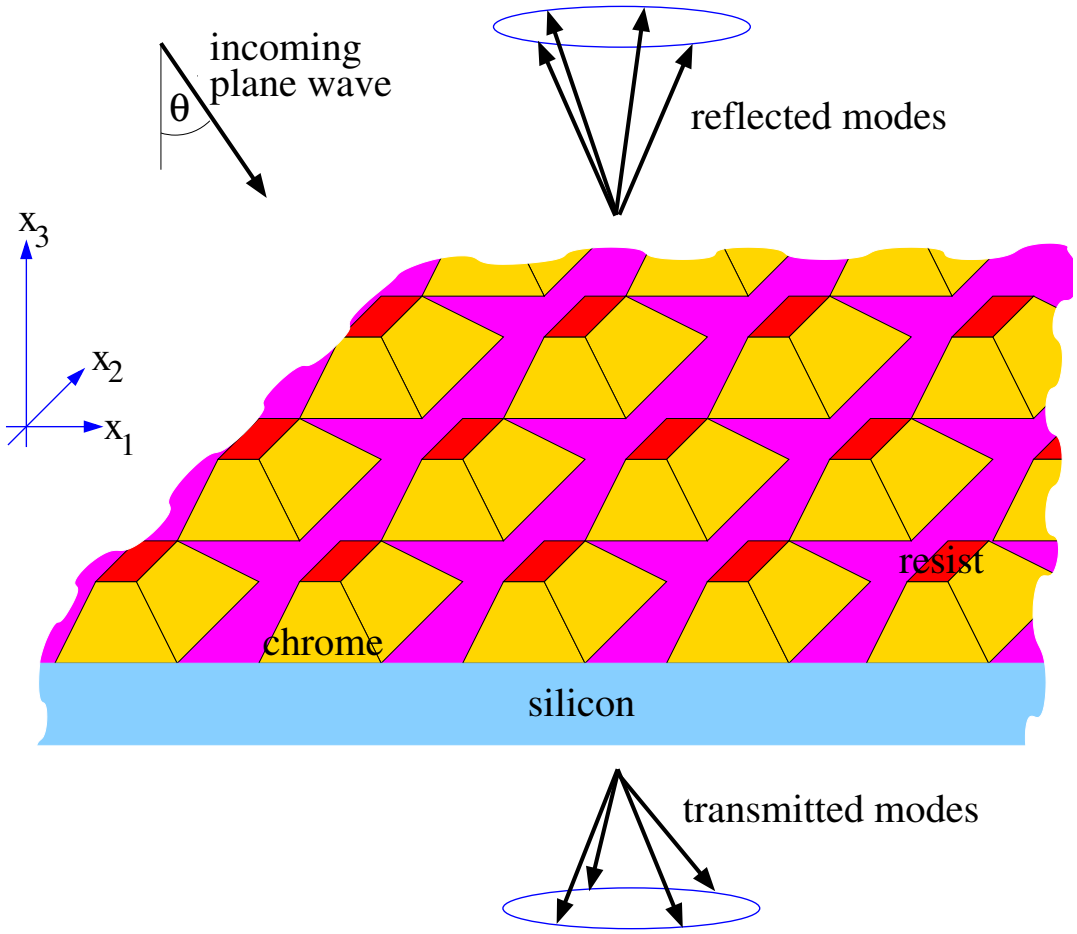
Periodic Grating



periodic gratings

details of surface geometry in size of wavelength

Biperiodic (Doubly Periodic) Grating



biperiodic surface structures (period d_j in x_j-direction, $j = 1, 2$)

details of surface geometry in size of wavelength

Partial Differential Equation

Plane-wave illumination. 3D time-harmonic Maxwell's equations reduce to:

– Curl-Curl equation for three-dimensional amplitude factor of time harmonic electric field (i.e. $E(x_1, x_2, x_3, t) = E(x_1, x_2, x_3)e^{-i\omega t}$)

$$\nabla \times \nabla \times E(x_1, x_2, x_3) - k^2 E(x_1, x_2, x_3) = 0$$

Here: wave number $k := \omega\sqrt{\mu\varepsilon}$

– scalar 2D Helmholtz equation if geometry is constant in x_3 direction and if direction of incoming plane wave is in $x_1 - x_2$ plane

$$\Delta v(x_1, x_2) + k^2 v(x_1, x_2) = 0, \; v = E_3, H_3$$

– two coupled scalar 2D Helmholtz equations if geometry is constant in x_3 direction but direction of incoming plane wave is not in $x_1 - x_2$ plane (conical diffraction)

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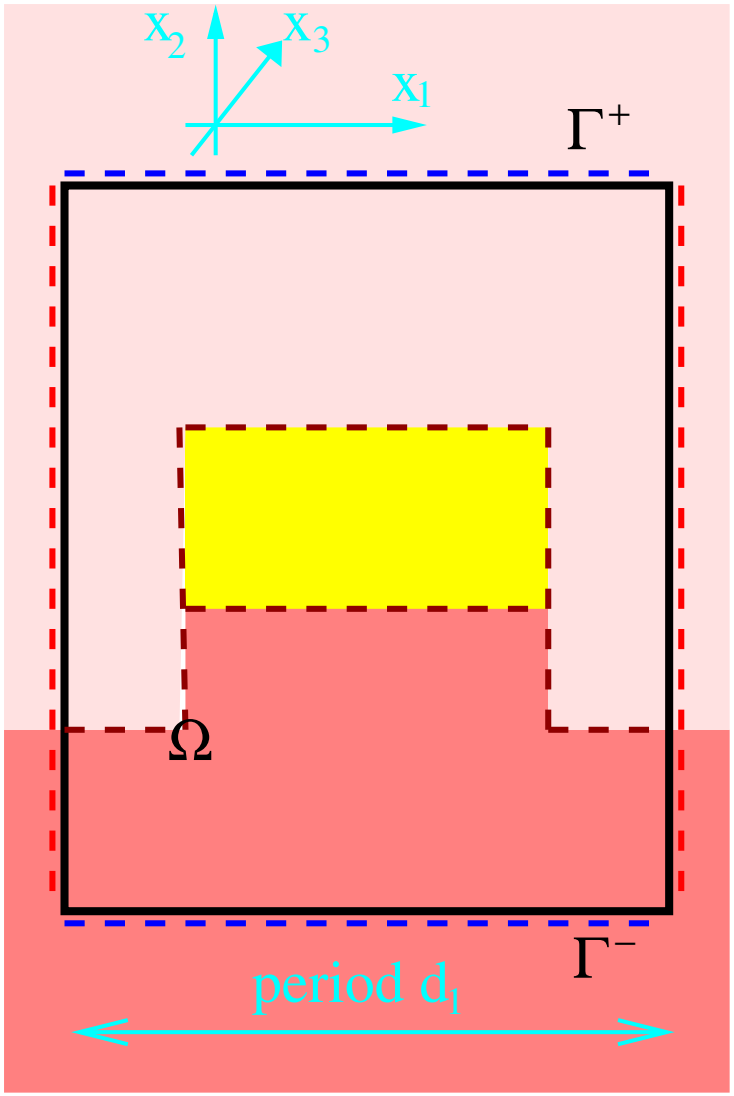

Domain of Computation:
Rectangular box over one periodic cell Ω

Non–local boundary condition:
Including radiation condition
Mortaring with Fourier mode sol.
(Coupling with boundary elements or Absorbing bound.cond. PML)

Transmission conditions:
Continuous tangential traces
Continuous tangential traces of curl

Quasiperiodic boundary condition:

$$v(x_1+d_1,x_2,x_3)=v(x_1,x_2,x_3)\exp(i\alpha d_1)$$
$$v(x_1,x_2+d_2,x_3)=v(x_1,x_2,x_3)\exp(i\beta d_2)$$

