ODE for Physicists - Homework 2

Due: April 19, 2005

4. (6 pts.) Find the differential equation associated with the following families of curves.

   (a) The family of curves having the property that the length along the tangent between the point of contact $P = (x, y)$ and the point Q of intersection of the tangent with the y-axis, is equal to the distance of Q to the origin (see figure).

   (b) The family of *cardiods*, $\rho(\theta) = a(1 - \cos\theta)$, where $a \in \mathbb{R}$ is a constant.

   (c) The family of parabolas with foci at the origin and axes along the y axis[1].

4. (6 pts.) Let the following two-parameter family of curves be given.

   (a) $y = Cx + D$,

   (b) $y = C\sin(x + D)$,

   (c) $y = e^x(Cx + D)$.

   Derive two more equations by twice differentiating, use these three equations for eliminating the parameters, and you obtain a second-order ODE. Give an interpretation of this ODE.

6. (4 pts.) For the following family of curves, state the differential equation of the orthogonal trajectories, find their general solution, and draw a picture.

   (a) $y^2 = x + C$,

   (b) $x^2 + 2y^2 = C$.
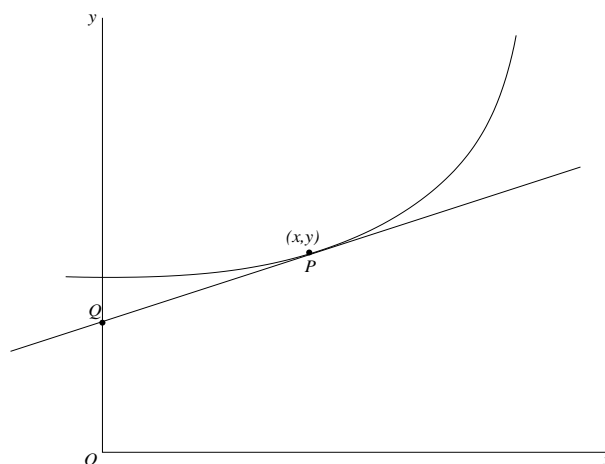

Figure 1: Problem 4.(a): $|PQ| = |OQ|$

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[1] Just a reminder: A parabola is the set of all points in the plane equidistant from a given line L (the directrix) and a given point F not on the line (the focus). The focal parameter (i.e., the distance between the directrix and focus) is therefore given by $p = 2a$, where a is the distance from the vertex to the directrix or focus. The axis of the parabola is the line through F, perpendicular to the directrix L. See `http://mathworld.wolfram.com/Parabola.html` .