Rayleigh Series Expansion Below/Above Computational Domain

incident plane wave (inspecting w.): $E^{\mathrm{inc}}(x_1, x_2, x_3)\ \exp(-\mathbf{i}\omega t)$,

$$E^{\mathrm{inc}}(x_1, x_2, x_3) := A^{\mathrm{inc}} \exp\left(\mathbf{i}\,\mathbf{k}^{\mathrm{inc}} \cdot (x_1, x_2, x_3)^\top\right)$$

$$\mathbf{k}^{\mathrm{inc}} = k^+(\sin\theta\cos\phi, \sin\theta\sin\phi, -\cos\theta)^\top$$

$$E(x_1, x_2, x_3) = \sum_{j,l=-\infty}^{\infty} A^-_{j,l} \exp\left(\mathbf{i}\,\mathbf{k}^-_{j,l} \cdot (x_1, x_2, x_3)^\top\right), \quad x_3 < x_{min},$$

$$\alpha_j := k^+ \sin\theta\cos\phi + \frac{2\pi}{d_1} j,$$

$$\mathbf{k}^-_{j,l} := (\alpha_j, \beta_l, \gamma^-_{j,l})^\top, \ \beta_l := k^+ \sin\theta\sin\phi + \frac{2\pi}{d_2} l,$$

$$\gamma^-_{j,l} := -\sqrt{[k^-]^2 - [\alpha_j]^2 - [\beta_l]^2}$$

Rayleigh series: Rayleigh coefficients $A^-_{j,l} \in \mathbb{C}^3$, $A^-_{j,l} \perp \mathbf{k}^-_{j,l}$, with $k = k^-$ for $x_3 < x_{min}$ (similarly, coefficients $A^+_{j,l}$ for expansion with $x_3 > x_{max}$, where $k = k^+$)

Efficiency $\mathcal{E}^\pm_{j,l}$ of $(j,l)-$th mode: ratio of energy radiated into direction of mode

$$\mathcal{E}^\pm_{j,l} := (\gamma^\pm_{j,l}/\gamma^+_{0,0})|A^\pm_{j,l}|^2$$

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Variational Equation 1

$$\int_\Omega \nabla\times E\cdot\overline{\nabla\times F} - \int_\Omega k^2 E\cdot\overline{F} - \int_{\Gamma^+} \nabla\times E^+\cdot\nu\times\overline{F} + \int_{\Gamma^+} \nu\times(E-E^+)\cdot\nabla\times\overline{F}^+$$
$$- \int_{\Gamma^-} \nabla\times E^-\cdot\nu\times\overline{F} + \int_{\Gamma^-} \nu\times(E-E^-)\cdot\nabla\times\overline{F}^- + \text{finite rank terms}$$
$$= \int_{\Gamma^+} \nabla\times E^{\text{inc}}\cdot\nu\times\overline{F} + \int_{\Gamma^-} \nu\times E^{\text{inc}}\cdot\nabla\times\overline{F}^+ + \text{finite rank term}, \quad \forall F,\ \forall F^\pm$$

$$\mathbf{a}\Big((E,E^+,E^-),(F,F^+,F^-)\Big) = \mathcal{F}\Big((F,F^+,F^-)\Big), \quad \forall F\in H(\text{curl},\Omega),$$
$$\forall F^\pm \text{Rayleigh expansions}$$

with solution (E,E^+,E^-): $E\in H(\text{curl},\Omega)$ and $E^\pm$ Rayleigh expansions in $H_{\text{loc}}(\text{curl})$
Ω domain of computation, $\Gamma^\pm$ upper/lower boundary face, ν normal at $\Gamma^\pm$
$H(\text{curl},\Omega) := \{F\in[L^2(\Omega)]^3 :\ F \text{ quasi-per.},\ \text{piecewise } \nabla\times F\in[L^2(\Omega)]^3,$
$\nu\times F \text{ continuous over interfaces}\}$

Theorem (Hu, R. '12)

For profile gratings with perfectly conducting substrate: operator corresponding to sesqui-linear form $\mathbf{a}$ *takes the form* $\begin{pmatrix} A_0 & 0 \\ 0 & -A_1 \end{pmatrix} + T$, *where* A_0, A_1 *strongly elliptic and* T *compact.*

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finite rank term to cope with the case of plane-wave modes in Rayleigh expansion propagating in the direction of the $x_1 - x_2$ plane,
(i.e., for $k^{\pm} := \omega\sqrt{\mu\varepsilon^{\pm}}$, there exist indices j, l with $\gamma^{\pm}_{j,l} = 0$)

$$\text{finite rank term} := \eta \sum_{(j,l)\in\Upsilon^{\pm}} \int_{\Gamma^{\pm}} \nu \times (E - E^{\pm}) \cdot (\nu \times \overline{U}_{j,l}) \; \overline{\int_{\Gamma^{\pm}} \nu \times (E - E^{\pm}) \cdot (\nu \times \overline{U}_{j,l})}$$

where: $\Upsilon^{\pm}$ is any fixed finite index set containing all (j, l) with $\gamma^{\pm}_{j,l} = 0$

η is a fixed constant

$U_{j,l}$ is special mode in Rayleigh expansion defined by

$$U_{j,l}(x_1, x_2, x_3) := \frac{1}{\sqrt{\alpha_j^2 + \beta_l^2}} (-\beta_l, \alpha_j, 0)^{\top} \; \exp\Big(\mathbf{i}\,\mathbf{k}^{\pm}_{j,l} \cdot (x_1, x_2, x_3)^{\top}\Big)$$

Variational Equation 3

$$\int_\Omega \nabla\times E\cdot\overline{\nabla\times F} - \int_\Omega k^2 E\cdot\overline{F} - \int_{\Gamma^+}\nabla\times E^+\cdot\nu\times\overline{F} + \int_{\Gamma^+}\nu\times(E-E^+)\cdot\nabla\times\overline{F}^+$$
$$+\eta\int_{\Gamma^+}\nu\times(E-E^+)\cdot\nu\times(\overline{F}-\overline{F}^+)$$
$$-\int_{\Gamma^-}\nabla\times E^-\cdot\nu\times\overline{F} + \int_{\Gamma^-}\nu\times(E-E^-)\cdot\nabla\times\overline{F}^-$$
$$+\eta\int_{\Gamma^-}\nu\times(E-E^-)\cdot\nu\times(\overline{F}-\overline{F}^-)$$
$$= \int_{\Gamma^+}\nabla\times E^{\mathrm{inc}}\cdot\nu\times\overline{F} + \int_{\Gamma^-}\nu\times E^{\mathrm{inc}}\cdot\nabla\times\overline{F}^+ + \eta\int_{\Gamma^+}\nu\times E^{\mathrm{inc}}\cdot\nu\times(\overline{F}-\overline{F}^+),\ \forall F, F^\pm$$

$$\mathbf{a}\Big((E,E^+,E^-),(F,F^+,F^-)\Big) = \mathcal{F}\Big((F,F^+,F^-)\Big), \quad \forall (F,F^+,F^-)\in H_b(\mathrm{curl})$$

theorem now true for $H_b(\mathrm{curl})$: $E\in H(\mathrm{curl},\Omega)$, $E^\pm$ Rayleigh expansions in $H_{\mathrm{loc}}(\mathrm{curl})$, and $\nu\times(E-E^\pm)|_{\Gamma^\pm}\in L^2_t(\Gamma^\pm)^3$

modification of mortar method by Huber/Schoeberl/Sinwel/Zaglmayr (analysis of FEM open)

Finite Element Methods and Alternatives

- **2D theory:** H. Urbach, G. Bao, D.C. Dobson, J.A. Cox, J. Elschner, R. Hinder, and G. Schmidt
- generalized finite elements (trial space of piecewise Helmholtz solutions) for faster convergence and for highly oscillating solutions

- **3D theory:** G. Schmidt for the approximation of the divergence free magnetic field
- Edge elements of Nédélec (cf. P. Monk, S. Burger, J. Pomplun, A. Schädle, L. Zschiedrich, F. Schmidt, M. Huber, J. Schöberl, A. Sinwel, S. Zagelmayr)

- **alternatives:** rigorous coupled wave analysis (RCWA) and integral equation methods (BEM, e.g., B. Bugert and G. Schmidt)

Outline

Inverse Problem

- Given: the diffraction properties of grating (e.g. efficiencies $\mathcal{E}^{\pm}_{j,l}$)
- Seek: the grating, i.e., the geometry of the domains filled with different materials and the refractive indices of these materials

- severely ill-posed problem: small errors in data lead to large errors for the solution
- contributions to 2D case by F. Hettlich, A. Kirsch, G. Bruckner, J. Elschner, G. Schmidt, D.C. Dobson, G. Bao, A. Friedman, M. Yamamoto, J. Cheng, K. Ito, F. Reitich T. Arens, and G. Hu
- contributions to 3D case by H. Ammari, G. Bao, Z. Zhou, H. Zhang, and J. Zou,

perfectly conducting profile grating (PCPG):

two materials, perfectly conducting substrate, dielectric cover material, interface graph $\{(x', x_3) \in \mathbb{R}^3 : \; x_3 = f(x')\}$ of a function f

- PCPG with small f with finite number of Fourier modes, Rayleigh coefficients measured for propagating plane-wave modes: uniqueness by Ammari
- general grating, Rayleigh coefficients measured for propagating reflected plane-wave modes (even for all angles of incidence): no reconstruction, 2D counter example by Kirsch
- PCPG with small and smooth function, electric field measured over plane above interface: certain uniqueness by Bao/Zhou
- PCPG with polyhedral function, single incoming plane wave, electric field measured over plane above interface: interface not identifiable if in one of 7 classes of Bao/Zhang/Zou

Theorem (Bao, Zhang, Zou '11)

Two polyhedral PCPGs. One interface has two faces forming an angle different from $\pi/4$, $\pi/3$, $\pi/2$, and $2\pi/3$. For single incoming wave, corresponding reflected fields coincide over plane above interfaces. Then: PCPGs coincide.

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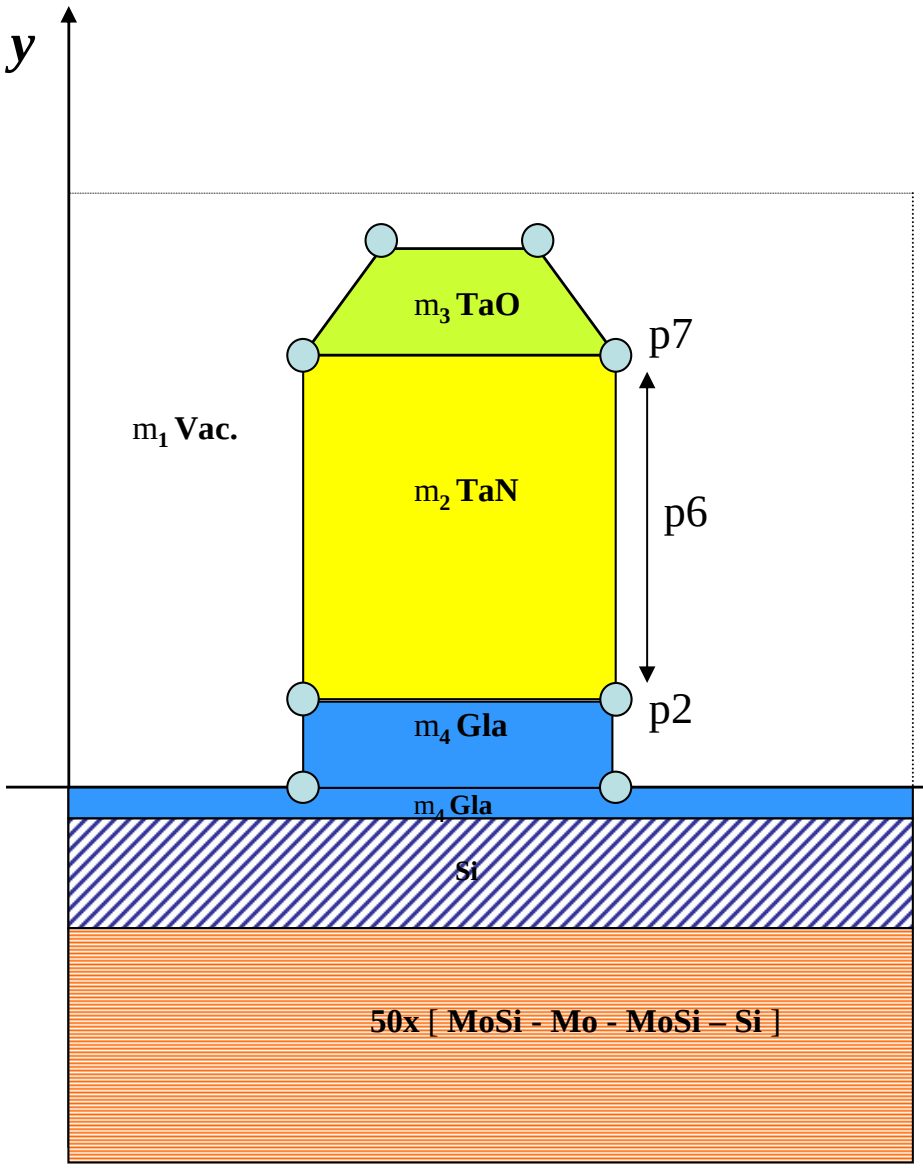
Class of Gratings

avoid full inverse problems by using more a priori information on the grating:

- seek grating in class defined by a few parameters $h = (h_j)_{j=1}^J$ (parameter identification)

Class of Gratings



Reasonable initial values for optimizations

bottomCD: ~	**140**	nm	
hTaO:	**12**	nm	SWA: **82,6°**
hTaN:	**54.9**	nm	SWA: **90°**
hGla(buff):	**8**	nm	SWA: **90°**

hGla(capping): 1.246 nm

hSi(capping): 12.536 nm

50x h [0.5–2.259– 1.263– 3.077 nm]

mask for EUV lithography: wavelength of inspecting light 13 nm

Equivalent Optimization Problem

measured data, efficiencies or phase shifts: $\mathcal{E}^{meas} = (\mathcal{E}_m^{meas})_{m\in\mathcal{M}}$
computed data corresponding to parameters h: $\mathcal{E}(h) = (\mathcal{E}_m(h))_{m\in\mathcal{M}}$
minimize objective functional

$$\boxed{\mathcal{F}\Big(\mathcal{E}_m(h)\Big) \longrightarrow \inf}$$

$$\boxed{\mathcal{F}(E) := \mathcal{F}\Big(\mathcal{E}(h)\Big) := \sum_{m\in\mathcal{M}} \omega_m \, |\mathcal{E}_m(h) - \mathcal{E}_m^{meas}|^2, \; \omega_m := \frac{1}{\sigma(\mathcal{E}_m^{meas})^2}}$$

standard deviation of measurement: $\sigma(\mathcal{E}_m^{meas})$
box constraints $h_j^{min} \leq h_j \leq h_j^{max}$

use: conjugate gradients method, interior point method,
Levenberg-Marquardt algorithm or SQP

Convergence of Gauß-Newton iteration (SQP)

- similar to statistics: least squares estimator
- proposed by Drège, Al-Assad, Byrne for grating reconstruction without box constraints
- convergence properties:
  - limit satisfies KKT condition necessary for minimum
  - if measurement data is exact: $\|h^{k+1} - h^{sol}\| \leq c\|h^k - h^{sol}\|^2$
  - small error in measurement data: $\|h^k - h^{sol}\| \leq cq^k,\ 0 < q < 1$
  - large error ε in measurement data: $\|h^k - h^{sol}\| \leq c\varepsilon$ if $k \geq k_0$
- advantages in comparison to gradient based method for optimization of objective functional:
  - fast: small number of iterations, no line search
  - no trouble with computation of small gradients at iterates close to optimum
  - more robust w.r.t. nonoptimal scaling

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Outline

Material Derivative

- vector field on Ω vanishing on boundary $\partial\Omega$: $\chi(\vec{x})$
- generates family of automorphisms of domain Ω: $\Phi_t(\vec{x}) := \vec{x} + t\chi(\vec{x}), 0 \leq t < 1$
- generates family of solutions over transformed domain: E_t solution of time-harmonic Maxwell equation with coefficient k replaced by $k_t := k \circ \Phi_t^{-1}$
- material derivative of $\mathcal{F}(E)$ w.r.t. χ:

$$\boxed{\partial_t[\mathcal{F}(E_t)]|_{t=0} := \lim_{t\to 0} \frac{\mathcal{F}(E_t) - \mathcal{F}(E_0)}{t}}$$

If grating geometry is polyhedral,
if the only non-vanishing component of χ is χ_{x_i},
if χ_{x_i} is piecewise linear and linear over all faces of the grating geometry,
if $\chi_{x_i}(P) = 1$ for the vertex P, and
if $\chi_{x_i}(Q) = 0$ for all other vertices Q,
then
$\partial_t[\mathcal{F}(E_t)]|_{t=0}$ is the shape derivative of $\mathcal{F}(E)$ w.r.t. the x_i-component of the vertex P

Derivation of Formula

- classical techniques of shape derivatives (cf. e.g. Sokolowski/Zolesio)
- unfortunately, energy space $H(\mathrm{curl}, \Omega)$ not invariant w.r.t. transform Φ_t
- fortunately, magnetic permeability is constant and magnetic field is in $H^1(\Omega)$ over each subdomain filled with one material
- derive shape derivative for the magnetic solution by classical method and, in the final formula, replace magnetic field by electric field

Shape-Derivative Formula

$$\mathbf{a}_1(E,F) := \int_\Omega k^2 E\cdot\overline{F}\ \nabla\cdot\chi - \int_\Omega \nabla\times E\cdot\overline{\nabla\times F}\ \nabla\cdot\chi$$
$$+\int_\Omega k^2 E\cdot\overline{\Big(\nabla_{x_1}\cdot F,\nabla_{x_2}\cdot F,\nabla_{x_3}\cdot F\Big)^\top} + \int_\Omega k^2\Big(\nabla\chi_{x_1}\cdot F,\nabla\chi_{x_2}\cdot F,\nabla\chi_{x_3}\cdot F\Big)^\top\cdot\overline{F}$$
$$+\int_\Omega [\nabla\times E]\cdot\overline{\Big[[\nabla\times F]_{x_1}\nabla\chi_{x_1}+[\nabla\times F]_{x_2}\nabla\chi_{x_2}+[\nabla\times F]_{x_3}\nabla\chi_{x_3}\Big]}$$
$$+\int_\Omega \Big[[\nabla\times E]_{x_1}\nabla\chi_{x_1}+[\nabla\times E]_{x_2}\nabla\chi_{x_2}+[\nabla\times E]_{x_3}\nabla\chi_{x_3}\Big]\cdot\overline{[\nabla\times F]}$$

differentiated sesqui-linear form

$$\mathbf{a}\Big((E,E^+,E^-),(F_{\mathrm{ad}},F_{\mathrm{ad}}^+,F_{\mathrm{ad}}^-)\Big) = \int_{\Gamma^+}[\nu\times E]\cdot\overline{\psi},\quad \forall E,E^+,E^-$$
$$\psi := \sum_{m\in\mathcal{M}} c_m\omega_m\sqrt{\mathcal{E}_m}\Big[\mathcal{E}_m-\mathcal{E}_m^{meas}\Big]$$

adjoint solution $(F_{\mathrm{ad}},F_{\mathrm{ad}}^+,F_{\mathrm{ad}}^-)$ (here c_m certain constants)

final formula: $\partial_t[\mathcal{F}(E_t)]|_{t=0} = -\Re\mathrm{e}\,\mathbf{a}_1(E_0,F_{\mathrm{ad}})$

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$$\mathbf{a}_1(E,F) := \int_\Omega k^2 E\cdot\overline{F}\ \nabla\cdot\chi - \int_\Omega \nabla\times E\cdot\overline{\nabla\times F}\ \nabla\cdot\chi$$
$$+\int_\Omega k^2 E\cdot\overline{\Big(\nabla_{x_1}\cdot F,\nabla_{x_2}\cdot F,\nabla_{x_3}\cdot F\Big)^\top} + \int_\Omega k^2\Big(\nabla\chi_{x_1}\cdot F,\nabla\chi_{x_2}\cdot F,\nabla\chi_{x_3}\cdot F\Big)^\top\cdot\overline{F}$$
$$+\int_\Omega [\nabla\times E]\cdot\overline{\Big[[\nabla\times F]_{x_1}\nabla\chi_{x_1}+[\nabla\times F]_{x_2}\nabla\chi_{x_2}+[\nabla\times F]_{x_3}\nabla\chi_{x_3}\Big]}$$
$$+\int_\Omega \Big[[\nabla\times E]_{x_1}\nabla\chi_{x_1}+[\nabla\times E]_{x_2}\nabla\chi_{x_2}+[\nabla\times E]_{x_3}\nabla\chi_{x_3}\Big]\cdot\overline{[\nabla\times F]}$$

differentiated sesqui-linear form

$$\mathbf{a}\Big((E,E^+,E^-),(F_{\mathrm{ad}},F_{\mathrm{ad}}^+,F_{\mathrm{ad}}^-)\Big) = \int_{\Gamma^+}[\nu\times E]\cdot\overline{\psi},\quad \forall E,E^+,E^-$$
$$\psi := \sum_{m\in\mathcal{M}} c_m\omega_m\sqrt{\mathcal{E}_m}\Big[\mathcal{E}_m-\mathcal{E}_m^{meas}\Big]$$

adjoint solution $(F_{\mathrm{ad}},F_{\mathrm{ad}}^+,F_{\mathrm{ad}}^-)$ (here c_m certain constants)

final formula: $\partial_t[\mathcal{F}(E_t)]|_{t=0} = -\Re\mathrm{e}\,\mathbf{a}_1(E_0,F_{\mathrm{ad}})$

Shape-Derivative Formula

$$\mathbf{a}_1(E,F) := \int_\Omega k^2 E\cdot\overline{F}\ \nabla\cdot\chi - \int_\Omega \nabla\times E\cdot\overline{\nabla\times F}\ \nabla\cdot\chi$$
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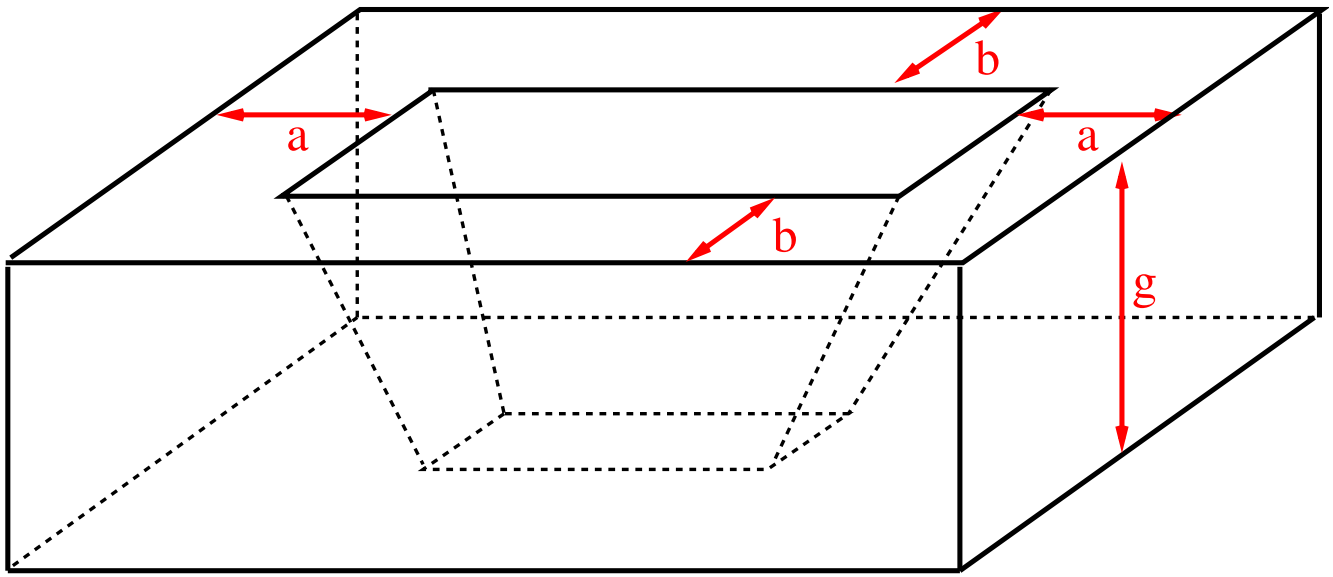
Outline

Parameters to be Reconstructed

reconstruct the three parameters a, b, and c for array of contact holes, contact hole symmetric frustum of pyramid in middle of periodic cell Ω



$$3.15\,\mathrm{nm} \le a \le 3.85\,\mathrm{nm},\ \ 4.55\,\mathrm{nm} \le b \le 5.25\,\mathrm{nm},\ \ 3.85\,\mathrm{nm} \le c \le 4.55\,\mathrm{nm}$$

period in x- and y- direction: 16.8 nm and 14 nm

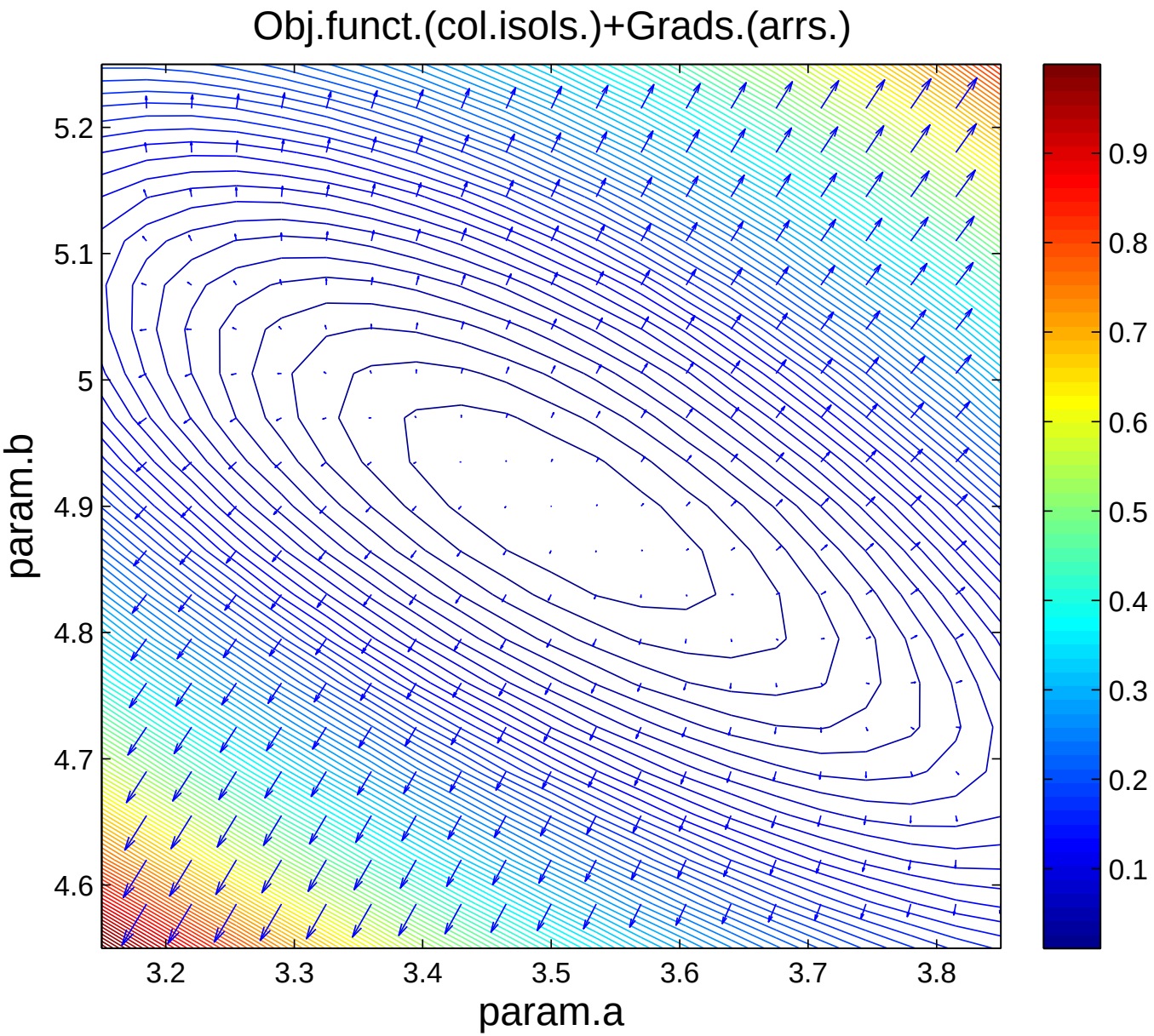
inspecting wave length: 13.7 nm

side material SiO_2, substrate layer Ta, lower multilayer system Mo and Si

normal incidence, five “measured” efficiencies $\mathcal{E}_m$ of order (-1,0), (0,-1), (0,0), (0,1), (1,0)

corresponding weights ω_m: 1000, 2000, 4, 2000, 1000

Objective Functional $\mathcal{F}$ and Numerical Gradients



Convergence of Reconstruction Algorithm

mesh size	nmb.of its.	a in nm	b in nm	g in nm
initial vals.		3.90000	4.60000	3.90000
2.95	8	3.52643	4.89211	4.20601
1.47	5	3.49339	4.90394	4.20270
0.73	5	3.49984	4.90018	4.20011
exact		3.50000	4.90000	4.20000

Number of iterations and reconstruction of parameters for contact hole.

Outline

Analysis Done for 2D Case

- Sensitivity analysis: Optimize choice of measurement data and weights using the Jacobians
- Given distribution and variance of measurement values: estimate uncertainties (standard deviation) of reconstructed values
  - covariance matrix estimates by inverse Jacobians
  - Monte Carlo methods
- Estimate the impact of model errors on the reconstruction, Maximum-likelihood methods
  - Stochastic deviations of the widths in the multilayer system
  - Corner roundings and footage
  - Linewidth roughness (LWR) and lineedge roughness (LER) of line-space structures

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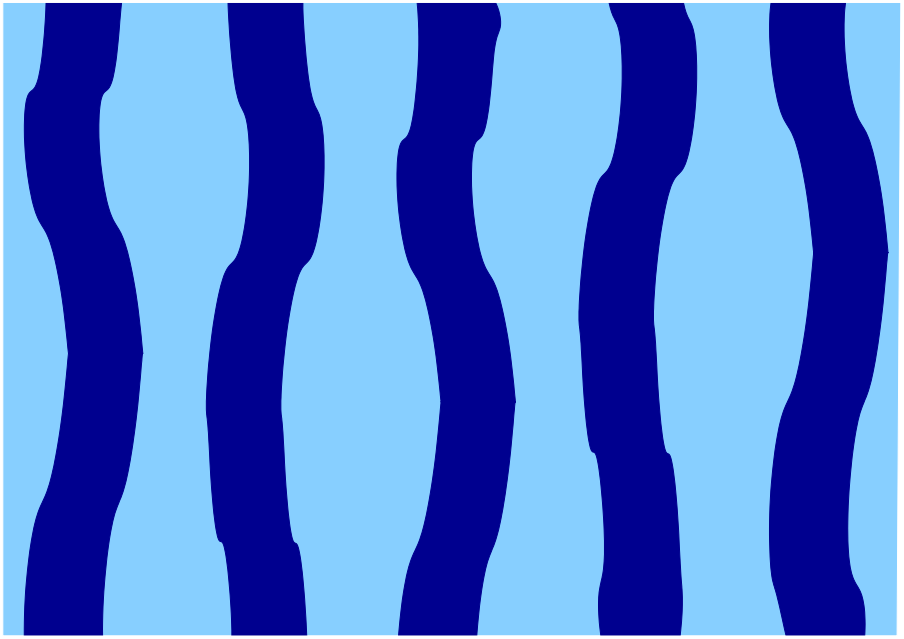
Lineedge Roughness and Linewidth Roughness

stochastic perturbation in line space structure

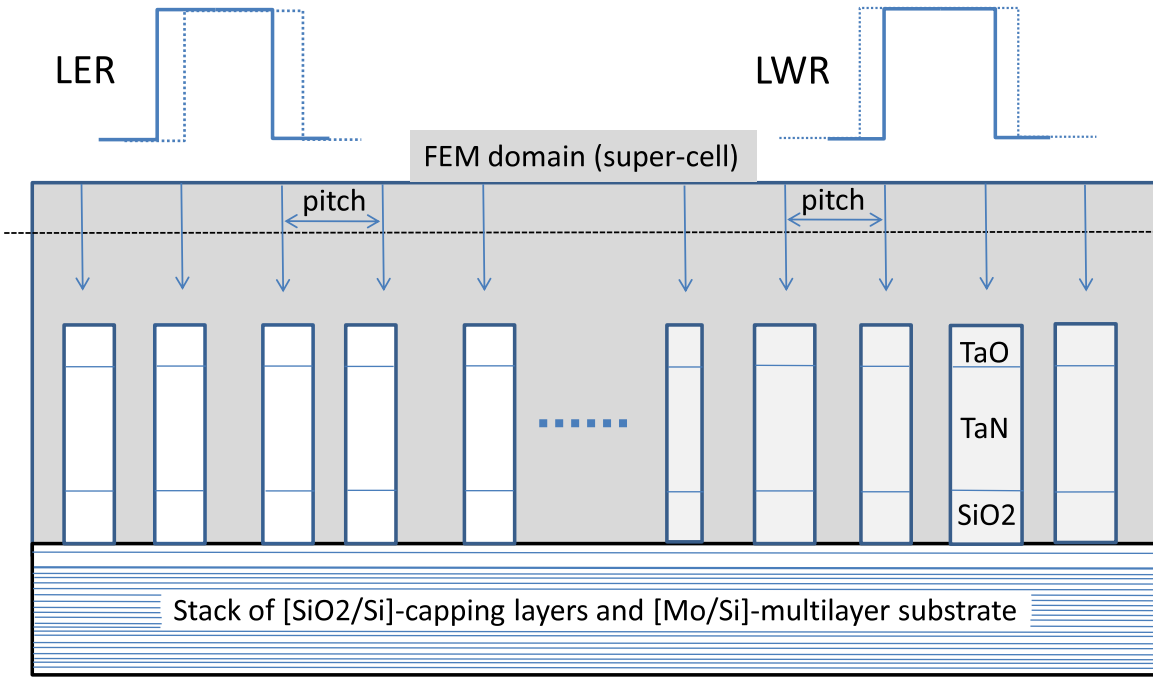


Lineedge roughness (LER)

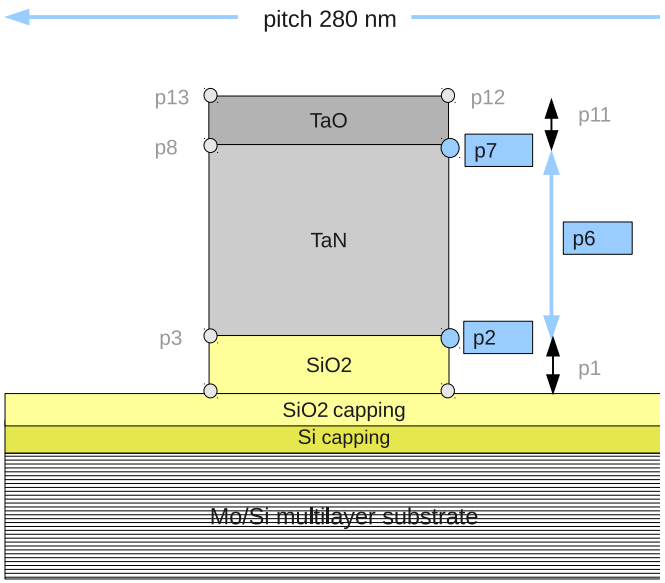
Linewidth roughness (LWR)

Simple 2D Simulation of LWR/LER

- simplifying assumption: assume perturbation of lines is slow in the direction of the lines
- locally: like geometry of perfect parallel lines with perturbations in line widths and distances between lines
- approximated by periodic grating: super cell with large period containing many (e.g. 48) lines with variable line widths and distances



cross section of super cell



cross section of single line

Simple 2D Simulation of LWR/LER

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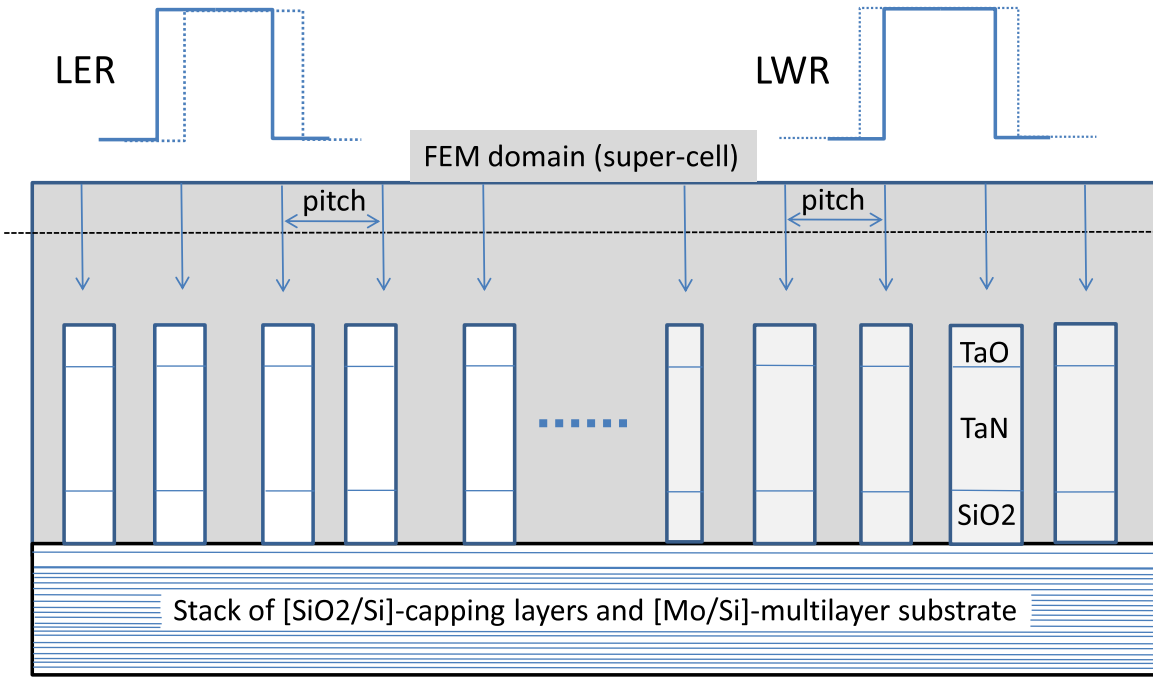
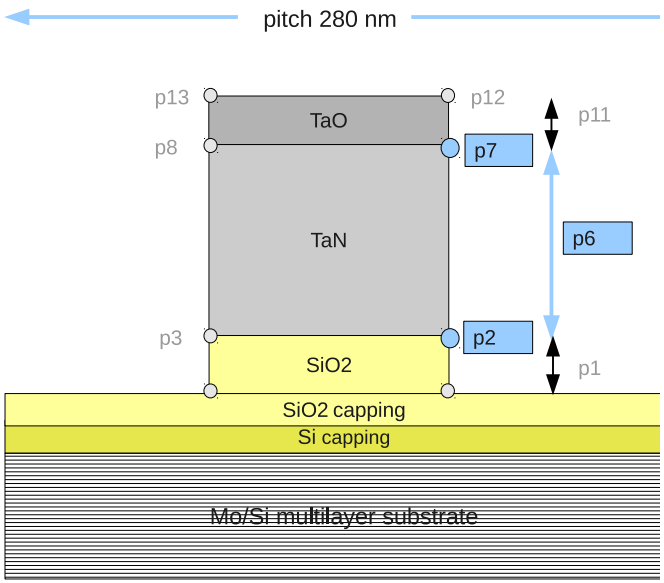
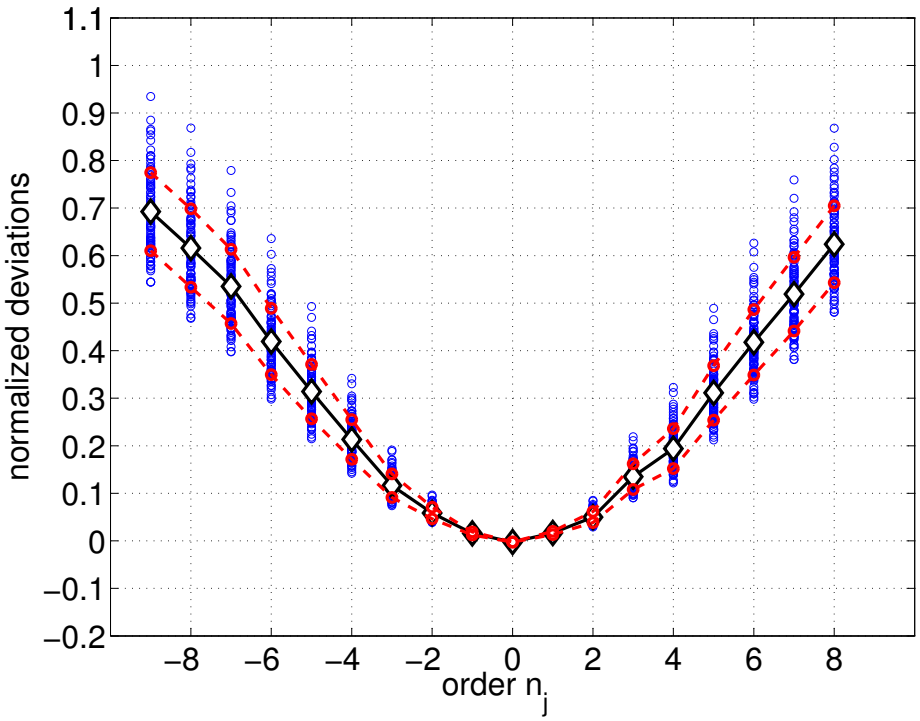


cross section of super cell

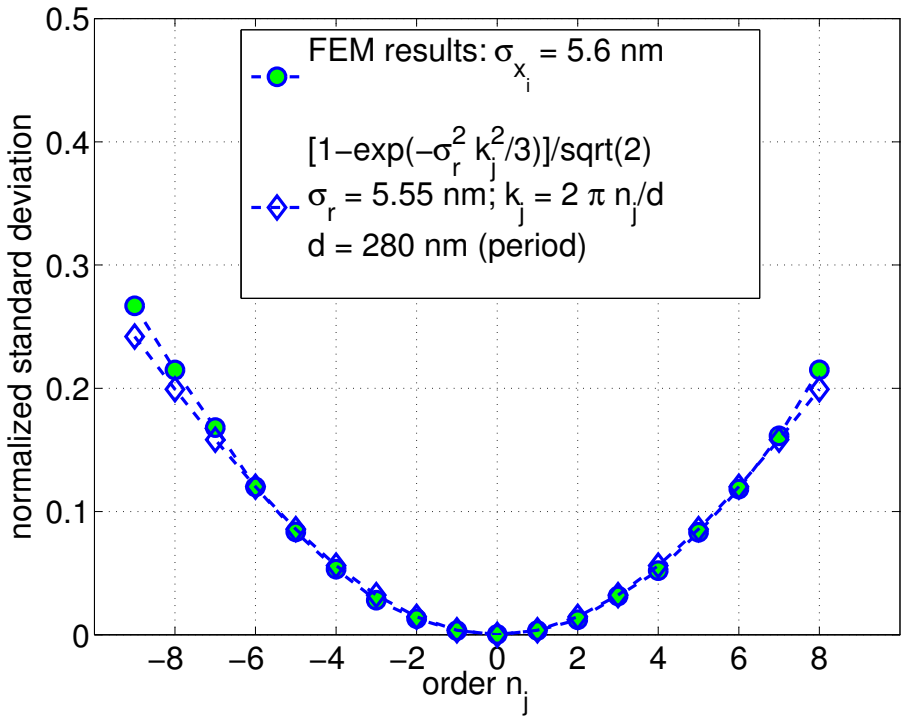


cross section of single line

Monte-Carlo Results for LER: Systematic Error



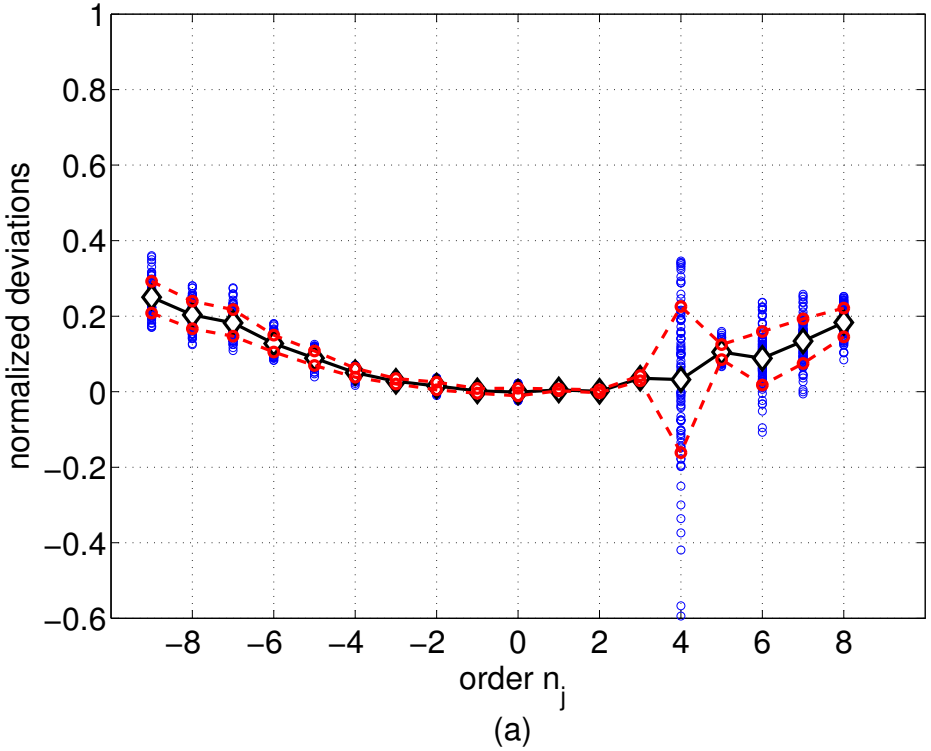
(a)

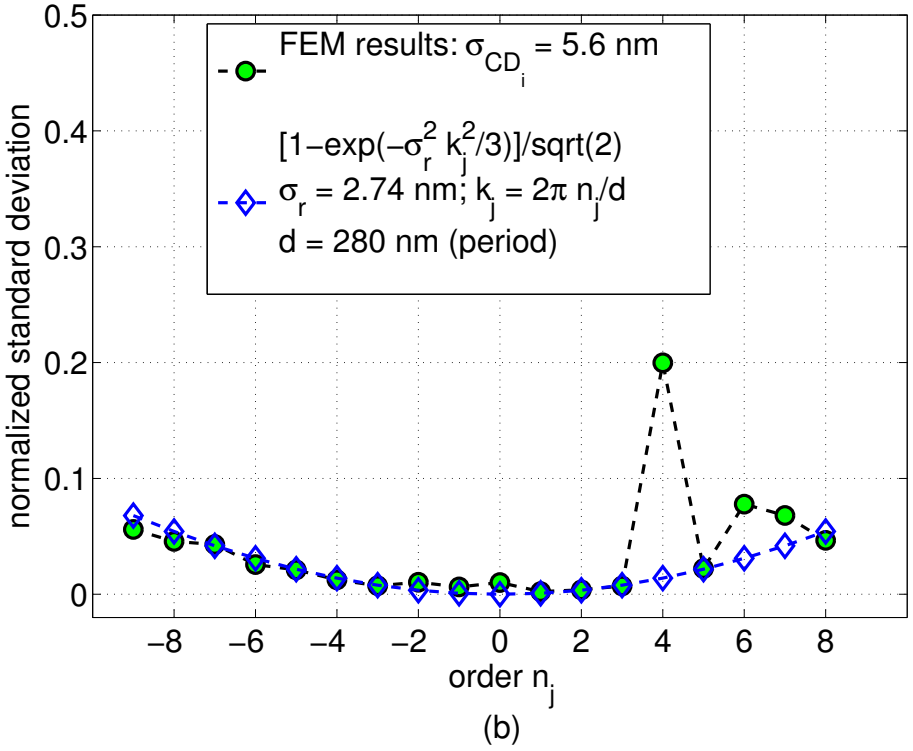


(b)

(a) Normalized deviations from the efficiencies of the unperturbed reference line structure, depicted as circles; diamonds represent the mean over the deviations of all 100 samples; dashed lines indicate the mean efficiency $\pm$ standard deviation; random perturbations of the center positions (LER): $\sigma_{x_i} = 5.6$ nm. (b) Standard deviations relative to the mean perturbed efficiencies shown as circles for the given example; diamonds depict best approximation by an exponential function.

Monte-Carlo Results for LWR: Systematic Error





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Correction Factor for LER/LWR Perturbations

A. Kato and F. Scholze: formula for impact of LER/LWR if Fraunhofer approximation applies

$$\boxed{\mathcal{E}^{+}_{j,\mathrm{pert}} \approx \exp\left(-\left[\frac{2\pi j}{d}\right]^2 \sigma\right) \mathcal{E}^{+}_{j,\mathrm{no\,pert}},}$$

where: d period,

j order of mode,

σ is the standard deviation of the stochastic perturbation of the line edges.

- similar exponential function for variances
- by our experiments: formulas confirmed for rigorous Maxwell's equations and slow perturbation along the lines
- knowing the LER/LWR perturbation: incorporate formula into numerical reconstruction algorithm

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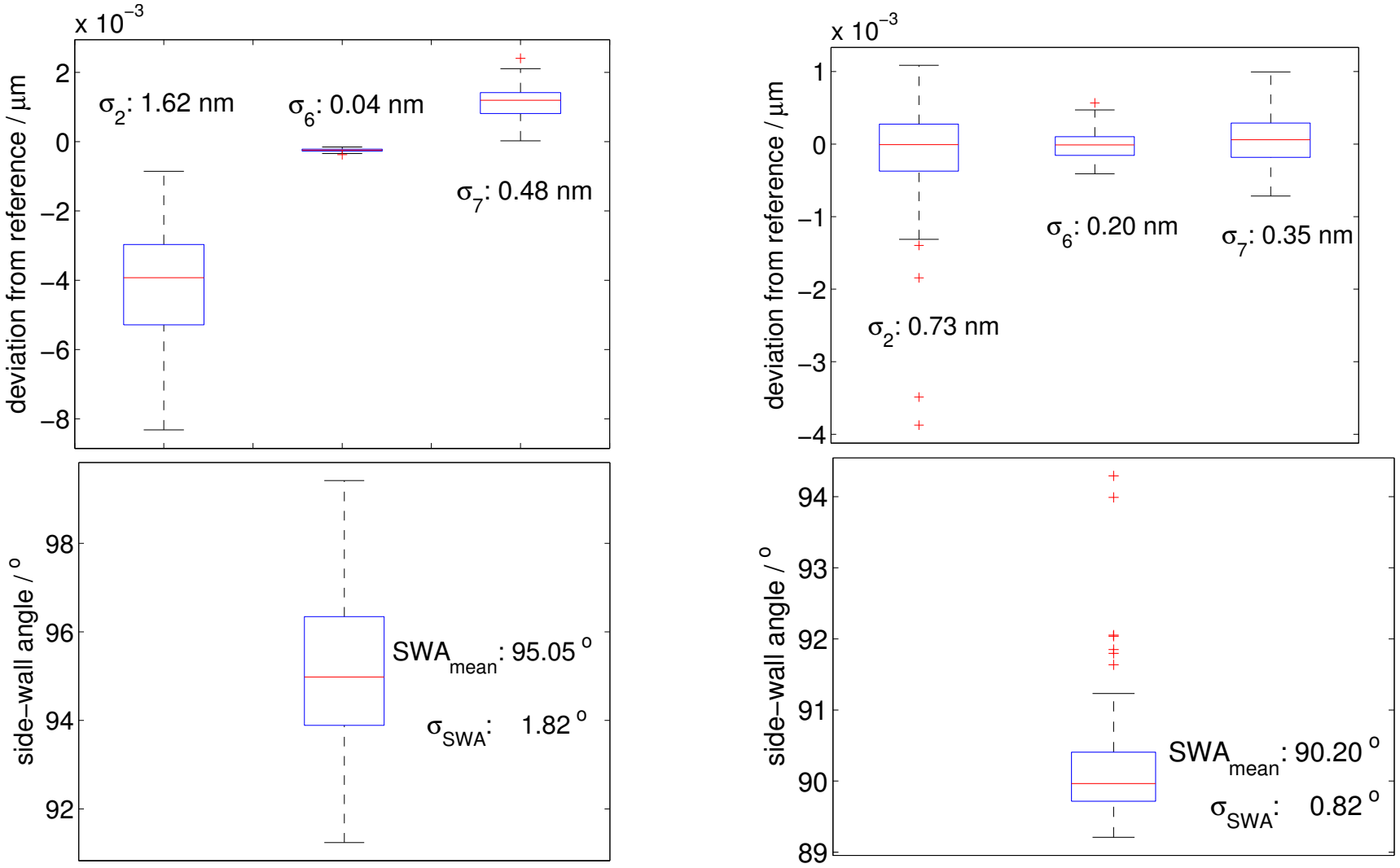
Improved Numerical Reconstruction



1% standard deviation of edges with LER/LWR, left reconstruction without correction factor, right with correction factor, deviation around nominal value

Outline

Conclusions

- Simulation of scattered field by biperiodic grating possible with FEM and mortar method (unfortunately: slow)
- Shape derivatives can be computed by FEM
- Reconstruction of geometric parameters possible (beyond diffraction limit)
- Uncertainties of reconstructed parameters can be estimated even by fast covariance-matrix method
- To get results closer to those obtained by alternative measurement methods: model errors need to be analyzed, e.g., LER, LWR

For more details:

S. Heidenreich: today's talk, M10, Salon B, 16:50–17:15

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Some References

[1] M.-A. Henn, R. Model, M. Bär, M. Wurm, B. Bodermann, A. Rathsfeld, and H. Gross, On numerical reconstructions of lithographic masks in DUV scatterometry, *Proc. SPIE*, 7390 (2009), 73900Q.

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[4] A. Rathsfeld, Shape derivatives for the diffraction intensities scattering by biperiodic structures, *WIAS Preprint*, Berlin, 1640 (2011).

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Thank you for your attention!